DEPARTMENT OF COMPUTER GRAPHICS AND INTERACTION

INTERSECTIONS OF LINE SEGMENTS AND POLYGONS

PETR FELKEL

FEL CTU PRAGUE

felkel@fel.cvut.cz

https://cw.felk.cvut.cz/doku.php/courses/a4m39vg/start

Based on [Berg], [Mount], [Kukral], and [Drtina]

Version from 13.11.2014

Talk overview

- Intersections of line segments (Bentley-Ottmann)
  - Motivation
  - Sweep line algorithm recapitulation
  - Sweep line intersections of line segments
- Intersection of polygons or planar subdivisions
  - See assignment [21] or [Berg, Section 2.3]
- Intersection of axis parallel rectangles
  - See assignment [26]

Geometric intersections – what are they for?

One of the most basic problems in computational geometry

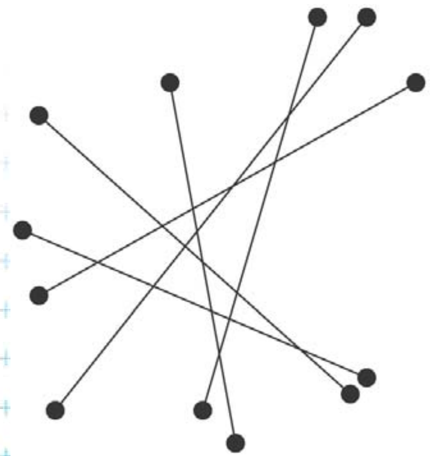
- Solid modeling
  - Intersection of object boundaries in CSG
- Overlay of subdivisions, e.g. layers in GIS
  - Bridges on intersections of roads and rivers
  - Maintenance responsibilities (road network X county boundaries)
- Robotics
  - Collision detection and collision avoidance
- Computer graphics
  - Rendering via ray shooting (intersection of the ray with objects)
- …

Line segment intersection

- Intersection of complex shapes is often reduced to simpler and simpler intersection problems
- Line segment intersection is the most basic intersection algorithm
- Problem statement:

  Given n line segments in the plane, report all points where a pair of line segments intersect.

- Problem complexity
  - Worst case – $I = O(n^2)$ intersections
  - Practical case – only some intersections
  - Use an output sensitive algorithm
    - $O(n \log n + I)$ optimal randomized algorithm
    - $O(n \log n + I \log n)$ sweep line algorithm - %



[Berg]

Plane sweep line algorithm recapitulation

- Horizontal line (sweep line, *scan line*) ℓ moves top-down (or vertical line: left to right) over the set of objects
- The move is not continuous, but ℓ jumps from one event point to another
  - Event points are in priority queue or sorted list (~y)
  - The top-most event point is removed first
  - New event points may be created
    (usually as interaction of neighbors on the sweep line)
    and inserted in the queue
- Scan-line status
  - Stores information about the objects intersected by ℓ
  - It is updated while stopping on event point

Line segment intersection - Sweep line alg.

- Avoid testing of pairs of segments far apart
- Compute intersections of neighbors on the sweep line only
- $O(n \log n + I \log n)$ time in $O(n)$ memory
  - $2n$ steps for end points,
  - I steps for intersections,
  - $\log n$ search the status tree
- Ignore "nasty cases" (most of them will be solved later on)
  - No segment is parallel to the sweep line
  - Segments intersect in one point and do not overlap
  - No three segments meet in a common point

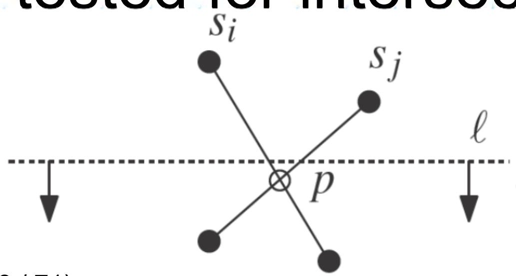
Line segment intersections

- *Status* = ordered sequence of segments intersecting the sweep line ℓ
- *Events* (waiting in the priority queue)
  = points, where the algorithm actually does something
  - Segment *end-points*
    - known at algorithm start
  - Segment *intersections* between neighboring segments along SL
    - Discovered as the sweep executes

Detecting intersections

- Intersection events must be detected and inserted to the event queue before they occur
- Given two segments *a, b* intersecting in a point *p*, there must be a placement of sweep line ℓ prior to *p*, such that segments *a, b* are adjacent along ℓ (only adjacent will be tested for intersection)
  - segments *a, b* are not adjacent when the alg. starts
  - segments *a, b* are adjacent just before *p*

  => there must be an event point when *a,b* become adjacent and therefore are tested for intersection

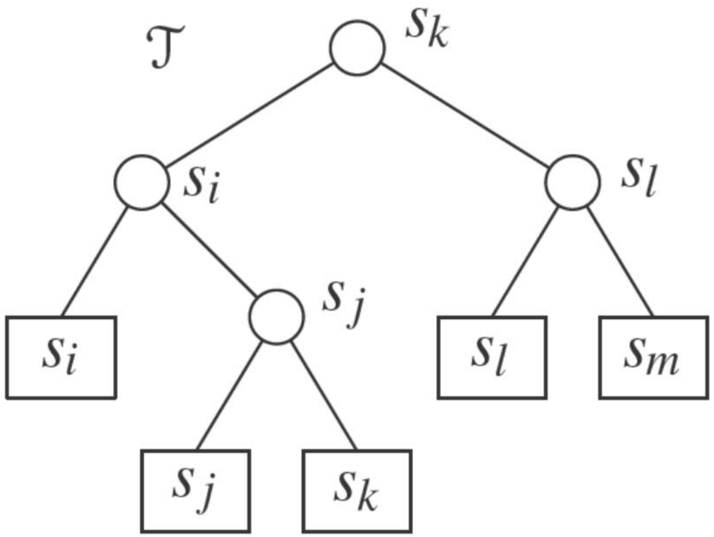


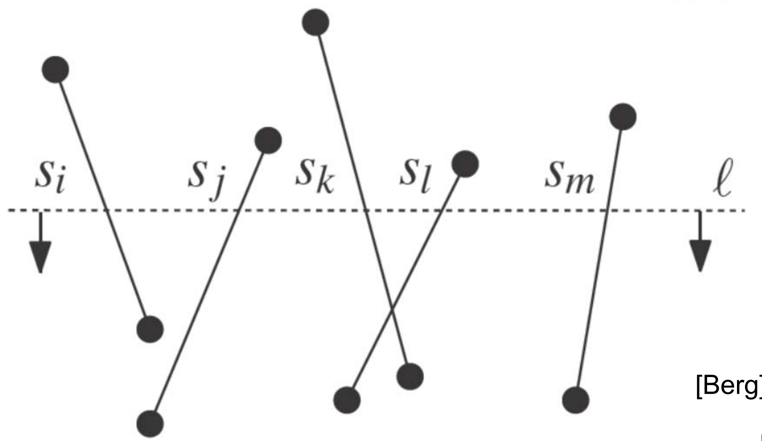
[Berg]

Data structures

Sweep line ℓ status = order of segments along ℓ

- Balanced binary search tree of segments
- Coords of intersections with ℓ vary as ℓ moves
  => store pointers to line segments in tree nodes
  - Position of ℓ is plugged in the $y=mx+b$ to get the x-key

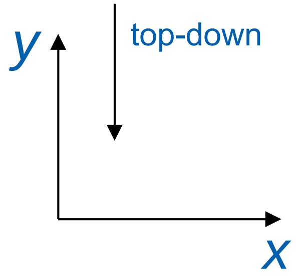




[Berg]

Data structures

Event queue (postupový plán, časový plán)



- Define: Order $\prec$ (top-down, lexicographic)

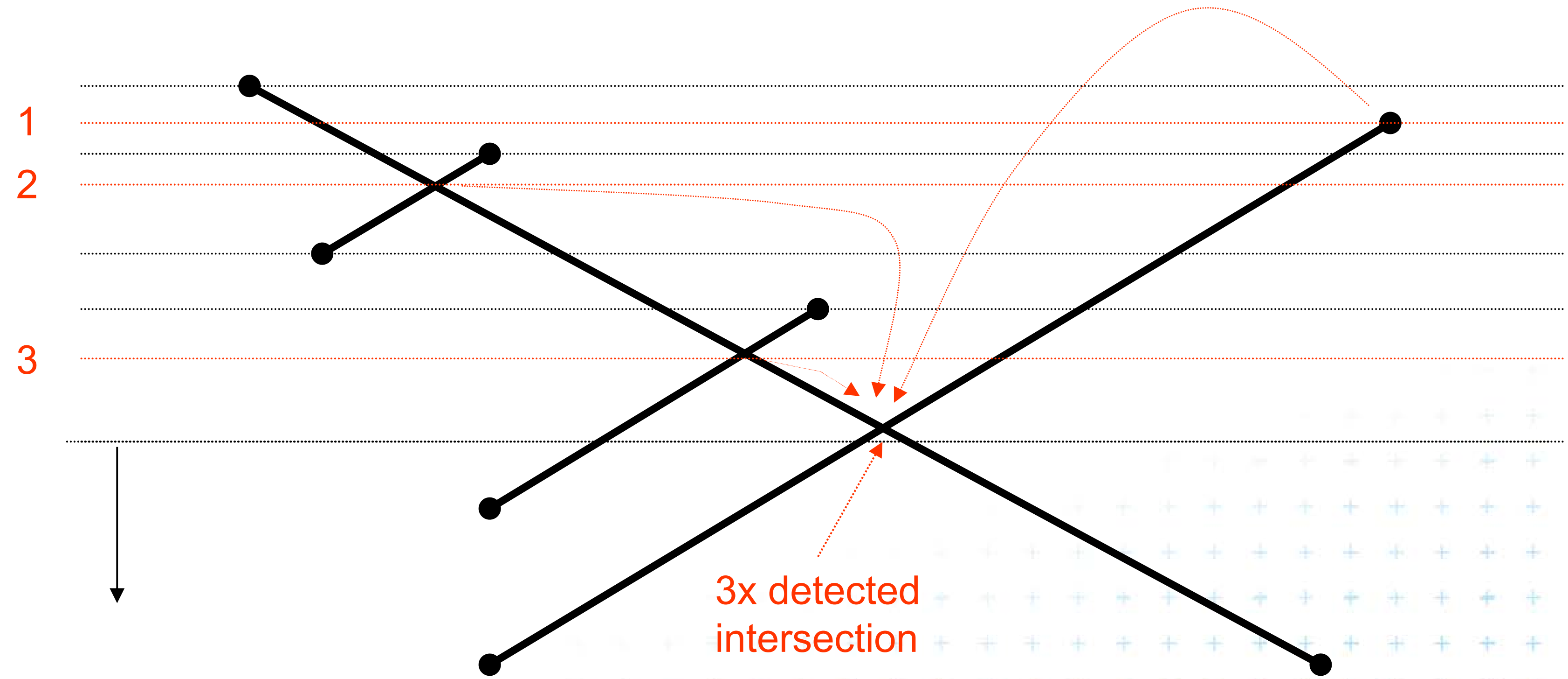
  $p \prec q$ **iff** $p_y > q_y$ **or** $p_y = q_y$ and $p_x < q_x$

  top-down, left-right approach

  (points on ℓ treated left to right)

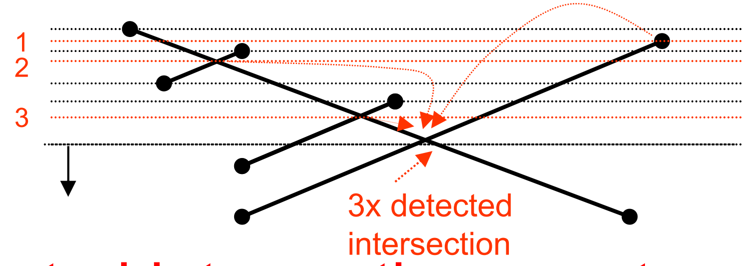
- Operations
  - Insertion of computed intersection points
  - Fetching the next event (highest *y* below ℓ)
  - Test, if the segment is already present in the queue
  - (Delete intersection event in the queue)

Problem with duplicities of intersections



Data structures

Event queue data structure



- Heap
  - Problem: can not check duplicated intersection events (reinvented more than once)
  - Intersections processed twice or even more
  - Memory complexity up to O(n^2)
- Ordered dictionary (balanced binary tree)
  - Can check duplicated events (adds just constant factor)
  - Nothing inserted twice
  - If non-neighbor intersections are deleted
    i.e., if only intersections of neighbors along ℓ are stored
    then memory complexity just O(n)

Line segment intersection algorithm

FindIntersections(*S*)

Input: A set *S* of line segments in the plane

Output: The set of intersection points + pointers to segments in each

1. init an empty event queue *Q* and insert the segment endpoints
2. init an empty status structure *T*
3. **while** Q in not empty
4.   remove next event *p* from *Q*
5.   handleEventPoint(*p*)

Upper endpoint

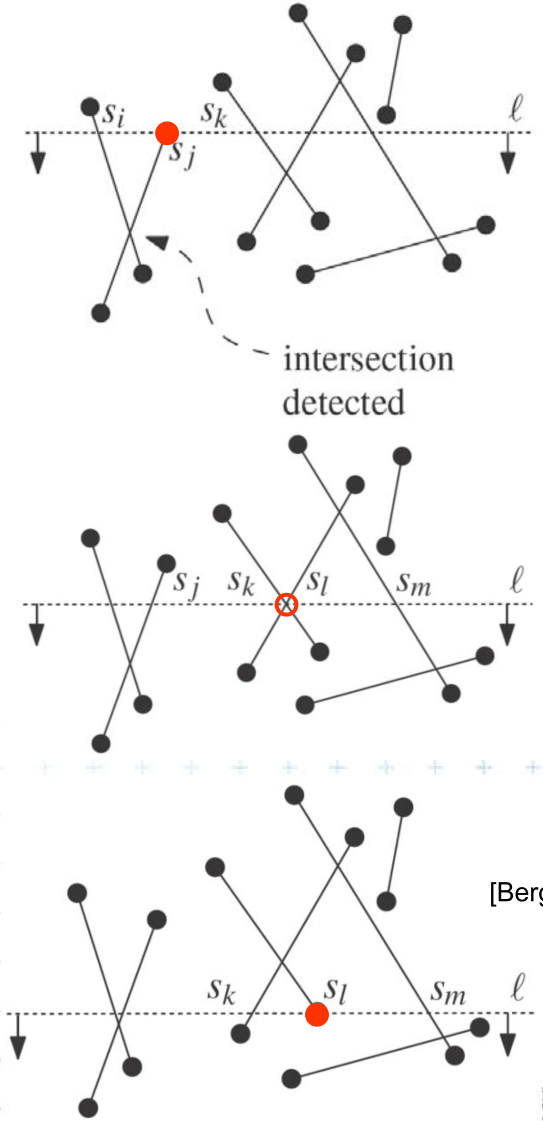
Intersection

Lower endpoint

Note: Upper-end-point events store info about the segment

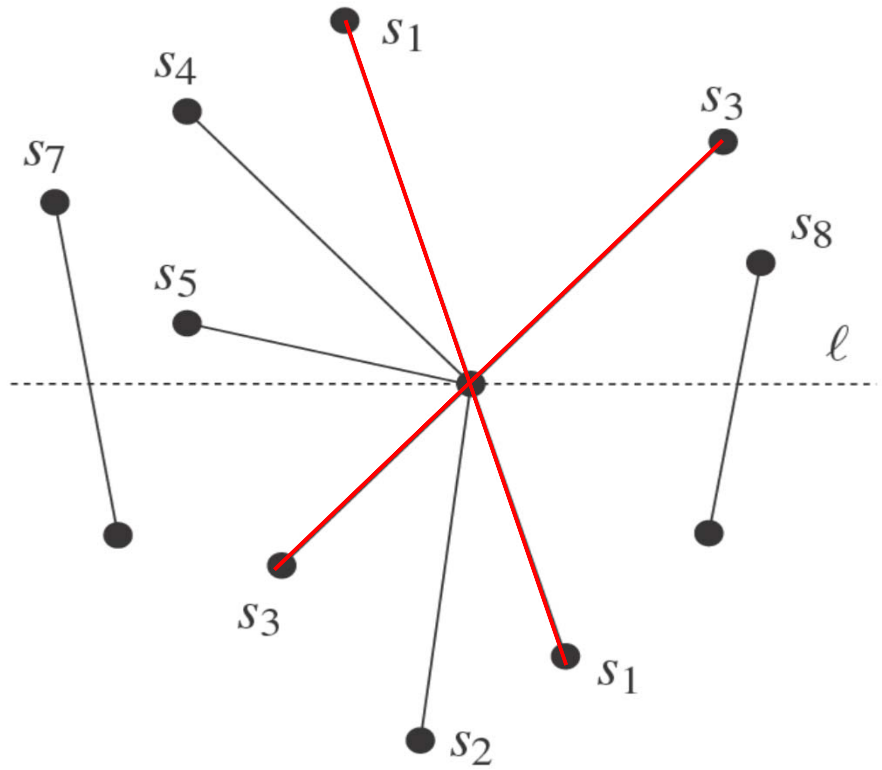
handleEventPoint principle

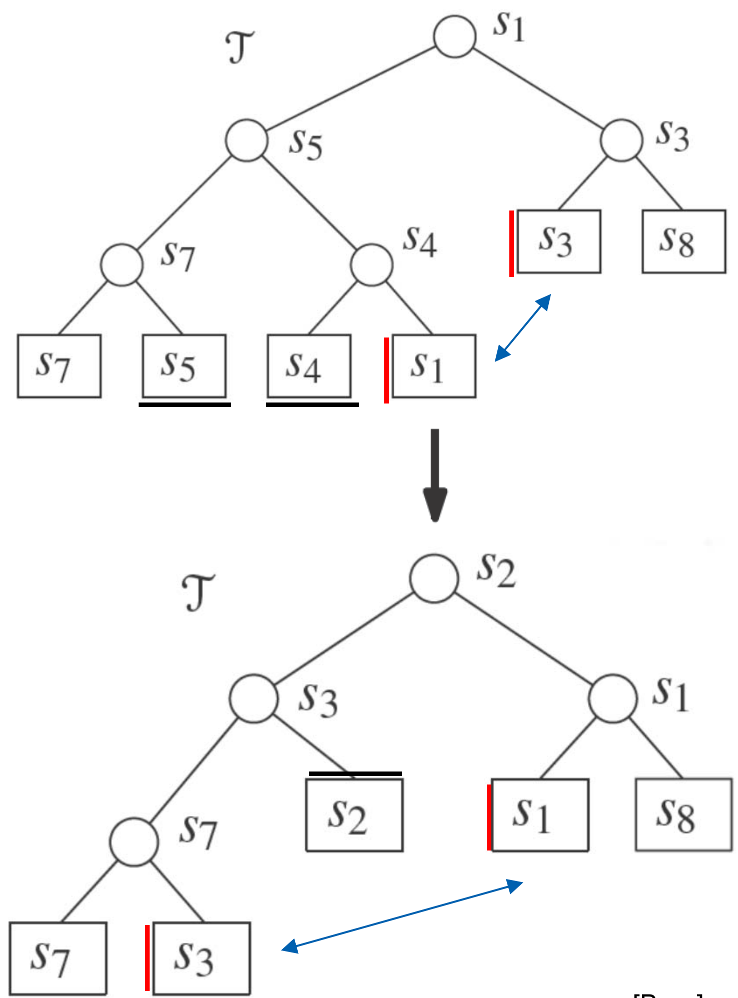
- Upper endpoint *U*(*p*)
  - insert *p* (on s_j) to status *T*
  - add intersections with left and right neighbors to *Q*
- Intersection *C*(*p*)
  - switch order of segments in *T*
  - add intersections of left and right neighbors to *Q*
- Lower endpoint *L*(*p*)
  - remove *p* (on s_l) from *T*
  - add intersections of left and right neighbors to *Q*



[Berg]

More than two segments incident





$U(p) = \{s_2\}$ start here

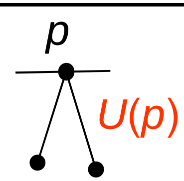
$C(p) = \{s_1, s_3\}$ cross on ℓ

$L(p) = \{s_4, s_5\}$ end here

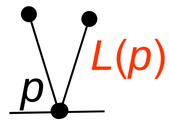
[Berg]

Handle Events [Berg, page 25]

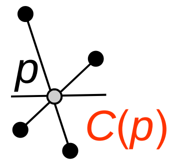
handleEventPoint(p)



1. Let $U(p)$ = set of segments whose Upper endpoint is p.
   These segmets are stored with the event point p (will be added to T)
2. Search T for all segments $S(p)$ that contain p (are adjacent in T):
   Let $L(p) \subset S(p)$ = segments whose Lower endpoint is p
   Let $C(p) \subset S(p)$ = segments that Contain p in interior



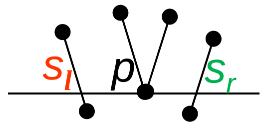
3. **if**($L(p) \cup U(p) \cup C(p)$ contains more than one segment)
4. report p as intersection together with $L(p)$, $U(p)$, $C(p)$



5. Delete the segments in $L(p) \cup C(p)$ from T
6. Insert the segments in $U(p) \cup C(p)$ into T

Reverse order of $C(p)$ in T

(order as below ℓ, horizontal segment as the last)



7. **if**($U(p) \cup C(p) = \emptyset$) then findNewEvent(s_l, s_r, p) // left & right neighbors
8. **else** s' = leftmost segment of $U(p) \cup C(p)$; findNewEvent(s_l, s', p)
   s'' = rightmost segment of $U(p) \cup C(p)$; findNewEvent(s'', s_r, p)

Detection of new intersections

findNewEvent(s_l, s_r, p) **// with handling of horizontal segments**

Input: two segments (left & right from *p* in *T*) and a current event point *p*

Output: updated event queue *Q* with new intersection

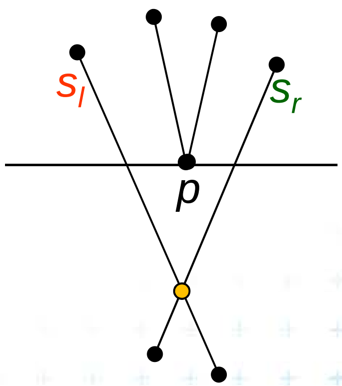
1. **if** [**(** s_l and s_r intersect below the sweep line ℓ) or

   (intersect on ℓ and to the right of *p*)] and // horizontal segments

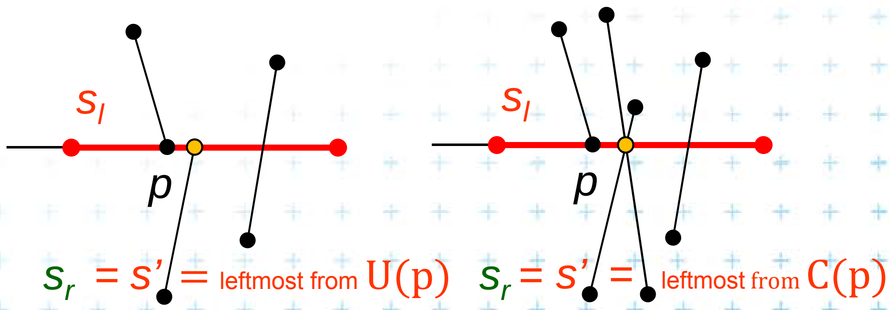
   (the intersection is not present in *Q*)
2. **then**

   insert *p* as a new event into *Q*

s' = leftmost segment of $U(p) \cup C(p)$;



s_l and s_r intersect below



s_l and s_r = s' intersect on ℓ

and to the right of *p*

Line segment intersections

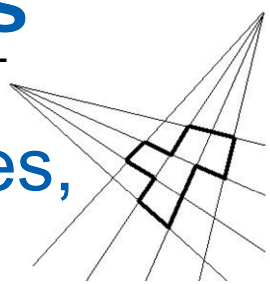
- Memory $O(I) = O(n^2)$ with duplicities in Q
  or $O(n\ )$ with duplicities in Q deleted
- Operational complexity
  - $n + I$ stops
  - $\log n$ each

  => $O(I + n) \log n$ total
- The algorithm is by Bentley-Ottmann

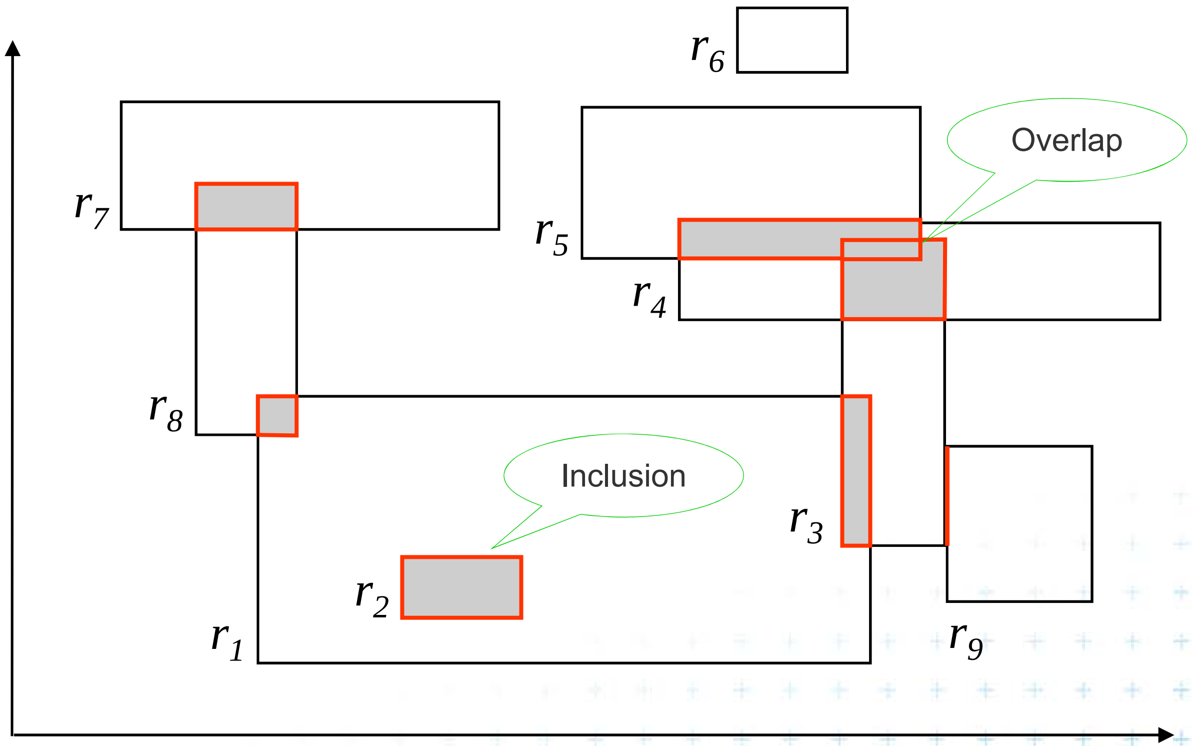
Bentley, J. L.; Ottmann, T. A. (1979), "Algorithms for reporting and counting geometric intersections", *IEEE Transactions on Computers* **C-28** (9): 643-647, doi:10.1109/TC.1979.1675432 .

