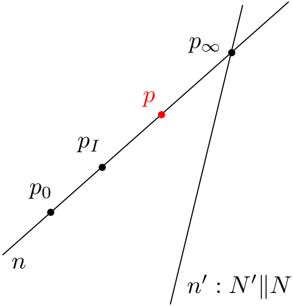
▶1D Projective Coordinates

The 1-D projective coordinate of a point P:

$$[P] = [P_\infty\, P_0\, P_I\, P] = [p_\infty\, p_0\, p_I\, p] = \frac{|p_\infty\, p_I|}{|p_0\, p_I|}\,\frac{|p_0\, p|}{|p_\infty\, p|}$$



P_0 – the origin $\quad [P_0] = 0$

P_I – the unit point $\quad [P_I] = 1$

P_∞ – the supporting point $\quad [P_\infty] = \pm\infty$

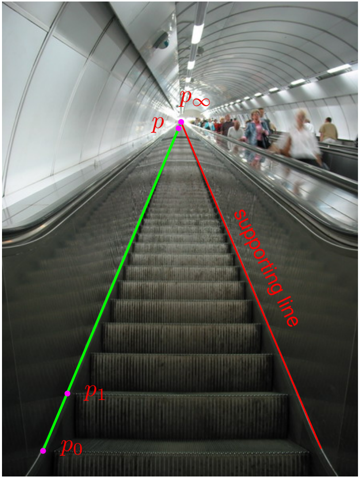
$[P]$ is equal to Euclidean coordinate along N

$[p]$ is its measurement in the image plane

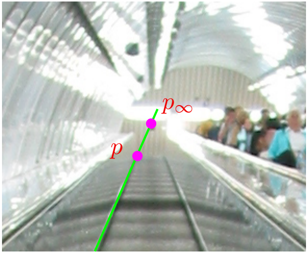
Applications

- Given the image of a line N, the origin, the unit point, and the vanishing point, then the Euclidean coordinate of any point $P \in N$ can be determined $\rightarrow$ see Slide 45
- Finding v.p. of a line through a regular object $\rightarrow$ see Slide 46

Application: Counting Steps



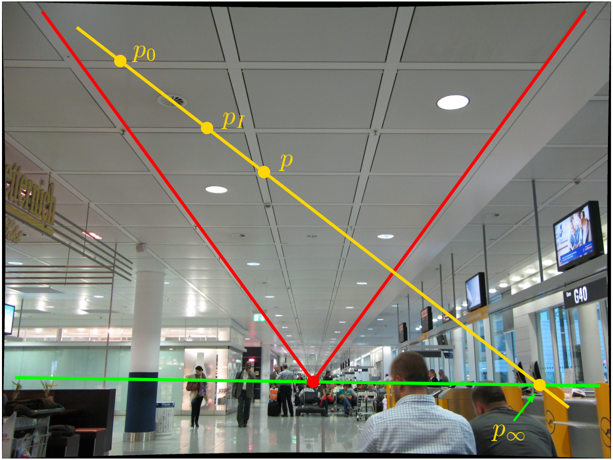
- Namesti Miru underground station in Prague

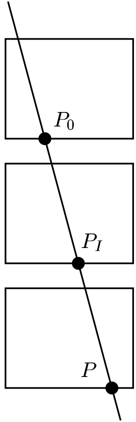


detail around the vanishing point

Result: $[P] = 214$ steps (correct answer is 216 steps) 4Mpx camera

Application: Finding the Horizon from Repetitions





in 3D: $|P_0 P| = 2|P_0 P_I|$ then [H&Z, p. 218] ⊛ P1; 1pt: How high is the camera above the floor?

