DCGI

DEPARTMENT OF COMPUTER GRAPHICS AND INTERACTION

WINDOWING

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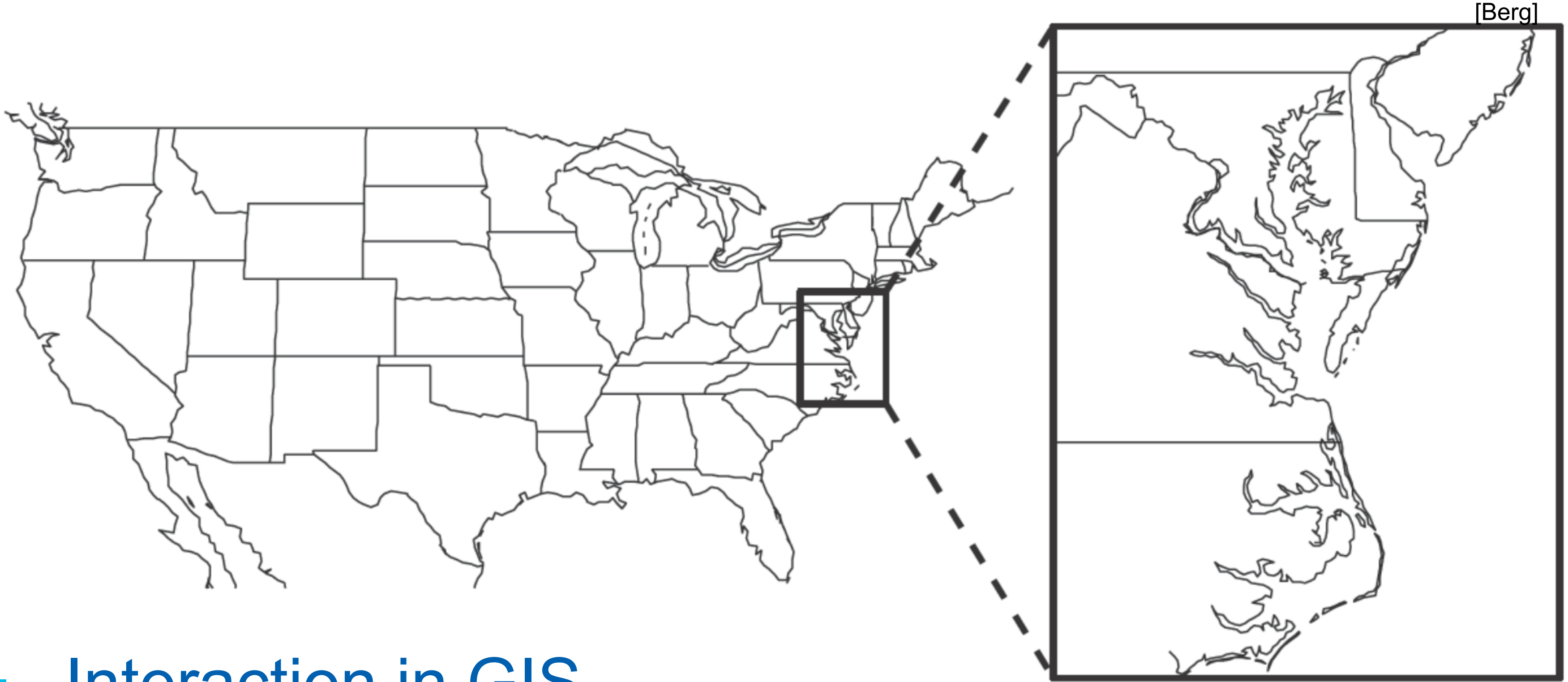
https://cw.felk.cvut.cz/doku.php/courses/a4m39vg/start

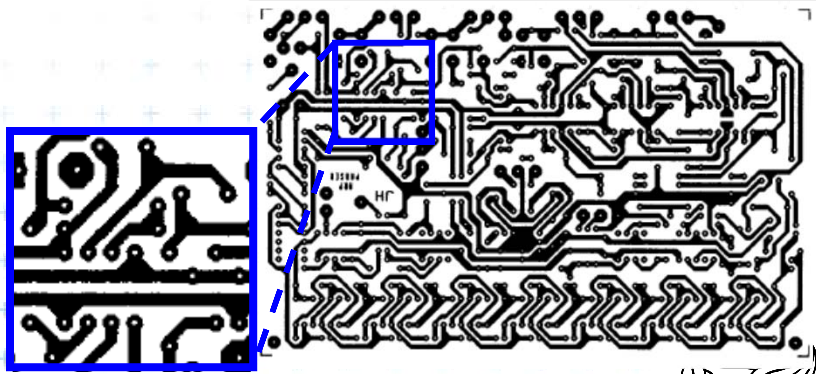
Based on [Berg], [Mount]

Version from 28.11.2013

Windowing queries - examples



- Interaction in GIS
  - Select subset by outlining
  - Zoom in and re-center
- Circuit board inspection,…

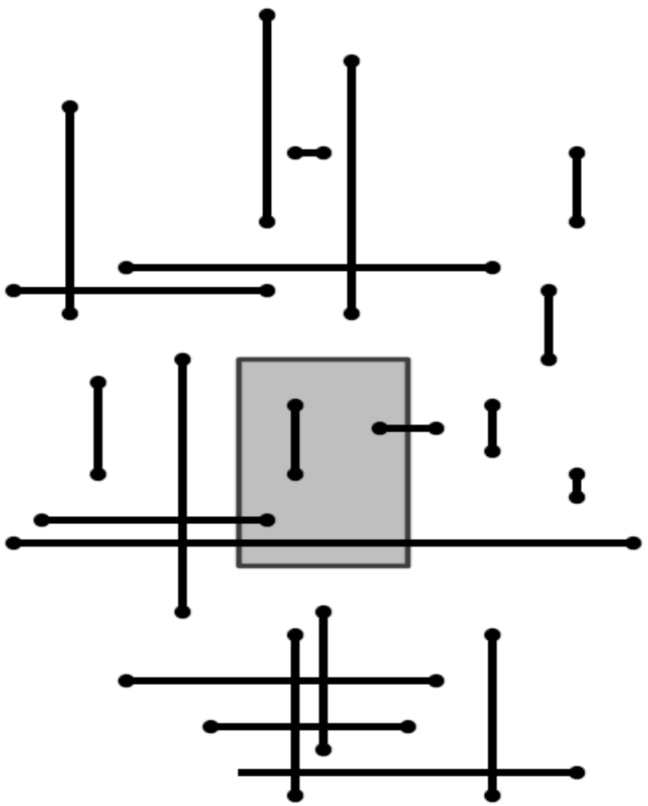


Windowing versus range queries

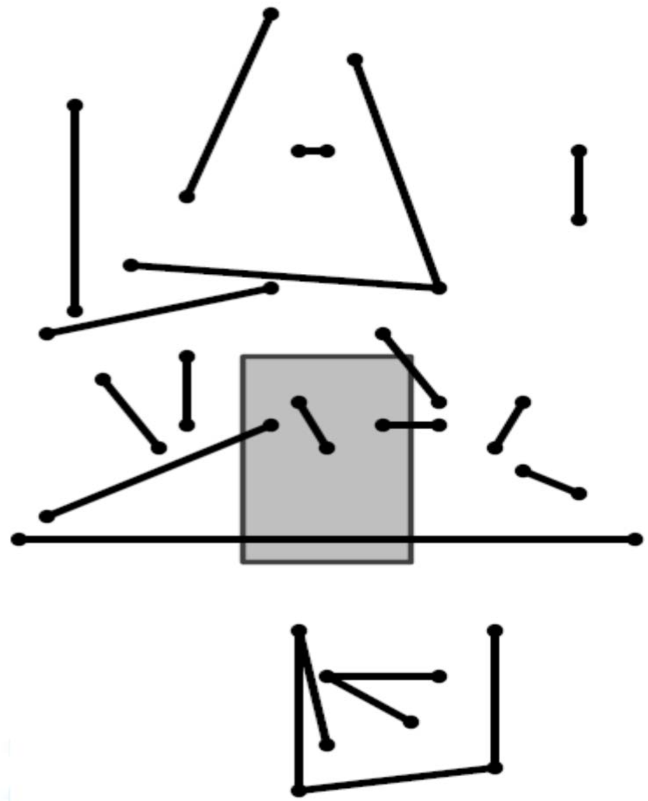
- Range queries (see range trees in Lecture 03)
  - Points
  - Often in higher dimensions
- Windowing queries
  - Line segments, curves, …
  - Usually in low dimension (2D, 3D)

- The goal for both:
  Preprocess the data into a data structure
  - so that the objects intersected by the query rectangle can be reported efficiently

Windowing queries on line segments



1. Axis parallel line segments



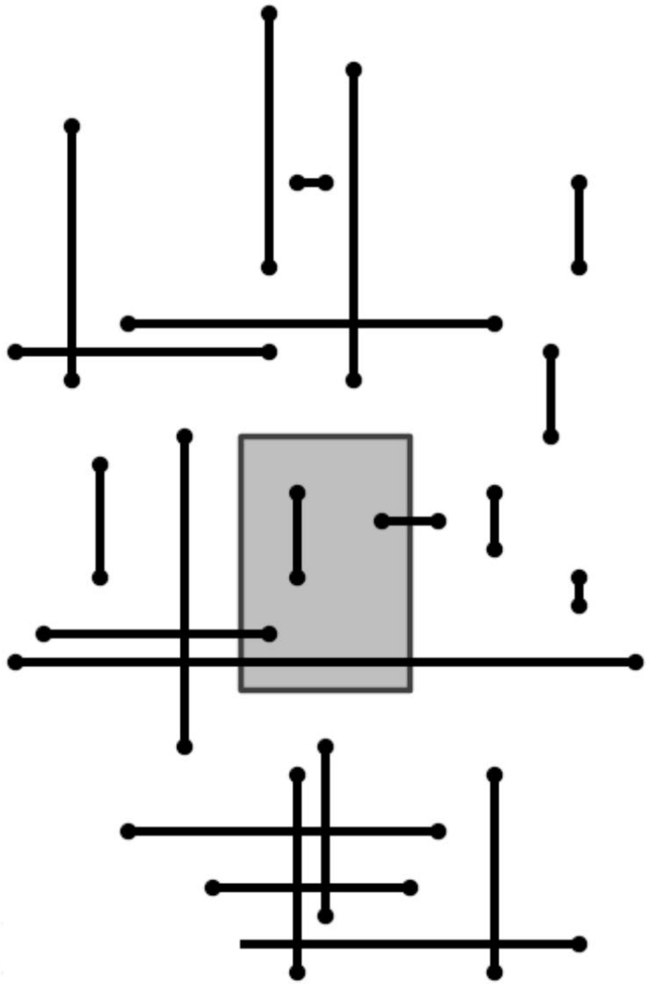
2. Arbitrary line segments (non-crossing)

[Vakken]

Talk overview

1. Windowing of axis parallel line segments in 2D (variants of *interval tree - IT*)
   a) Line stabbing (*IT* with *sorted lists*)
   b) Line segment stabbing (*IT* with *range trees*)
   c) Line segment stabbing (*IT* with *priority search trees*)
2. Windowing of line segments in general position
   – *segment tree*