See also http://wapedia.mobi/en/Bentley%E2%80%93Ottmann_algorithm

Intersection of axis parallel rectangles



- Given the collection of n *isothetic* rectangles, report all intersecting parts



Alternate sides belong to two pencils of lines
(trsy přímek)
(often used with points in infinity = axis parallel)

[?]

Answer: (r_1, r_2) (r_1, r_3) (r_1, r_8) (r_3, r_4) (r_3, r_5) (r_3, r_9) (r_4, r_5) (r_7, r_8)

Brute force intersection

Brute force algorithm

Input: set S of axis parallel rectangles
Output: pairs of intersected rectangles

1. For every pair (r_i, r_j) of rectangles $\in S$, $i \neq j$
2. if $(r_i \cap r_j \neq \emptyset)$ then
3. report (r_i, r_j)

Analysis

Preprocessing: None.

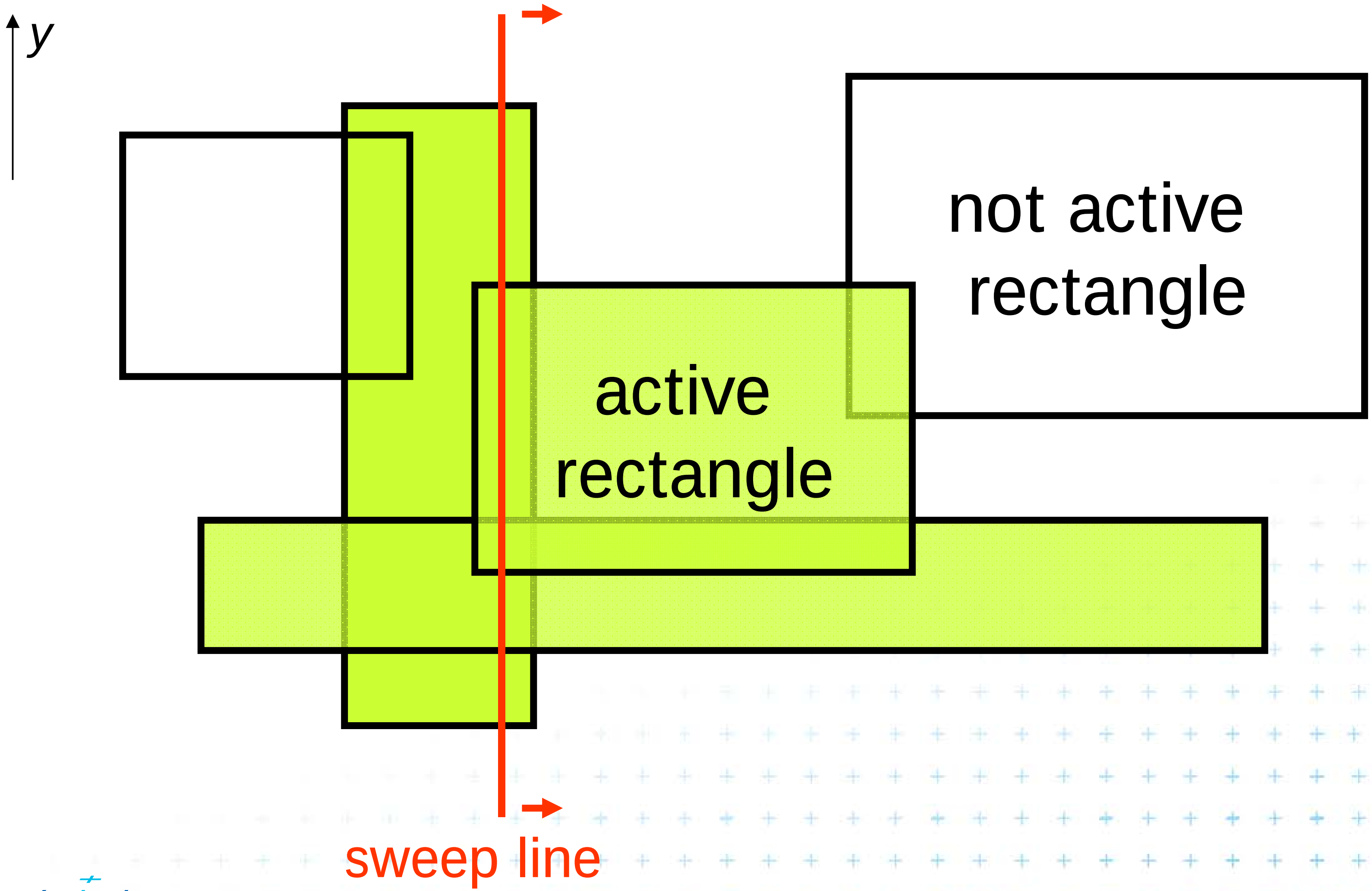
Query: $O(N^2)$ $\binom{N}{2} = \frac{N(N-1)}{2} \in O(N^2)$.

Storage: $O(N)$

Plane sweep intersection algorithm

- Vertical sweep line moves from left to right
- Stops at every x-coordinate of a rectangle (either its left side or its right side).
- active rectangles – a set
  = rectangles currently intersecting the sweep line
  - left side event of a rectangle
    => the rectangle is added to the active set.
  - right side
    => the rectangle is deleted from the active set.
- The active set used to detect rectangle intersection

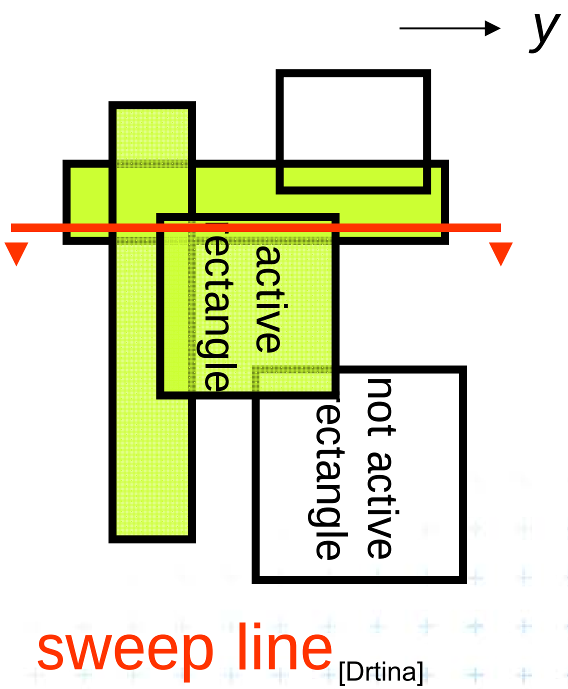
Example rectangles and sweep line



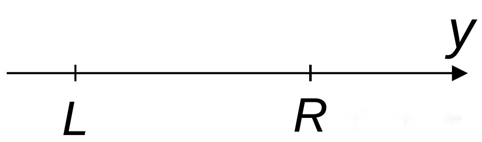
[Drtina]

Interval tree as sweep line status structure

- Vertical sweep-line => Only *y*-coordinates along it
- Turn our view in slides 90° right
- Sweep line (*y*-axis) will be drawn as horizontal



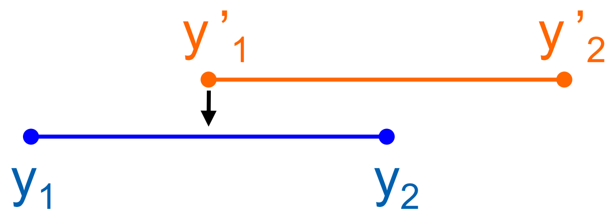
[Drtina]



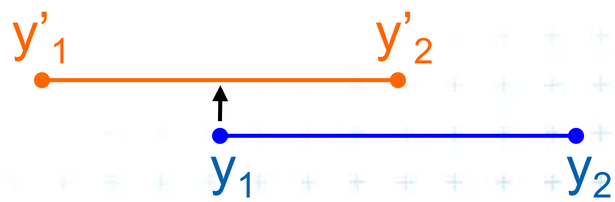
Intersection test – between pair of intervals

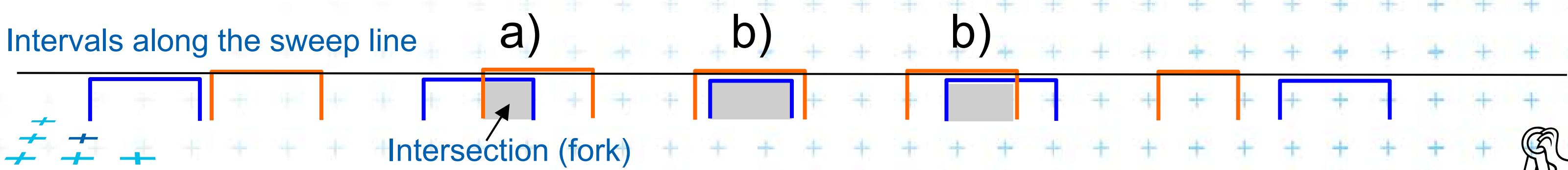
- Given two intervals $R = [y_1, y_2]$ and $R' = [y'_1, y'_2]$ the condition $R \cap R'$ is equivalent to one of these mutually exclusive conditions:

a) $y_1 \le y'_1 \le y_2$



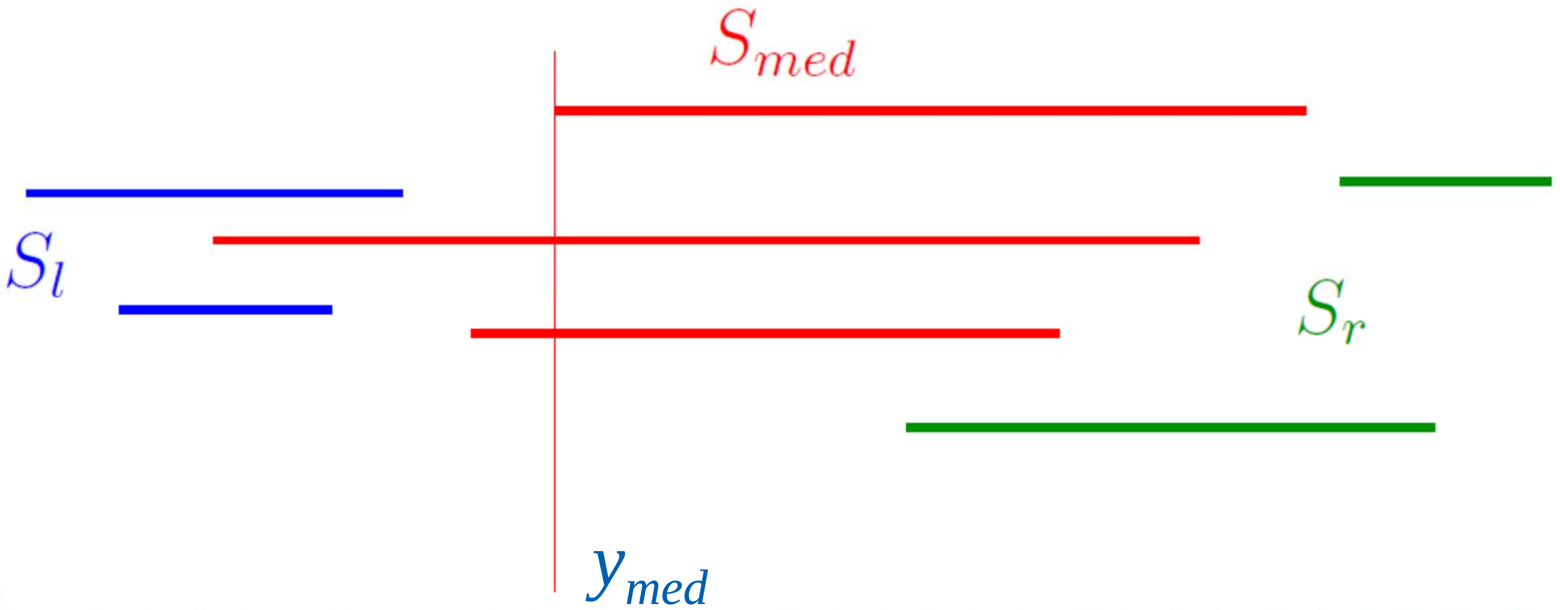
b) $y'_1 \le y_1 \le y'_2$





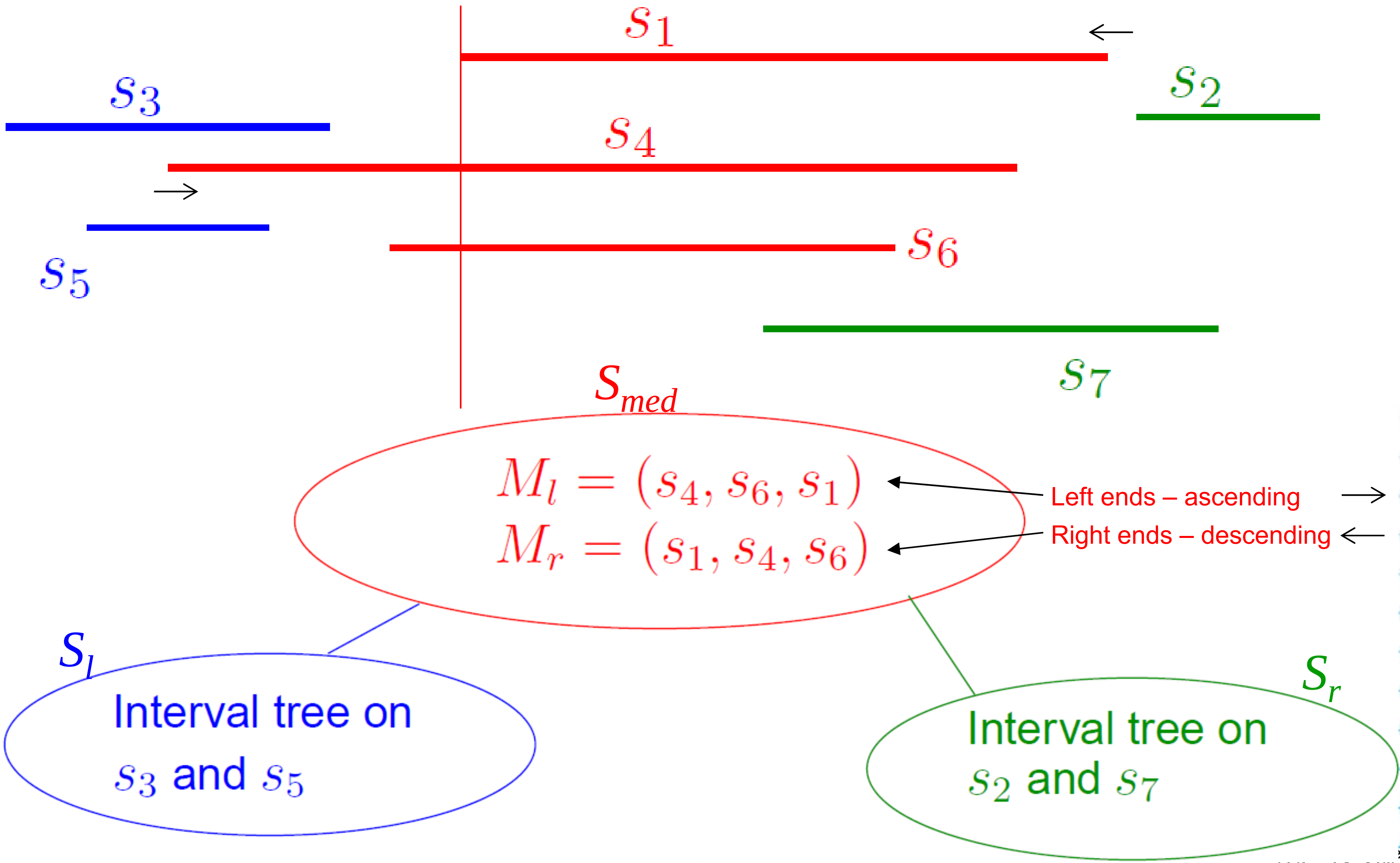
Static interval tree – stores all end points

- Let $v = y_{med}$ be the median of end-points of segments
- S_l : segments of S that are completely to the left of y_{med}
- S_{med} : segments of S that contain y_{med}
- S_r : segments of S that are completely to the right of y_{med}



[Vigneron]

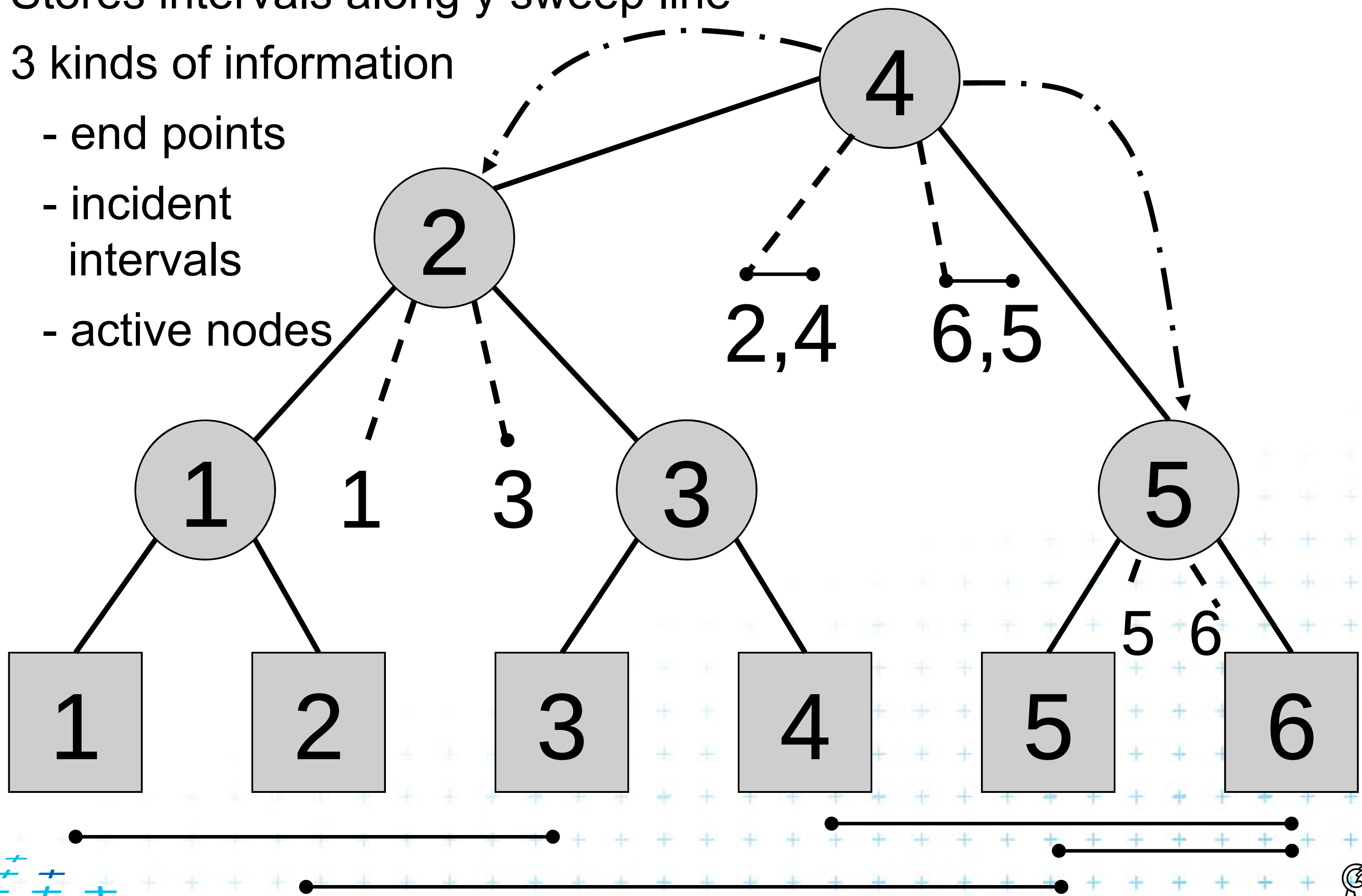
Static interval tree – Example



[Vigneron]

Static interval tree [Edelsbrunner80]

- Stores intervals along y sweep line
- 3 kinds of information
  - end points
  - incident intervals
  - active nodes

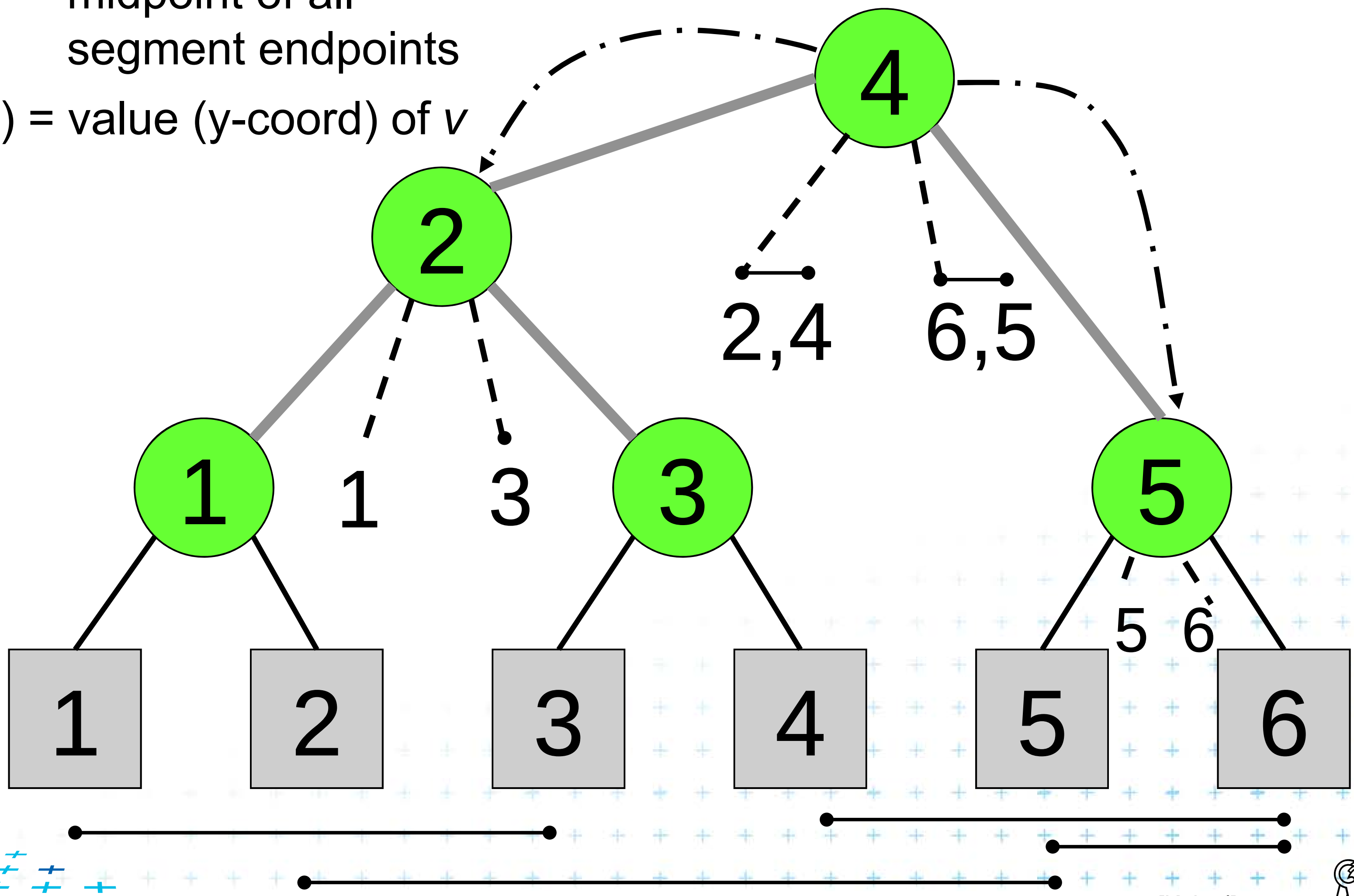


[Kukral]

Primary structure – static tree for endpoints

v = midpoint of all segment endpoints

H(v) = value (y-coord) of *v*

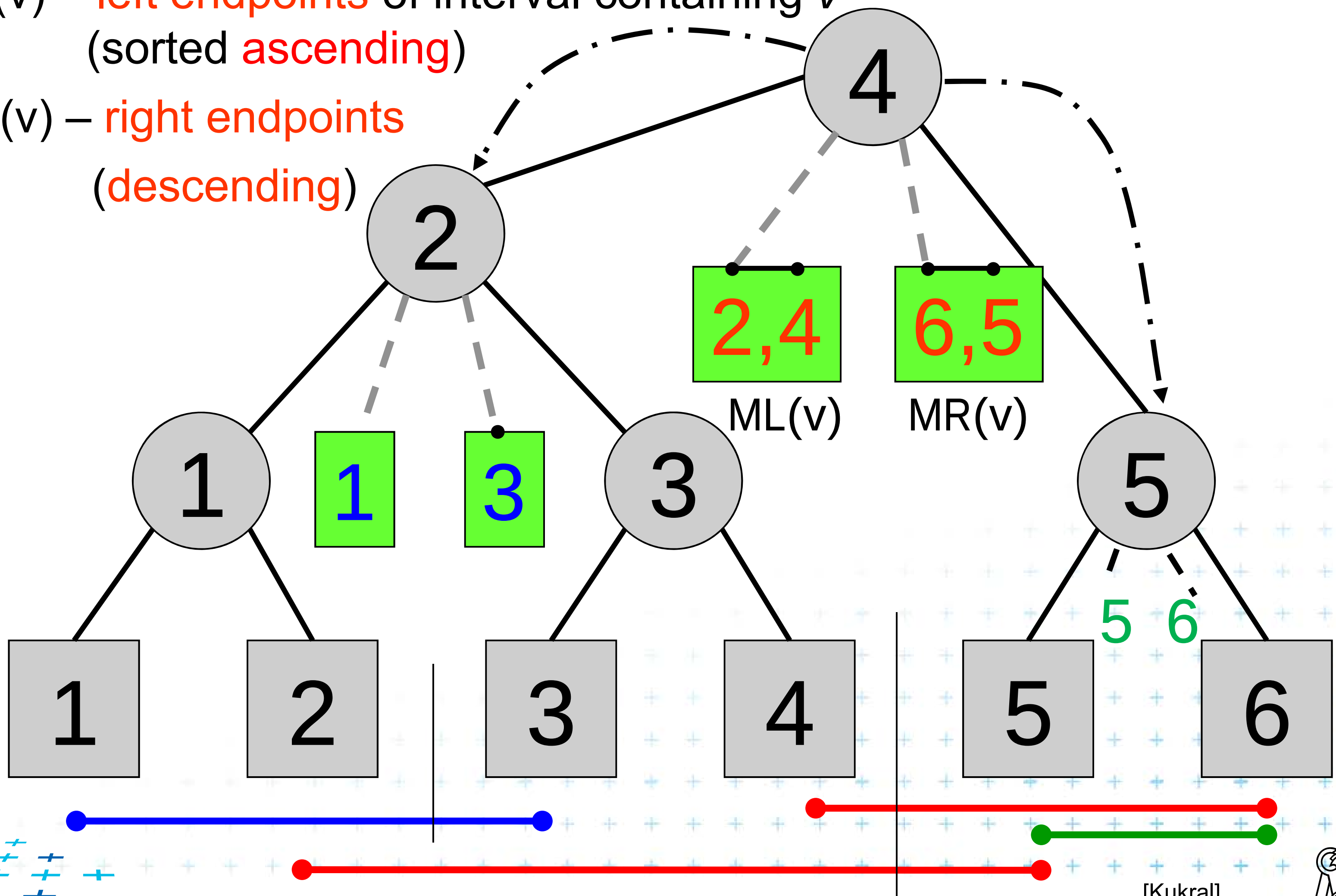


[Kukral]

Secondary lists of incident interval end-pts.

