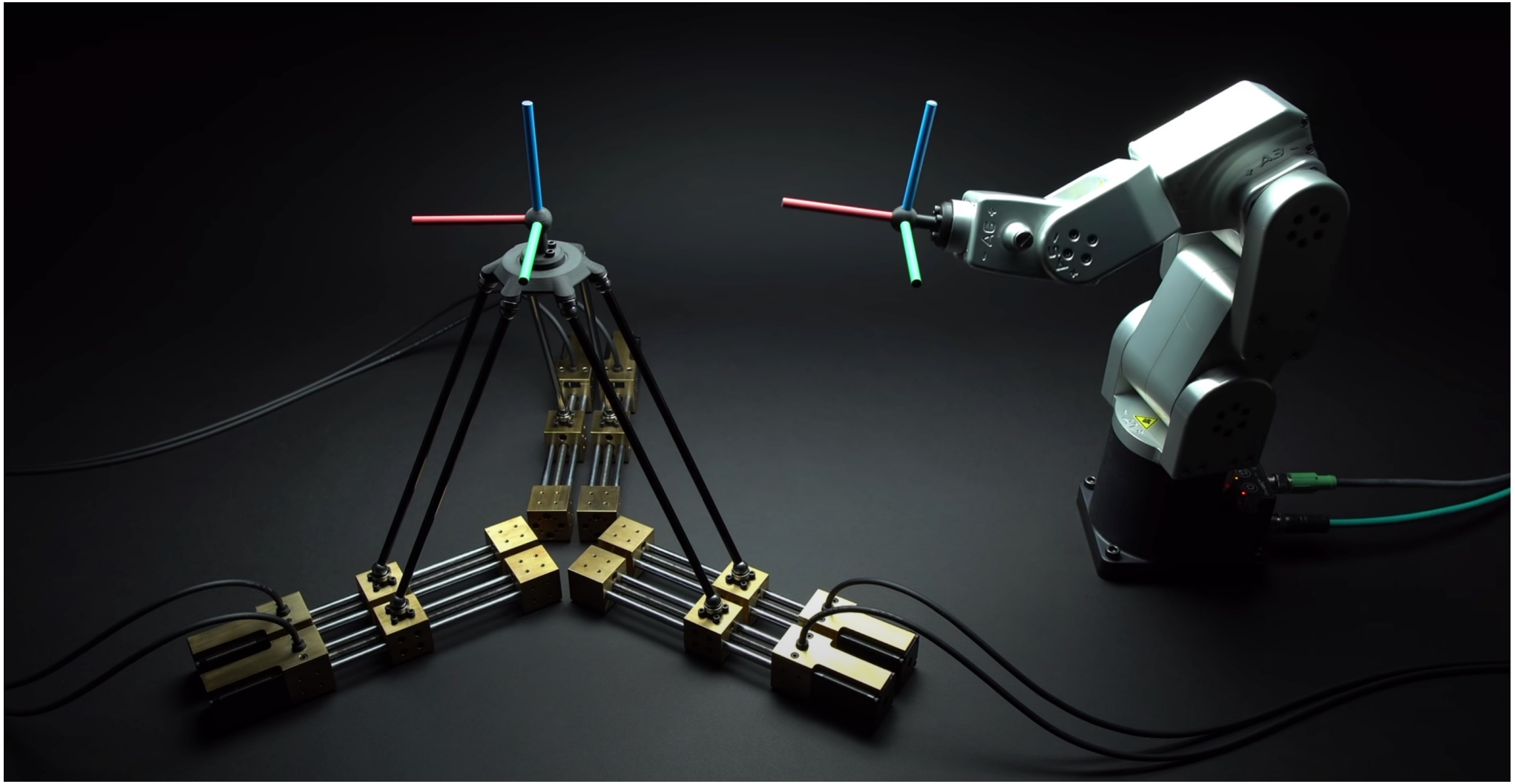
ADVANCED ROBOT KINEMATICS



Tomas Pajdla

2025

pajdla@cvut.cz

AAG – Applied Algebra & Geometry

Basic & Applied Research, 15 Members

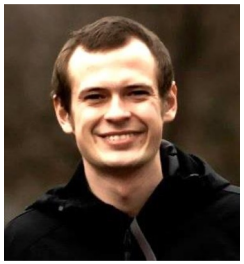
Tomas Pajdla
Group Leader

Vision
Robotics
Mathematics

Mircea Cimpoi
Postdoc
Machine Learning

Ludovic Magerand
Postdoc
3D Vision

Federica Arrigoni
Postdoc
3D Vision

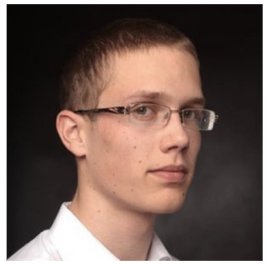


Čeněk Albl

PhD Student
Rolling Shutter

Michal Polic

PhD Student
3D Reconstruction



Pavel Trutman

PhD Student
Polynomial Optimization

Stanislav Steidl
PhD Student
Autonomous Driving

Research

We apply elements of

- Algebra
- Geometry
- Statistics
- Optimization in
- Computer Vision
- Robotics
- Machine Learning

Teaching

We teach Geometry of

- Computer Vision
- Robotics at

CTU CZECH TECHNICAL UNIVERSITY IN PRAGUE
CHARLES UNIVERSITY IN PRAGUE

FEE of the CTU in Prague
MFF of Charles University

Projects

We are funded by

LADIO H2020 EU
3D Reconstruction for Movies

IMPACT OP VaVPI
Intelligent Machine Perception

R4I OP VaVPI
Robotics for Industry 4.0

Industry

We collaborate with

Omni-Vision

Leica Photogrammetry

Magik Eye 3D sensing

Continental Camera calibration

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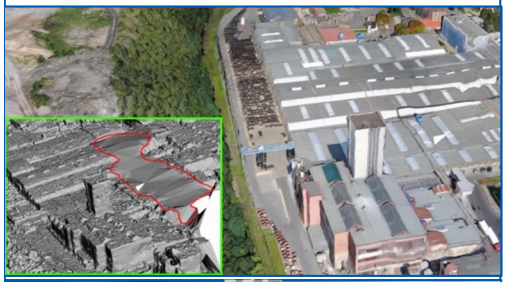
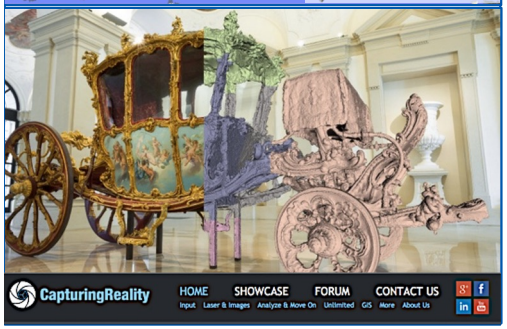
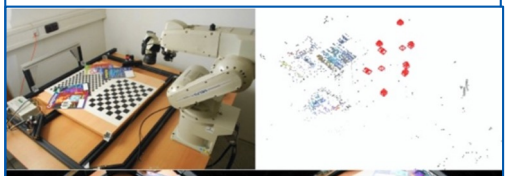
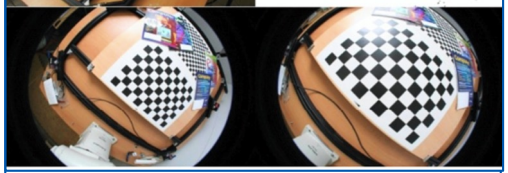
AAG – Research & Applications