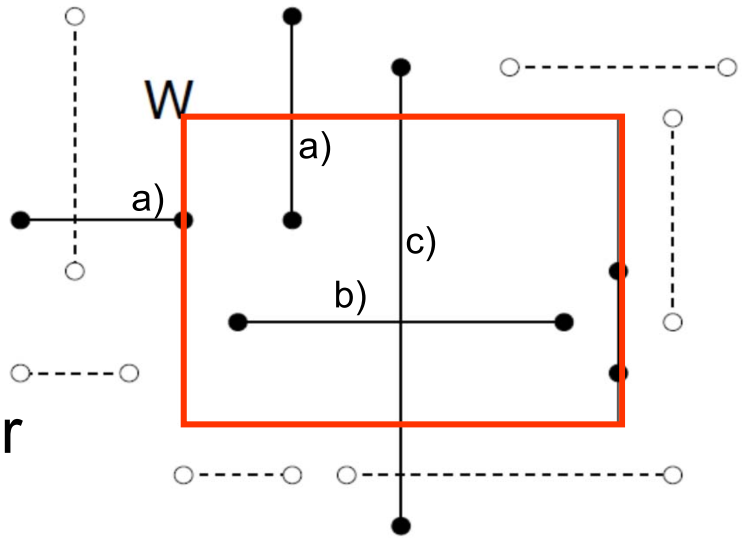
1. Windowing of axis parallel line segments



1. Windowing of axis parallel line segments

Window query

- Given
  - a set of orthogonal line segments S (preprocessed),
  - and orthogonal query rectangle $W = [\, x : x' \,] \circ [\, y : y' \,]$
- Count or report all the line segments of S that intersect W
- Such segments have
  a) 1 endpoint in
  b) 2 end points in – Included
  c) no end point in – Cross over

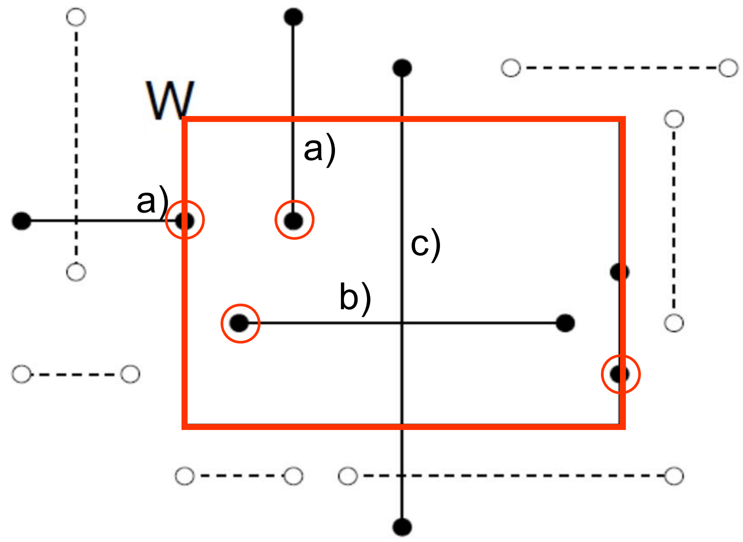


[Mount]

Line segments with 1 or 2 points inside

a) 1 point inside

- Use a range tree (Lesson 3)
- $O(n \log n)$ storage
- $O(\log^2 n + k)$ query time or
- $O(\log n + k)$ with fractional cascading

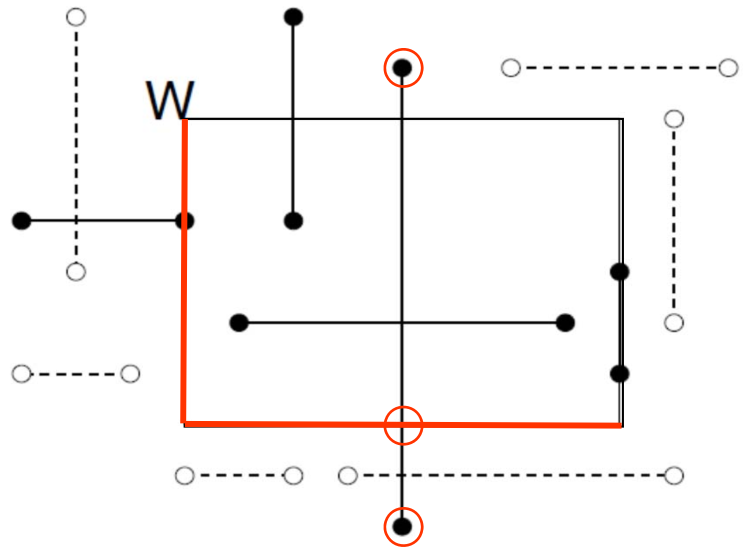


b) 2 points inside – as a) 1 point inside

- Avoid reporting twice
  1. Mark segment when reported (clear after the query)
  2. When end point found, check the other end-point. Report only the leftmost or bottom endpoint

Line segments that cross over the window

c) No points inside



- not detected using a range tree
- Cross the boundary twice
  or
  contain one boundary edge
- It is enough to detect segments intersected by the left and bottom boundary edges (not having end point inside)
- For left boundary: Report the segments intersecting vertical query *line segment* (B)
- Let's discuss vertical query *line* first (A)
- Bottom boundary is rotated 90°

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   c) Line segment stabbing (*IT* with *priority search trees*)
2. Windowing of line segments in general position
   – *segment tree*

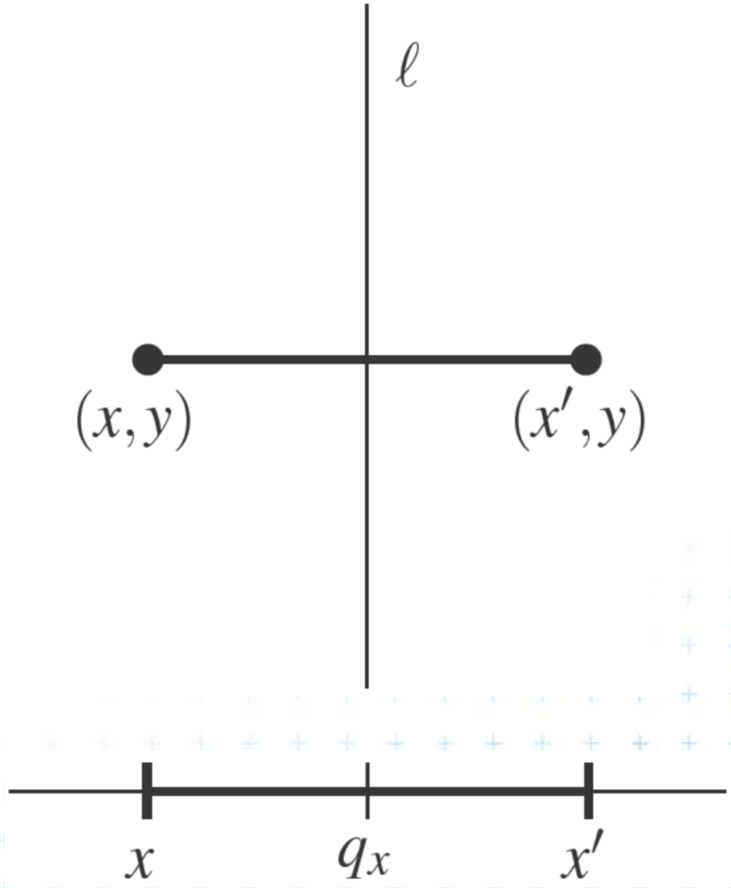
a) Segment intersected by vertical line->1D

- Query line $\ell := (x=q_x)$

  Report the segments stabbed by a vertical line

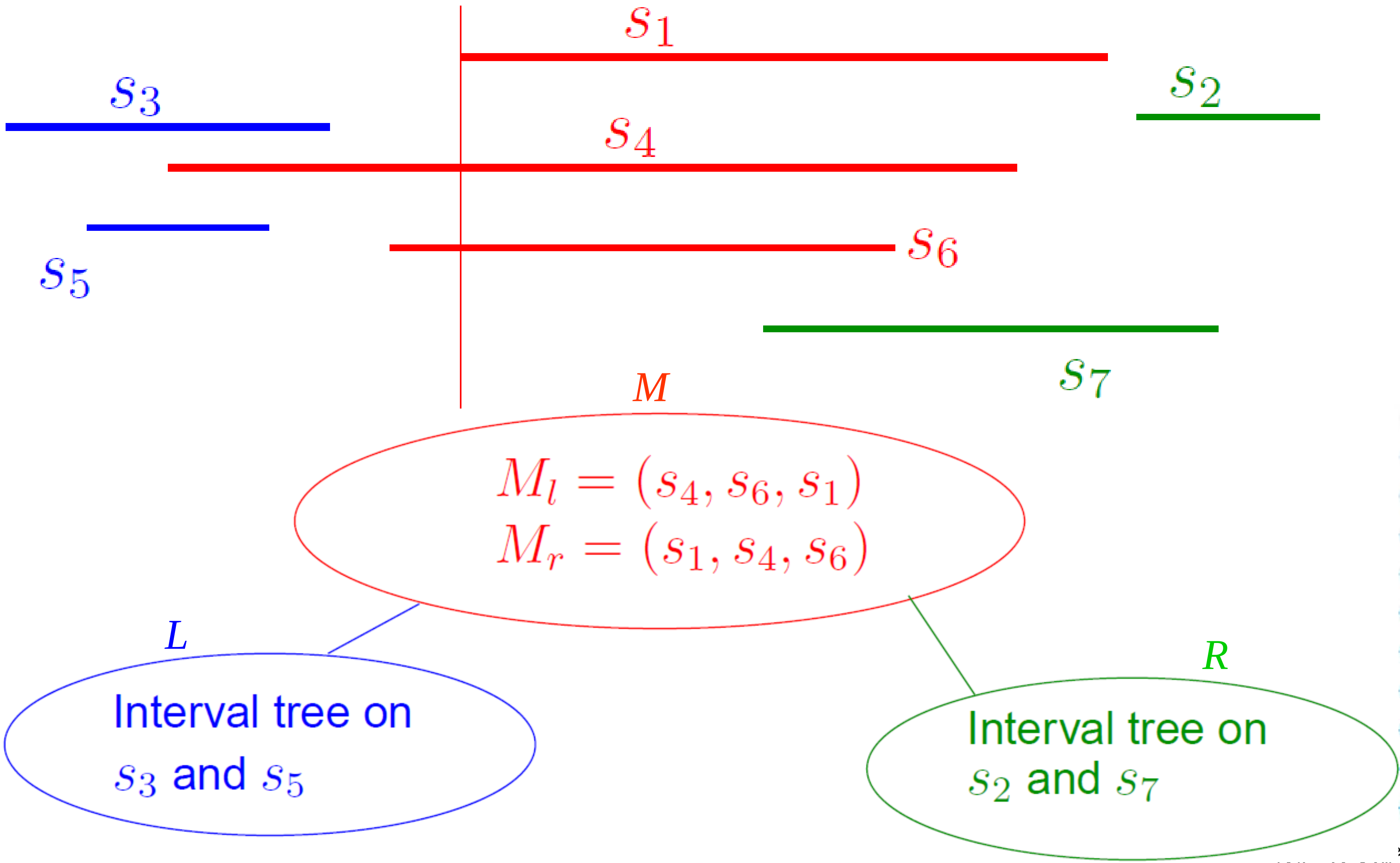
  = 1 dimensional problem

  (ignore y coordinate)