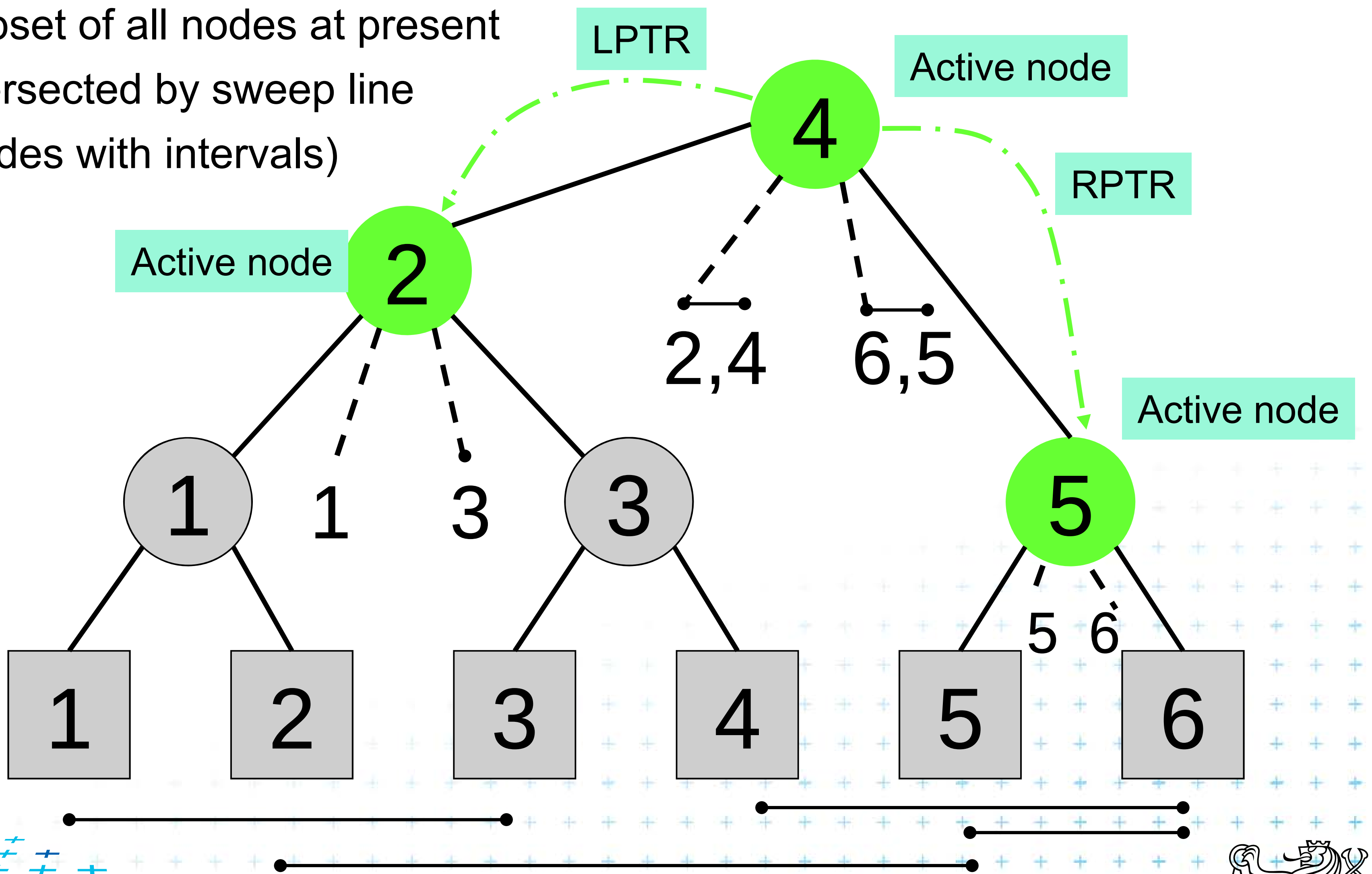
ML(v) – left endpoints of interval containing *v*
(sorted ascending)

MR(v) – right endpoints
(descending)



Active nodes – intersected by the sweep line

Subset of all nodes at present intersected by sweep line (nodes with intervals)



[Kukral]

Query = sweep and report intersections

Rectangle**Intersections****(*S*)**

Input: Set *S* of rectangles

Output: Intersected rectangle pairs

```
Preprocess( S )            // create the interval tree T (for y-coords)
                           // and event queue Q         (for x-coords)
while ( Q ≠ ∅ ) do
    Get next entry (x_i , y_il , y_ir , t) from Q       // t ∈ { left | right }
    if ( t = left )   // left edge
            a) QueryInterval ( y_il , y_ir, root(T))   // report intersections
            b) InsertInterval ( y_il , y_ir, root(T))  // insert new interval
    else              // right edge
            c) DeleteInterval ( y_il , y_ir, root(T) )
```

Preprocessing

Preprocess(S)
Input: Set *S* of rectangles
Output: Primary structure of the interval tree *T* and the event queue *Q*

1. *T* = PrimaryTree(*S*) // Construct the static primary structure
   // of the interval tree -> sweep line STATUS *T*

2. // Init event queue Q with vertical rectangle edges in ascending order ~x
   // Put the left edges with the same x ahead of right ones
3. for i = 1 to n
4. insert((x_{il} , y_{il} , y_{ir} , left), Q) // left edges of *i-th* rectangle
5. insert((x_{ir}, y_{il} , y_{ir} , right), Q) // right edges

Interval tree – primary structure construction

PrimaryTree(*S*) // **only the y-tree structure, without intervals**

Input: Set *S* of rectangles

Output: Primary structure of an interval tree *T*

1. S_y = Sort endpoints of all segments in *S* according to *y*-coordinate
2. *T* = BST(S_y)
3. **return** *T*

BST(S_y)

1. **if**($|S_y|$ = 0) **return** null
2. *yMed = median of* S_y
3. *L* = endpoints $p_y \leq yMed$
4. R = endpoints $p_y > yMed$
5. *t* = *new* IntervalTreeNode(*yMed*)
6. *t.left* = BST(*L*)
7. *t.right* = BST(*R*)
8. **return** *t*

Interval tree – search the intersections

QueryInterval (*b, e, T*)

Input: Interval of the edge and current tree *T*

Output: Report the rectangles that intersect [*b, e*]

if(*T* = null) **return**

i=0; **if**(b < H(v) < e) // forks at this node

    while ~~(*MR*(v).[i] >= b) &&~~ (i < Count(v)) .. Report all intervals inM

        ReportIntersection; i++

    QueryInterval(*b,e,T.LPTR*)

    QueryInterval(*b,e,T.RPTR*)

else if (H(v) ≤ b < e) // search RIGHT (←)

    while (*MR*(v).[i] >= b) && (i < Count(v))

        ReportIntersection; i++

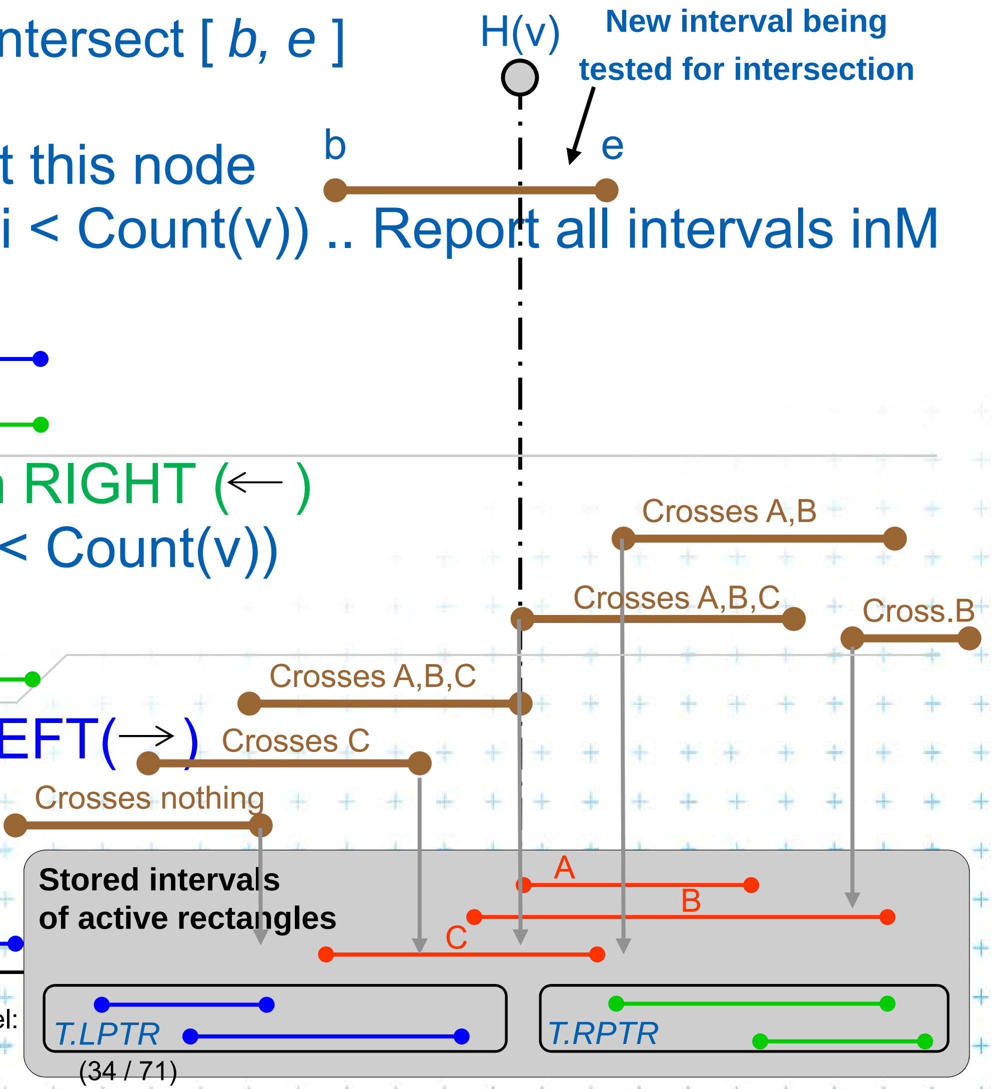
    QueryInterval(*b,e,T.RPTR*)

else // b < e ≤ H(v) //search LEFT(→)

    while (*ML*(v).[i] <= e)

        ReportIntersection; i++

    QueryInterval(*b,e,T.LPTR*)



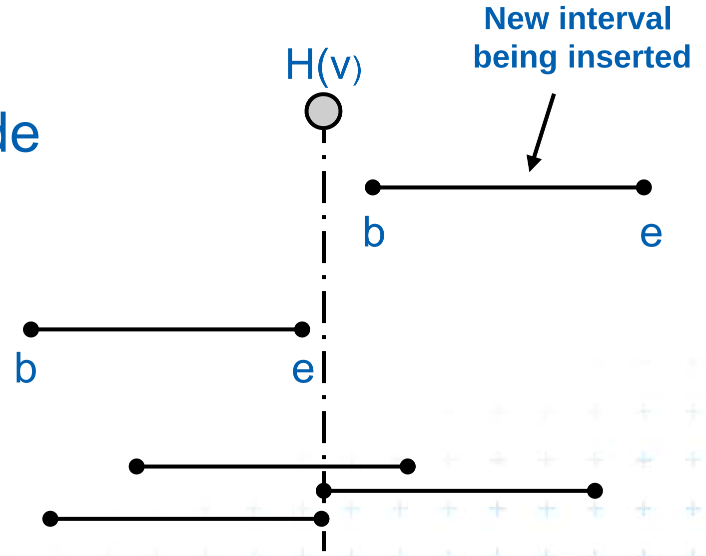
Interval tree - interval insertion

InsertInterval (*b, e, T*)

Input: Interval [*b*,*e*] and interval tree *T*

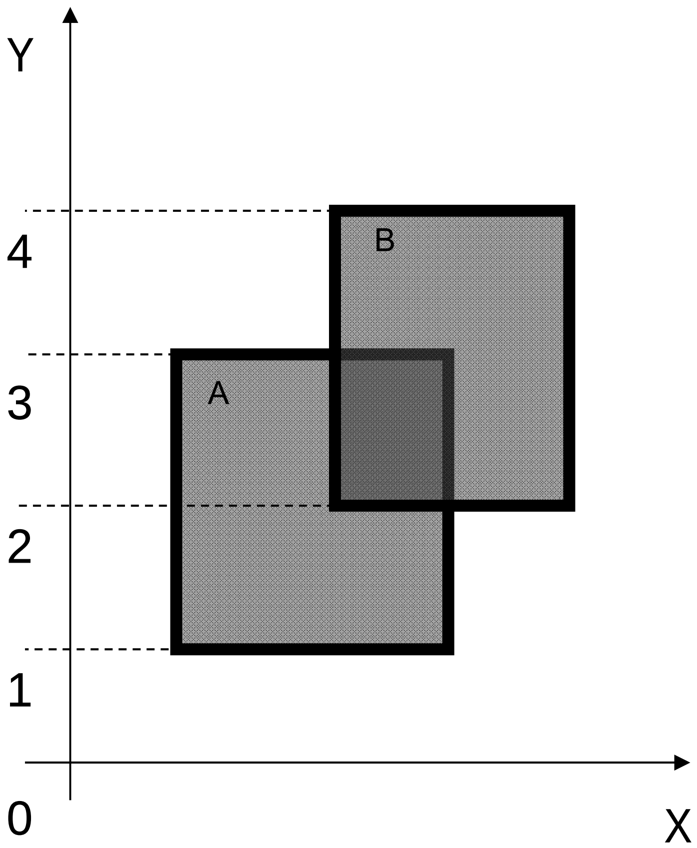
Output: *T* after insertion of the interval

```
v = root(T )
while( v != null )        // find the fork node
    if (H(v) < b < e)
        v = v.right       // continue right
    else if (b < e < H(v))
        v = v.left        // continue left
    else  // b ≤ H(v) ≤ e  // insert interval
        set v node to active
        connect LPTR resp. RPTR to its parent
        insert [b,e] into list ML(v) – sorted in ascending order of b's
        insert [b,e] into list MR(v) – sorted in descending order of e's
        break
endwhile
return T
```

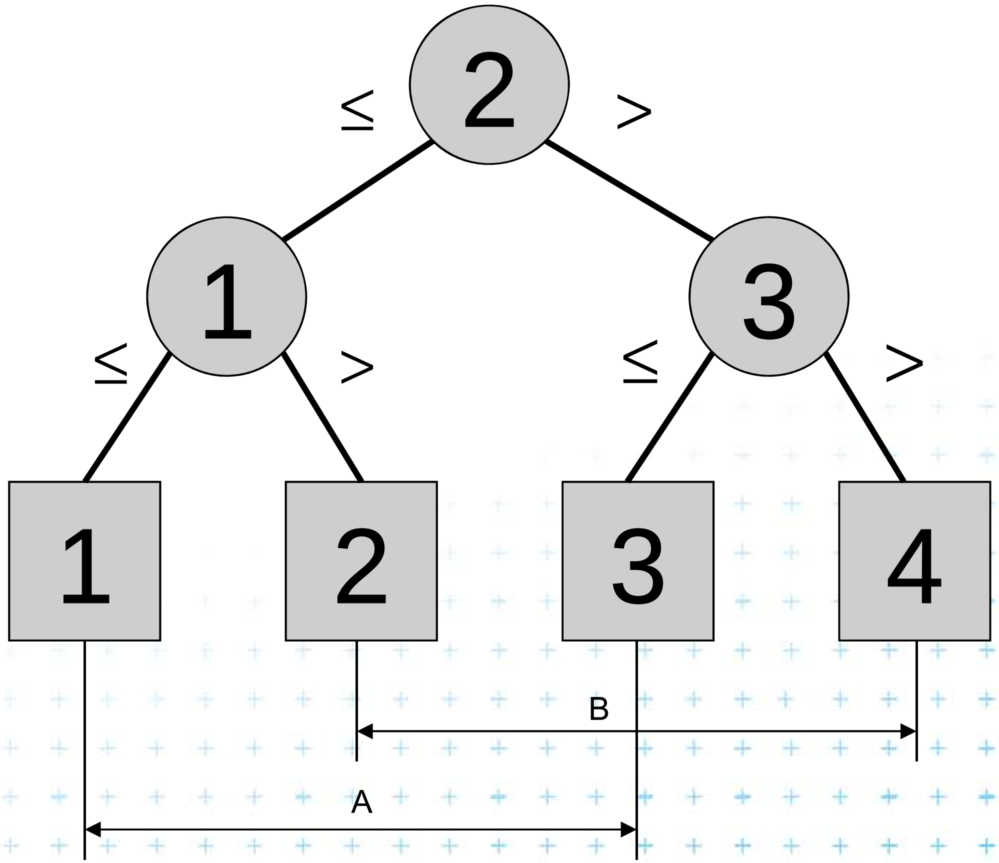


Example 1

Example 1 – static tree on endpoints



H(v) – value of node *v*



[Drtina]

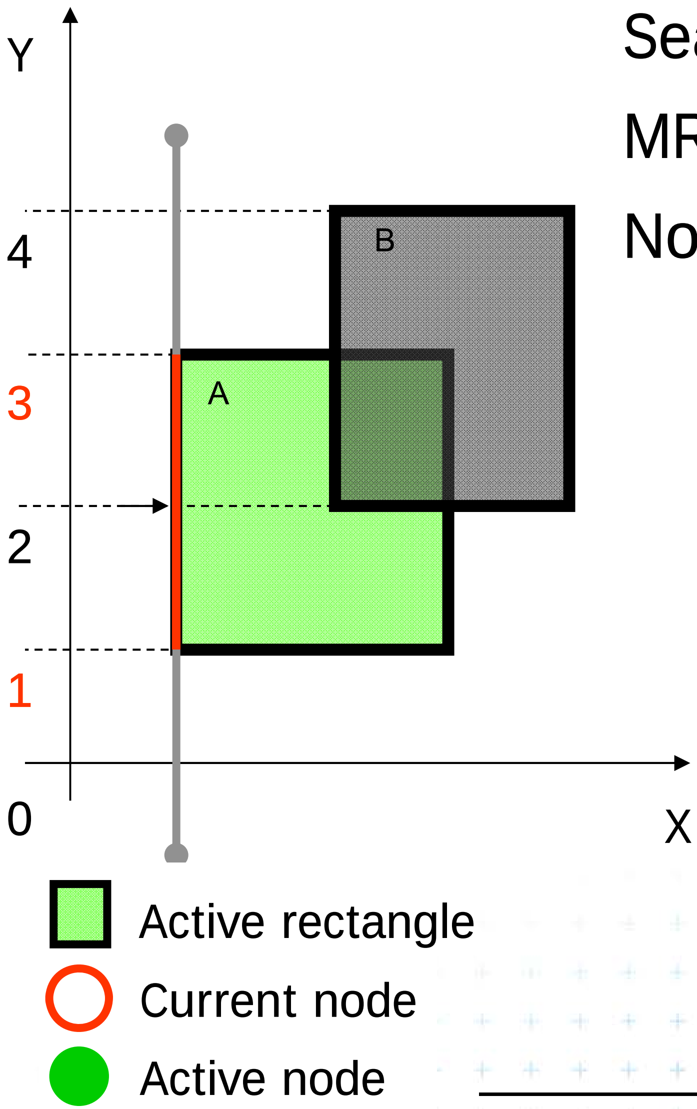
Interval insertion [1,3] a) Query Interval