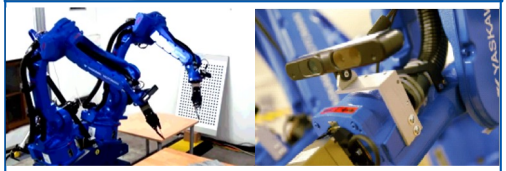
3D Reconstruction



Camera Geometry



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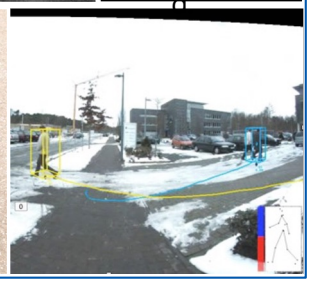
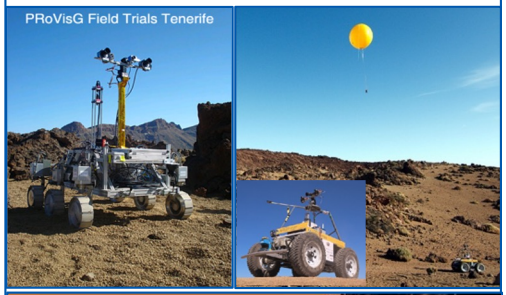
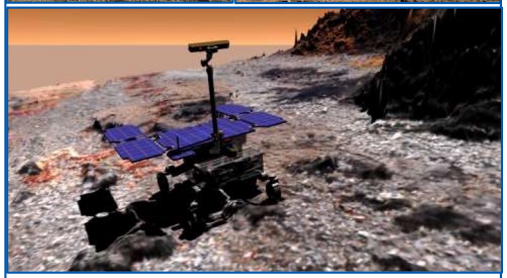
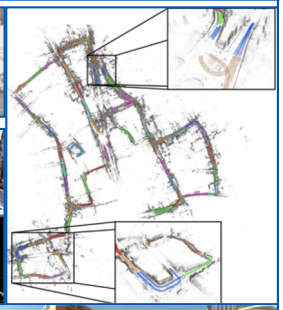
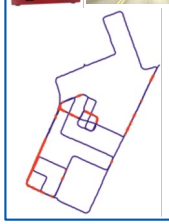


Visual localization








Applications

DAIMLER
Continental
Magik Eye
Leica
gisat
AZOS CZ s.r.o.
zentity
a touch of innovation

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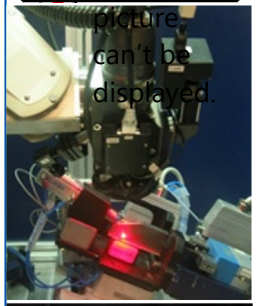





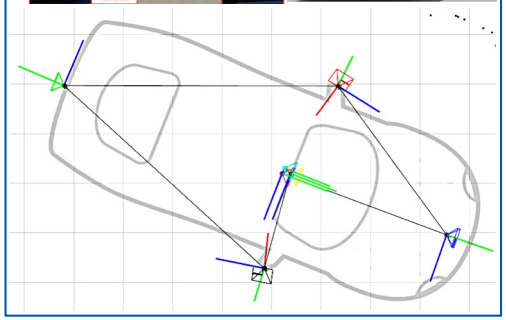

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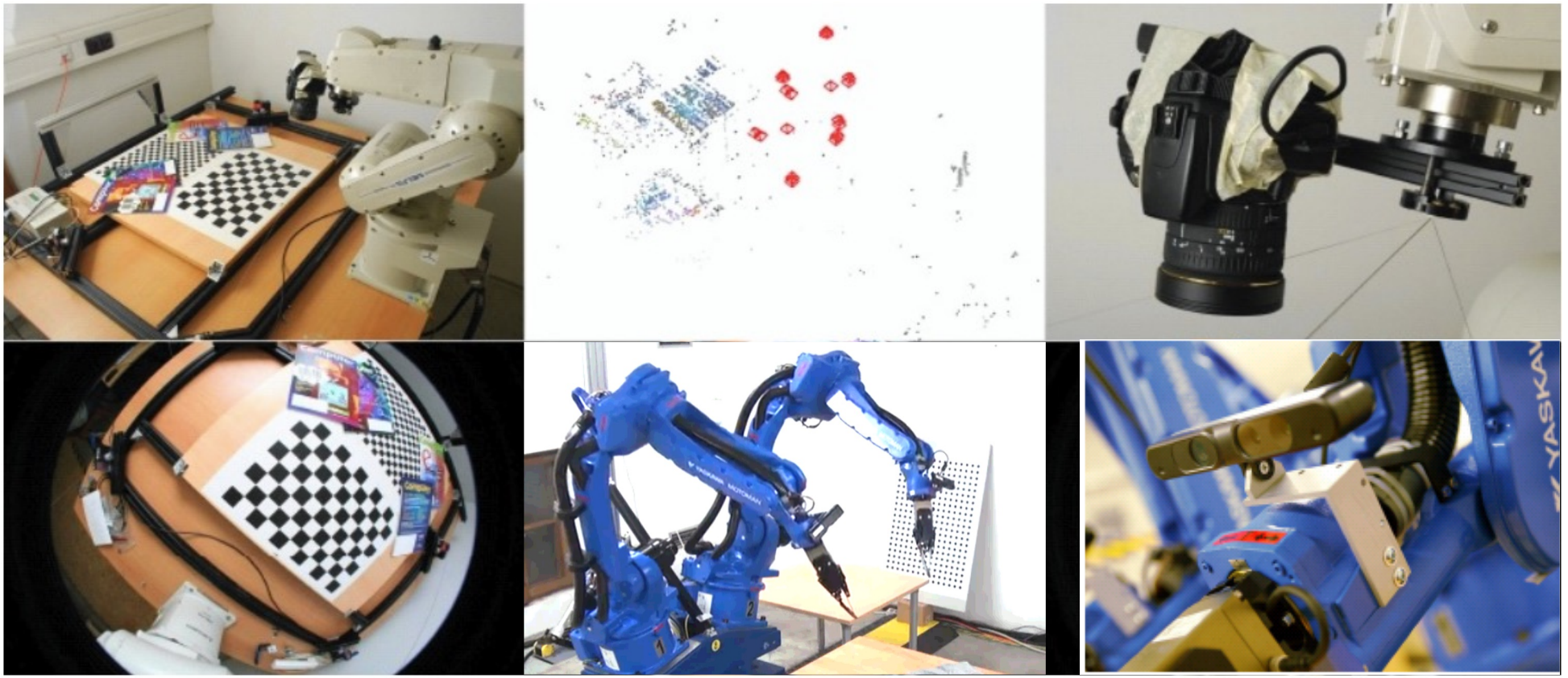
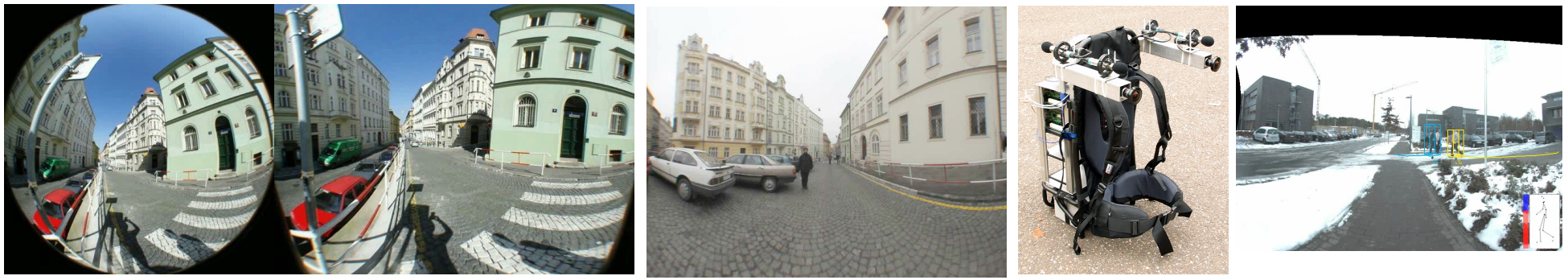
AAG - Geometry of Cameras and Robots





