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# Graphical probabilistic models – learning from data

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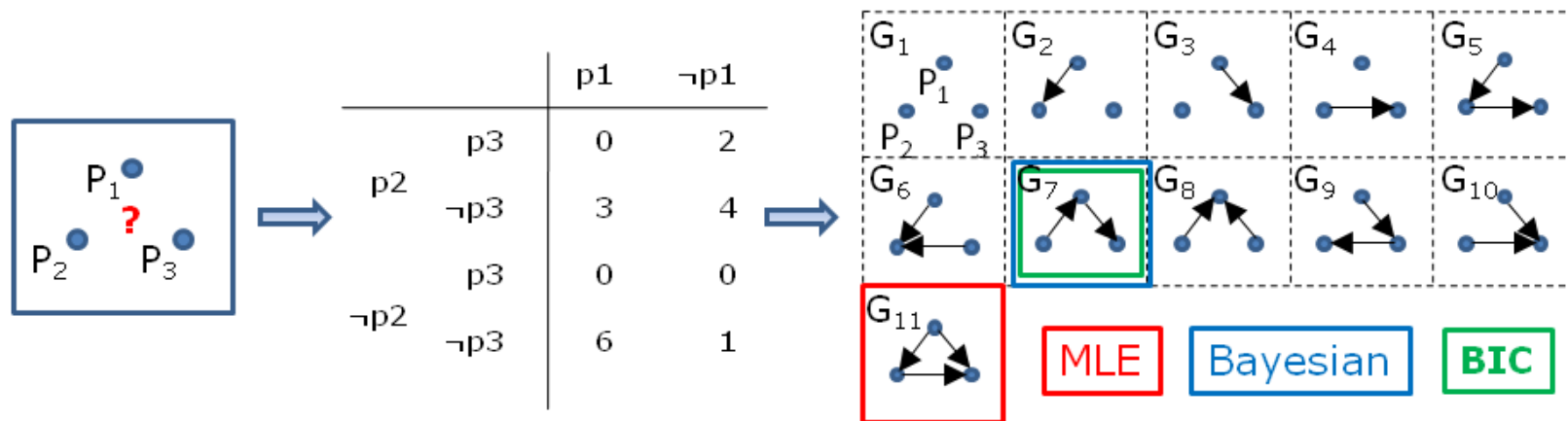






## Structure learning – 3DAG example

- let us concern a 3 node trial network and generate 16 samples of it,
- score a member of each Markov equivalence class (complete search, 11 graphs),
- apply 3 distinct criteria (max likelihood, Bayesian MAP and BIC) to identify the best model.



- $G_1$  gradually evaluated by three criteria:

– likelihood: ML parameters first  $Pr(p_1) = Pr(p_2) = \frac{9}{16}$ ,  $Pr(p_3) = \frac{1}{8}$

$$\begin{aligned} \ln L(G_1 : D) &= \sum_{m=1}^{16} Pr(d_m : G_1) = \\ &= 2 \ln\left(\frac{7}{16} \frac{9}{16} \frac{1}{8}\right) + 3 \ln\left(\frac{9}{16} \frac{9}{16} \frac{7}{8}\right) + 10 \ln\left(\frac{9}{16} \frac{7}{16} \frac{7}{8}\right) + \ln\left(\frac{7}{16} \frac{7}{16} \frac{7}{8}\right) = -27.96 \end{aligned}$$













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