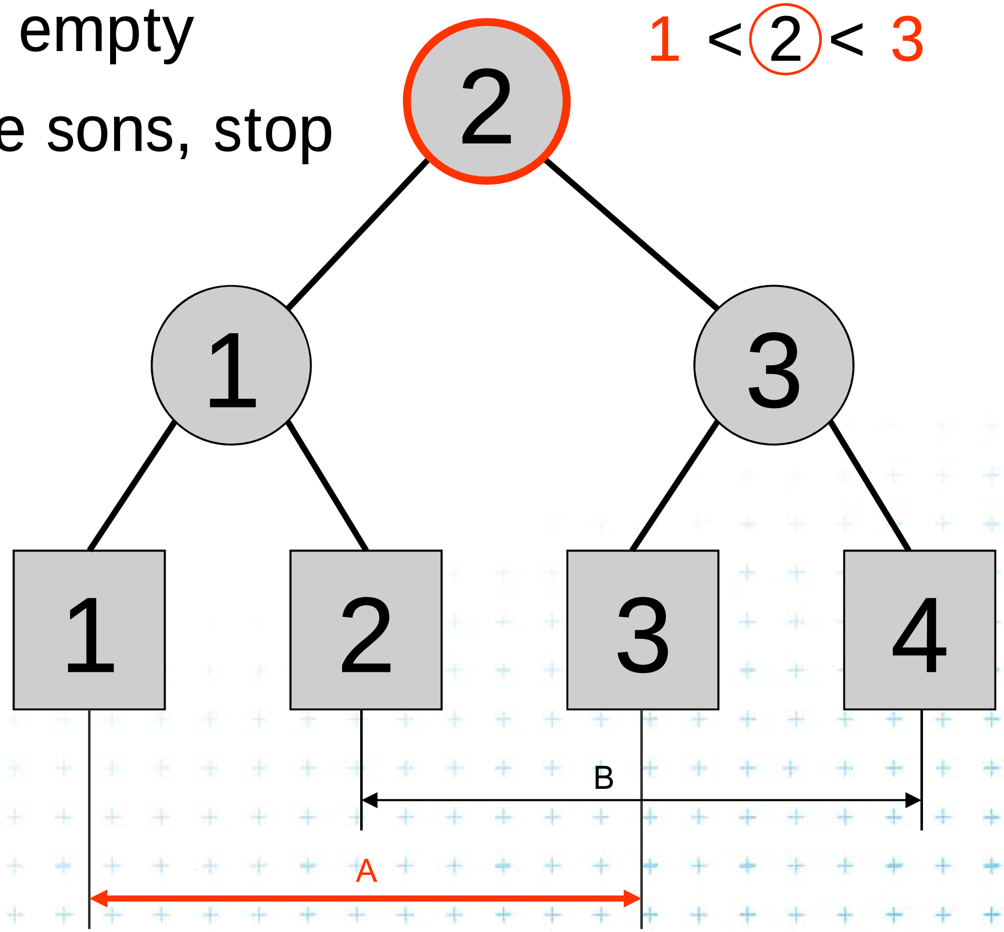
Search MR(v) or ML(v): ← $b < H(v) < e$

MR(v) is empty

No active sons, stop

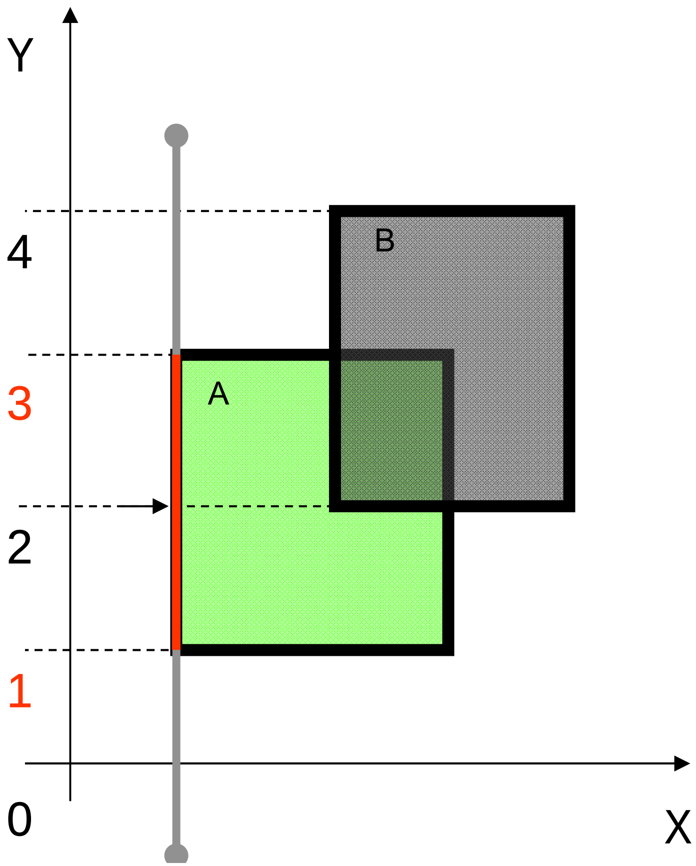
$1 < 2 < 3$





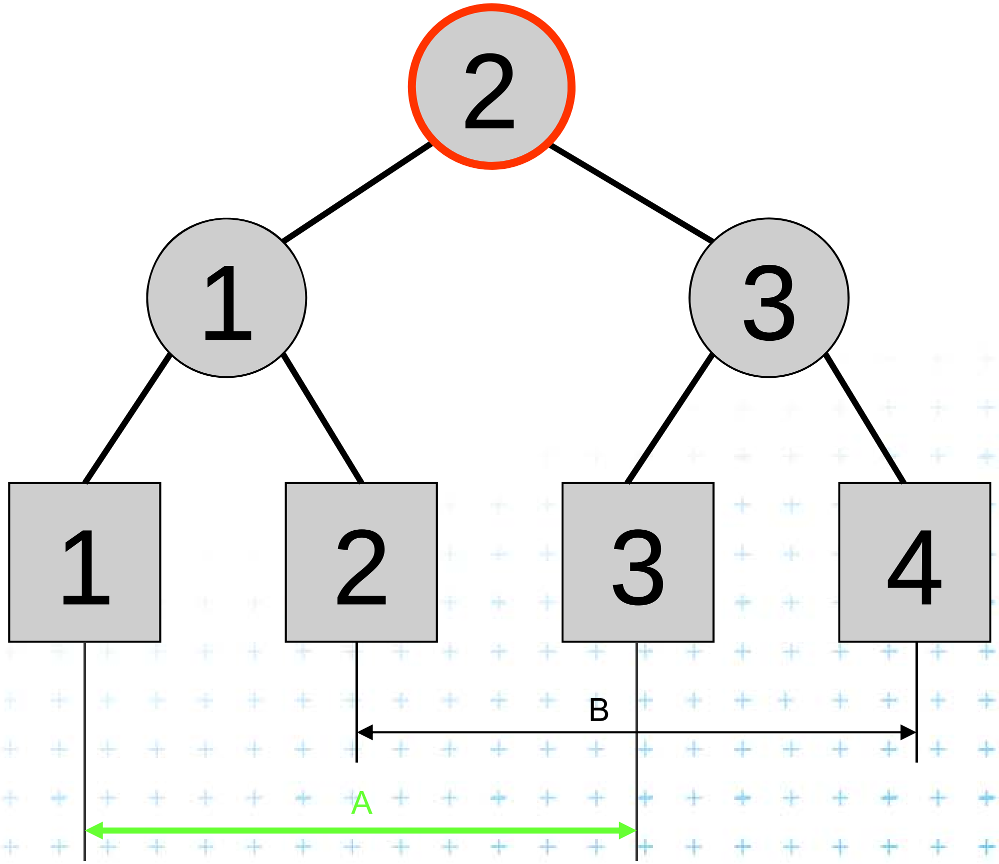
[Drtina]

Interval insertion [1,3] b) Insert Interval



$b \le H(v) \le e$

? 1 $\le$ 2 $\le$ 3 ?



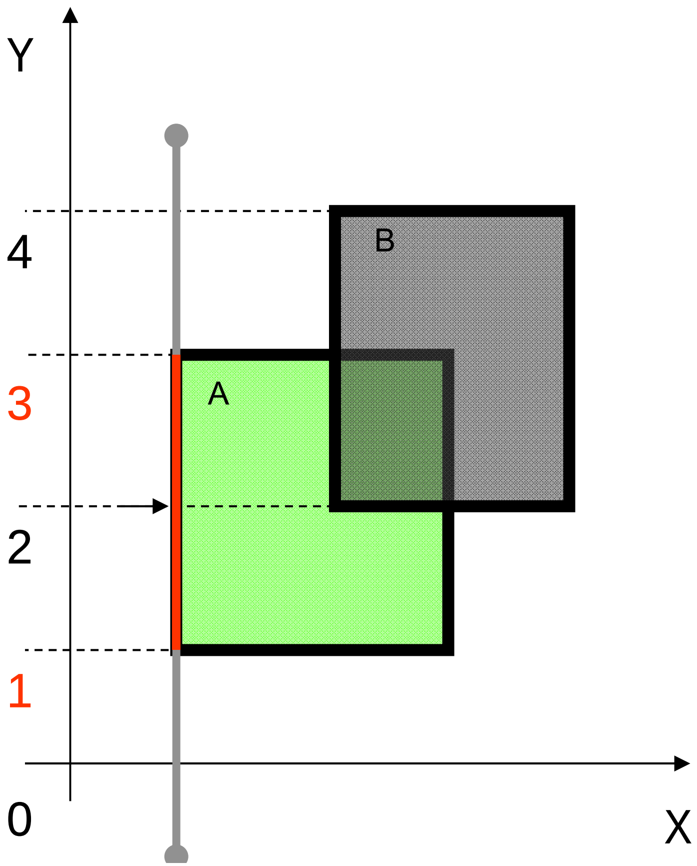
Active rectangle

Current node

Active node

[Drtina]

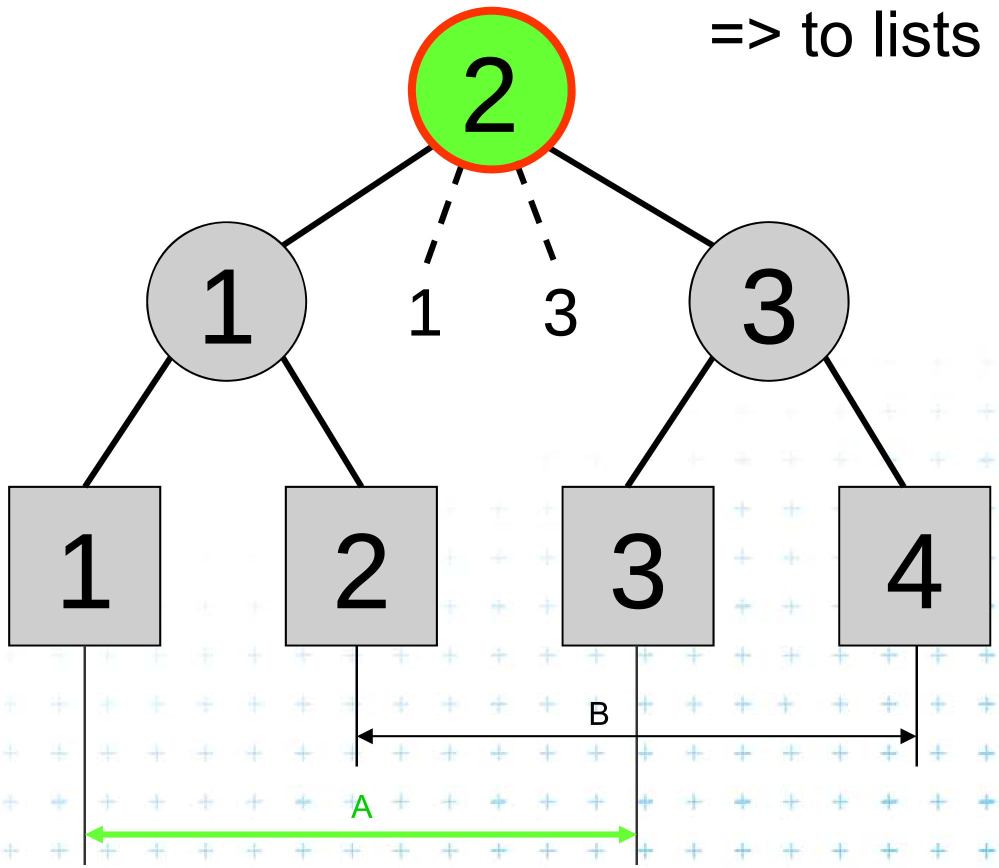
Interval insertion [1,3]   b) Insert Interval



$b \leq H(v) \leq e$

$1 \leq 2 \leq 3$

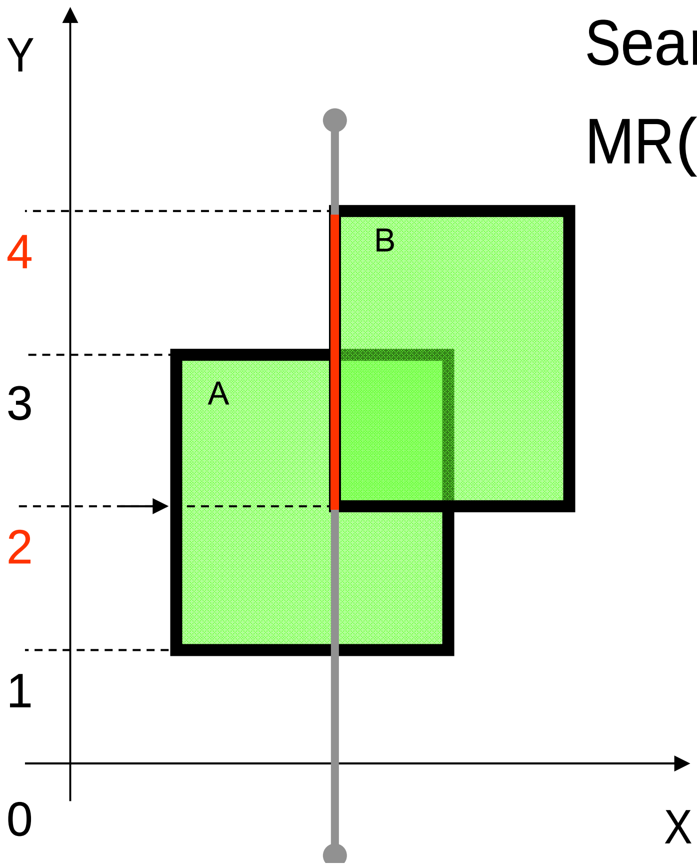
fork
=> to lists



Active rectangle
Current node
Active node

[Drtina]

Interval insertion [2,4] a) Query Interval

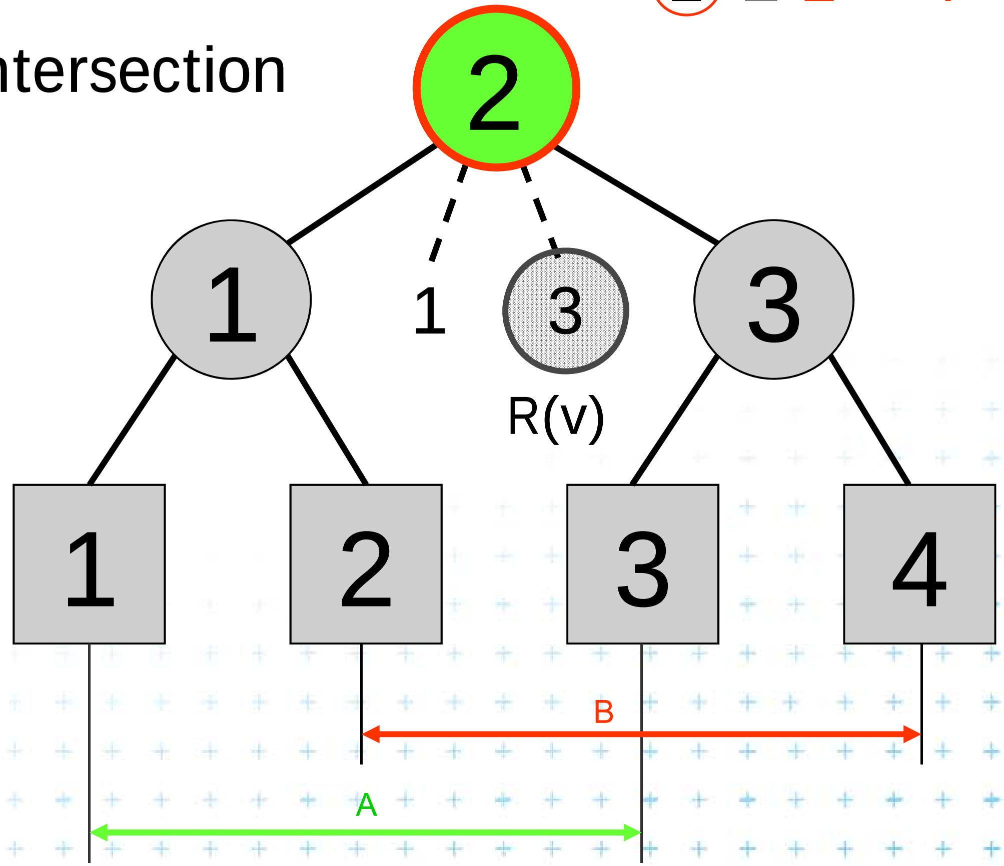


Search MR(v) only: $\longleftarrow$ $H(v) \leq b < e$

$MR(v)[1] = 3 \geq 2$?

$2 \leq 2 < 4$

=> intersection



- Active rectangle
- Current node
- Active node

[Drtina]

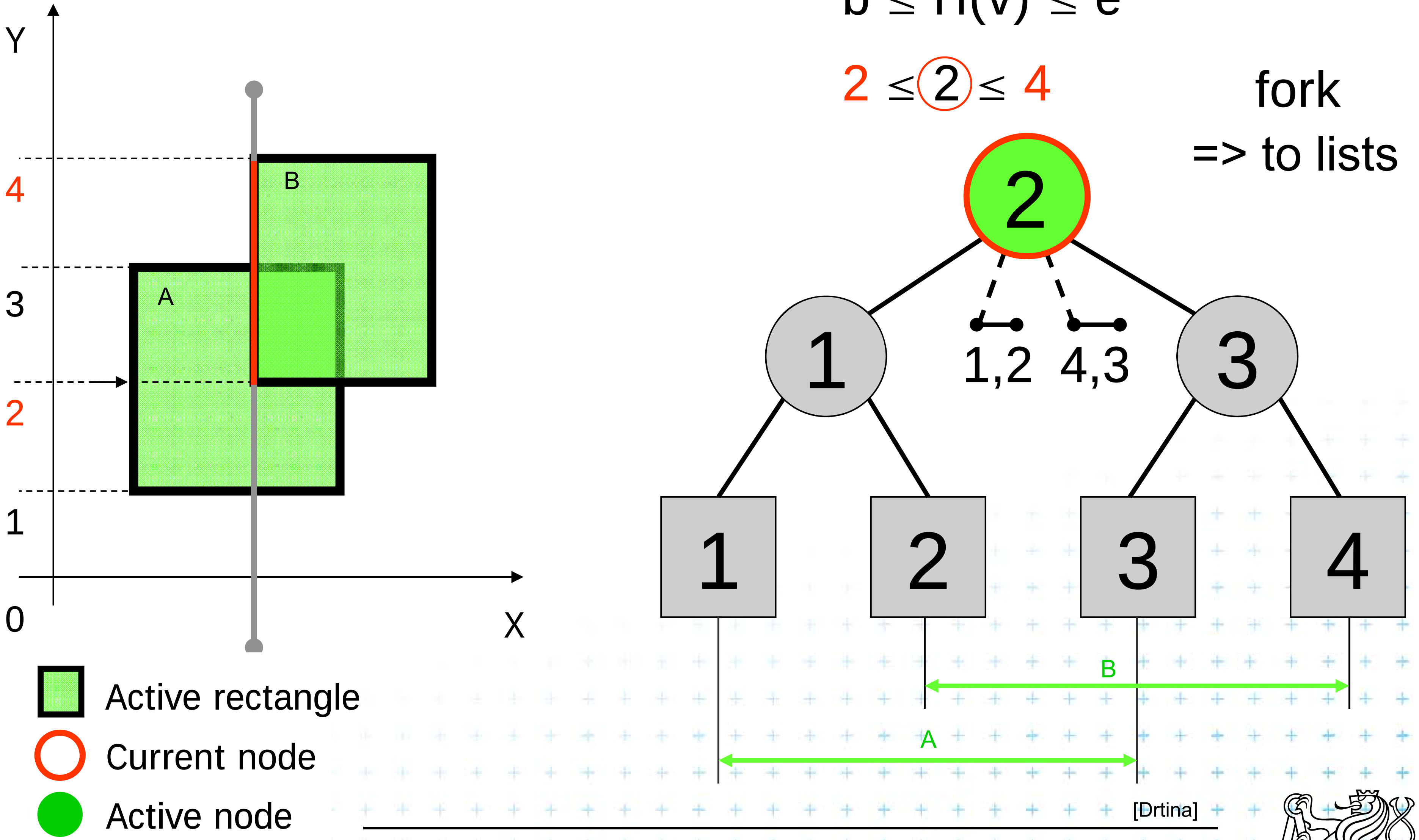
Interval insertion [2,4]   b) Insert Interval

$b \le H(v) \le e$

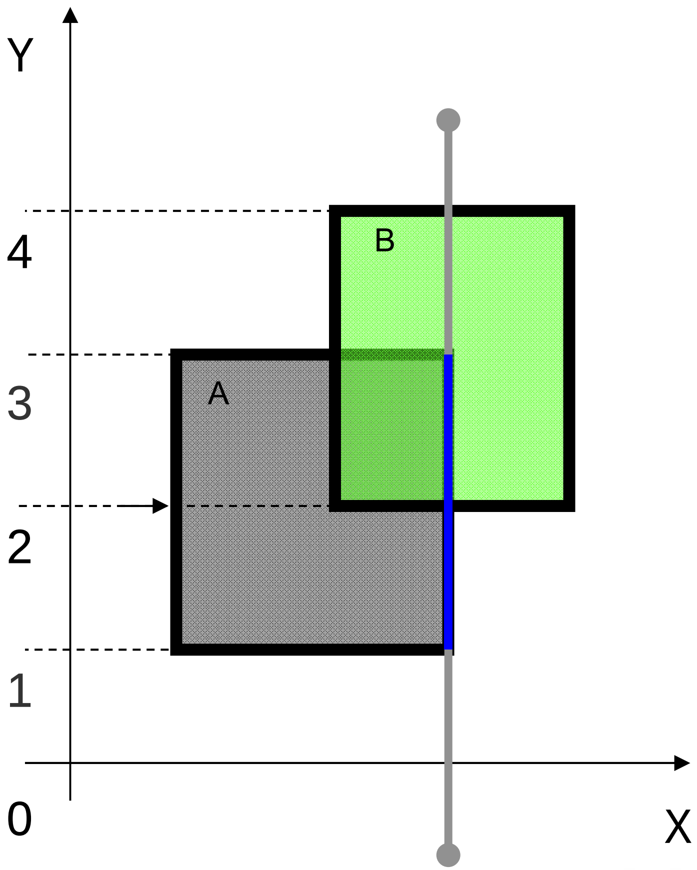
$2 \le 2 \le 4$

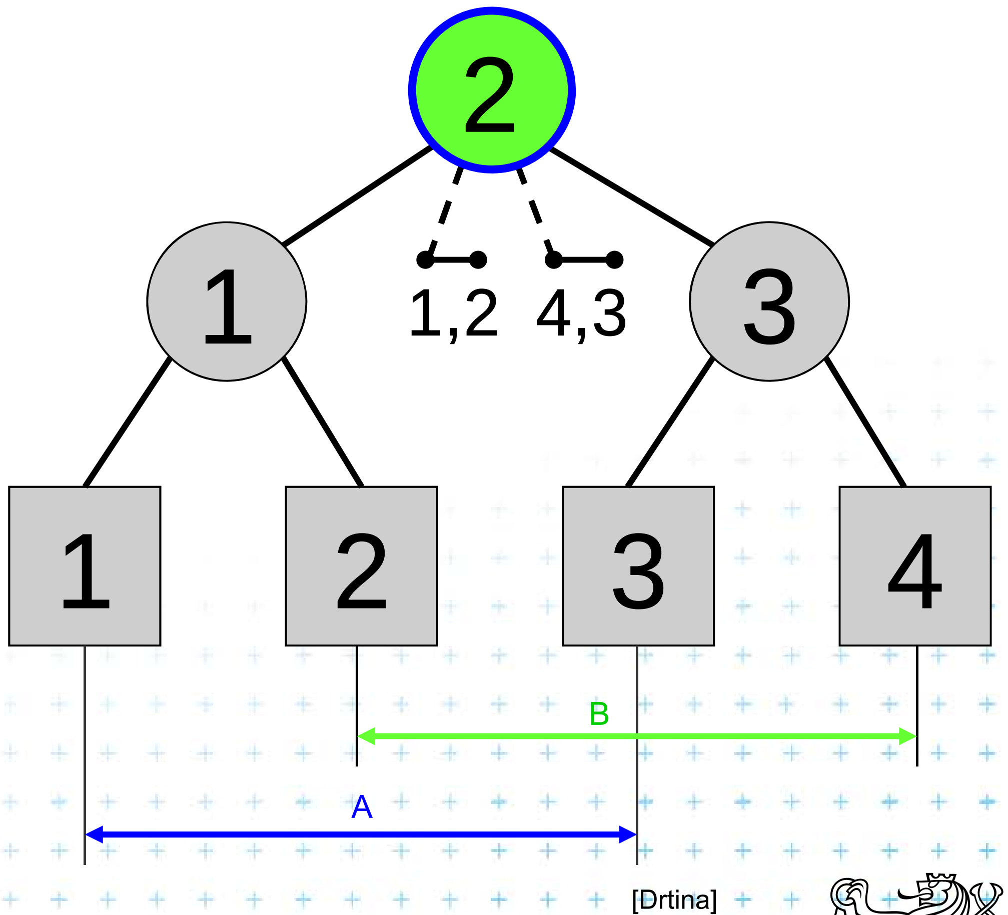
fork

=> to lists



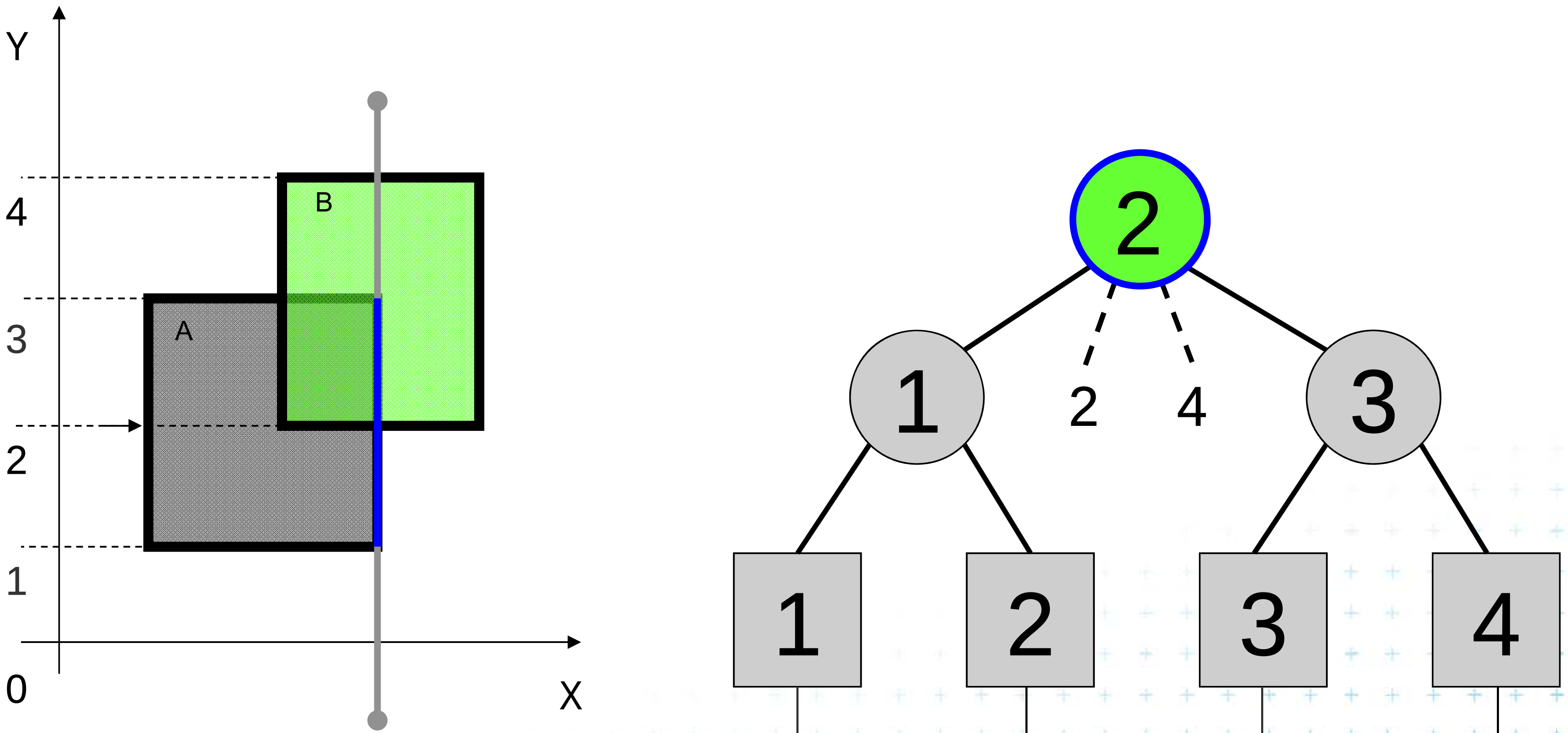
Interval delete [1,3]