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Advanced Robotics

Lecture 1

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We will study more advanced robot kinematics problems, e.g.,

1. solving inverse kinematics of a general 6 DOF manipulator
2. finding singular poses of a manipulator

with more advanced mathematical tools, such as

1. space rotation and motion and
2. solving algebraic equations

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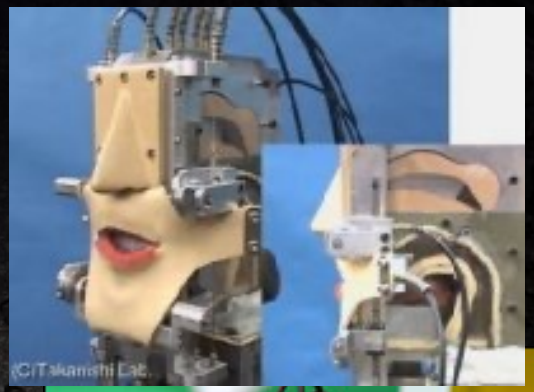
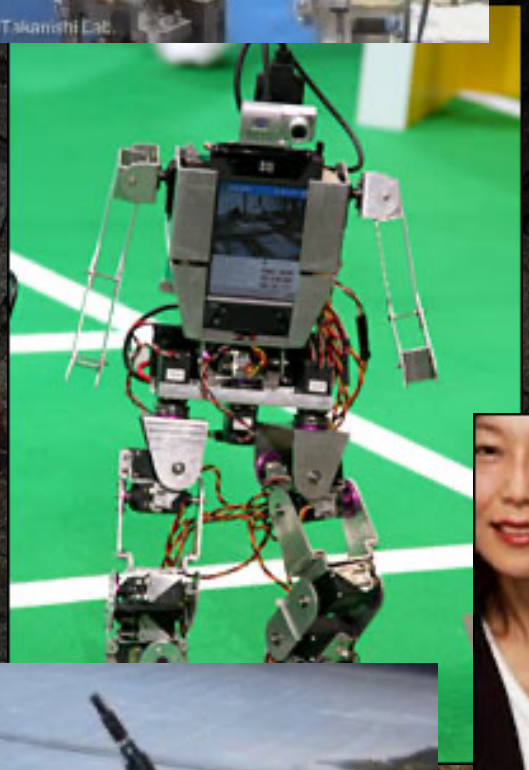
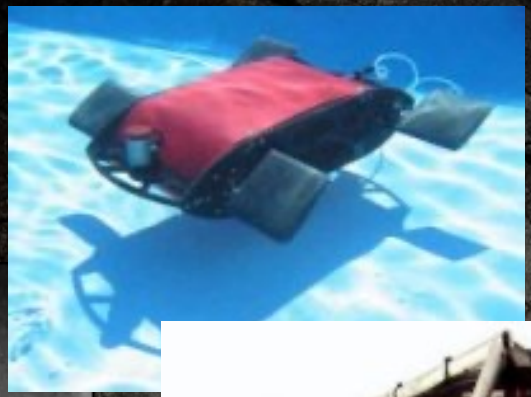
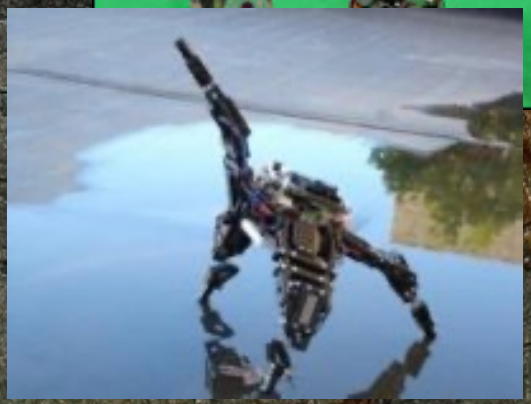
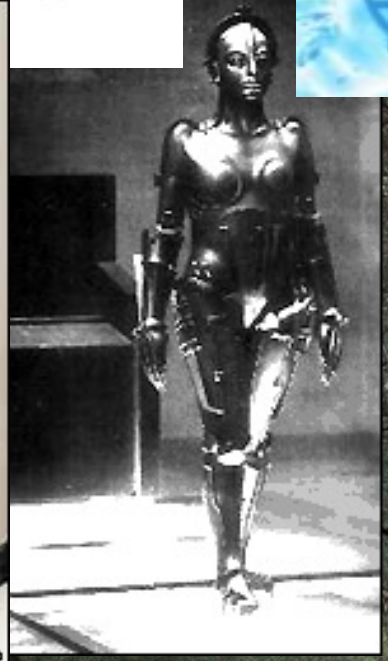

(C)Takanishi Lab.