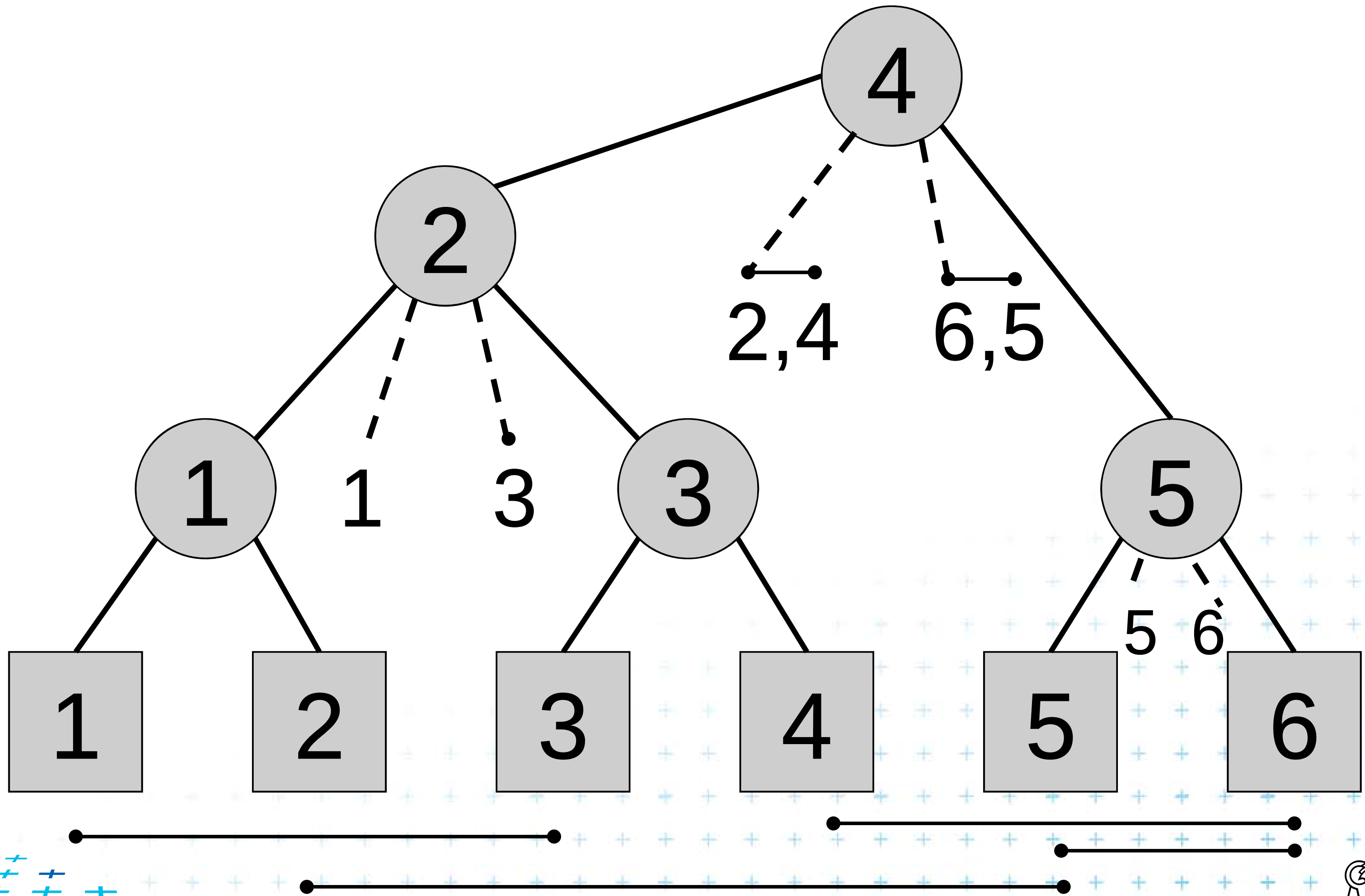
=> Report the interval containing query point q_x

Interval tree principle



[Vigneron]

Static interval tree [Edelsbrunner80]

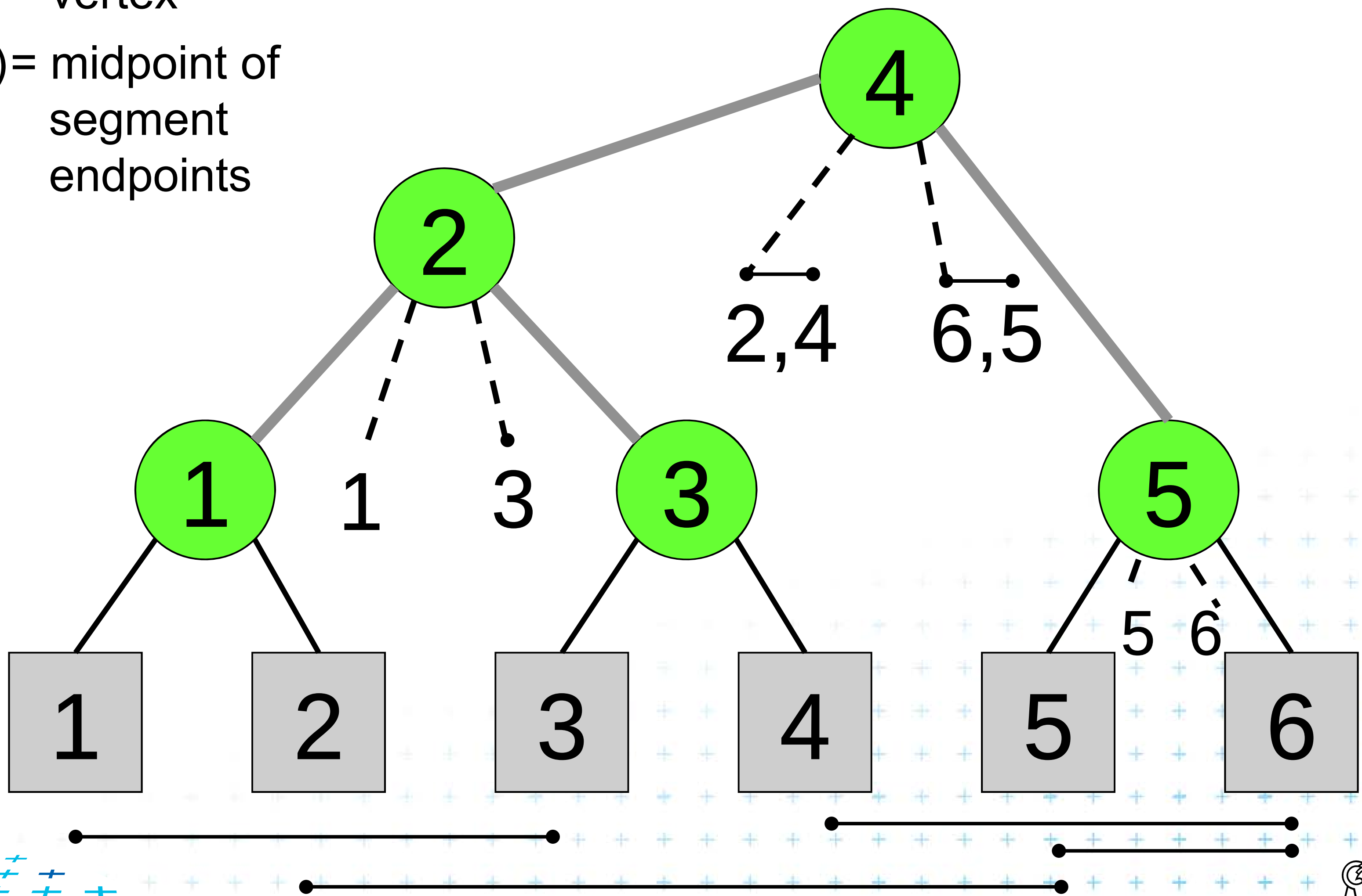


[Kukral]

Primary structure – static tree for endpoints

v = vertex

d(v)= midpoint of segment endpoints

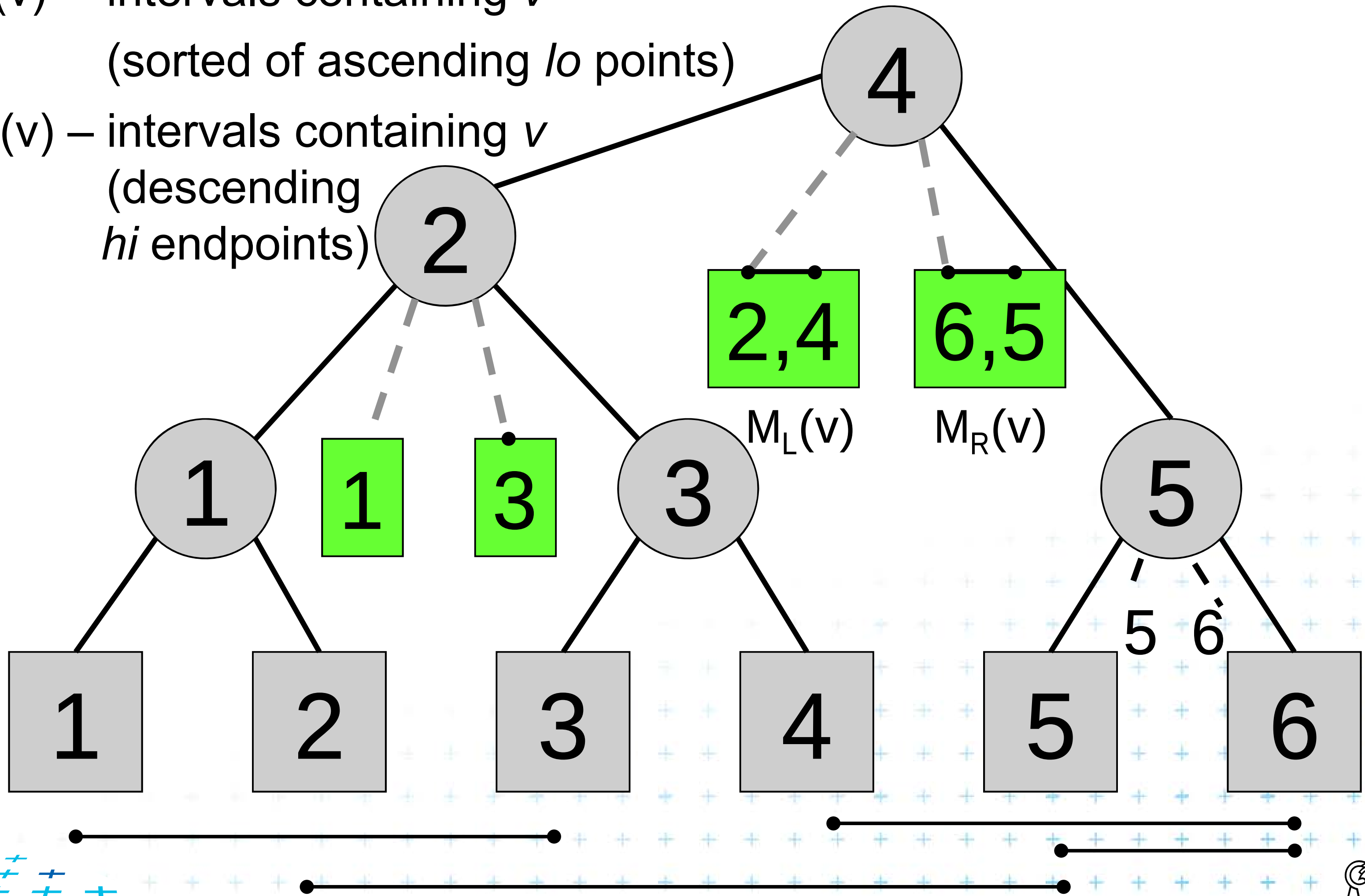


[Kukral]

Secondary lists – sorted segments in M

ML(v) – intervals containing *v*
(sorted of ascending *lo* points)

MR(v) – intervals containing *v*
(descending
hi endpoints)



[Kukral]

Interval tree construction (all intervals at once)

ConstructIntervalTree(*S*)   **// Intervals all active – no active lists**

Input: Set S of intervals on the real line – on *x-axis*

Output: The root of an interval tree for *S*

```
if (|S| == 0) return null                                  // no more
else
    xMed = median endpoint of intervals in S               // median endpoint
    L = { [xlo, xhi] in S | xhi < xMed }                   // left of median
    R = { [xlo, xhi] in S | xlo > xMed }                   // right of median
    M = { [xlo, xhi] in S | xlo <= xMed <= xhi }           // contains median
    ML = sort M in increasing order of xlo                 // sort M
    MR = sort M in decreasing order of xhi
    t = new IntTreeNode(xMed, ML, MR)                      // this node
    t.left  = ConstructIntervalTree(L)                     // left subtree
    t.right = ConstructIntervalTree(R)                     // right subtree
    return t
```