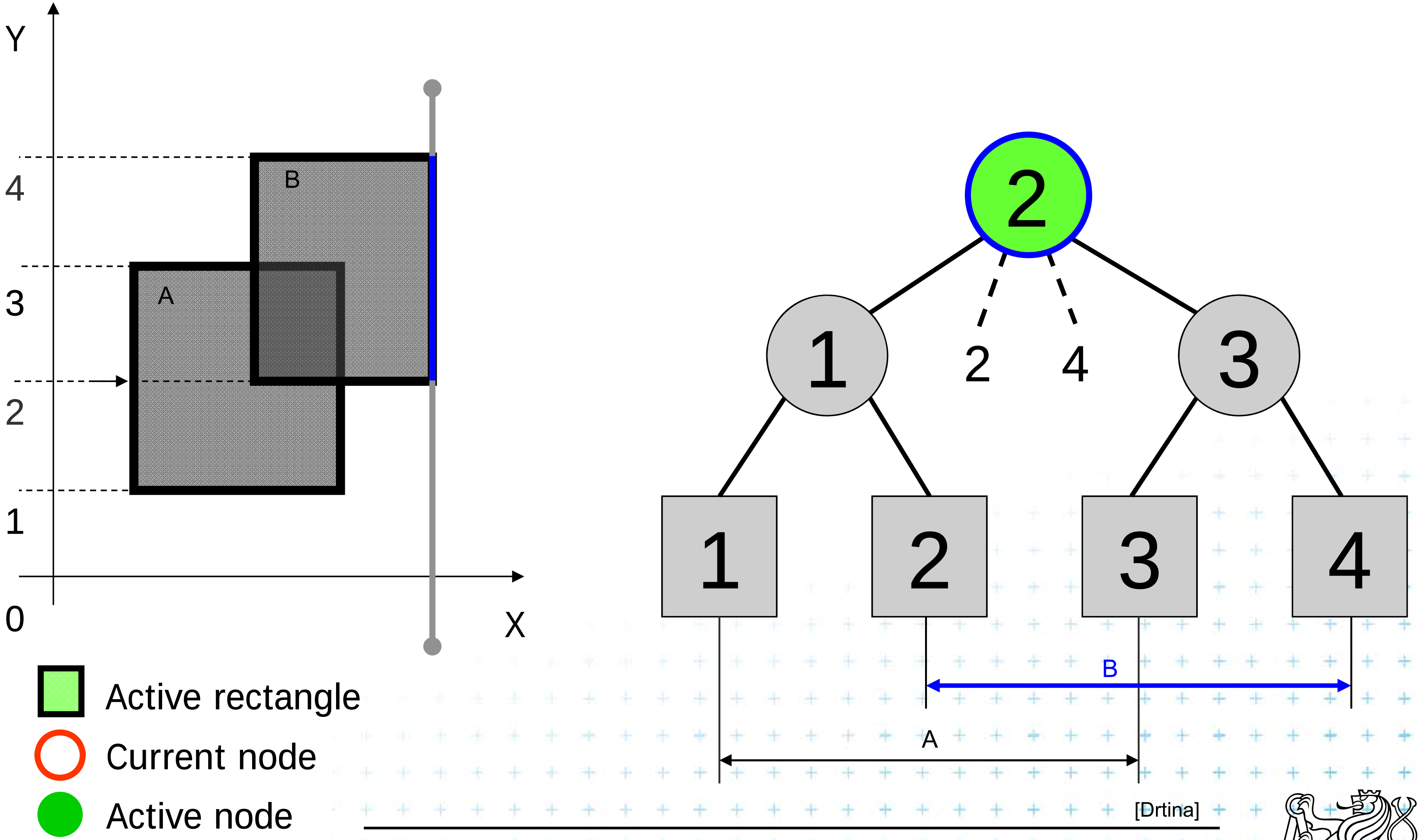
Active rectangle
Current node
Active node

Interval delete [1,3]



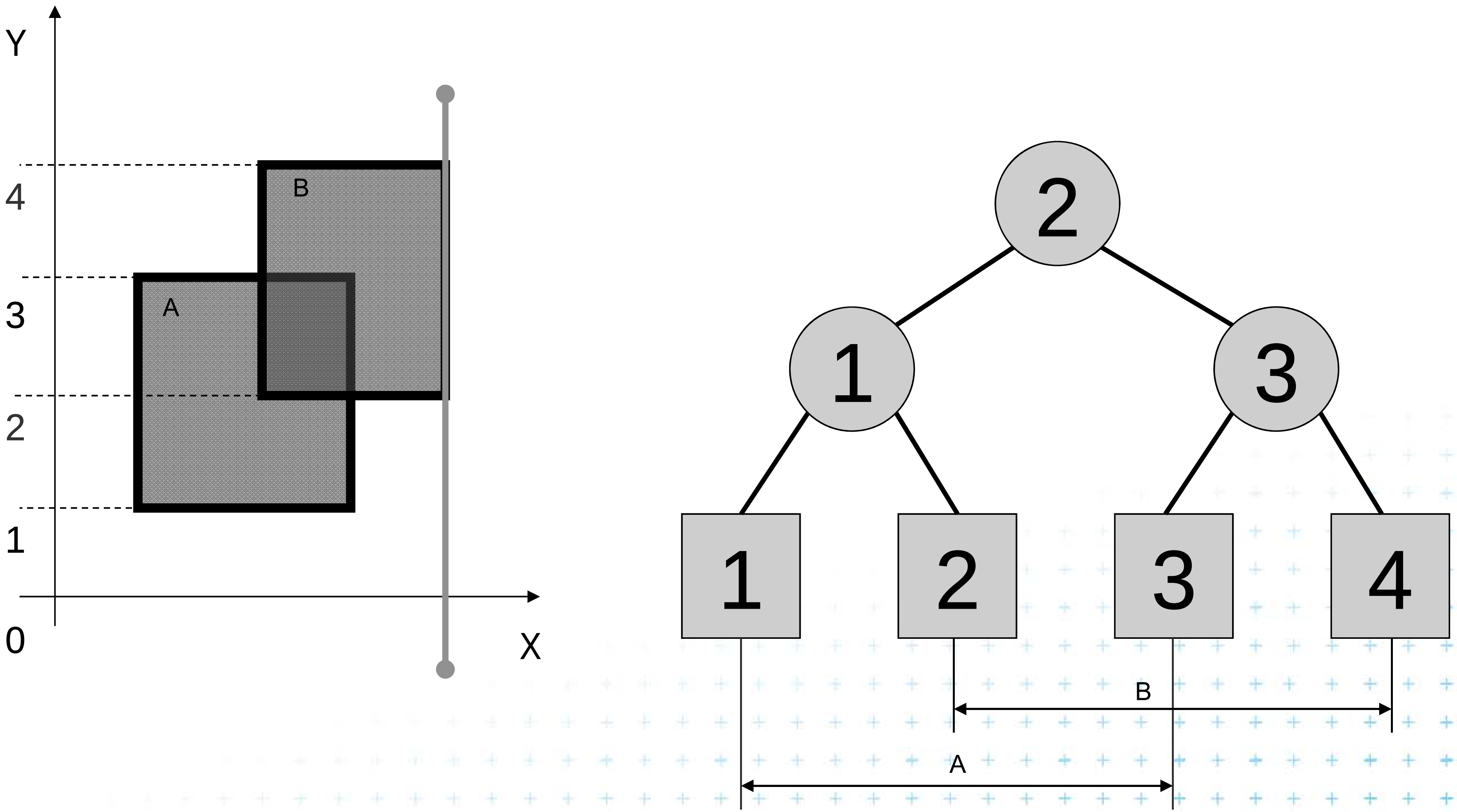
[Drtina]

Interval delete [2,4]



[Drtina]

Interval delete [2,4]



[Drtina]

Example 2

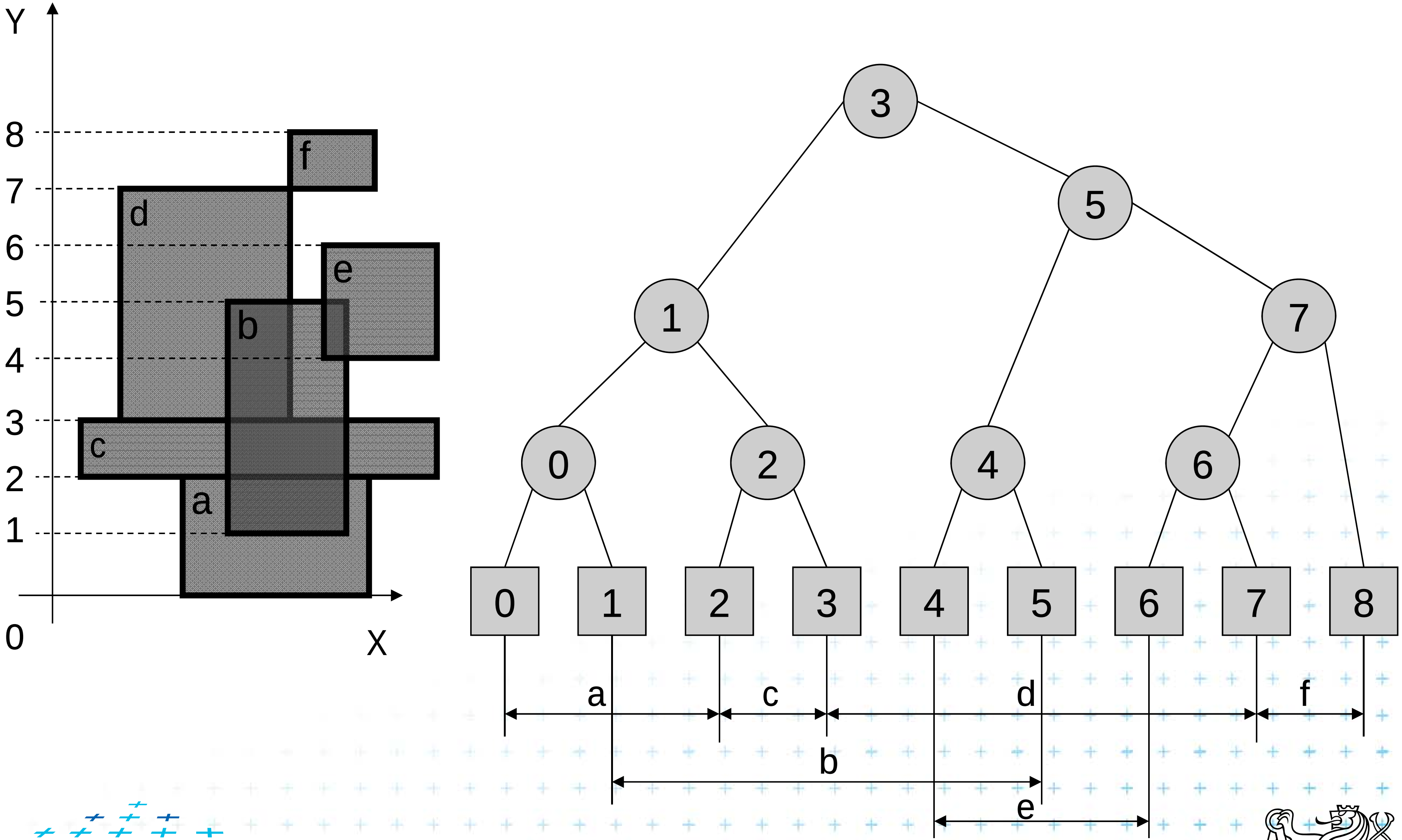
RectangleIntersections(*S*)
Input: Set *S* of rectangles
Output: Intersected rectangle pairs

// this is a copy of the slide before
// just to remember the algorithm

1. Preprocess(S) // create the interval tree *T* and event queue *Q*

2. **while** ($Q \neq \emptyset$) do
3. Get next entry ($x_{il}, y_{il}, y_{ir}, t$) from *Q* // $t \in$ { *left* | *right* }
4. **if** (t = left) // left edge
5. a) QueryInterval (y_{il}, y_{ir}, root(T)) // report intersections
6. b) InsertInterval (y_{il}, y_{ir}, root(T)) // insert new interval
7. **else** // right edge
8. c) DeleteInterval (y_{il}, y_{ir}, root(T))

Example 2



[Drtina]

Insert [2,3] – empty => b) Insert Interval

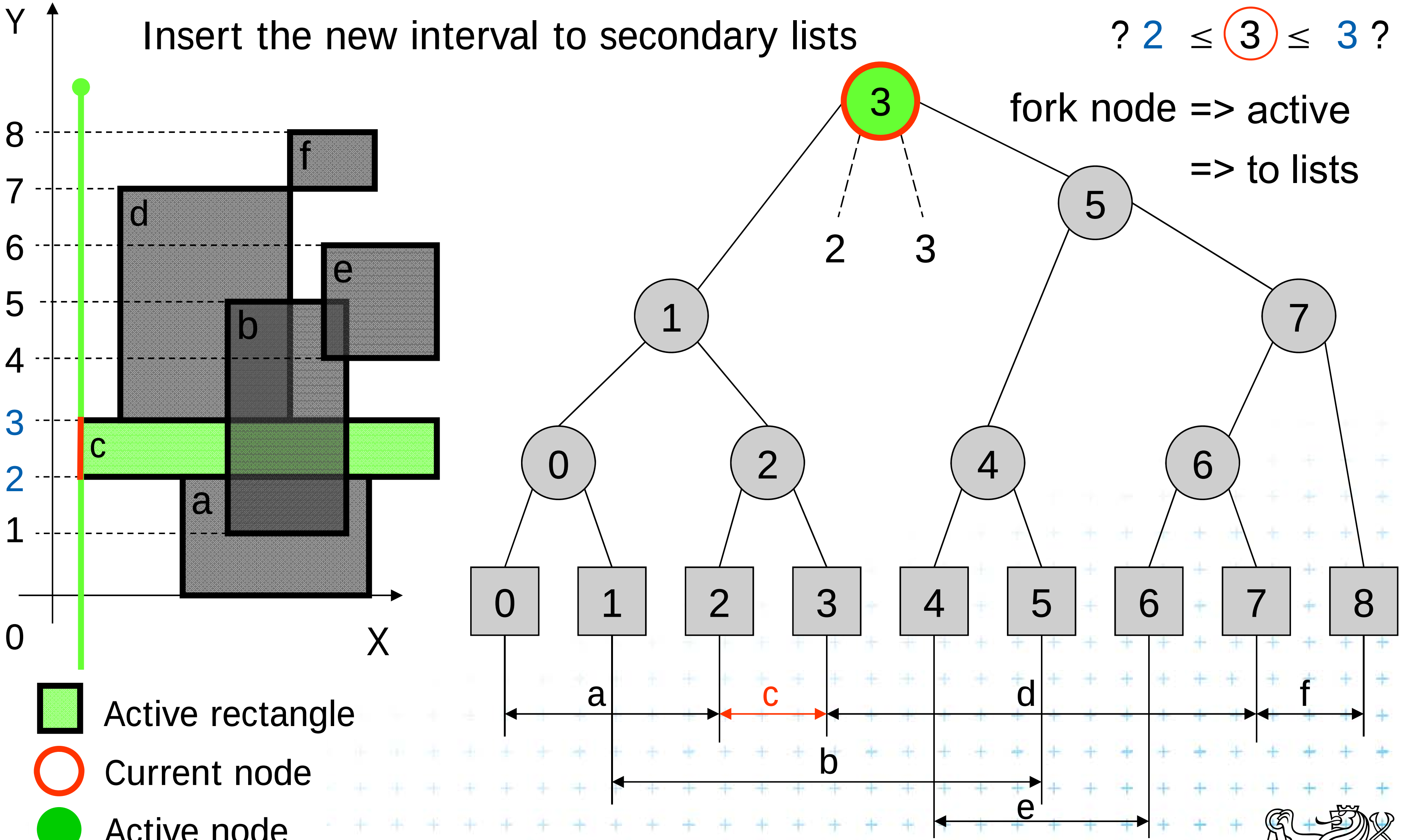
$b \le H(v) \le e$

Insert the new interval to secondary lists

? 2 ≤ 3 ≤ 3 ?

fork node => active

=> to lists



Active rectangle

Current node

Active node

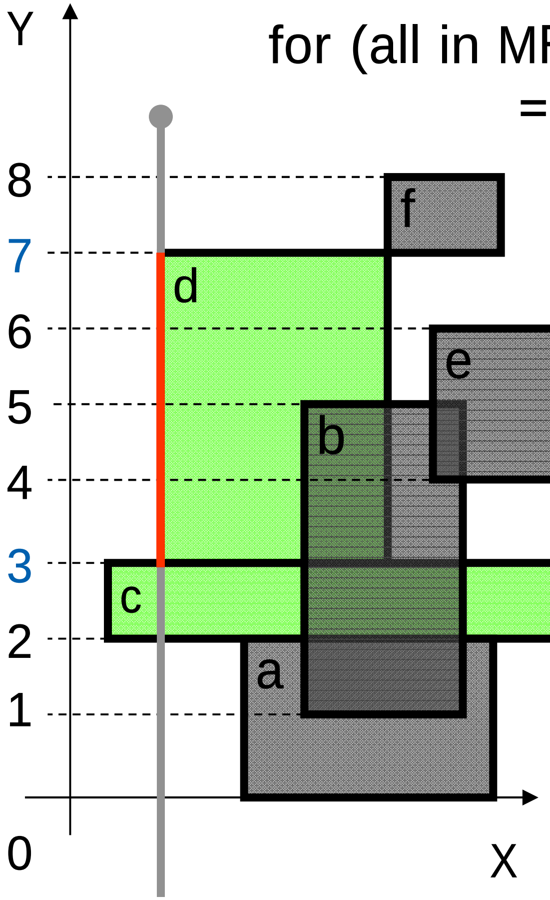
[Drtina]

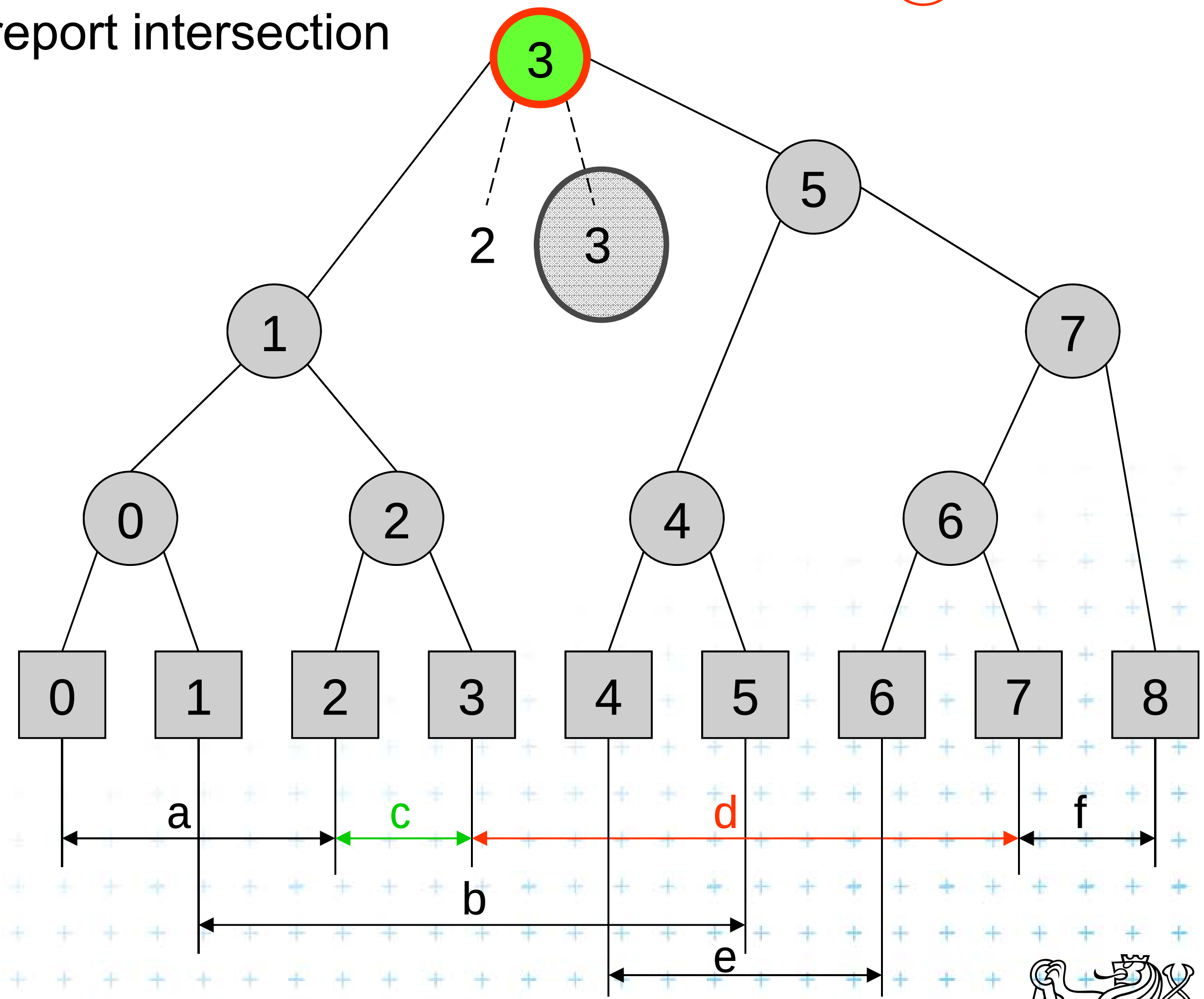
Insert [3,7] a) Query Interval

$H(v) \le b < e$

for (all in MR(v)) test MR(v)[i] >= 3
=> report intersection

? $3 \le 3 < 7$?





Active rectangle

Current node

Active node

[Drtina]

Insert [3,7] b) Insert Interval

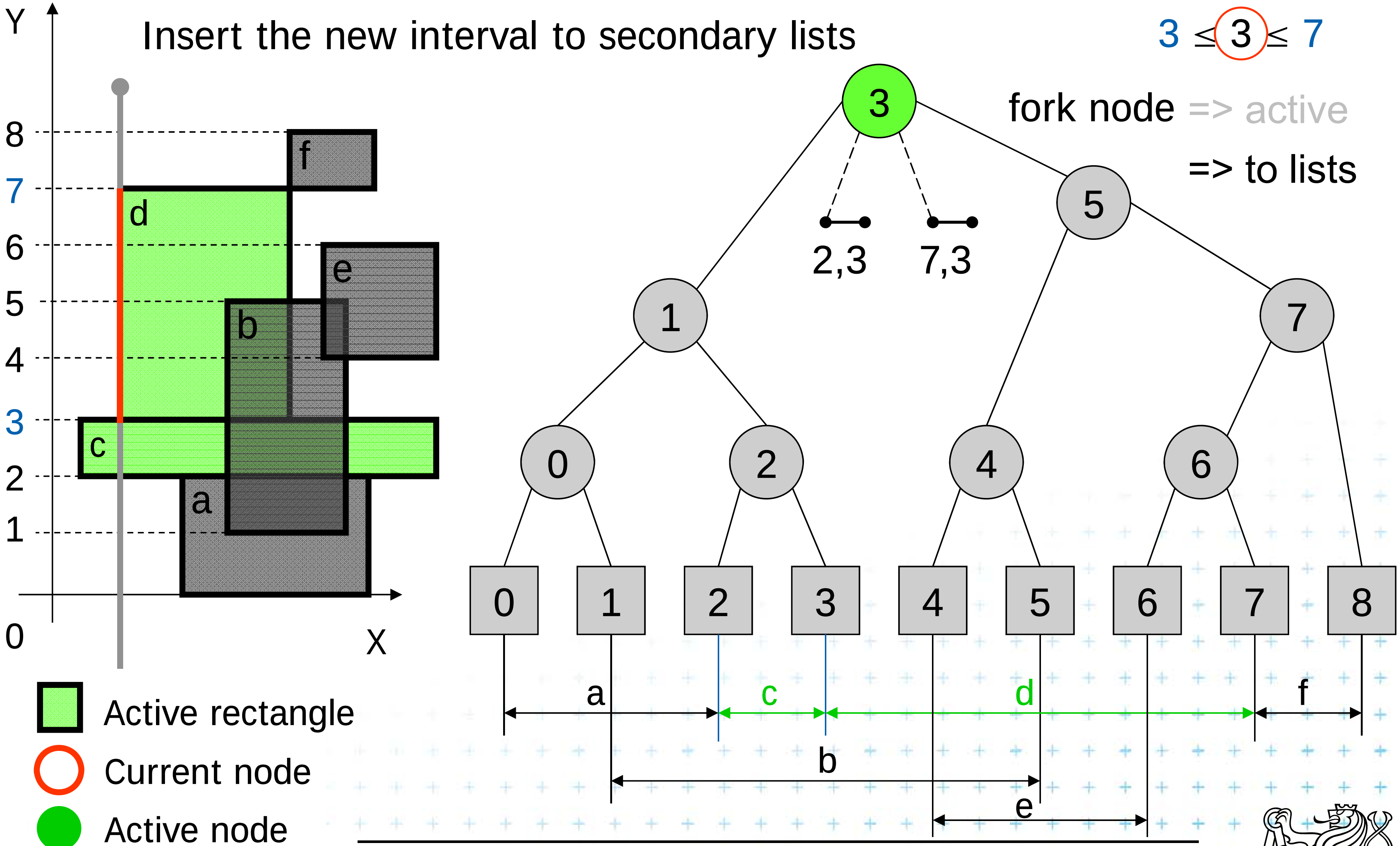
$b \le H(v) \le e$

Insert the new interval to secondary lists

$3 \le 3 \le 7$

fork node => active

=> to lists



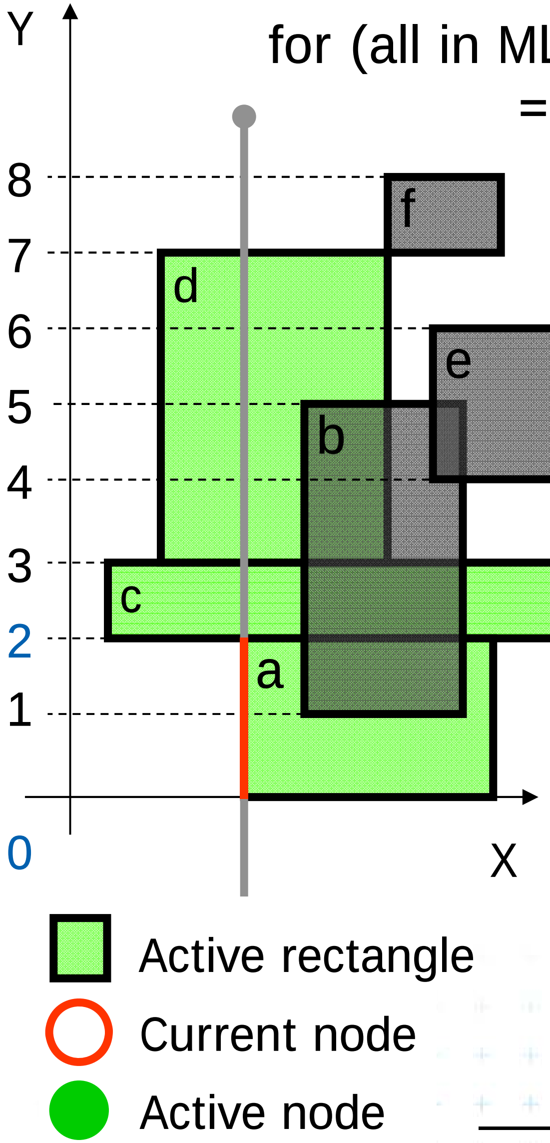
[Drtina]