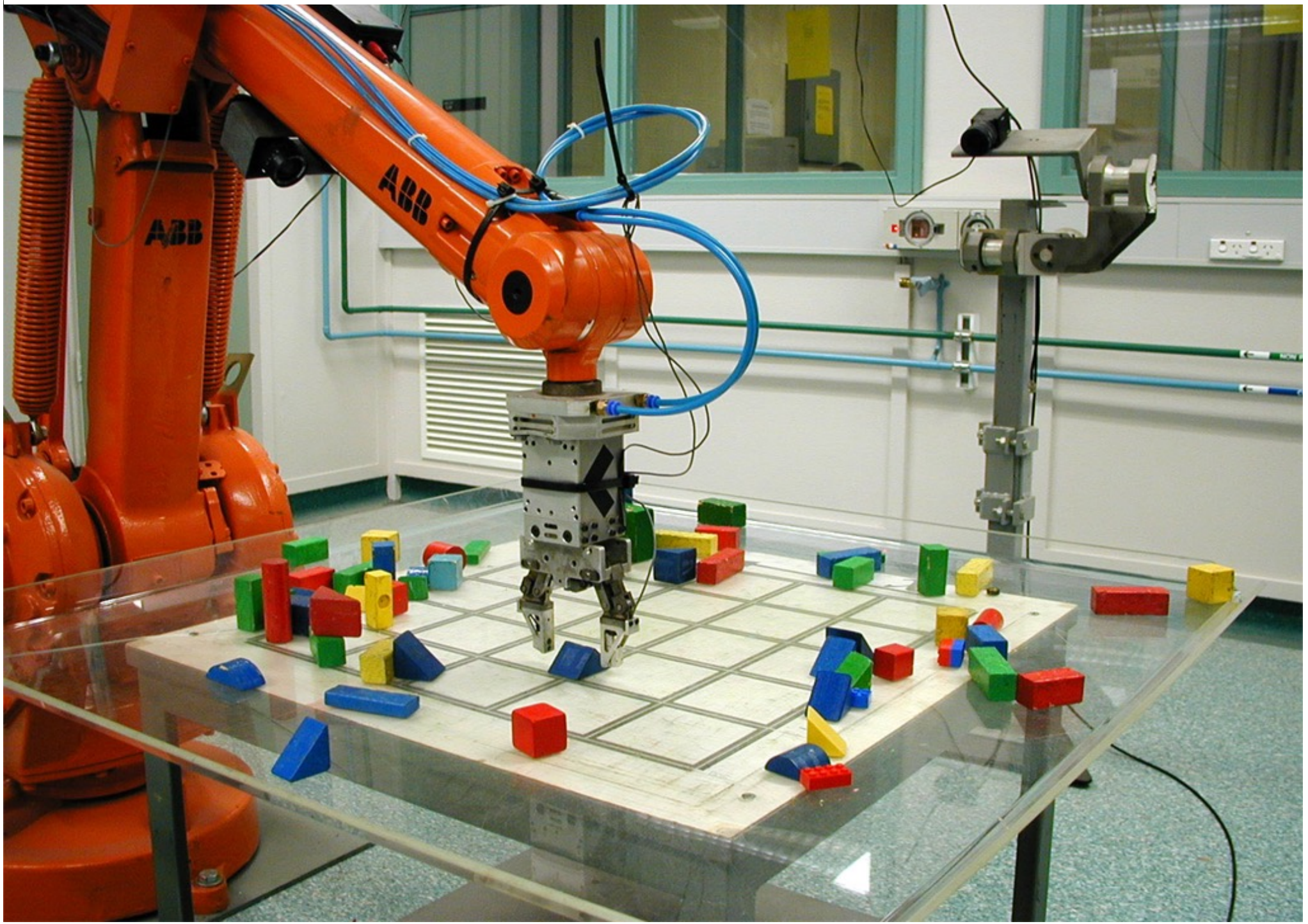








ROBOT = A GENERAL MANIPULATOR



Industrial Robotic Applications

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  - Robotics startpage
  - ⊞ Product range
  - ⊟ Application areas
    - Arc welding
    - Assembly
    - Foundry applications
    - Gluing and Sealing
    - ⊞ Material handling and Machine Tending
    - Packing
    - Palletizing
    - Picking
    - Painting and coating
    - Spot welding
    - Waterjet cutting

Application areas

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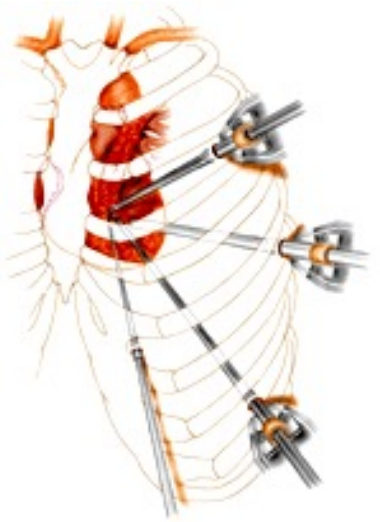
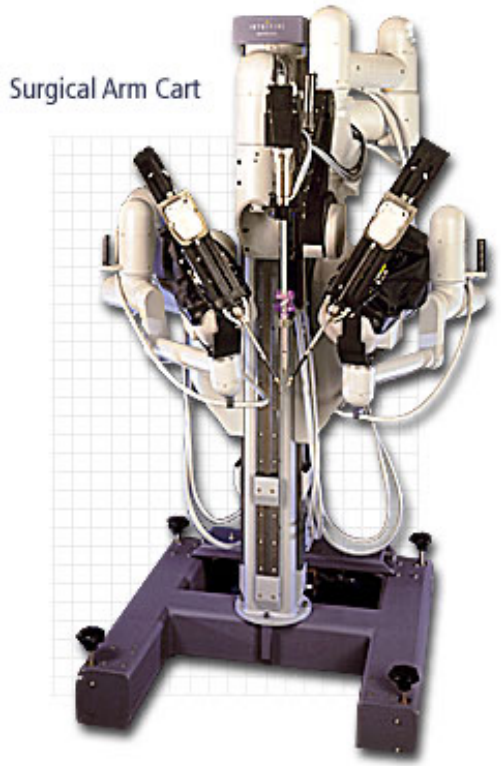
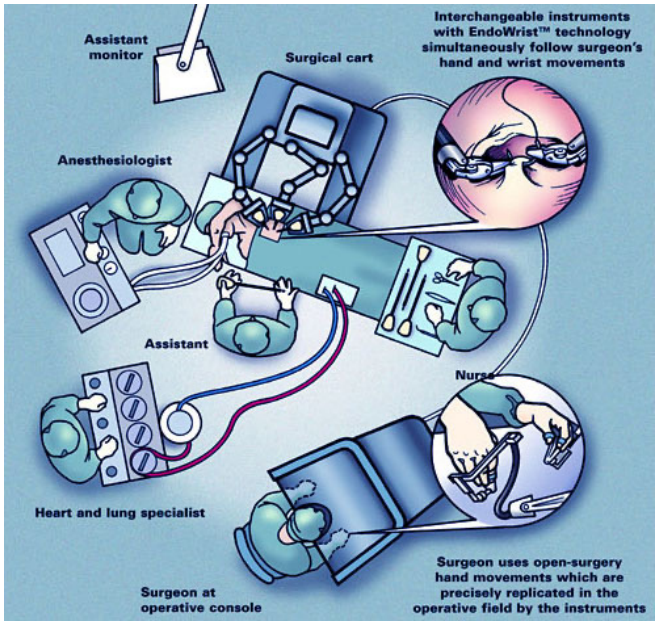
Precision for robotic surgery







http://www.cts.usc.edu/rsi-article-robotputsuscatforefront.html

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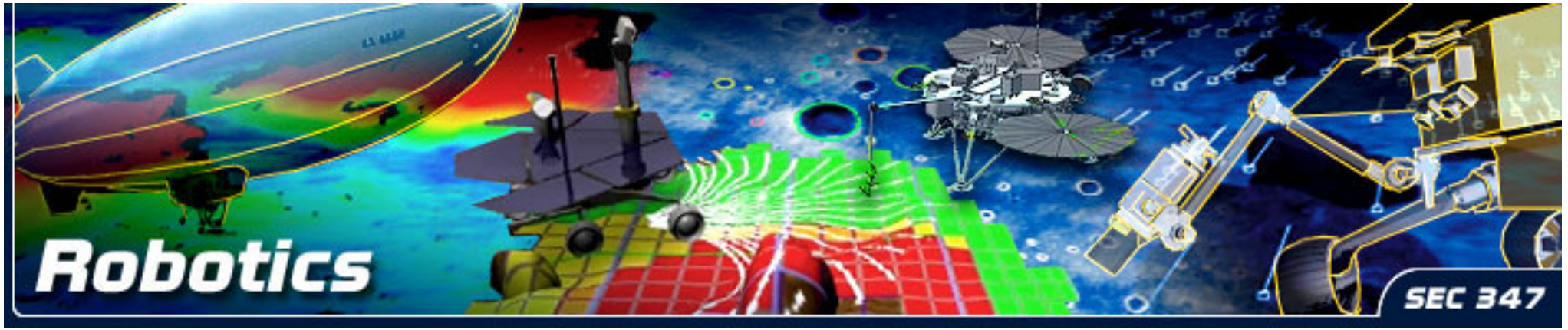