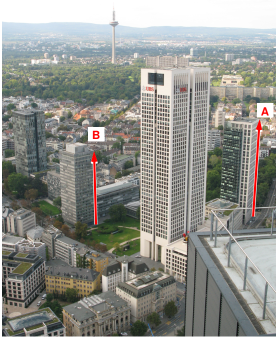
$$[P_\infty P_0 P_I P] = \frac{|P_0 P|}{|P_0 P_I|} = 2 \quad \Rightarrow \quad |p_\infty p_0| = \frac{|p_0 p_I| \cdot |p_0 p|}{|p_0 p| - 2|p_0 p_I|}$$

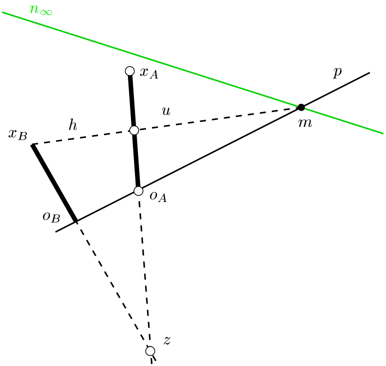
- could be applied to counting steps (Slide 45)

Homework Problem

⊛ H2; 3pt: What is the ratio of heights of Building A to Building B?

- expected: conceptual solution
- deadline: +2 weeks

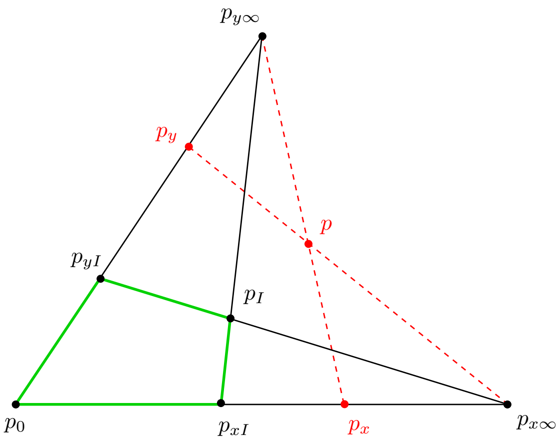




Hints

1. what are the properties of line h connecting the top of Buiding B with the point m at which the horizon is intersected with the line p joining the foots of both buildings? [1 point]
2. how do we actually get the horizon n_∞? [1 point] (we do not see it directly, there are hills there)
3. what tool measures the length? [formula = 1 point]

2D Projective Coordinates



$$[P_x] = [P_{x\infty}\ P_0\ P_{xI}\ P_x]$$
$$[P_y] = [P_{y\infty}\ P_0\ P_{yI}\ P_y]$$

Application: Measuring on the Floor (Wall, etc)

San Giovanni in Laterano, Rome

- measuring distances on the floor in terms of tile units
- what are the dimensions of the seal? Is it circular (assuming square tiles)?
- needs no explicit camera calibration

because we see the calibrating object (vanishing points)

▶Real Camera with Radial Distortion

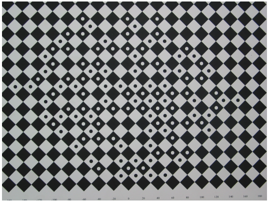
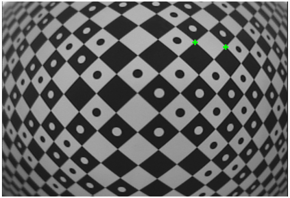


image with no radial distortion



an extreme case of radial distortion

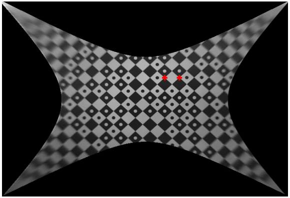
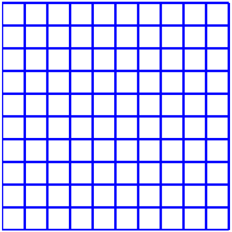
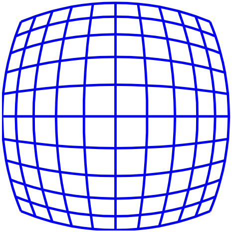


image undistorted by division model

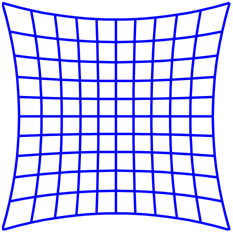
distortion types



none ($\lambda = 0$)

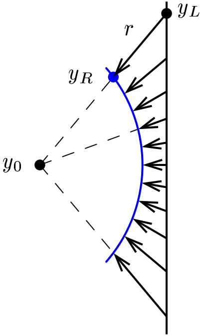


barrel ($\lambda = 0.3$)



pincushion ($\lambda = -0.3$)