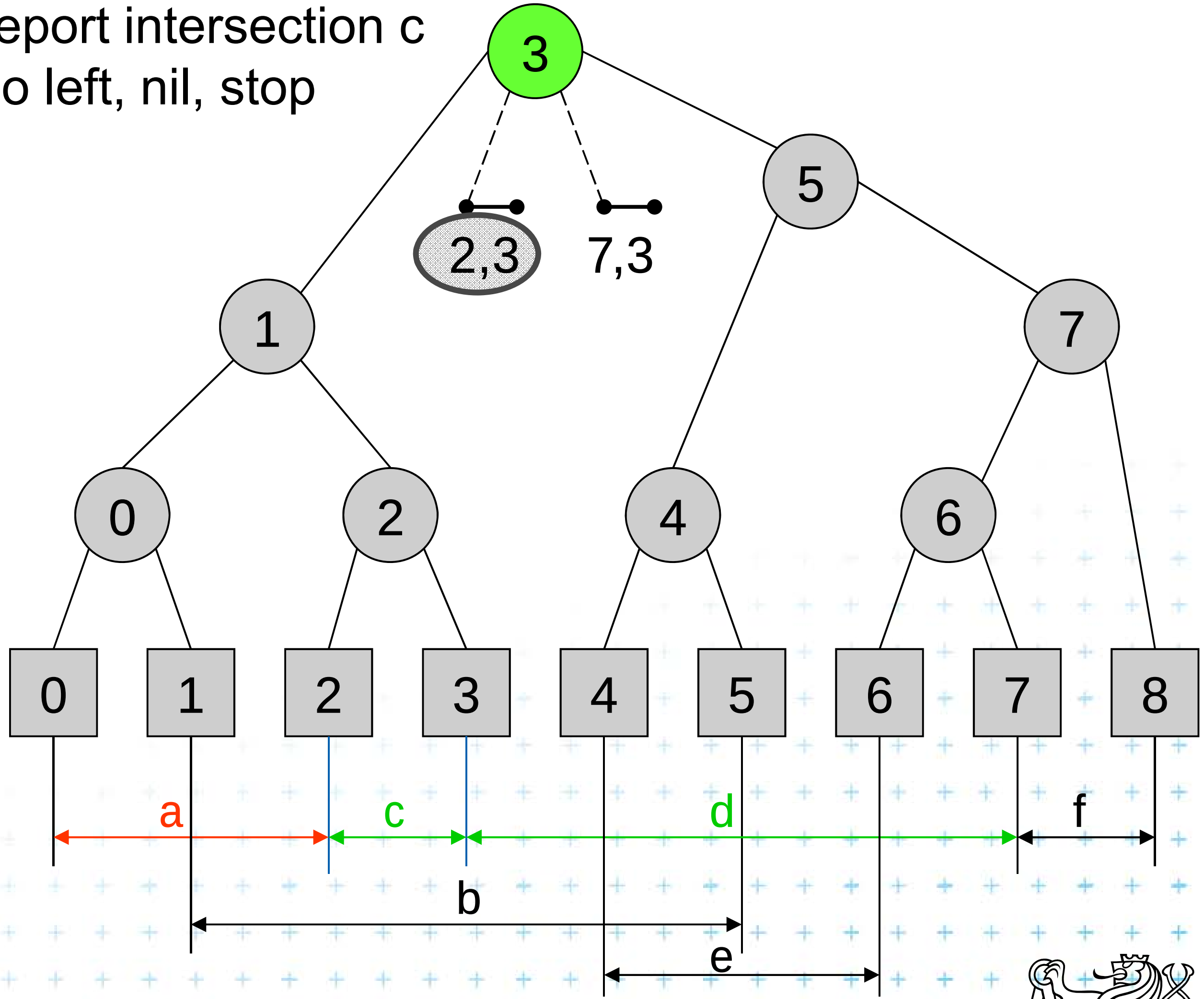
Insert [0,2] a) Query Interval

$b < e \leq H(v)$

for (all in ML(v)) test ML(v).[i] $\leq$ 2
=> report intersection c
go left, nil, stop

? $0 < 2 \leq 3$?



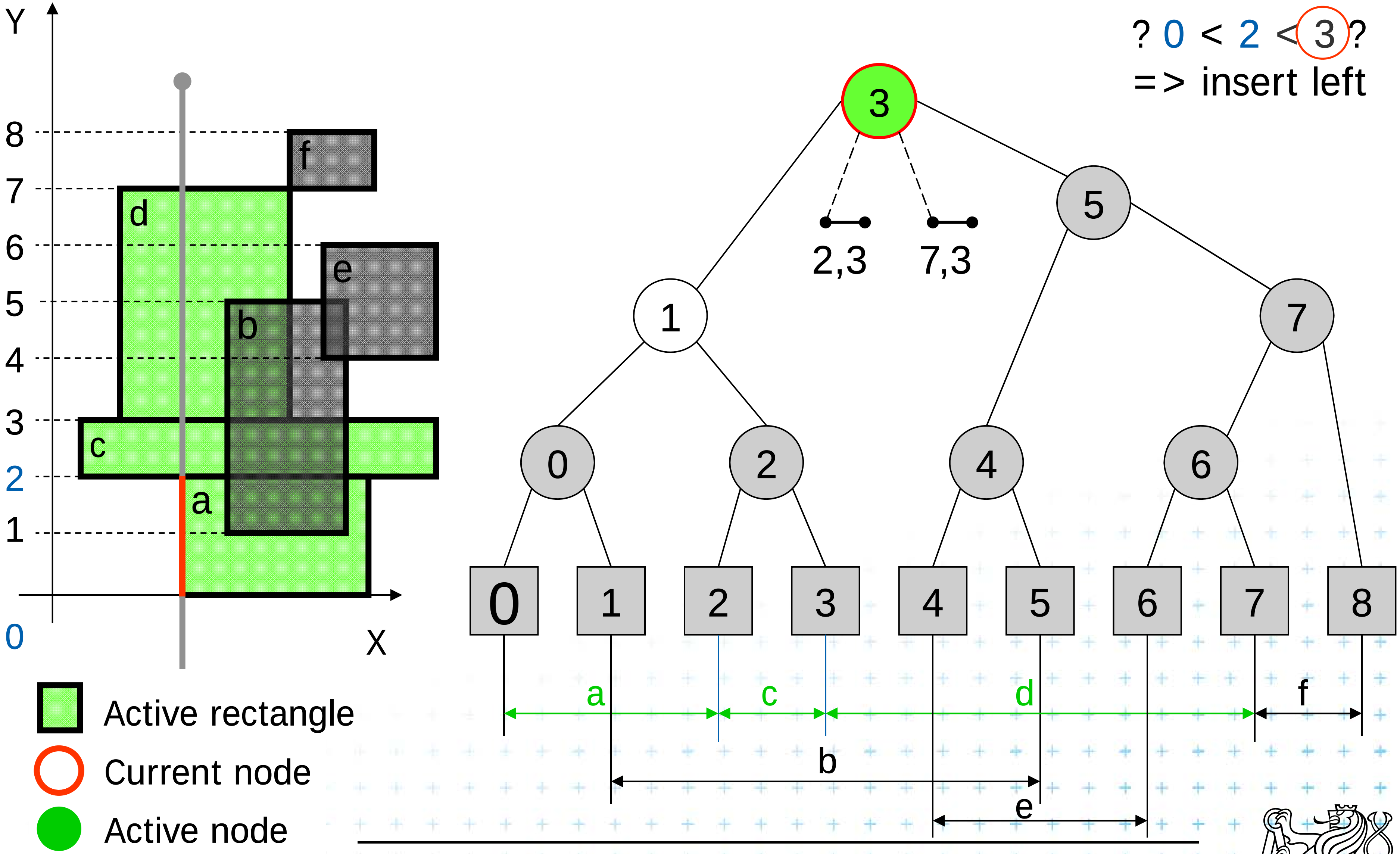


[Drtina]

Insert [0,2] b) Insert Interval 1/2

b < e <H(v)

? 0 < 2 < 3 ?
=> insert left



[Drtina]

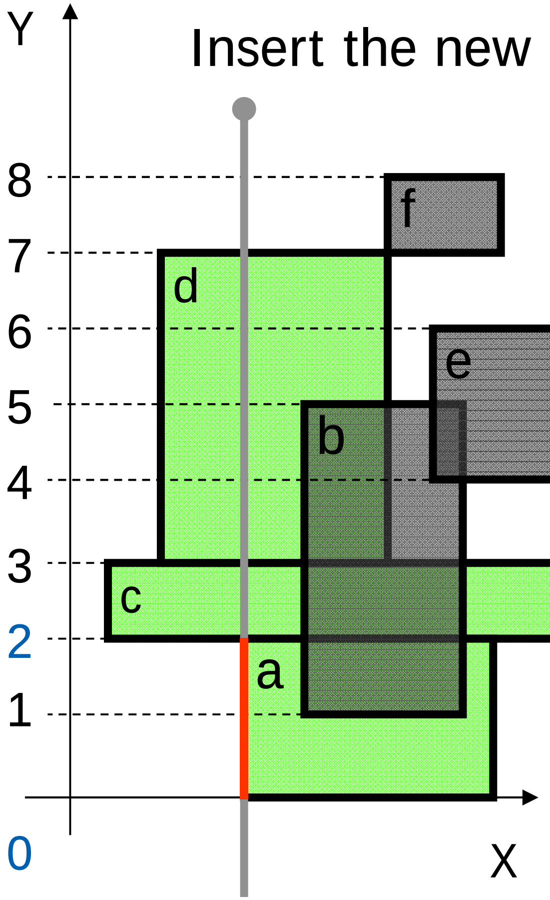
Insert [0,2] b) Insert Interval 2/2

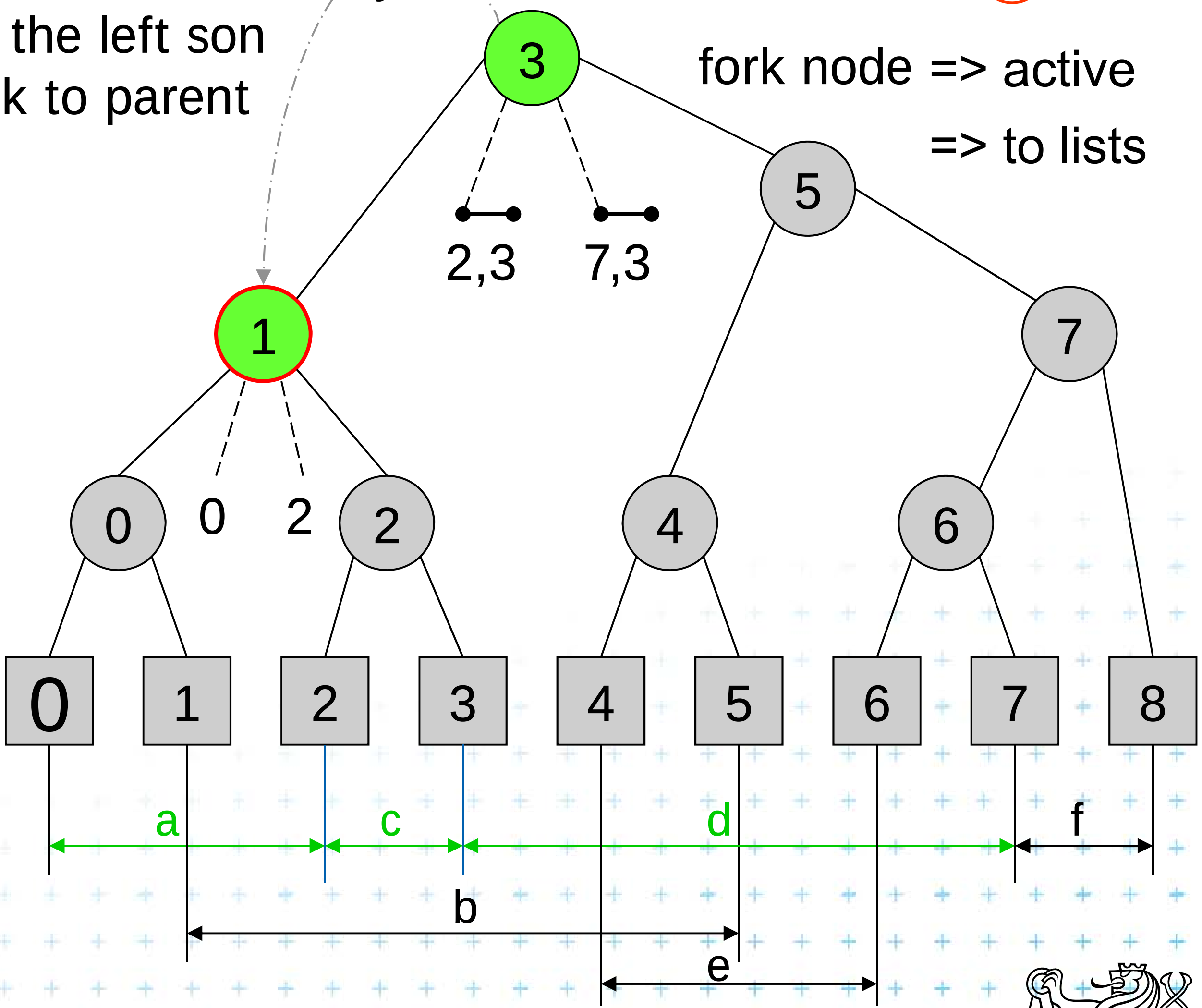
$b \le H(v) \le e$

? $0 \le 1 \le 2$?

Insert the new interval to secondary lists
of the left son
link to parent

fork node => active
=> to lists





Active rectangle
Current node
Active node

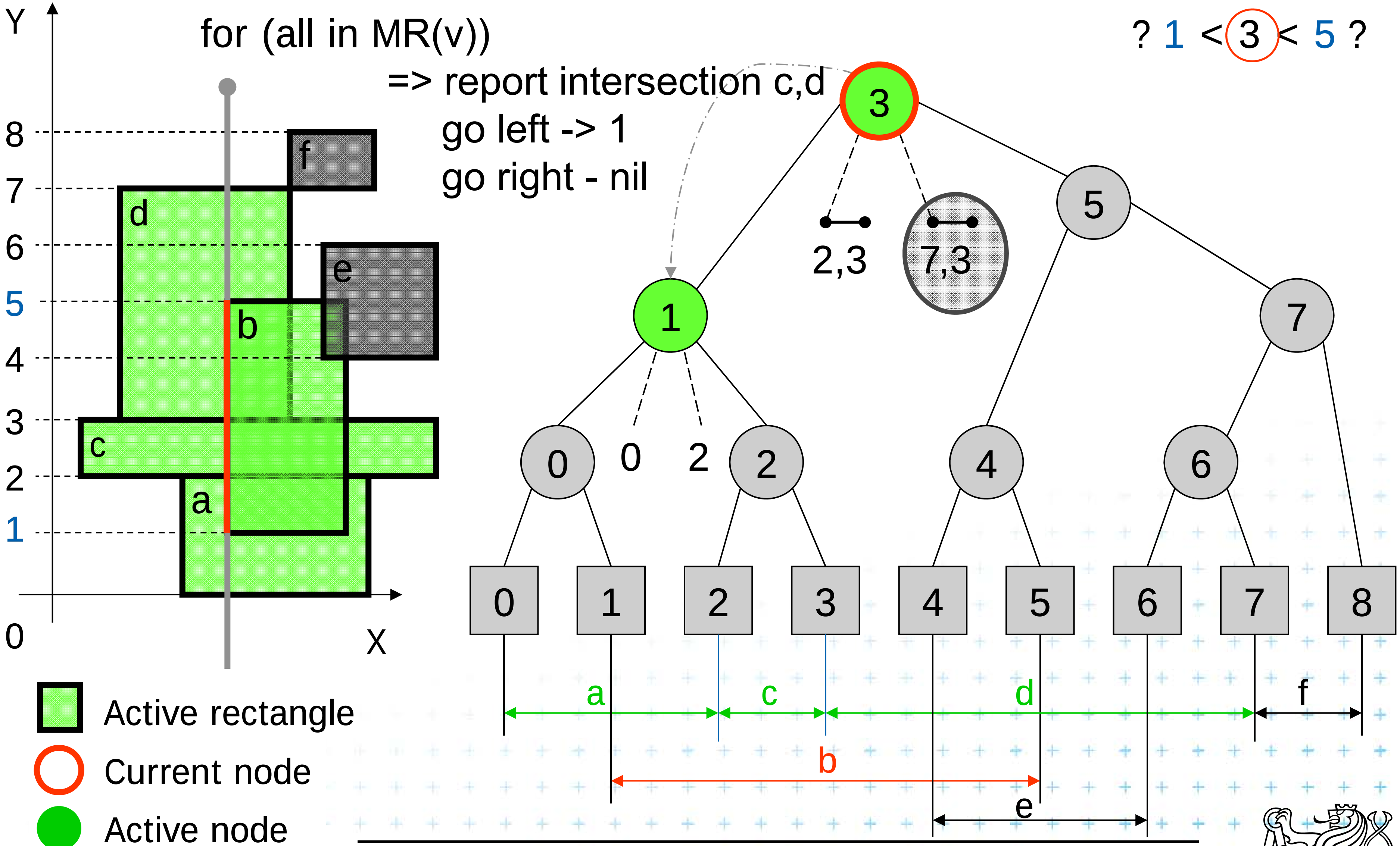
[Drtina]

Insert [1,5] a) Query Interval 1/2

b < H(v) < e

? 1 < 3 < 5 ?

for (all in MR(v))
=> report intersection c,d
go left -> 1
go right - nil

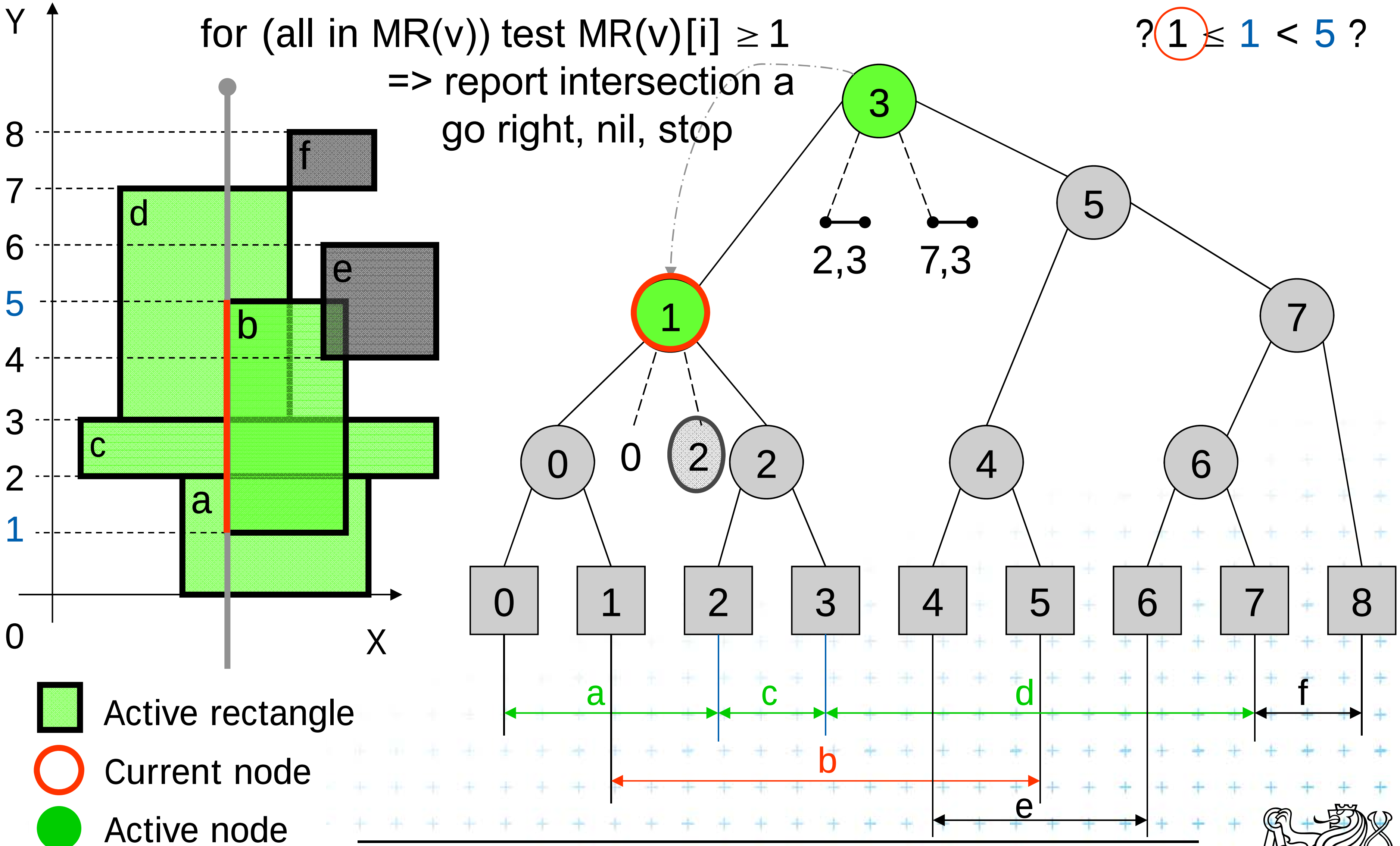


Insert [1,5] a) Query Interval 2/2

H(v) ≤ b < e

for (all in MR(v)) test MR(v)[i] ≥ 1
=> report intersection a
go right, nil, stop

? 1 ≤ 1 < 5 ?



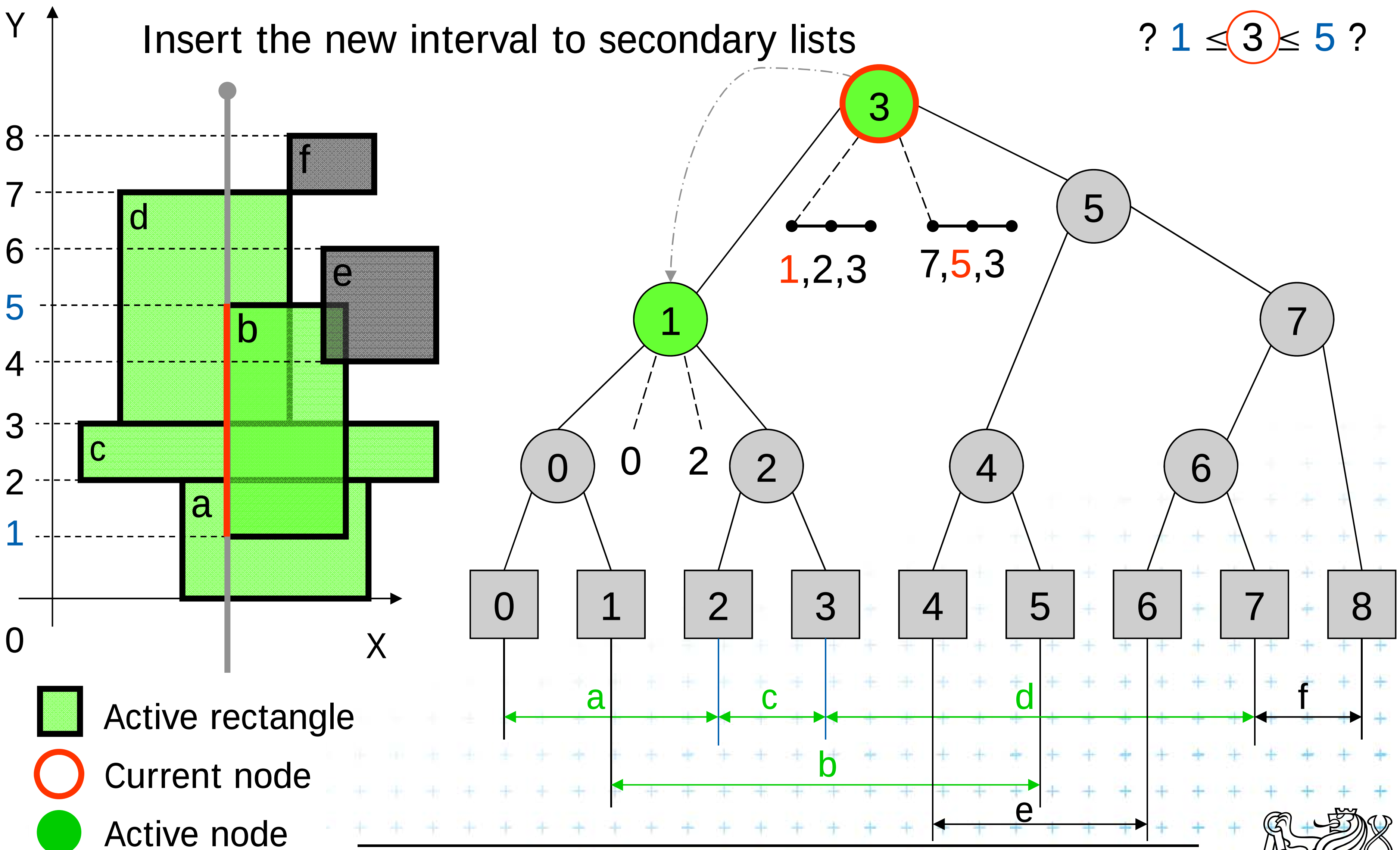
[Drtina]

Insert [1,5] b) Insert Interval

$b \le H(v) \le e$

Insert the new interval to secondary lists

? 1 ≤ 3 ≤ 5 ?