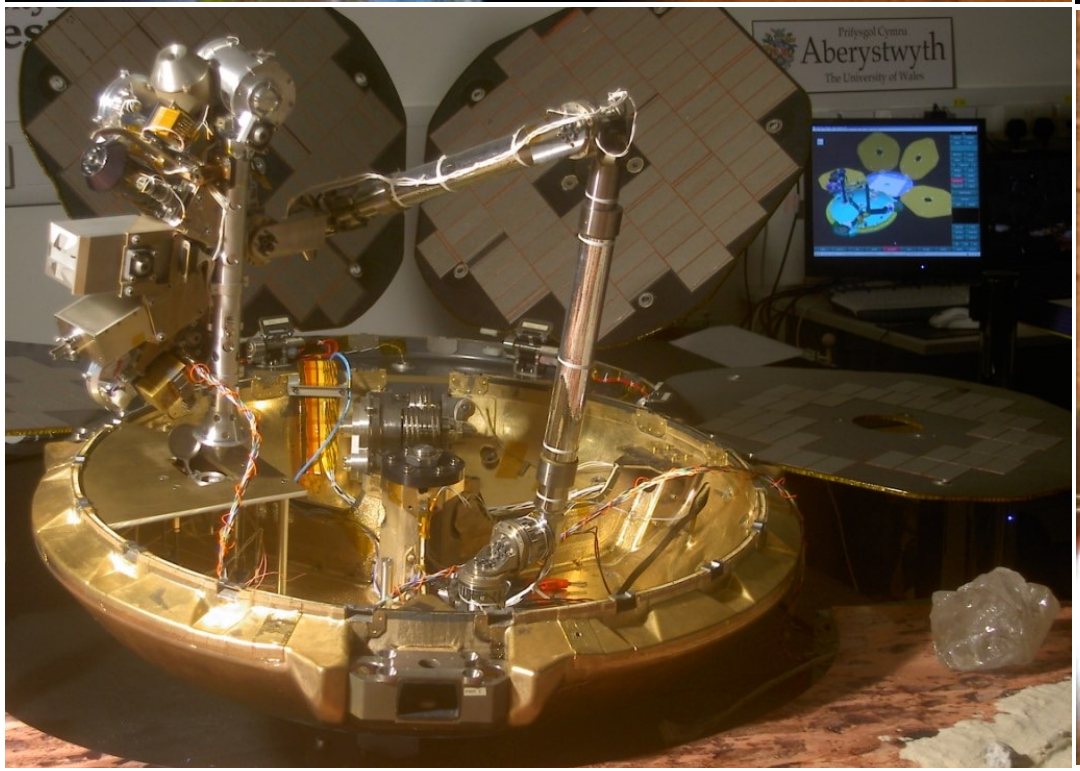

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Two kinds of manipulators

1. Serial manipulators
2. Parallel manipulators

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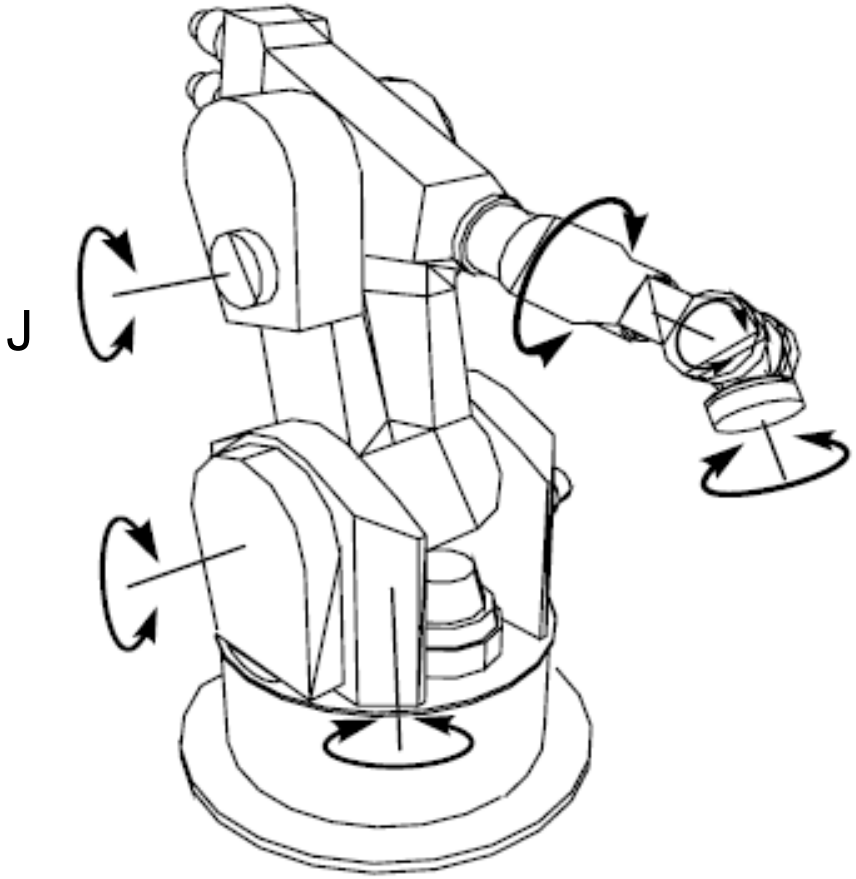
Serial manipulators



KUKA manipulator

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Serial manipulators

Stäubli (courtesy Neovision s.r.o.)

Mitsubishi (courtesy Neovision s.r.o.)

1. Direct kinematic task – easy
2. Inverse kinematic task – difficult

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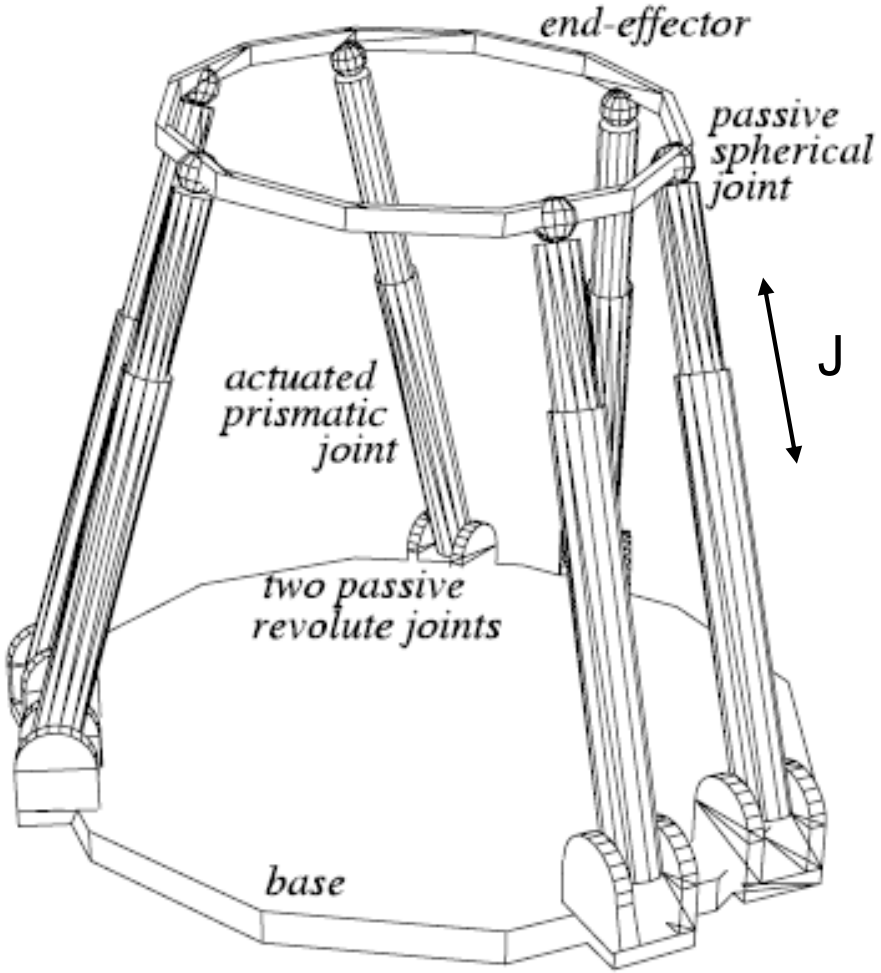
Parallel manipulators