▸The Radial Distortion Mapping

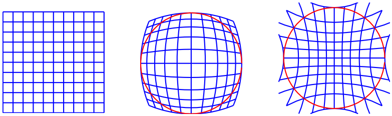


y_0 – center of radial distortion (usually principal point)

y_L – linearly projected point

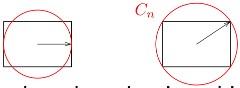
y_R – radially distorted point

- radial distortion r maps y_L to y_R along the radial direction
- magnitude of the transfer depends on the radius $\|y_L - y_0\|$ only



- circles centered at y_0 map to centered circles, lines incident on y_0 map on themselves
- the mapping $r()$ can be scaled to $a\,r()$ so that a particular circle C_n does not scale

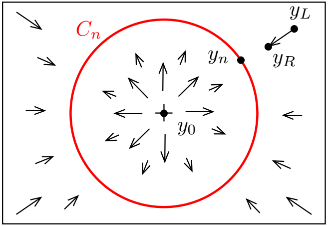
distortion	inside C_n	outside C_n
barrel	expanding	contracting
pincushion	contracting	expanding



in barrel in pincushion

- choose boundary point that preserves all image content within the same image size

▶Radial Distortion Models



- let $\mathbf{z} = \mathbf{y} - \mathbf{y}_0$ — non-homogeneous
- we have $\mathbf{z}_R = r(\mathbf{z}_L)$ — $\mathbf{z}_L$ – linear, $\mathbf{z}_R$ – distorted
- but are often interested in $\mathbf{z}_L = r^{-1}(\mathbf{z}_R)$

- $\mathbf{y}_n$ – a no-distortion point on C_n: $r(\mathbf{y}_n) = \mathbf{y}_n$
- $\mathbf{z}_n = \mathbf{y}_n - \mathbf{y}_0$

Division Model

single parameter $-1 \leq \lambda < 1$, has an analytic inverse, models even some fish-eye lenses

$$\mathbf{z}_R = \frac{\hat{\mathbf{z}}}{1 + \sqrt{1 + \lambda \frac{\|\hat{\mathbf{z}}\|^2}{\|\mathbf{z}_n\|^2}}}, \quad \text{where } \hat{\mathbf{z}} = \frac{2\,\mathbf{z}_L}{1-\lambda} \quad \text{and} \quad \boxed{\mathbf{z}_L = \frac{1-\lambda}{1 - \lambda \frac{\|\mathbf{z}_R\|^2}{\|\mathbf{z}_n\|^2}}\, \mathbf{z}_R}$$

$\lambda > 0$ – barrel distortion, $\lambda < 0$ – pincushion distortion

Polynomial Model

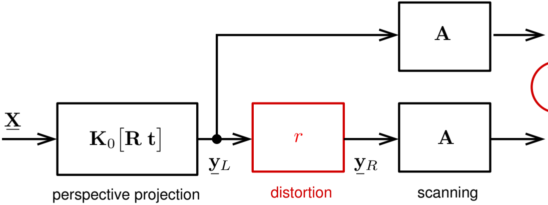
better fit for $n \geq 3$, no analytic inverse, may loose monotonicity, hard to calibrate

$$\boxed{\mathbf{z}_L = \frac{D(\mathbf{z}_R; \mathbf{z}_n, \mathbf{k})}{1 + \sum_{i=1}^{n} k_i}\, \mathbf{z}_R\,,} \qquad D(\mathbf{z}_R; \mathbf{z}_n, \mathbf{k}) = 1 + k_1\rho^2 + k_2\rho^4 + \cdots + k_n\rho^{2n}, \quad \rho = \frac{\|\mathbf{z}_R\|}{\|\mathbf{z}_n\|}, \quad \mathbf{k} = (k_i)$$

e.g. $k_i \geq 0$ – barrel distortion, $k_i \leq 0$ – pincusion distortion, $i = 1, \ldots, n$

Zernike polynomials R_i^0 are a better choice: $R_2^0(\rho) = 2\rho^2 - 1$, $R_4^0(\rho) = 6\rho^4 - 6\rho^2 + 1$, $R_6^0(\rho) = \cdots$