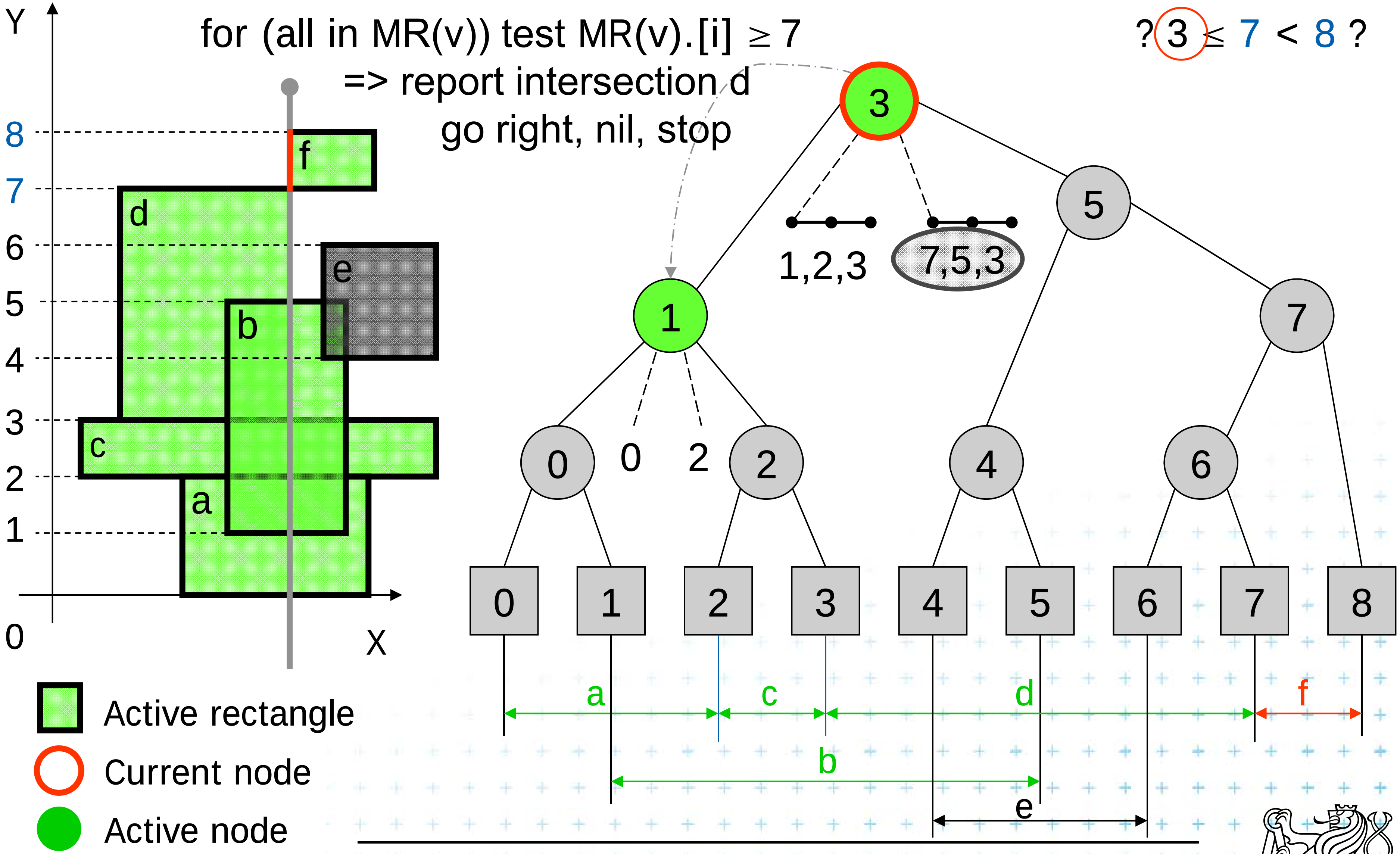
[Drtina]

Insert [7,8] a) Query Interval

$H(v) \le b < e$

for (all in MR(v)) test MR(v).[i] ≥ 7
=> report intersection d
go right, nil, stop

? $3 \le 7 < 8$?



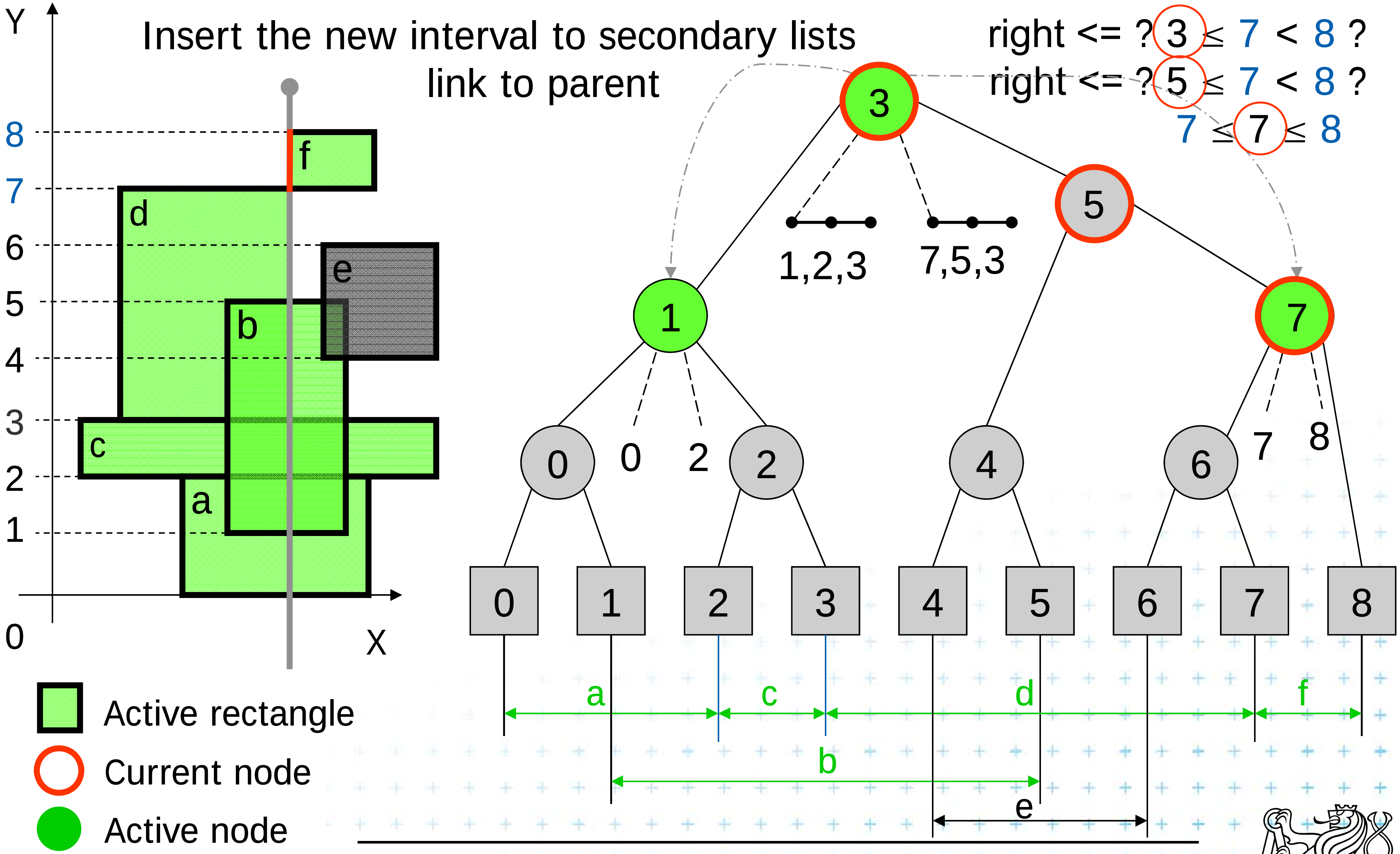
[Drtina]

Insert [7,8] b) Insert Interval

$b \le H(v) \le e$

Insert the new interval to secondary lists

link to parent



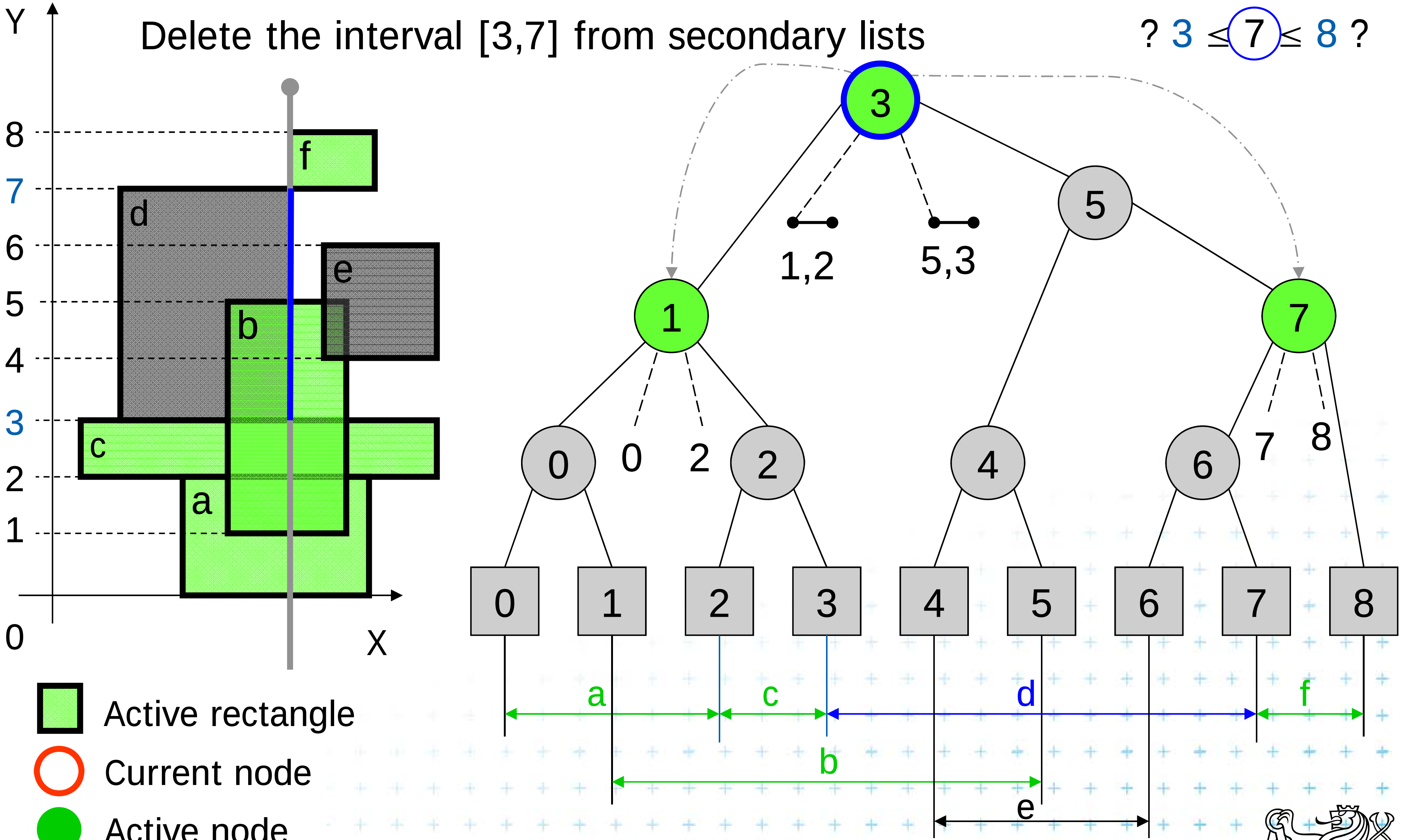
[Drtina]

Delete [3,7] Delete Interval

$b \le H(v) \le e$

Delete the interval [3,7] from secondary lists

? $3 \le 7 \le 8$?



Active rectangle

Current node

Active node

[Drtina]

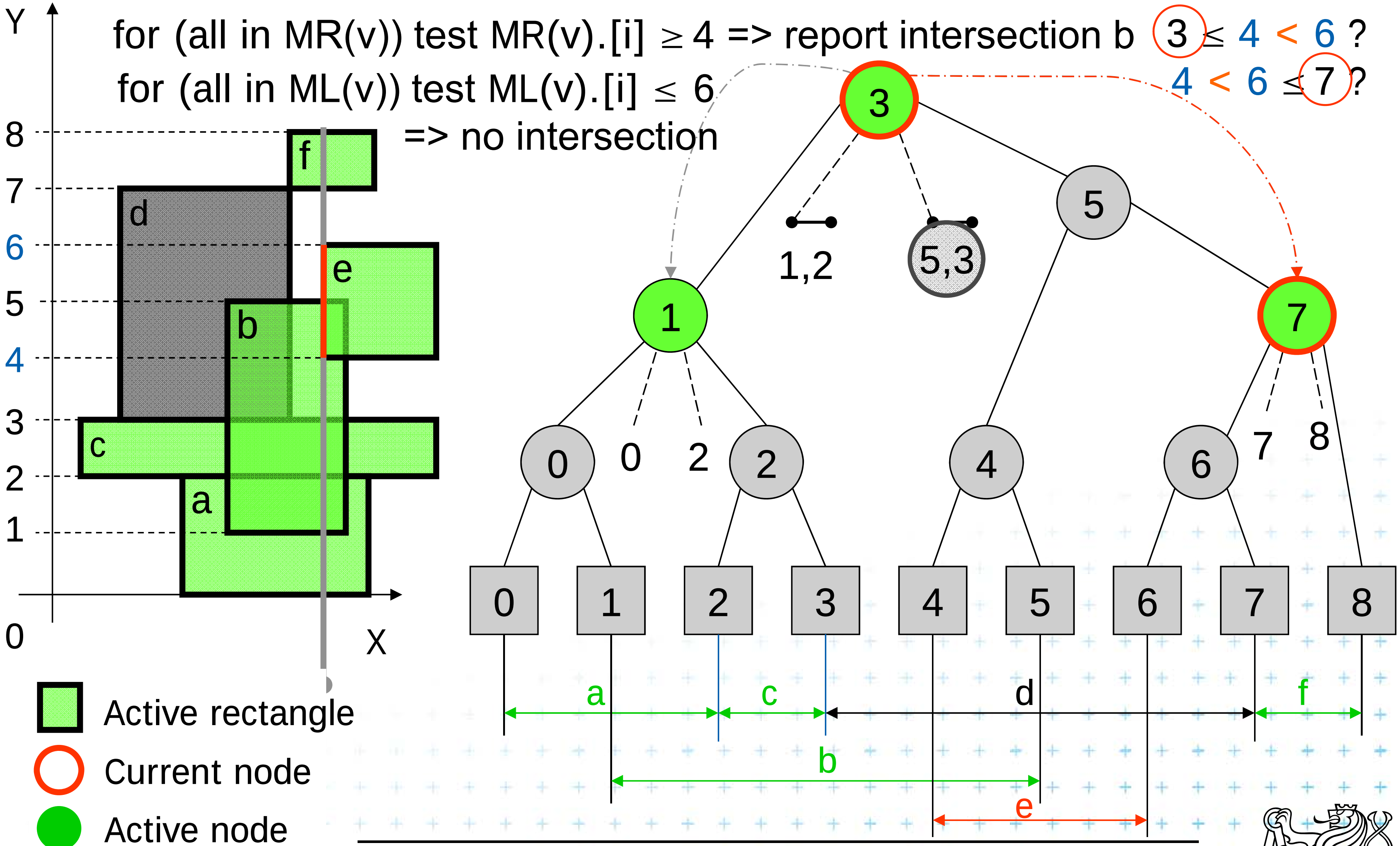
Insert [4,6] a) Query Interval

H(v) ≤ b < e

for (all in MR(v)) test MR(v).[i] ≥ 4 => report intersection b 3 ≤ 4 < 6 ?

for (all in ML(v)) test ML(v).[i] ≤ 6 4 < 6 ≤ 7 ?

=> no intersection



Active rectangle

Current node

Active node

[Drtina]

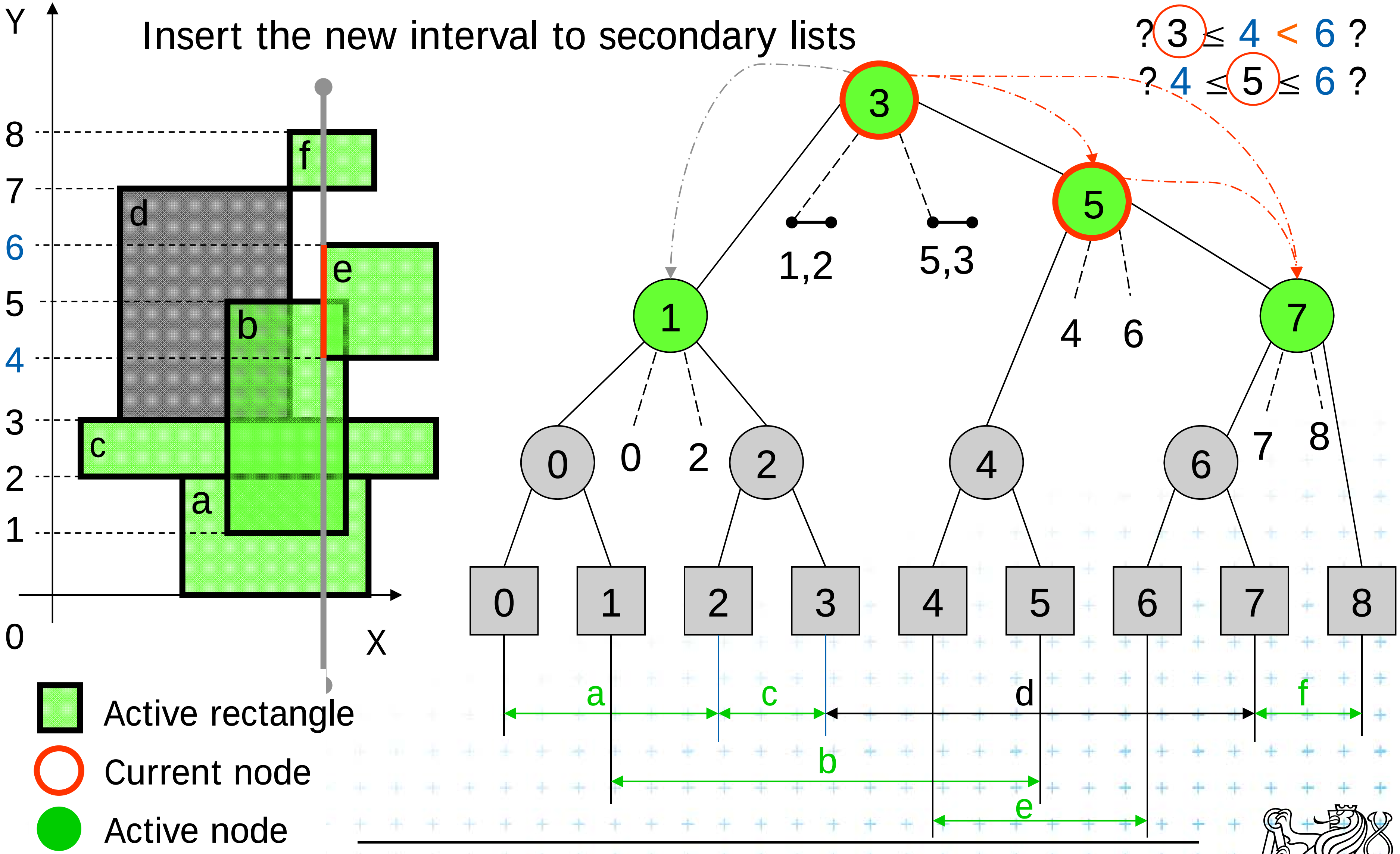
Insert [4,6] b) Insert Interval

$H(v) \le b < e$

Insert the new interval to secondary lists

? $3 \le 4 < 6$?

? $4 \le 5 \le 6$?



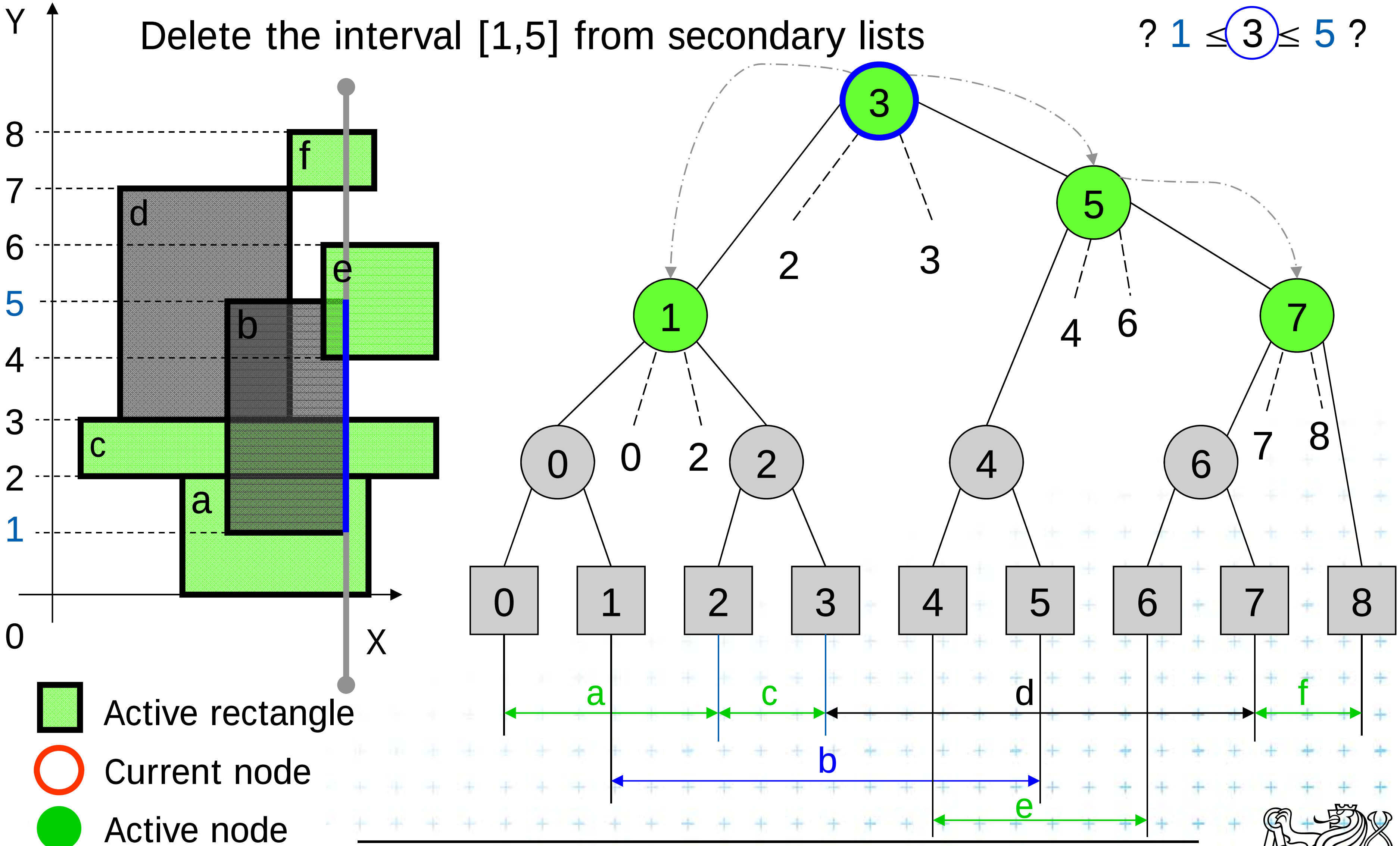
[Drtina]

Delete [1,5] Delete Interval

$b \le H(v) \le e$

Delete the interval [1,5] from secondary lists

? $1 \le 3 \le 5$?