Stewart-Gough Platform

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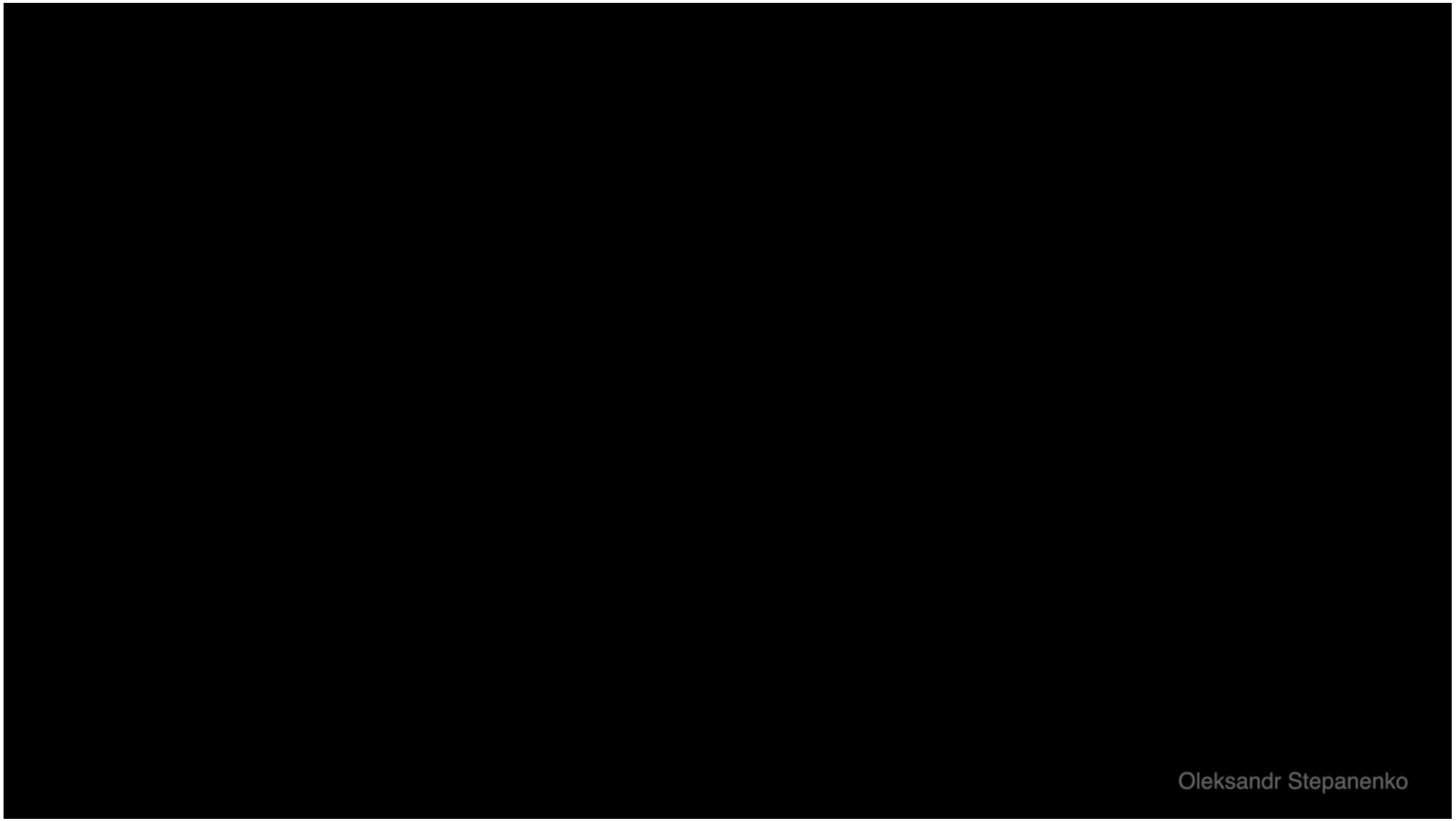
Parallel manipulators



https://youtu.be/xiECumcaEx0

Hexapod (Gough-Stewart platform) 6-axis parallel robot

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Kinematics in robotics

Three main problems

1. Direct kinematic task (přímá kinematická úloha)
2. Inverse kinematic task (inverzní kinematická úloha)
3. Manipulator singularity analysis

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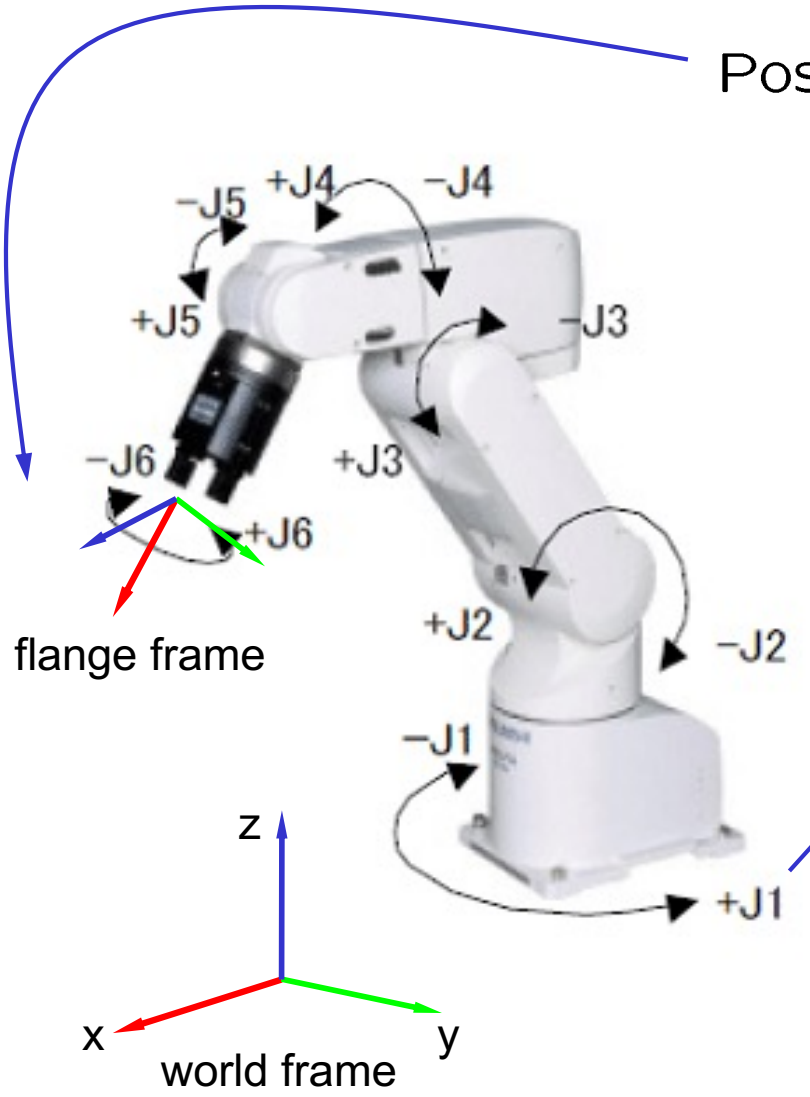
Direct kinematic task