[Mount]

Line stabbing query for an interval tree

Stab(t, xq)
Input: IntTreeNode t, Scalar xq
Output: prints the intersected intervals

```
if (t == null) return                          // fell out of tree
if (xq < t.xMed)                               // left of median?
    for (i = 0; i < t.ML.length; i++)          // traverse ML
            if (t.ML[i].lo <= xq) print(t.ML[i])   // ..report if in range
            else break                         // ..else done
    stab(t.left, xq)                           // recurse on left
else  // (xq – t.xMed)                          // right of or equal to
median
    for (i = 0; i < t.MR.length; i++) {        // traverse MR
            if (t.MR[i].hi – xq) print(t.MR[i])    // ..report if in range
            else break                         // ..else done
    stab(t.right, xq)                          // recurse on right
```

Note: Small inefficiency for xq == t.xMed – recurse on right

[Mount]

Complexity of line stabbing via interval tree

- Construction - $O(n \log n)$ time
  - Each step divides at maximum into two halves or less (minus elements of M) => tree height $O(\log n)$
  - If presorted the endpoints in three lists L,R,M then median in O(1) and copy to new L,R,M in $O(n)$]
- Vertical line stabbing query - $O(k + \log n)$ time
  - One node processed in O(1 + k'), k'=reported intervals
  - v visited nodes in $O(v + k)$, k=total reported intervals
  - v = tree height = $O(\log n)$
- Storage - $O(n)$
  - Tree has $O(n)$ nodes, each segment stored twice (two endpoints)

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2. Windowing of line segments in general position
   – *segment tree*

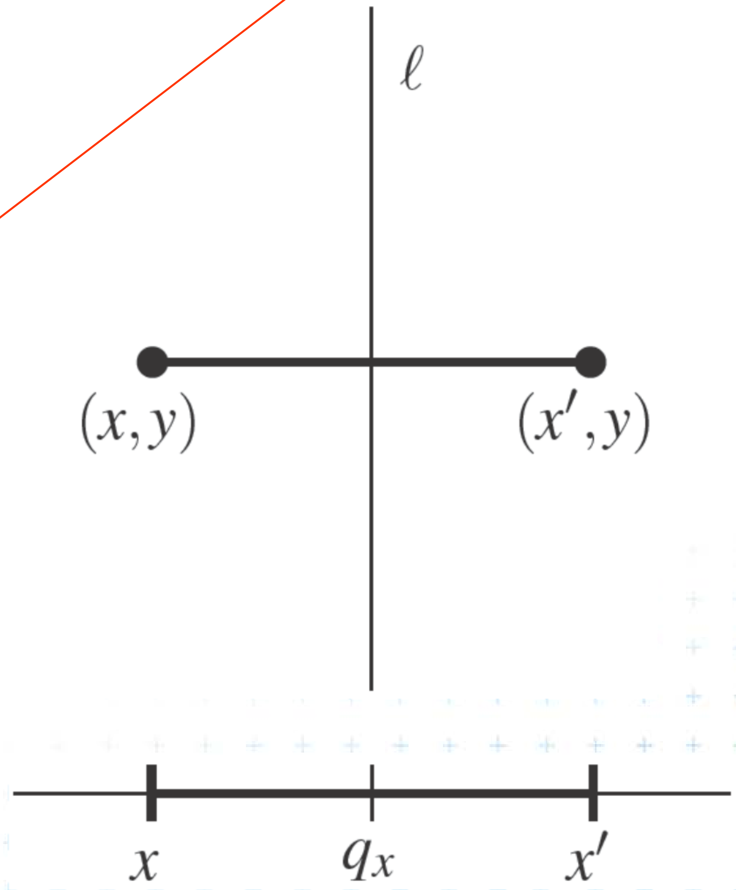
a) Segment intersected by vertical line - 1D

- Query line $\ell := (x = q_x)$

  Report the segments stabbed by a vertical line

  = 1 dimensional problem

  (ignore y coordinate)

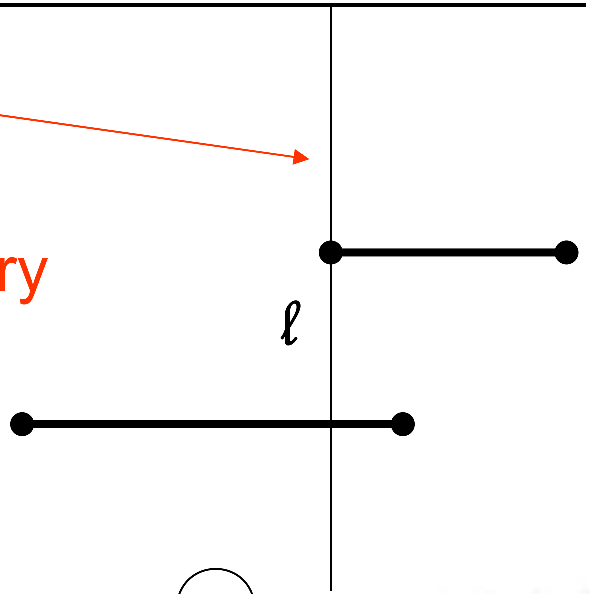


=> Report the interval containing query point q_x

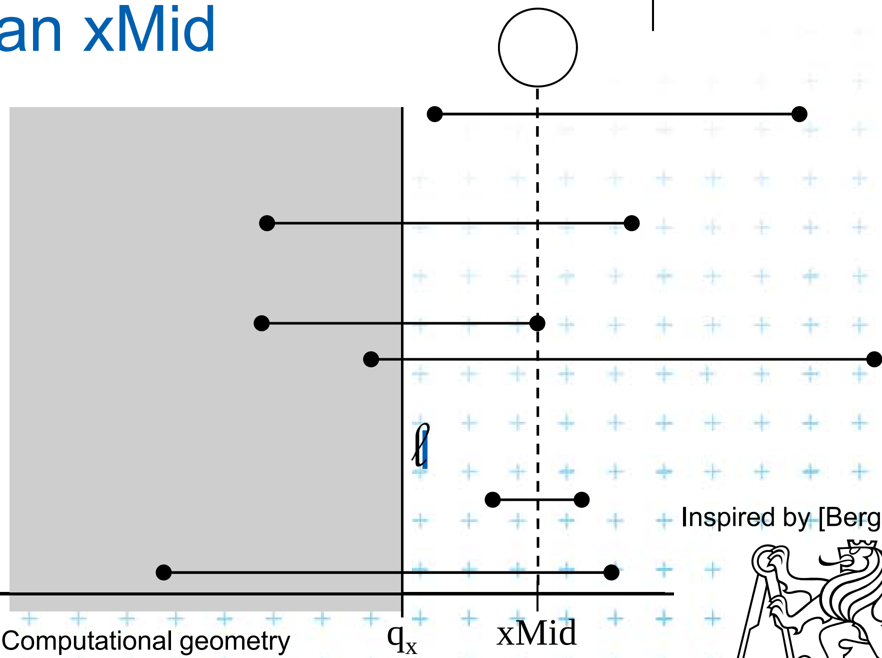
DS: Interval tree

a) Segment intersected by vertical line - 2D

- Query line $\ell := q_x \circ [— : —]$
  ;
- Horizontal segment of M stabs the query line ℓ **iff** its left endpoint lies in halph-space
  $(— : q_x] \circ [-— : —]$



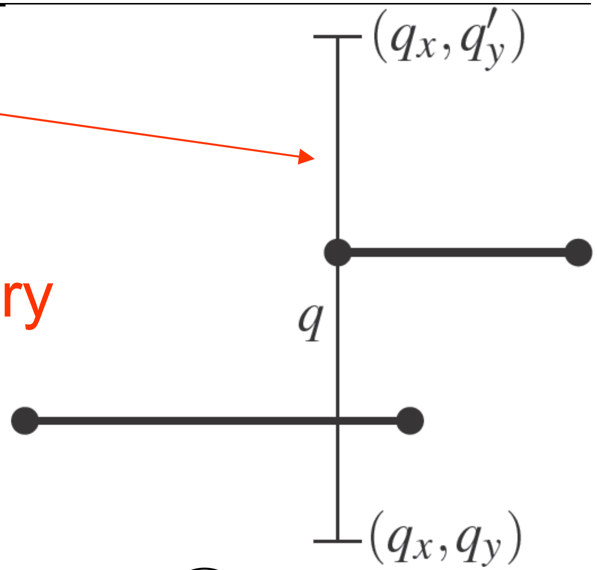
- In IT node with stored median xMid report all segments from M
  - whose left point lies in $(— : q_x]$
    if ℓ lies left from xMid
  - whose right point lies in $(q_x : +—]$
    if ℓ lies right from xMid



Inspired by [Berg]

b) Segment intersected by vertical line segment

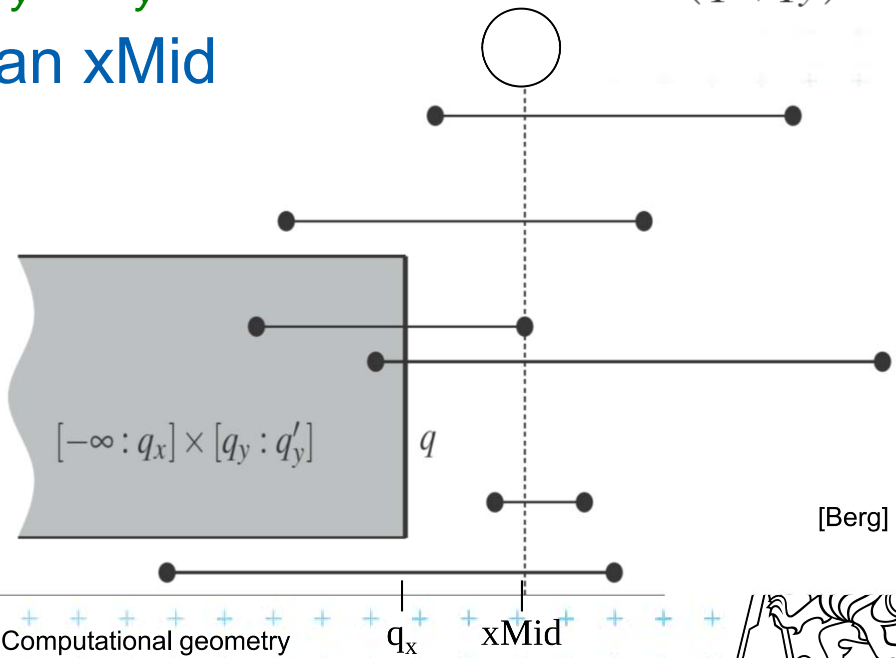
- Query segment $q := q_x \circ [q_y : q'_y]$



- Horizontal segment of *M* stabs the query segment *q* **iff** its left endpoint lies in semi-infinite rectangular region

$$(— : q_x] \circ [q_y ; q'_y]$$

- In IT node with stored median xMid report all segments
  - whose left point lies in $(— : q_x] \circ [q_y ; q'_y]$ if *q* lies left from xMid
  - whose right point lies in $(q_x : +—] \circ [q_y ; q'_y]$ if *q* lies right from xMid



[Berg]