▶Real and Linear Camera Models



$\underline{\mathbf{x}}_L \simeq \mathbf{A}\mathbf{K}_0 \begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \underline{\mathbf{X}}$ (linear camera)

undistortion: $\underline{\mathbf{x}}_L = \mathbf{A}\, r^{-1}(\mathbf{A}^{-1} \underline{\mathbf{x}}_R)$

$\underline{\mathbf{x}}_R \simeq \mathbf{A}\, r\Big(\mathbf{K}_0 \begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix} \underline{\mathbf{X}}\Big)$ (real camera)

$$\mathbf{K}_0 = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ 'ideal' calibration matrix $$\mathbf{A}\mathbf{K}_0 = \begin{bmatrix} f & s\,f & u_0 \\ 0 & a\,f & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & s & u_0 \\ 0 & a & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$ everything affecting radial distortion — center, skew, aspect ratio

r radial distortion function (here, it includes conversion from/to homogeneous representation!)

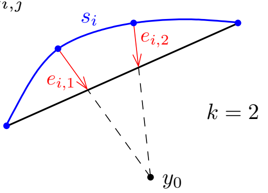
Notes

- assumption: the principal point and the center of radial distortion coincide
- f included in $\mathbf{K}_0$ to make radial distortion independent of focal length
- $\mathbf{A}$ makes radial lens distortion an elliptic image distortion
- it suffices in practice that r^{-1} is an analytic function (r need not be)

Calibrating Radial Distortion

- radial distortion calibration includes at least 5 parameters: $\lambda,\ u_0,\ v_0,\ s,\ a$

1. detect a set of straight line segment images $\{s_i\}_{i=1}^n$ from a calibration target
2. select a suitable set of k measurement points per segment — how to select k?
3. define invariant radial transfer error per measurement point $e_{i,j}$ and per segment $e_i^2 = \sum_{j=1}^{k-2} e_{i,j}^2$ — invariant to rotation, translation



4. minimize total radial transfer error: $$\arg \min_{\lambda,\, u_0,\, v_0,\, s,\, a} \sum_{i=1}^{n} e_i^2$$

- line segments from real-world images requires segmentation to inliers/outliers

inliers = lines that are straight in reality

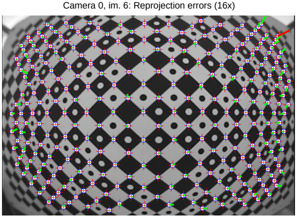
- marginalisation over the hidden label gives a 'robust' error, e.g.

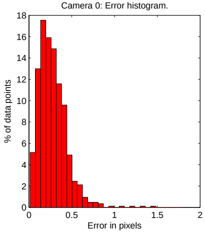
$$\varepsilon_i^2 = -\log\left(e^{-\frac{e_i^2}{2\sigma^2}} + t\right), \qquad t > 0$$

- direct optimization usually suffices but in general such optimization is unstable

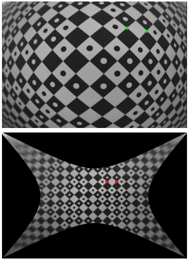
Example Calibrations

Low-resolution (VGA) wide field-of-view (130°) camera

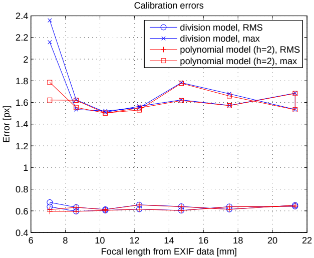


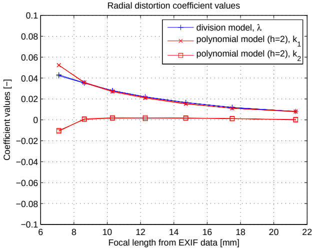


Cam	0
RMS [px]	0.33
max [px]	1.97
f [px]	94.59
a [-]	1.0951
u_0 [px]	243.26
v_0 [px]	353.37
(poly) k_1	0.8256
k_2	-0.2261
k_3	1.2524