Position of the flange reference frame in the world reference frame

joint angles

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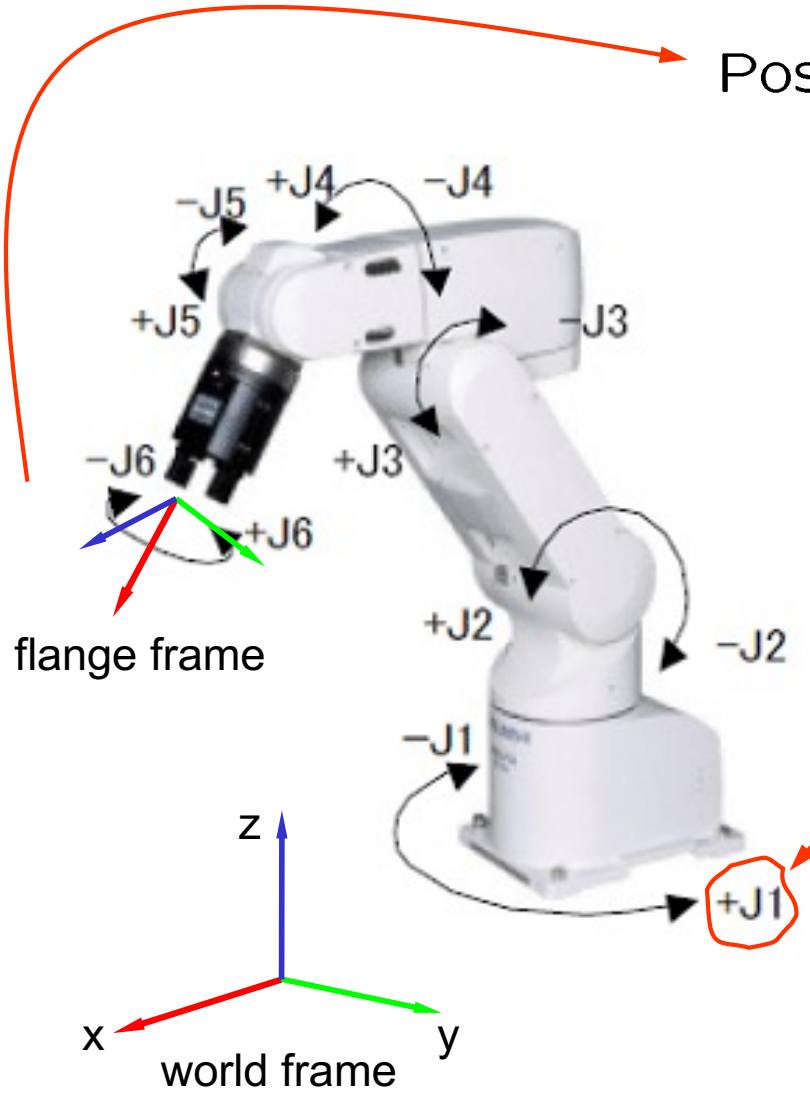
Inverse kinematic task



Position of the flange reference frame in the world reference frame

joint angles

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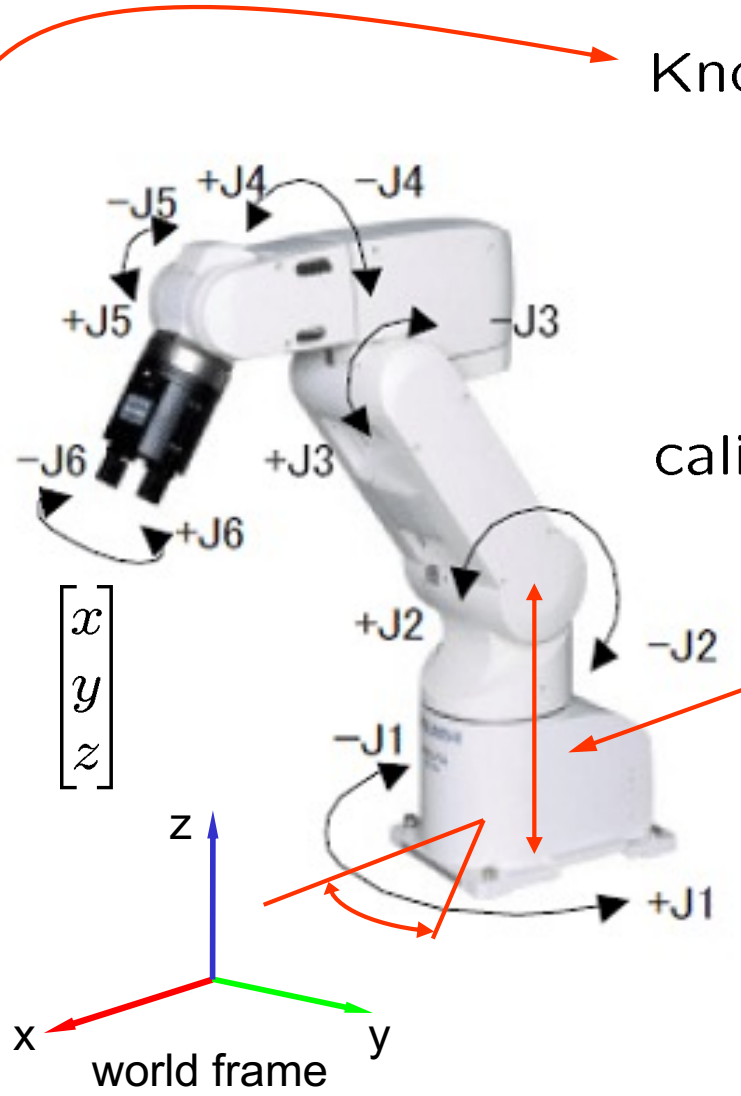
Kinematic calibration



Known position of a point on the flange in the world reference frame

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

calibration parameters (lengths, offsets)