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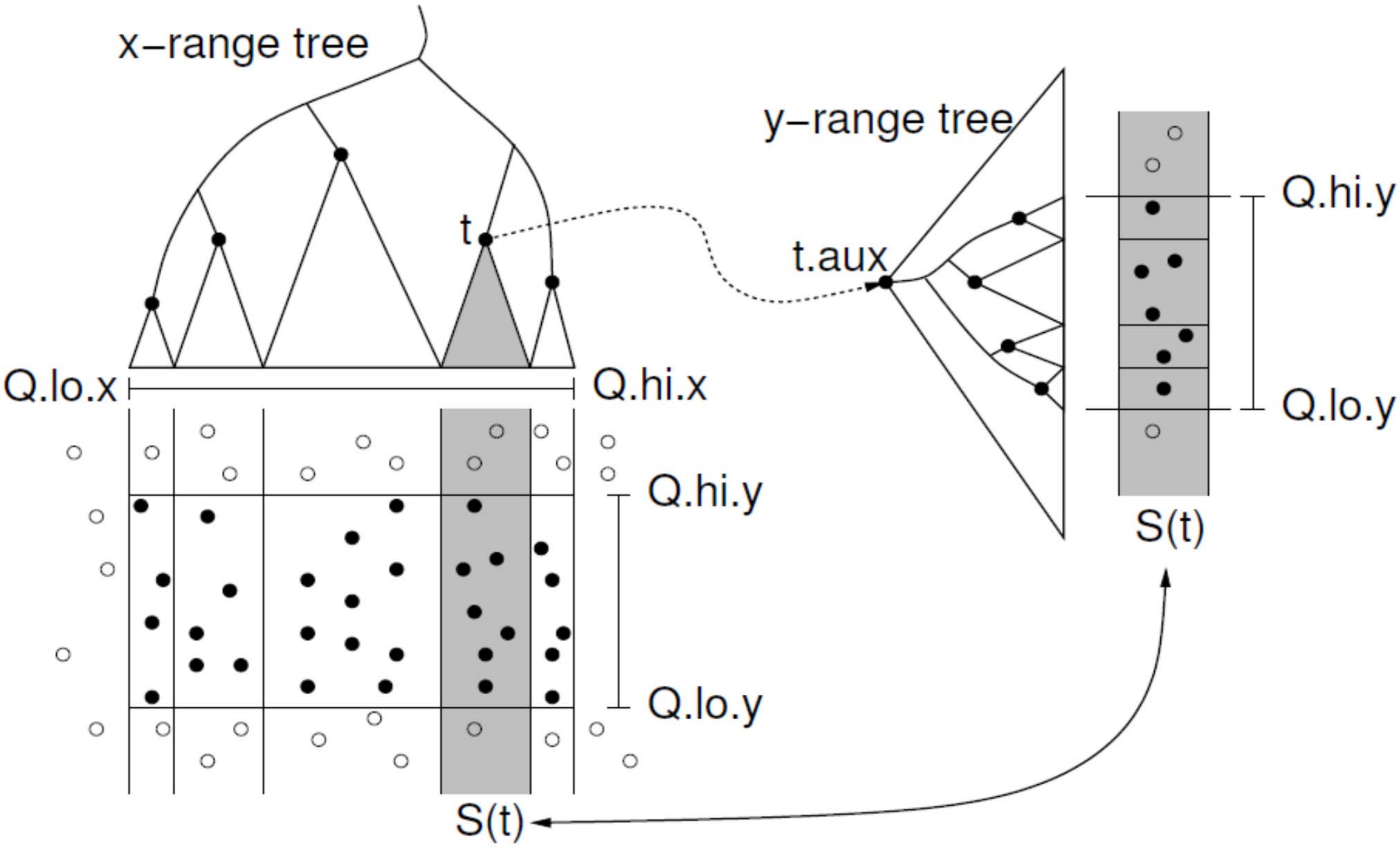
Data structure for endpoints

- Storage of ML and MR
  - Sorted lists not enough for line segments
  - Use two range trees
- Instead O(n) sequential search in ML and MR perform O(log n) search in range tree with fractional cascading

2D range tree (without fractional casc. - see more in Lecture 3)



Complexity of line segment stabbing

- Construction - $O(n \log n)$ time
  - Each step divides at maximum into two halves L,R or less (minus elements of M) => tree height $O(\log n)$
  - If the range trees are efficiently build in $O(n)$
- Vertical line segment stab. q. - $O(k + \log^2 n)$ time
  - One node processed in $O(\log n + k')$, k'=reported inter.
  - v visited nodes in $O(v \log n + k)$, k=total reported inter.
  - v = tree height = $O(\log n)$
  - $O(k + \log^2 n)$ time - range tree with fractional cascading
  - $O(k + \log^3 n)$ time - range tree without fractional casc.
- Storage - $O(n \log n)$
  - Dominated by the range trees

Talk overview

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c) Priority search trees [McCreight85]

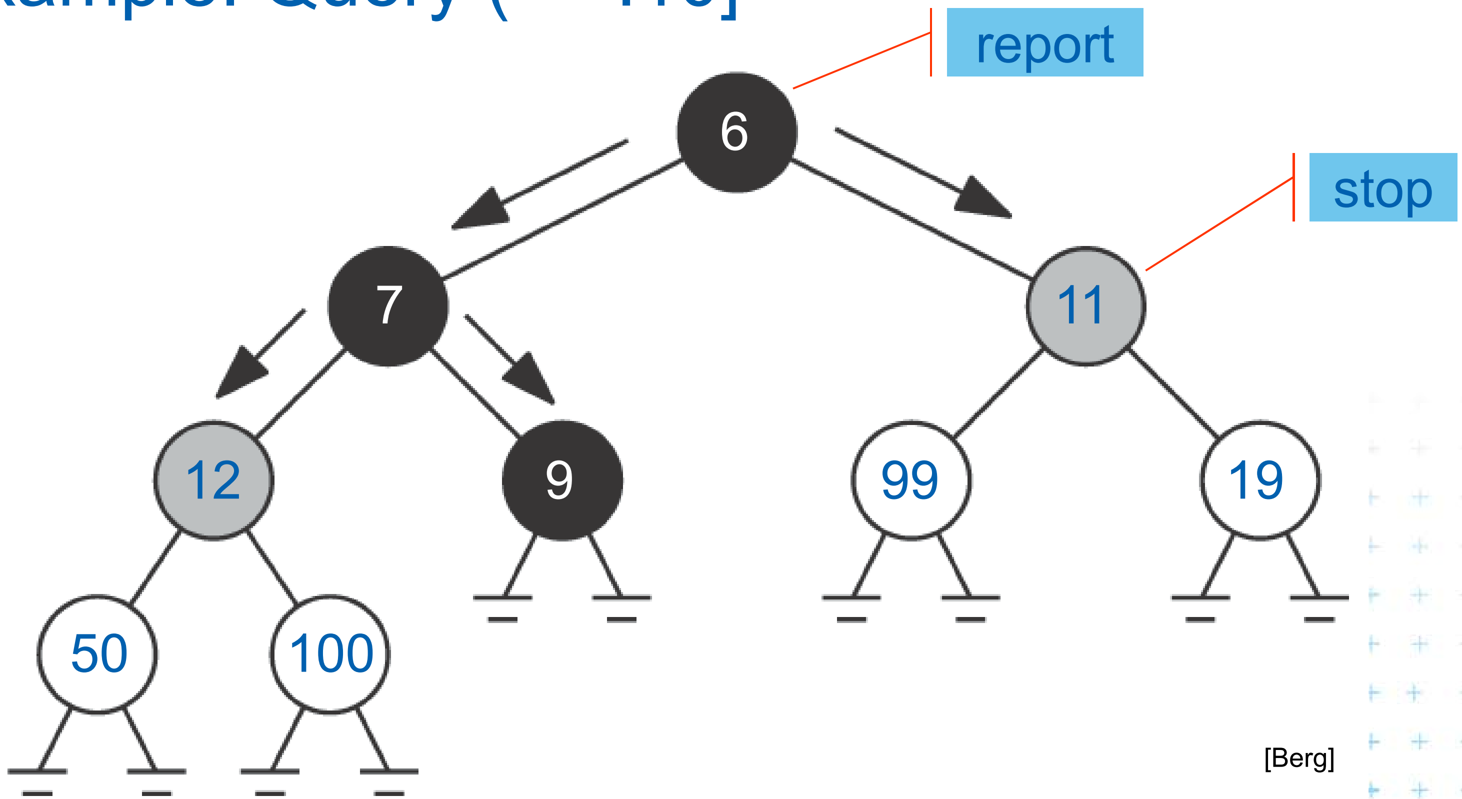
- Priority search trees – in case c) on slide 8
  - Exploit the fact that query rectangle in range queries is unbounded
  - Can be used as secondary data structures for both left and right endpoints (ML and MR) of segments (intervals) in nodes of interval tree
  - Improve the storage to O(n) for horizontal segment intersection with window edge (Range tree has O(n log n))
- For cases a) and b) - O(n log n) remains
  - we need range trees for windowing segment endpoints

Rectangular range queries variants

- Let $P = \{ p_1, p_2, \ldots, p_n \}$ is set of points in plane
- Goal: rectangular range queries of the form
  $(— : q_x] \circ [q_y ; q'_y]$
- In 1D: search for nodes v with v_x µ $(— : q_x]$
  - range tree O(log n + k) time
  - ordered list O(1 + k) time
    (start in the leftmost, stop on v with $v_x > q_x$)
  - use heap O(1 + k) time
    (traverse all children, stop when $v_x > q_x$)
- In 2D – use heap for points with x µ $(— : q_x]$
  + integrate information about y-coordinate

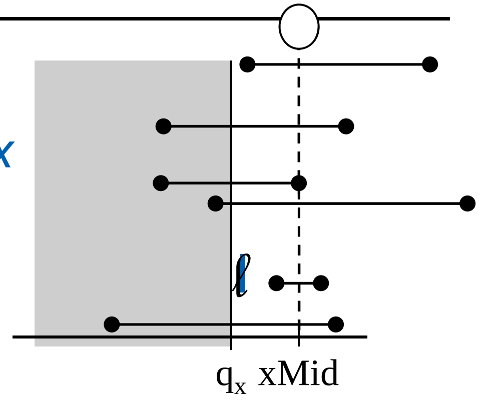
Heap for 1D unbounded range queries

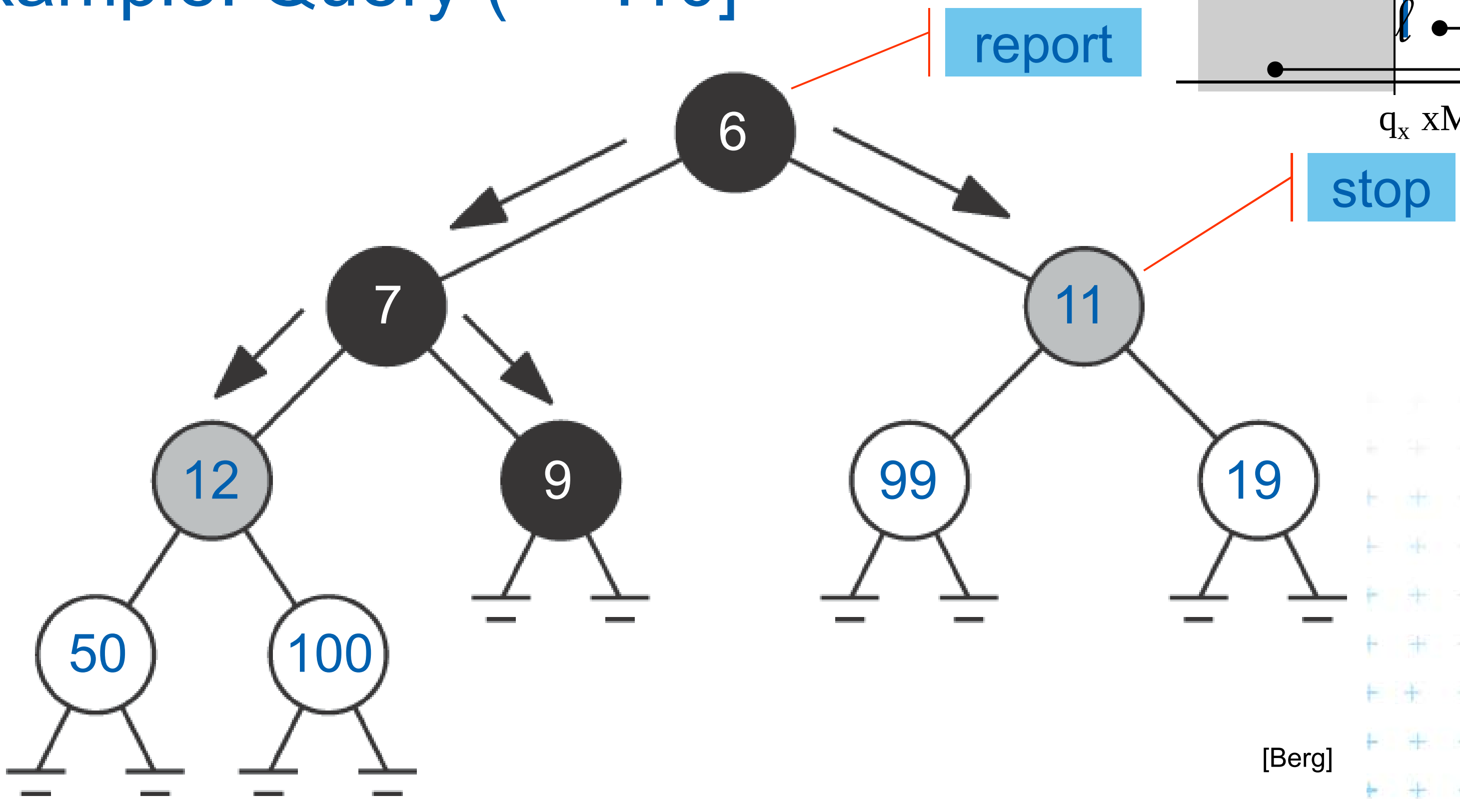
- Traverse all children, stop when $v_x > q_x$
- Example: Query (— :10]