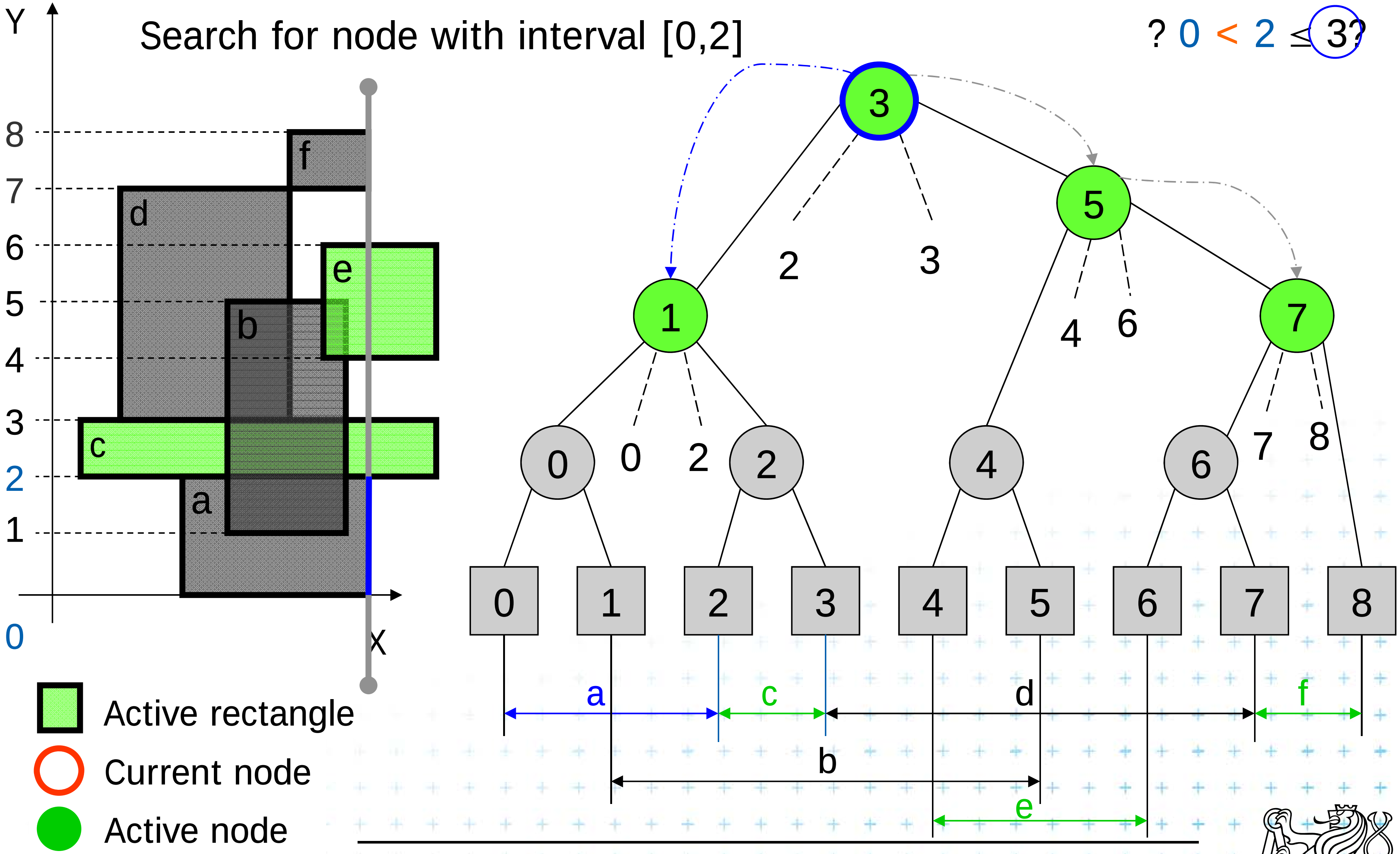


Delete [0,2] Delete Interval 1/2

$b < e \leq H(v)$

Search for node with interval [0,2]

? $0 < 2 \leq 3$?

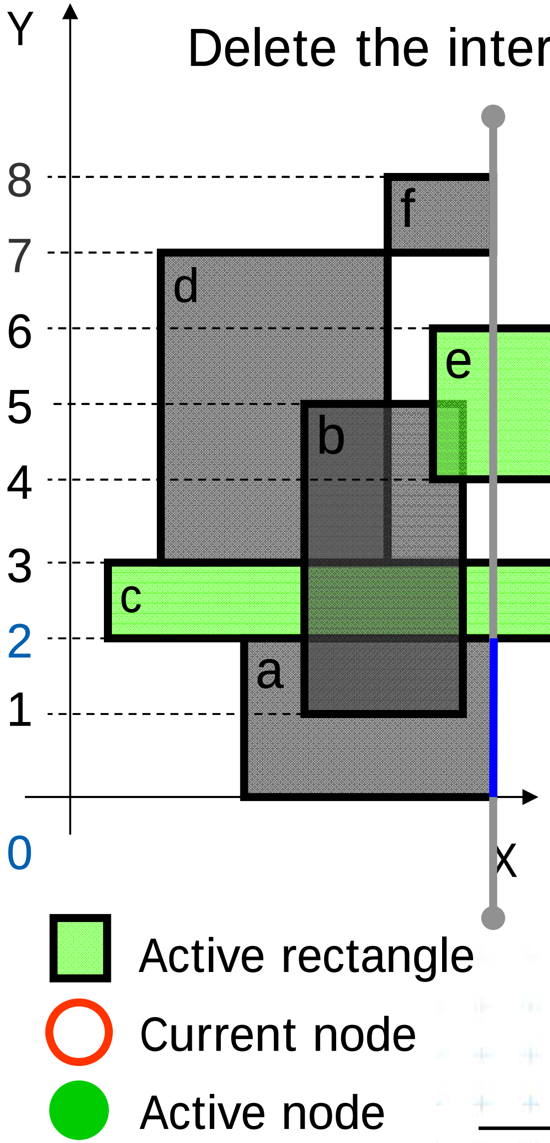


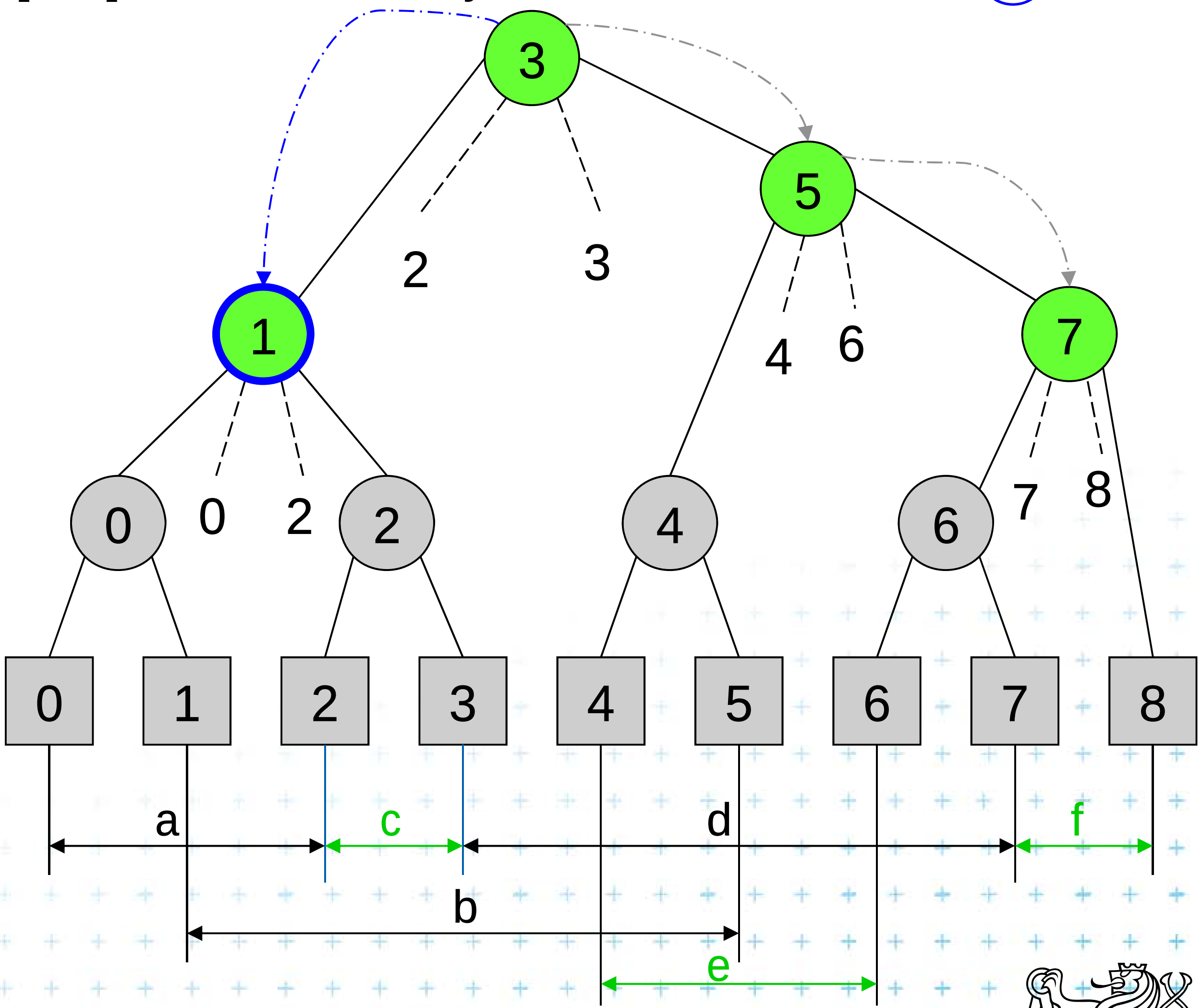
[Drtina]

Delete [0,2] Delete Interval 2/2

$b \le H(v) \le e$

Delete the interval [0,2] from secondary lists of node 1 ? $0 \le 1 \le 2$?





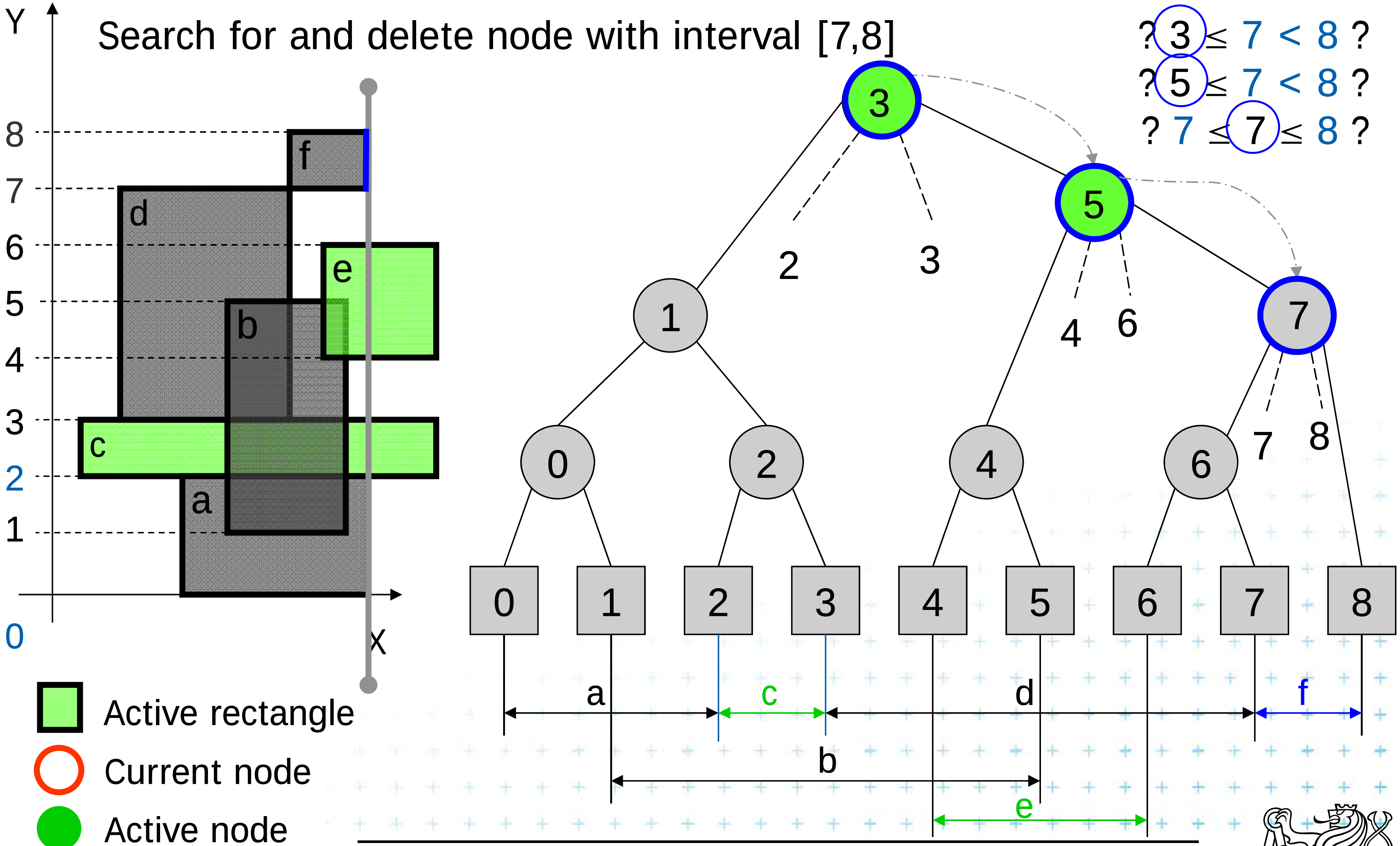
[Drtina]

Delete [7,8] Delete Interval

$b \le H(v) \le e$

Search for and delete node with interval [7,8]

? 3 ≤ 7 < 8 ?
? 5 ≤ 7 < 8 ?
? 7 ≤ 7 ≤ 8 ?



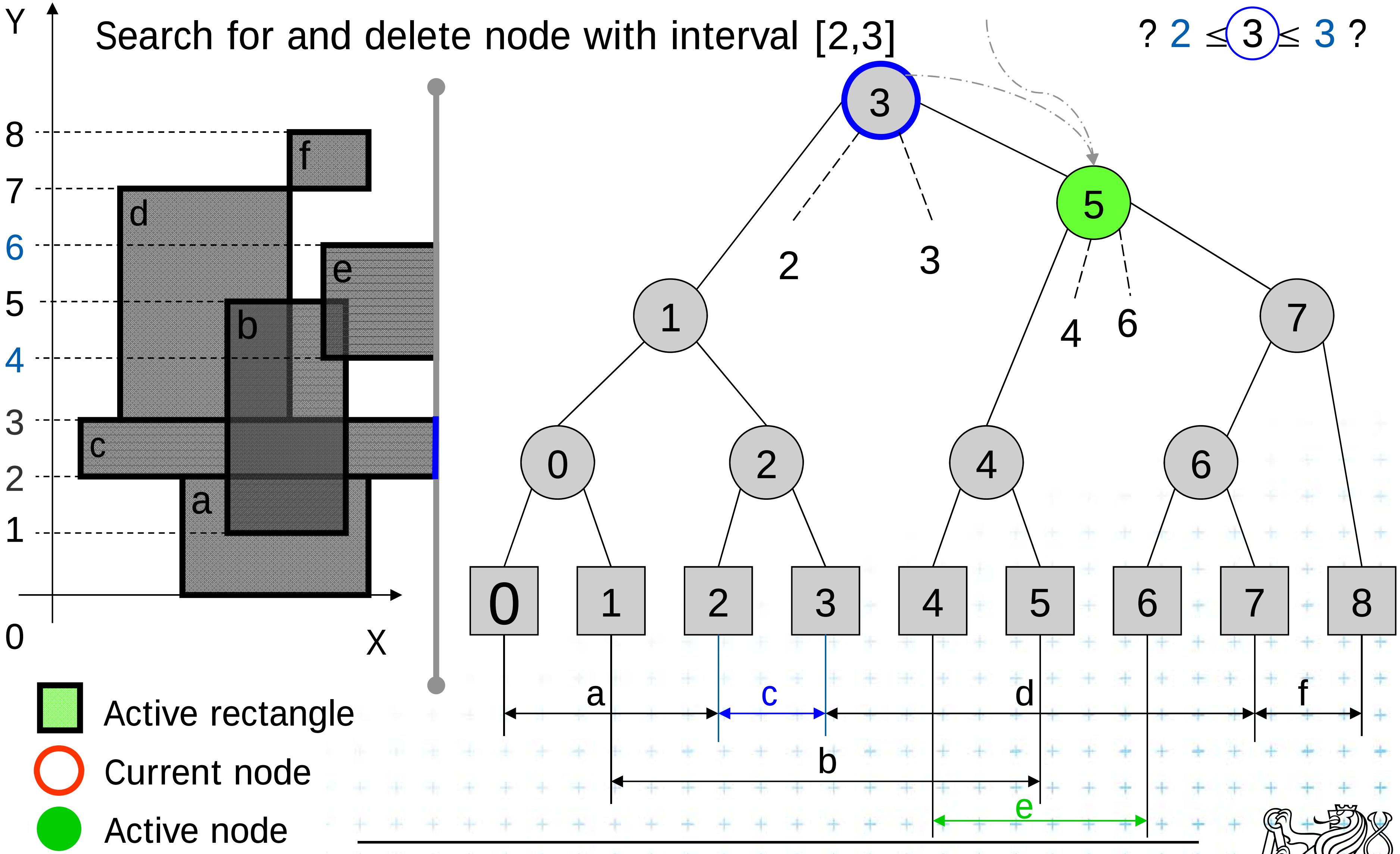
[Drtina]

Delete [2,3] Delete Interval

$b \le H(v) \le e$

Search for and delete node with interval [2,3]

? 2 ≤ 3 ≤ 3 ?

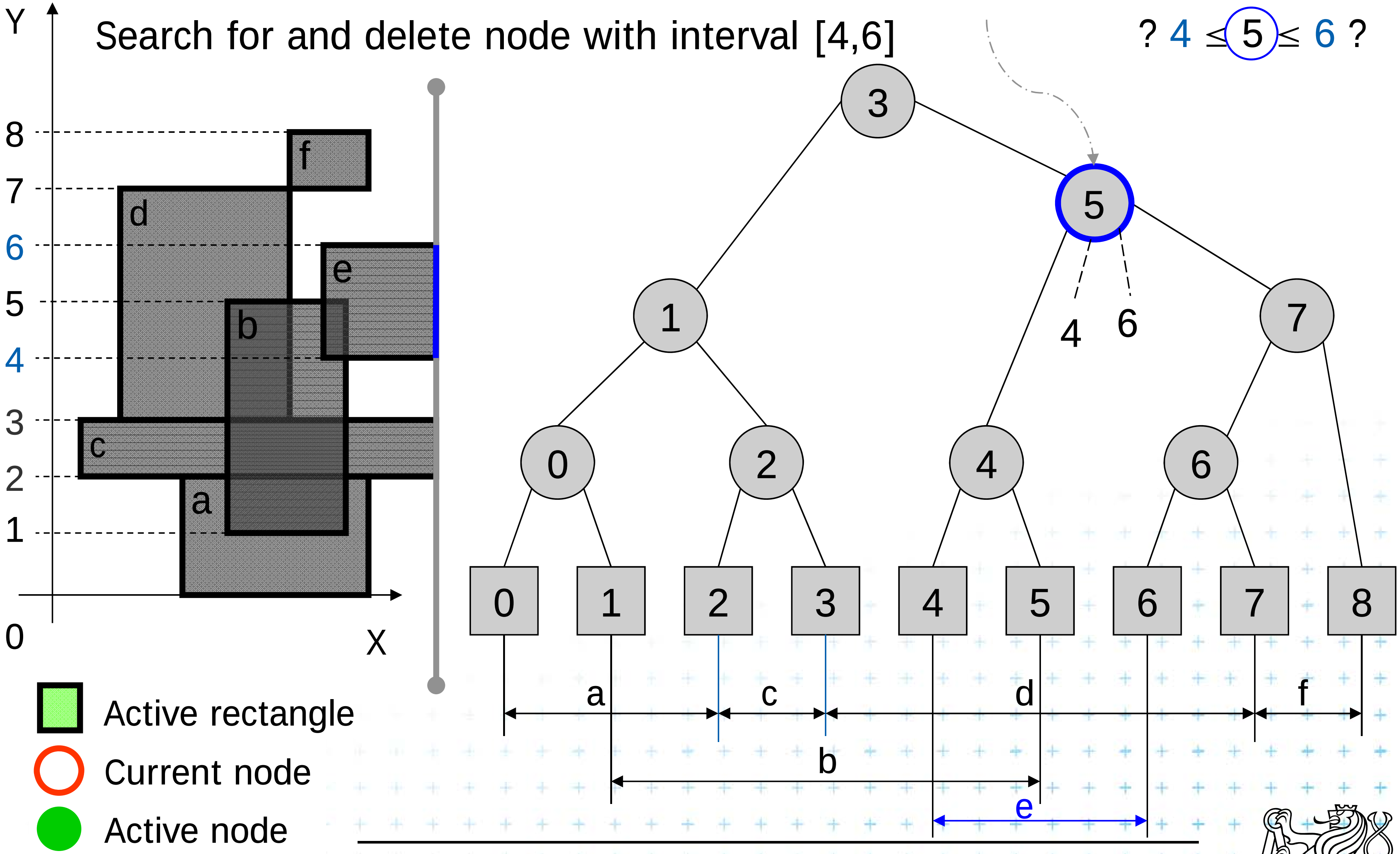


Delete [4,6] Delete Interval

$b \leq H(v) \leq e$

Search for and delete node with interval [4,6]

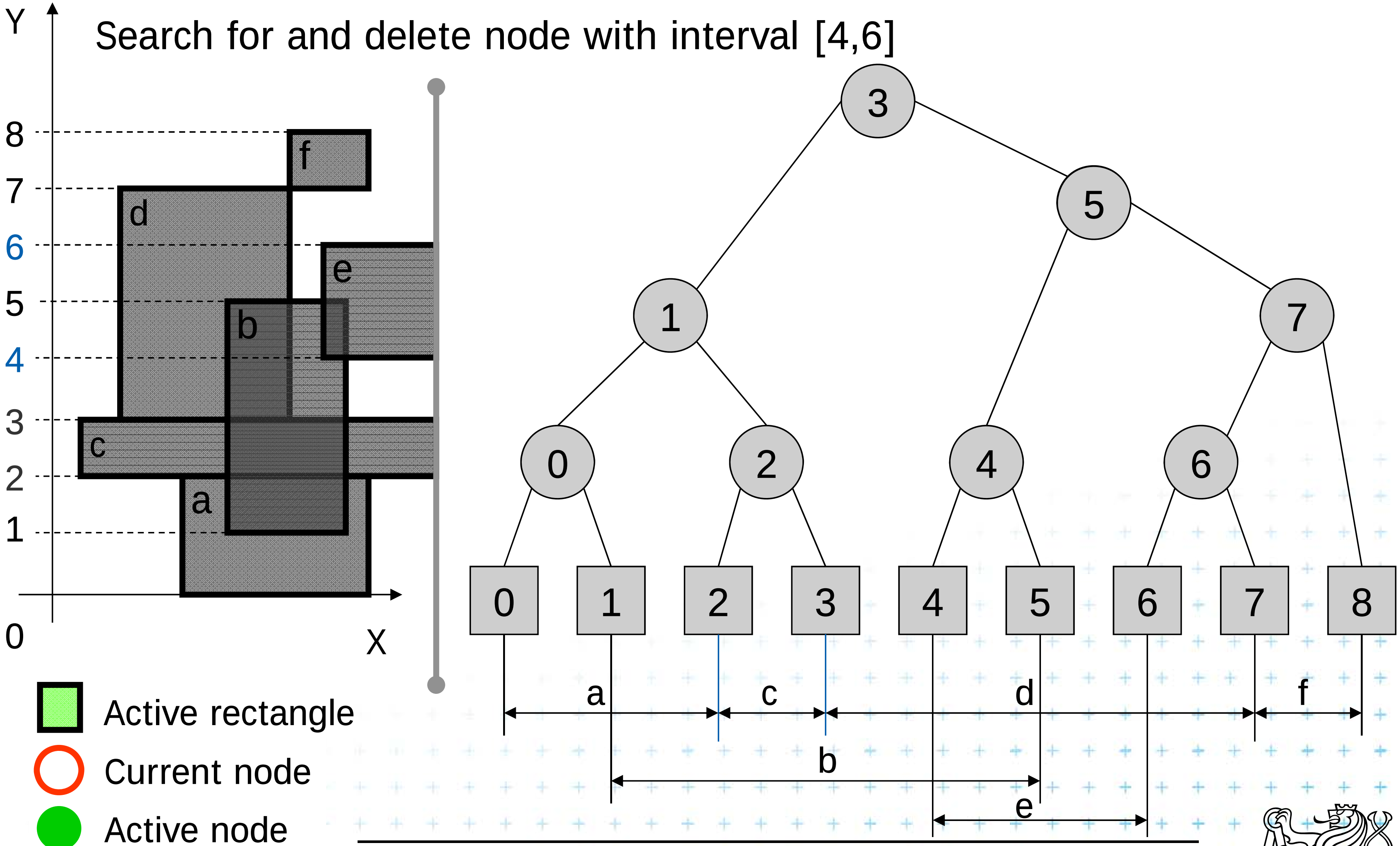
? 4 ≤ 5 ≤ 6 ?



[Drtina]

Empty tree

Search for and delete node with interval [4,6]



Active rectangle

Current node

Active node

Complexities of rectangle intersections

- *n* rectangles, *s* intersected pairs found
- O(*n* log *n*) preprocessing time to separately sort
  - x-coordinates of the rectangles for the plane sweep
  - the y-coordinates for initializing the interval tree.
- The plane sweep itself takes O(*n* log *n* + *s*) time, so the overall time is O(*n* log *n* + *s*)
- O(*n*) space
- This time is optimal for a decision-tree algorithm (i.e., one that only makes comparisons between rectangle coordinates).

References

[Berg] Mark de Berg, Otfried Cheong, Marc van Kreveld, Mark Overmars: Computational Geometry: Algorithms and Applications, Springer-Verlag, 3rd rev. ed. 2008. 386 pages, 370 fig. ISBN: 978-3-540-77973-5, Chapters 3 and 9, http://www.cs.uu.nl/geobook/

[Mount] David Mount, - CMSC 754: Computational Geometry, Lecture Notes for Spring 2007, University of Maryland, Lectures 7,22, 13,14, and 30. http://www.cs.umd.edu/class/spring2007/cmsc754/lectures.shtml

[Rourke] Joseph O´Rourke: .: Computational Geometry in C, Cambridge University Press, 1993, ISBN 0-521- 44592-2 http://maven.smith.edu/~orourke/books/compgeom.html

[Drtina] Tomáš Drtina: Intersection of rectangles. Semestral Assignment. Computational Geometry course, FEL CTU Prague, 2006

[Kukral] Petr Kukrál: Intersection of rectangles. Semestral Assignment. Computational Geometry course, FEL CTU Prague, 2006

[Vigneron] Segment trees and interval trees, presentation, INRA, France, http://w3.jouy.inra.fr/unites/miaj/public/vigneron/cs4235/slides.html