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Singularities of Manipulators

www.mecademic.com/en/what-are-singularities-in-a-six-axis-robot-arm

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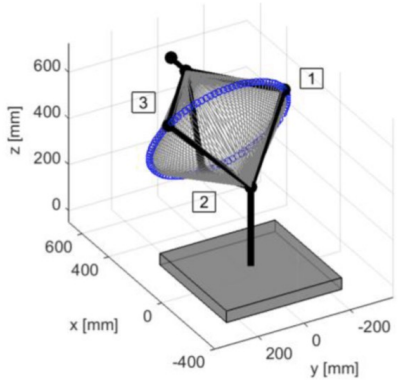
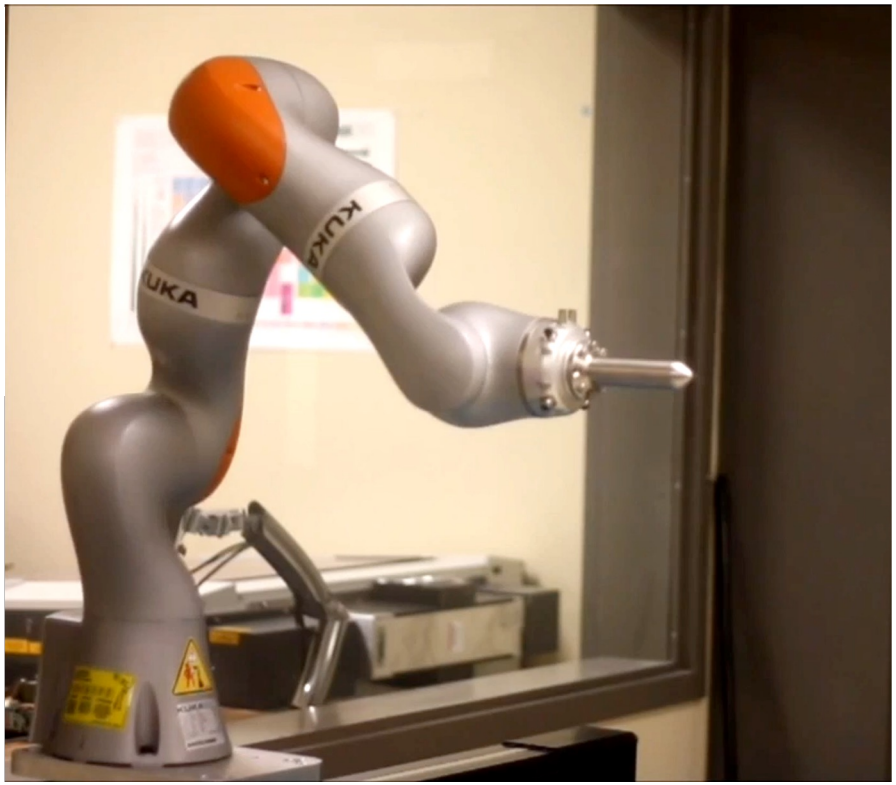
Algebraic Analysis of Manipulator Kinematics

KUKA LBR iiwa



- 7 revolute joints $\rightarrow$ 7 DOF
- Solutions of IKT can be parametrized by one parameter
- Task: to find an optimal solution



Nicholas Nadeau (https://www.youtube.com/watch?v=JhCzt1KAWMY)

Figure: Manipulator KUKA LBR iiwa.

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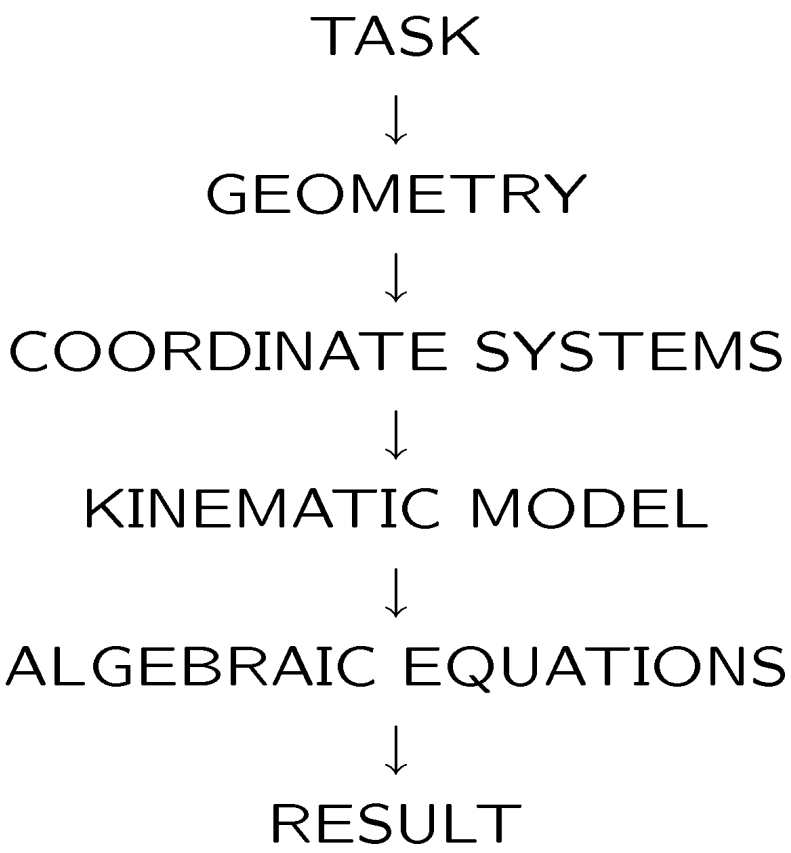
Solving kinematic tasks



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Solving kinematic tasks

1968 Donald L. Pieper (Ph.D. thesis)

The inverse kinematics of any serial manipulator with six revolute joints, and with three consecutive joints intersecting, can be solved in closed-form, i.e., analytically.

1989 M. Raghavan, B. Roth. *Kinematic Analysis of the 6R Manipulator of General Geometry.* Int. Symp. Robotics. Research. Pp. 314-320, Tokyo 1989/1990.

A general technique for computing inverse kinematics for any serial manipulator with six revolute joints.

… leads to solving an algebraic equation of degree 16.

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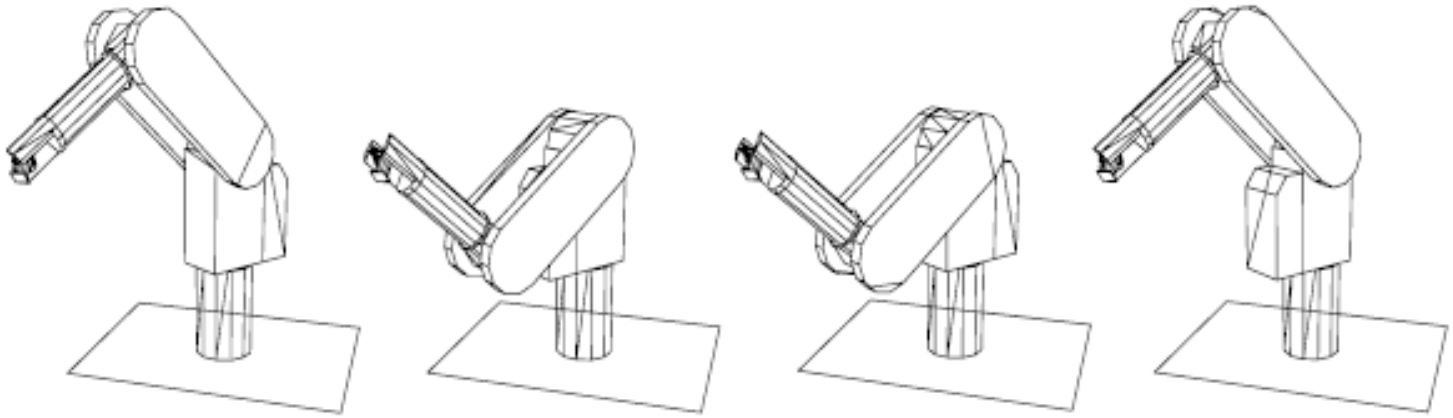
Solving kinematic tasks

Algebraic equation of degree 16 … up to 16 solutions

4 typical solutions



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- E N D -

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