Heap for 1D unbounded range queries

- Traverse all children, stop when $v_x > q_x$
- Example: Query (— :10]



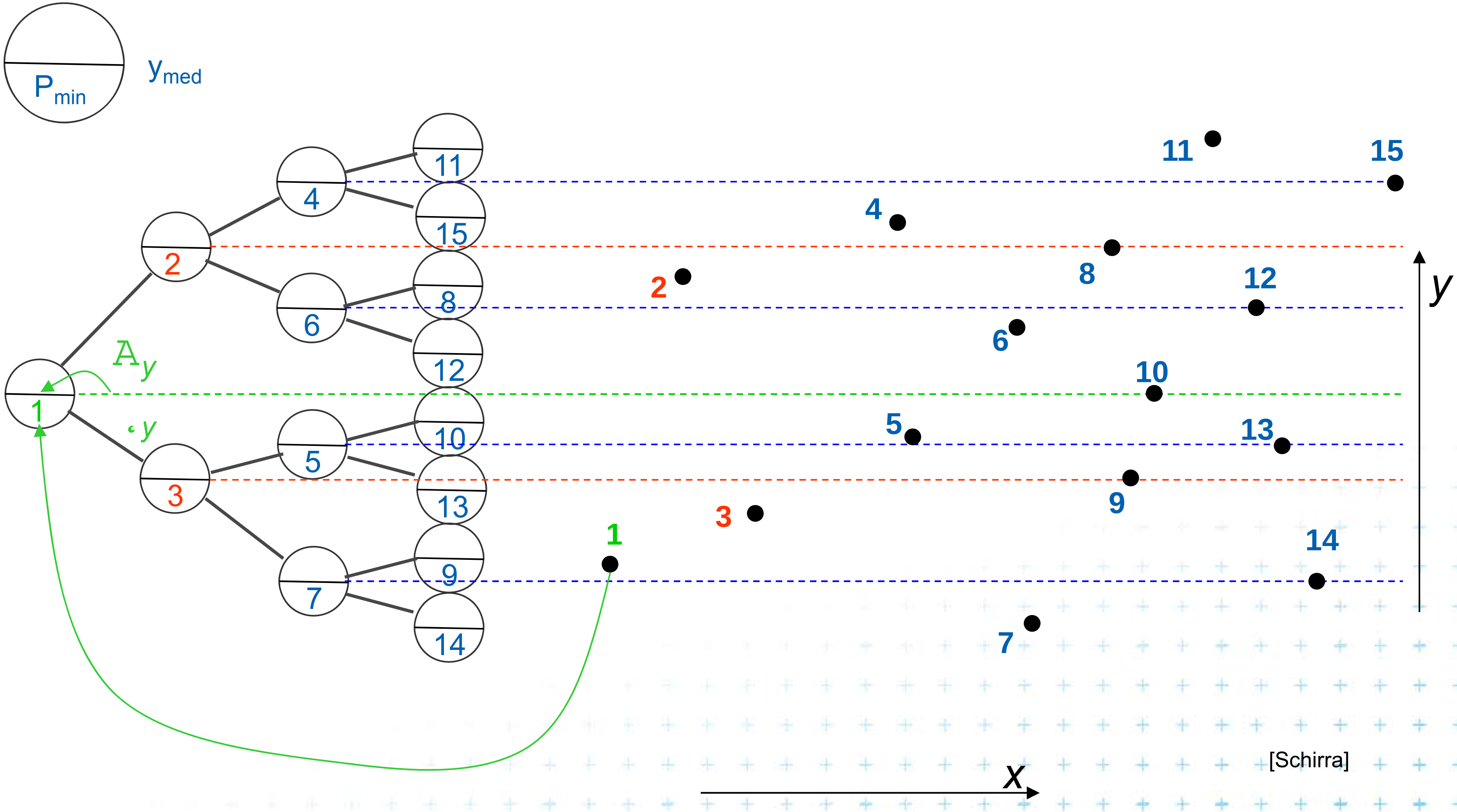


[Berg]

Priority search tree (PST)

- Heap in 2D can incorporate info about both *x,y*
  - BST on *y*-coordinate (horizontal slabs) ~ range tree
  - Heap on *x*-coordinate (minimum *x* from slab along *x*)
- If *P* is empty, PST is empty leaf
- else
  - p_{min} = point with smallest x-coordinate in *P* --- a heap root
  - y_{med} = *y*-coord. median of points $P \setminus \{p_{min}\}$ --- BST root
  - $P_{below} := \{ p \in P \setminus \{p_{min}\} : p_y \le y_{med} \}$
  - $P_{above} := \{ p \in P \setminus \{p_{min}\} : p_y > y_{med} \}$
- Point p_{min} and scalar y_{med} are stored in the PST root
- The left subtree is PST of P_{below}
- The right subtree is PST of P_{above}

Priority search tree construction example



Priority search tree definition

PrioritySearchTree(*P*)

Input: set *P* of points in plane
Output: priority search tree *T*

```
if P=∅ then PST is an empty leaf
else
    p_min = point with smallest x-coordinate in P      // heap on x root
    y_med = y-coord. median of points P \ {p_min}       // BST on y root
    Split points P \ {p_min} into two subsets – according to y_med
        P_below := { p ∈ P \ {p_min} : p_y ≤ y_med }
        P_above := { p ∈ P \ {p_min} : p_y > y_med }
    T = newTreeNode()
    T.p = p_min       // point [ x, y ]
    T.y = y_mid       // skalar
    T.left  = PrioritySearchTree( P_below )
    T.rigft = PrioritySearchTree( P_above )
```

Notation in alg:
… p(v)
… y(v)
… lc(v)
… rc(v)

O(*n* log *n*) , but O(*n*) if presorted on *y*-coordinate and bottom up

Query Priority Search Tree

QueryPrioritySearchTree(T, $(-\infty : q_x] \times [q_y ; q'_y])$)

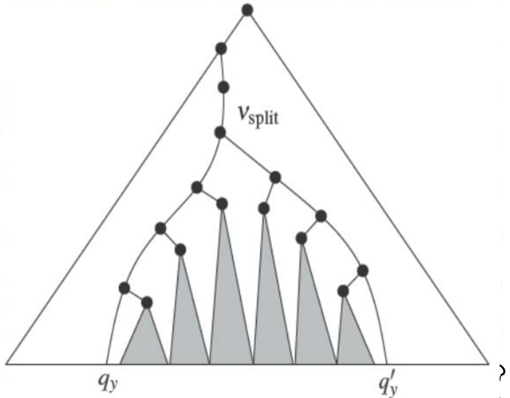
Input: A priority search tree and a range, unbounded to the left

Output: All points lying in the range

1. Search with q_y and q'_y in T // BST on y-coordinate – select y range
   Let v_{split} be the node where the two search paths split (split node)

2. for each node v on the search path of q_y or q'_y // points along the paths
3. if $p(v) \in (-\infty : q_x] \times [q_y ; q'_y]$ then report $p(v)$ // starting in tree root

4. for each node v on the path of q_y in the left subtree of v_{split} // inner trees
5. if the search path goes left at v
6. ReportInSubtree($rc(v)$, q_x) // report right sul
7. for each node v on the path of q'_y in right subtree o
8. if the search path goes right at v
9. ReportInSubtree($lc(v)$, q_x) // rep. left subtre



Reporting of subtrees between the paths

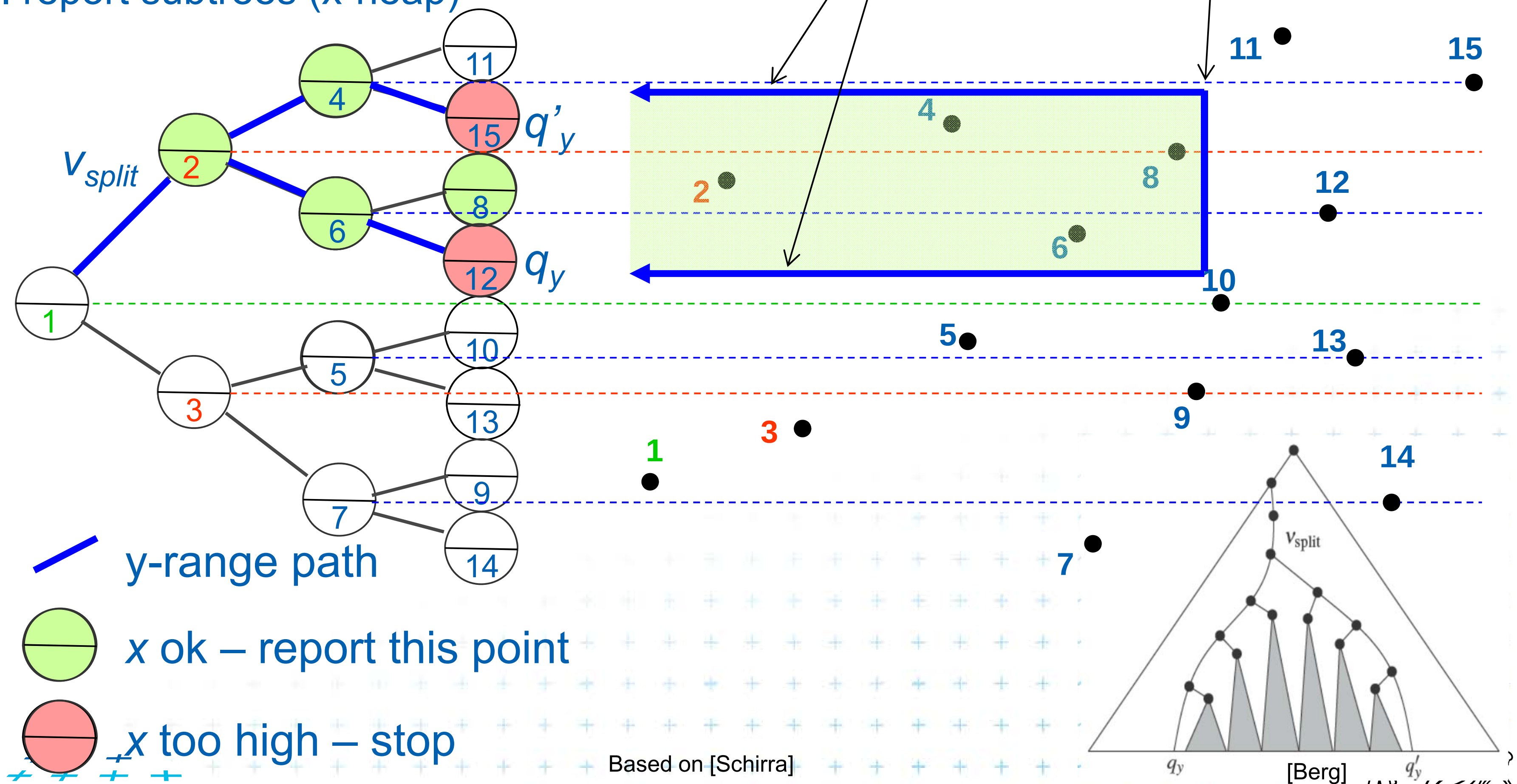
ReportInSubtree(*v*, q_x)

Input: The root *v* of a subtree of a priority search tree and a value q_x.
Output: All points in the subtree with *x*-coordinate at most q_x.