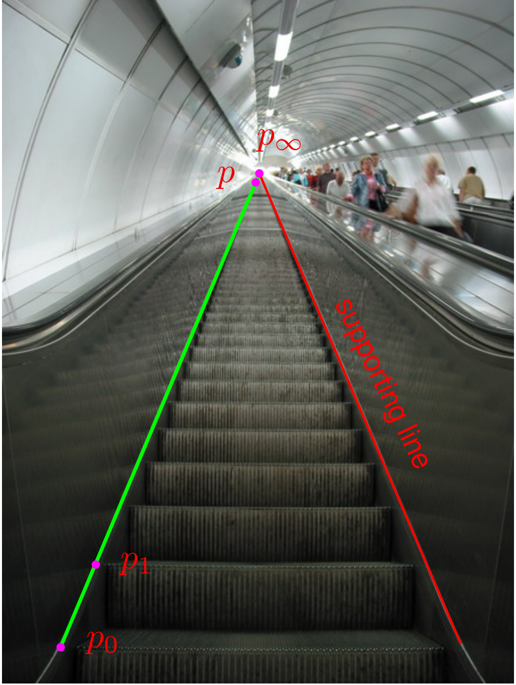
4 Mpix consumer camera

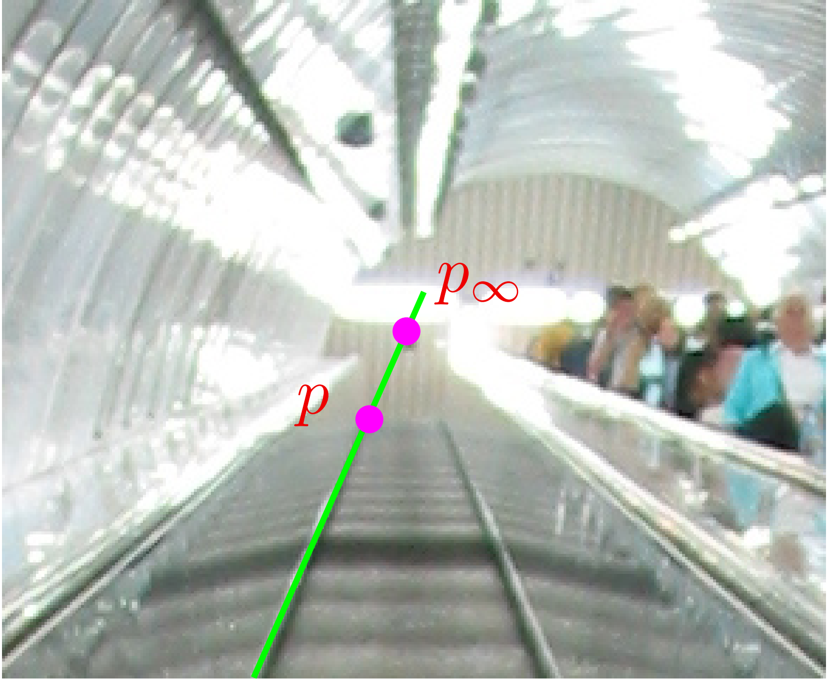


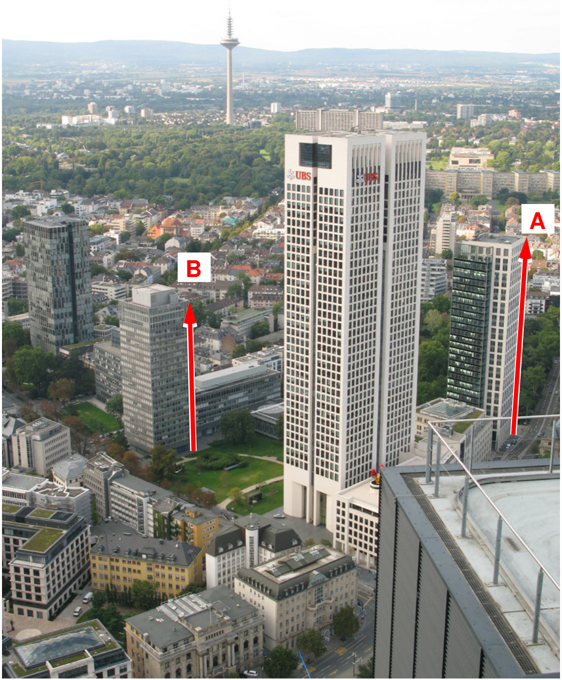


- radial distortion is slightly dependend on focal length

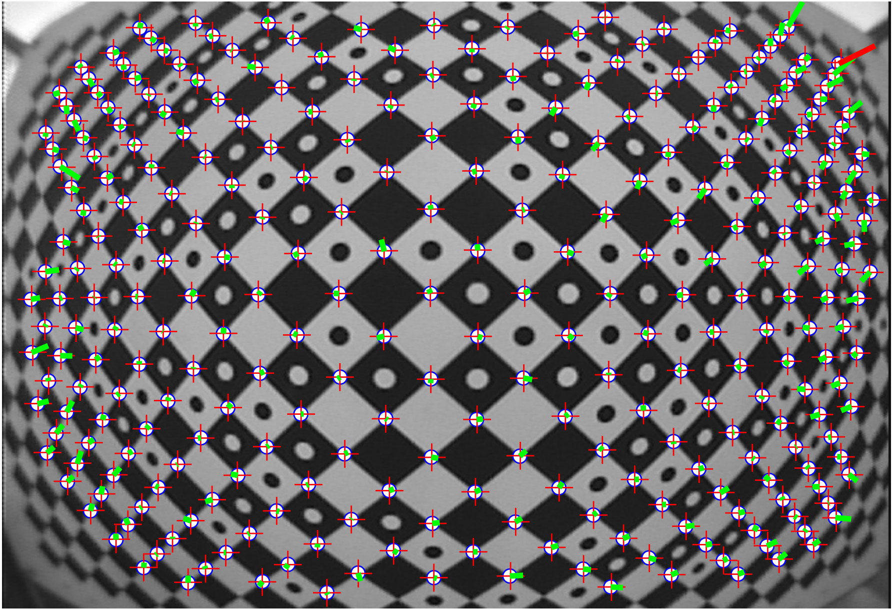
Thank You

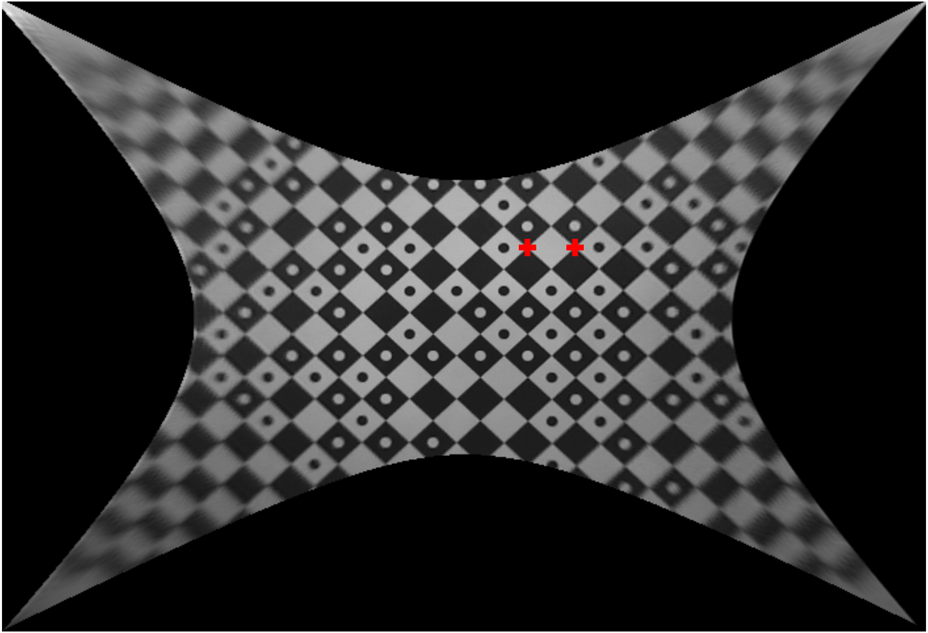

p_∞
p
supporting line
p_1
p_0

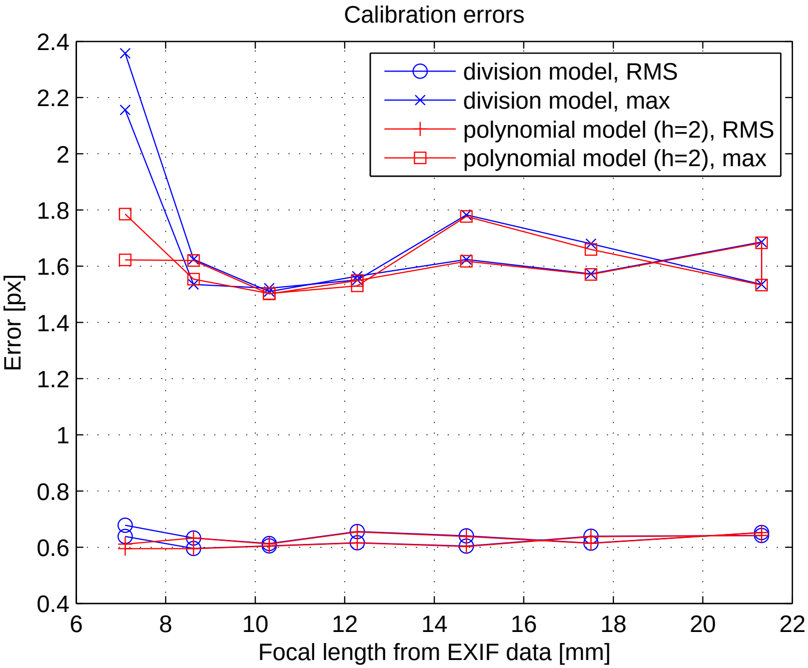

p_∞
p

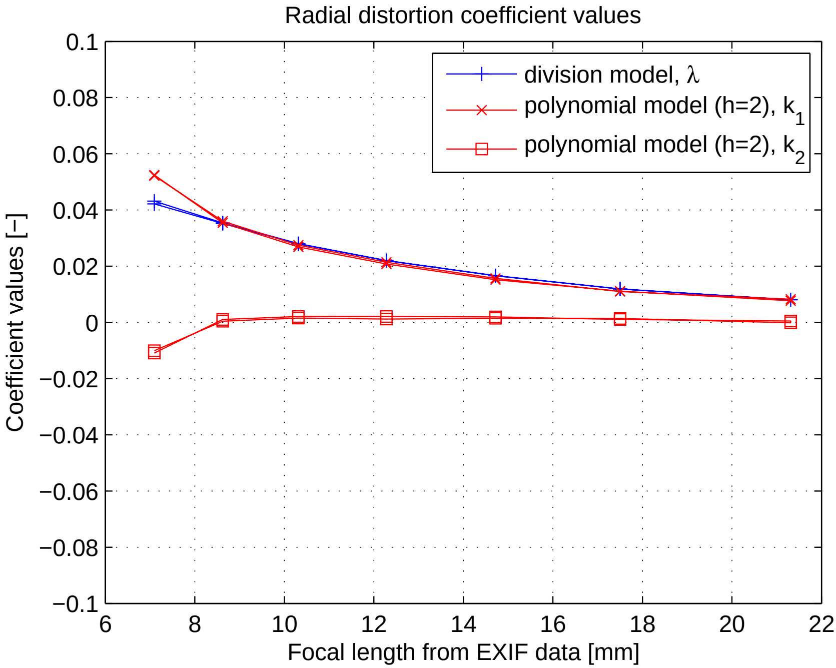

UBS
UBS
A
B

Camera 0, im. 6: Reprojection errors (16x)






Calibration errors
division model, RMS
division model, max
polynomial model (h=2), RMS
polynomial model (h=2), max
Error [px]
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
6
8
10
12
14
16
18
20
22
Focal length from EXIF data [mm]



Radial distortion coefficient values
division model, λ
polynomial model (h=2), k_1
polynomial model (h=2), k_2
Coefficient values [−]
Focal length from EXIF data [mm]
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
−0.08
−0.1
6
8
10
12
14
16
18
20
22