1. if *v* is not a leaf and $x(\,p(v)\,) \leq q_x$ // $x \in (-\infty : q_x]$ -- heap condition
2. Report $p(v)$.
3. ReportInSubtree($lc(v)$, q_x)
4. ReportInSubtree($rc(v)$, q_x)

Priority search tree query

1. select y range (y-BVS~ 1D range tree)
2. report points on paths (x-heap)
3. report subtrees (x-heap)



Based on [Schirra]

[Berg]

Priority search tree complexity

For set of *n* points in the plane

- Build O(*n* log *n*)
- Storage O(*n*)
- Query O(*k* + log *n*)
  - points in query range $(\text{—} : q_x] \circ [q_y ; q'_y])$
  - *k* is number of reported points

- Use PST as associated data structure for interval trees for storage of M

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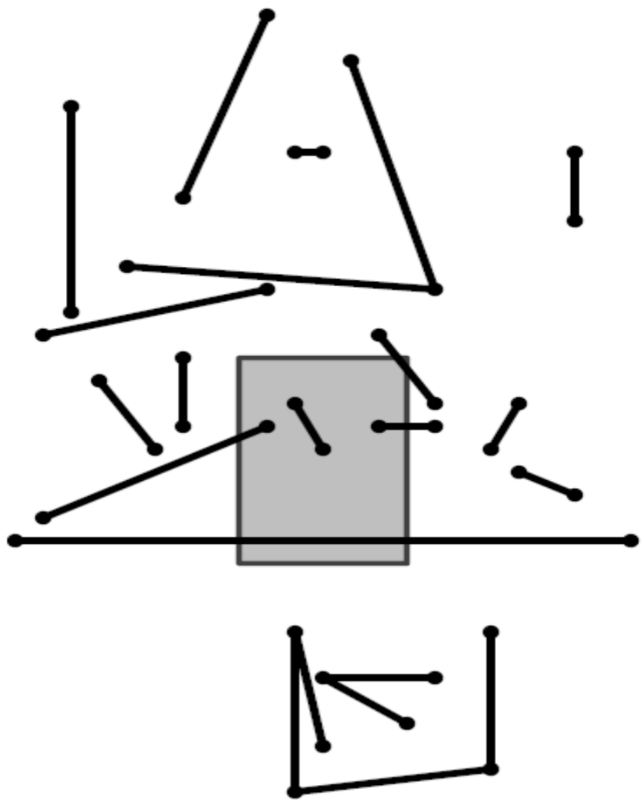
2. Windowing of line segments in general position



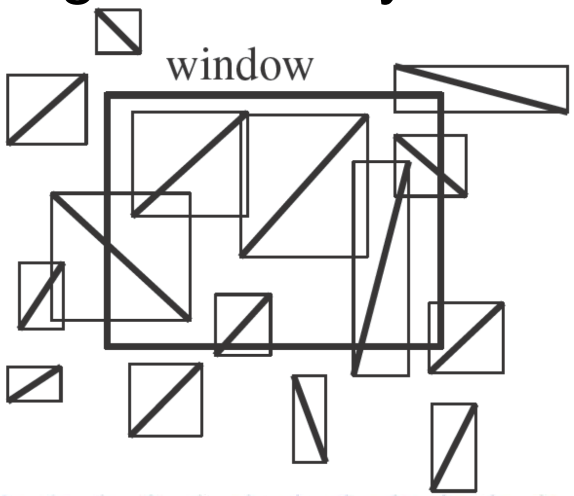
Windowing of arbitrary oriented line segments

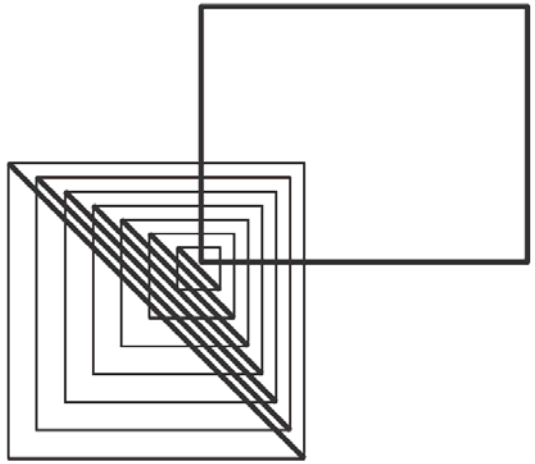
- Two cases of intersection

  a,b) Endpoint inside the query window => range tree

  c) Segment intersects side of query window => ???

- Intersection with BBOX (segment bounding box)?
  - Intersection with 4n sides
  - But segments may not intersect the window –> query y





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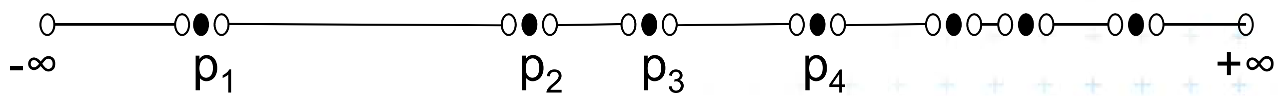
Segment tree

[Bentley, 1977]

- Exploits locus approach
  - Partition parameter space into regions of same answer
  - Localization of such region = knowing the answer
- For given set S of n intervals (segments) on real line
  - Finds m elementary intervals (induced by interval end-points)
  - Partitions 1D parameter space into these elementary intervals



$$(-\infty : p_1), [p_1 : p_1], (p_1 : p_2), [p_2 : p_2], \ldots,$$
$$(p_{m-1} : p_m), [p_m : p_m], (p_m : +\infty)$$

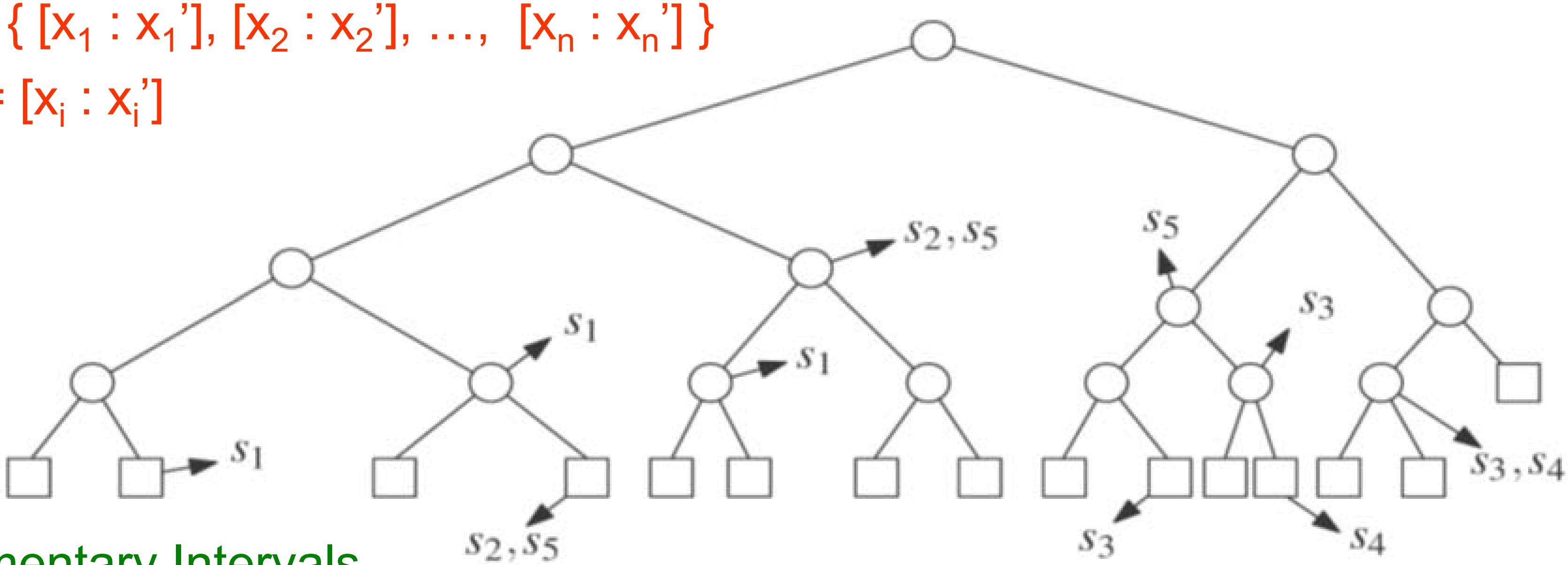
  - Stores intervals s_i with the elementary intervals
  - Reports the intervals s_i containing query point q_x.

Segment tree example

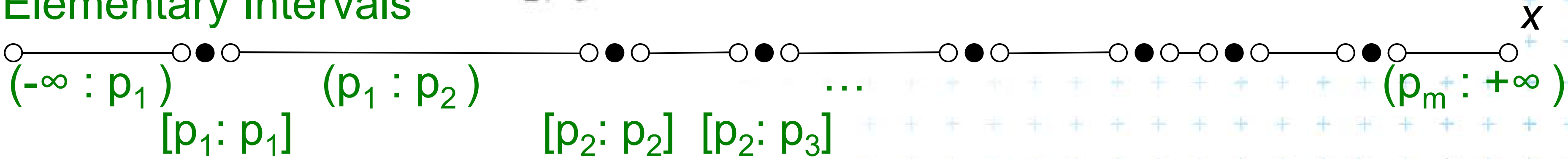
Intervals

$S = \{ [x_1 : x_1'], [x_2 : x_2'], \ldots, [x_n : x_n'] \}$

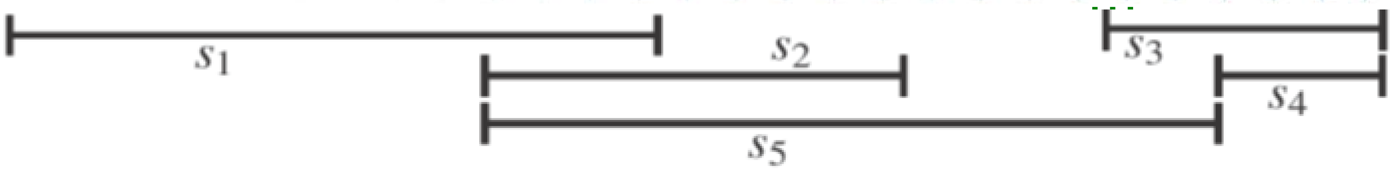
$s_i = [x_i : x_i']$



Elementary Intervals



Intervals



Segment tree definition

Segment tree

- Skeleton is a balanced binary tree T
- Leaves ~ elementary intervals Int(v)
- Internal nodes v
  ~ union of elementary intervals of its children
  - Store: 1. interval Int(v) = union of elementary intervals of its children
    2. canonical set $S(v)$ of intervals $[x : x']$ µ S segments s_i
  - Holds Int(v) Å $[x : x']$ and Int(parent(v)) † $[x : x']$ (node interval is not larger than a segment)
  - Intervals $[x : x']$ are stored as high as possible, such that Int(v) is completely contained in the segment

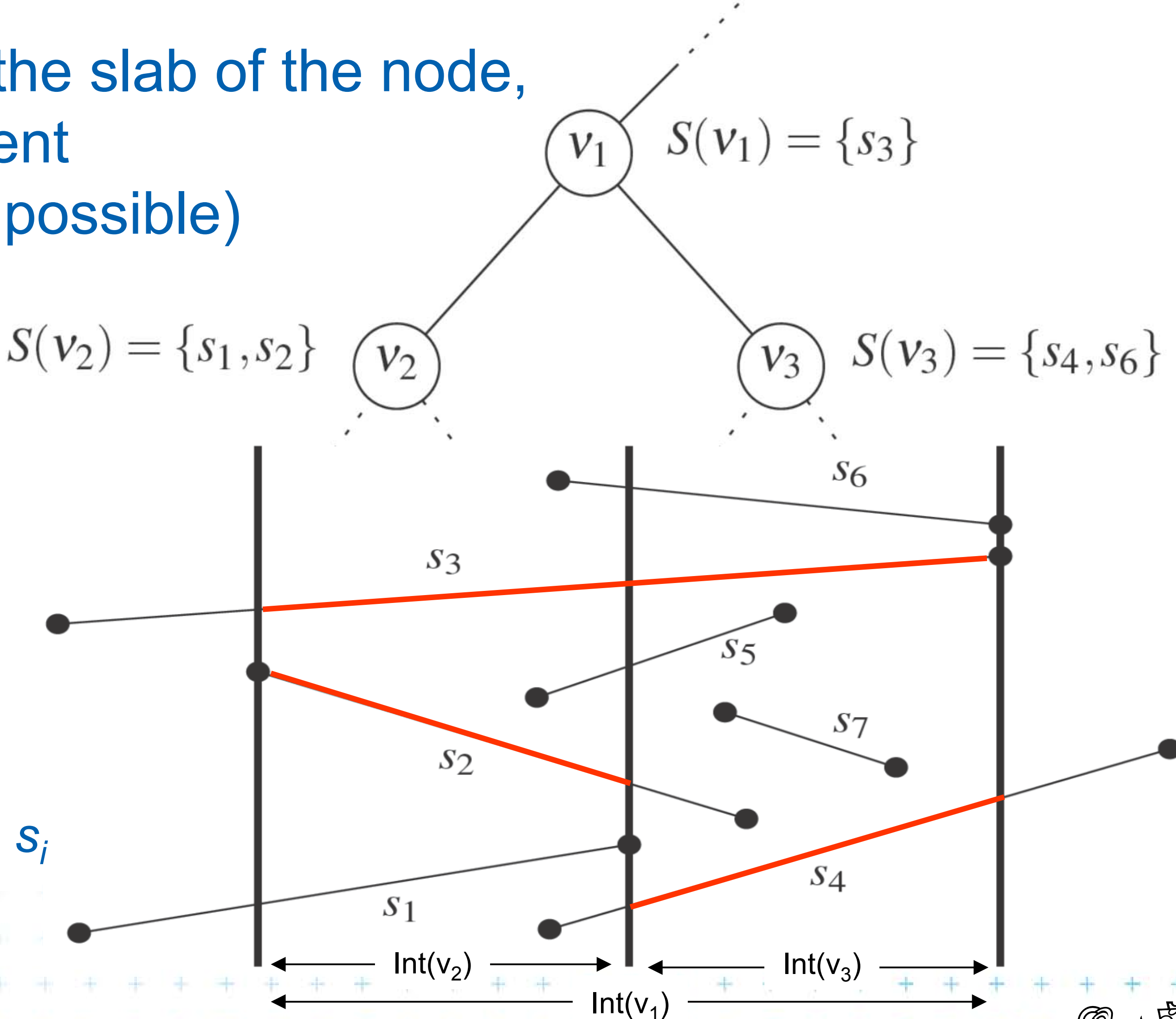
Segments span the slab

Segments span the slab of the node,
but not of its parent
(stored as up as possible)

Int(v_j) Å s_i

and

Int(parent(v_j)] † s_i



Query segment tree

QuerySegmentTree(v, q_x)
Input: The root of a (subtree of a) segment tree and a query point q_x
Output: All intervals in the tree containing q_x.

```
Report all the intervals s_i in S(v).        // current node
if v is not a leaf
    if q_x µ Int( lc(v) )                    // go left
        QuerySegmentTree( lc(v), q_x )
    else                                     // or go right
        QuerySegmentTree( rc(v), q_x )
```

Query time O(log n + k), where k is the number of reported intervals
Height O(log n), O(1 + k_v) for node
Storage O(n log n)

Segment tree construction

ConstructSegmentTree(*S*)
Input: Set of intervals *S* - segments
Output: segment tree

1. Sort endpoints of segments in *S* -> get elemetary intervals …O(n log n)
2. Construct a binary search tree *T* on elementary intervals …O(n) (bottom up) and determine the interval Int(v) it represents
3. Compute the canonical subsets for the nodes (lists of their segments):
4. v = root(*T*)
5. for all segments $s_i = [x : x']$ µ S
6. InsertSegmentTree(v, $[x : x']$)

Segment tree construction – interval insertion

InsertSegmentTree(v, [x : x'])

Input: The root of (a subtree of) a segment tree and an interval.

Output: The interval will be stored in the subtree.

if Int(v) Å [x : x'] *// Int(v) contains* s_i = [x : x']
    store [x : x'] at v
else if Int($lc(v)$) ∩ [x : x'] û þ
        InsertSegmentTree($lc(v)$, [x : x'])
    if Int($rc(v)$) ∩ [x : x'] û þ
        InsertSegmentTree($rc(v)$, [x : x'])

One interval is stored at most twice in one level =>
Single interval insert O(log n)
Construction total O(n log n)

Segment tree complexity

A segment tree for set S of n intervals in the plane,

- Build $O(n \log n)$
- Storage $O(n \log n)$
- Query $O(k + \log n)$
  - Report all intervals that contain a query point
  - k is number of reported intervals

Segment tree versus Interval tree

- Segment tree
  - O(n log n) storage x O(n) of Interval tree
  - But returns exactly the intersected segments s_i, interval tree must search the lists ML and/or MR
- Good for
  1. extensions (allows different structuring of intervals)
  2. stabbing counting queries
     - store number of intersected intervals in nodes
     - O(n) storage and O(log n) query time = optimal
  3. higher dimensions – multilevel segment trees

     (Interval and priority search trees do not exist in ^dims)

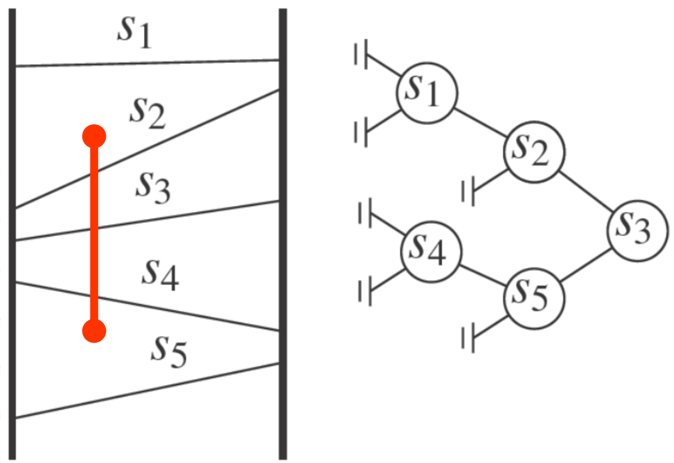
Talk overview

1. Windowing of axis parallel line segments in 2D (variants of *interval tree - IT*)
   a) Line stabbing (*IT* with *sorted lists*)
   b) Line segment stabbing (*IT* with *range trees*)
   c) Line segment stabbing (*IT* with *priority search trees*)
2. Windowing of line segments in general position
   – *segment tree*
   – the algorithm

Windowing of **arbitrary oriented line segments**

- Let S be a set of arbitrarily oriented line segments in the plane.
- Report the segments intersecting a vertical query segment $q := q_x \circ [q_y : q'_y]$
- Segment tree T on x intervals of segments in S
  - node v of T corresponds to vertical slab Int(v) ∘ (-— : –)
  - segments span the slab of the node, but not of its parent
  - segments do not intersect

  => segments in the slab (node) can be vertically ordered – BST

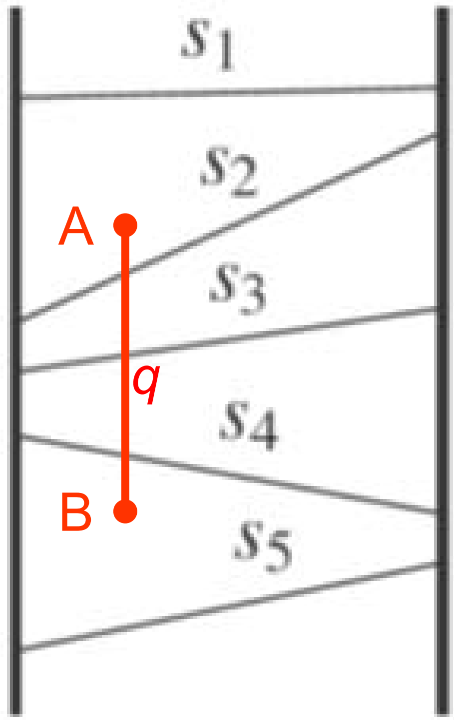


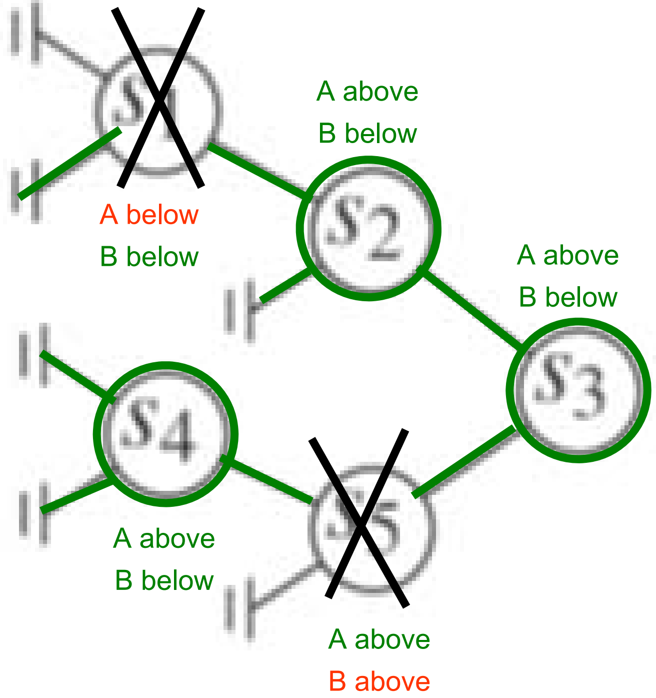
Segments between vertical segment endpoints

- Segments (in the slab) do not mutually intersect
  => segments can be vertically ordered and stored in BST
  - Each node v of the x segment tree has an associated y BST
  - BST $T(v)$ of node v stores the canonical subset $S(v)$ according to the vertical order
  - Intersected segments can be found by searching $T(v)$ in $O(k_v + \log n)$, k_v is the number of intersected segments

Segments between vertical segment endpoints

- Segment *s* is intersected by vert.query segment *q* iff
  - The lower endpoint (B) of *q* is below *s* and
  - The upper endpoint (A) of *q* is above *s*





Windowing complexity

Structure associated to node (BST) uses storage linear in the size of $S(v)$

- Build $O(n \log n)$
- Storage $O(n \log n)$
- Query $O(k + \log^2 n)$
  - Report all segments that contain a query point
  - k is number of reported segments

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