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# Min-Hashing and Geometric min-Hashing

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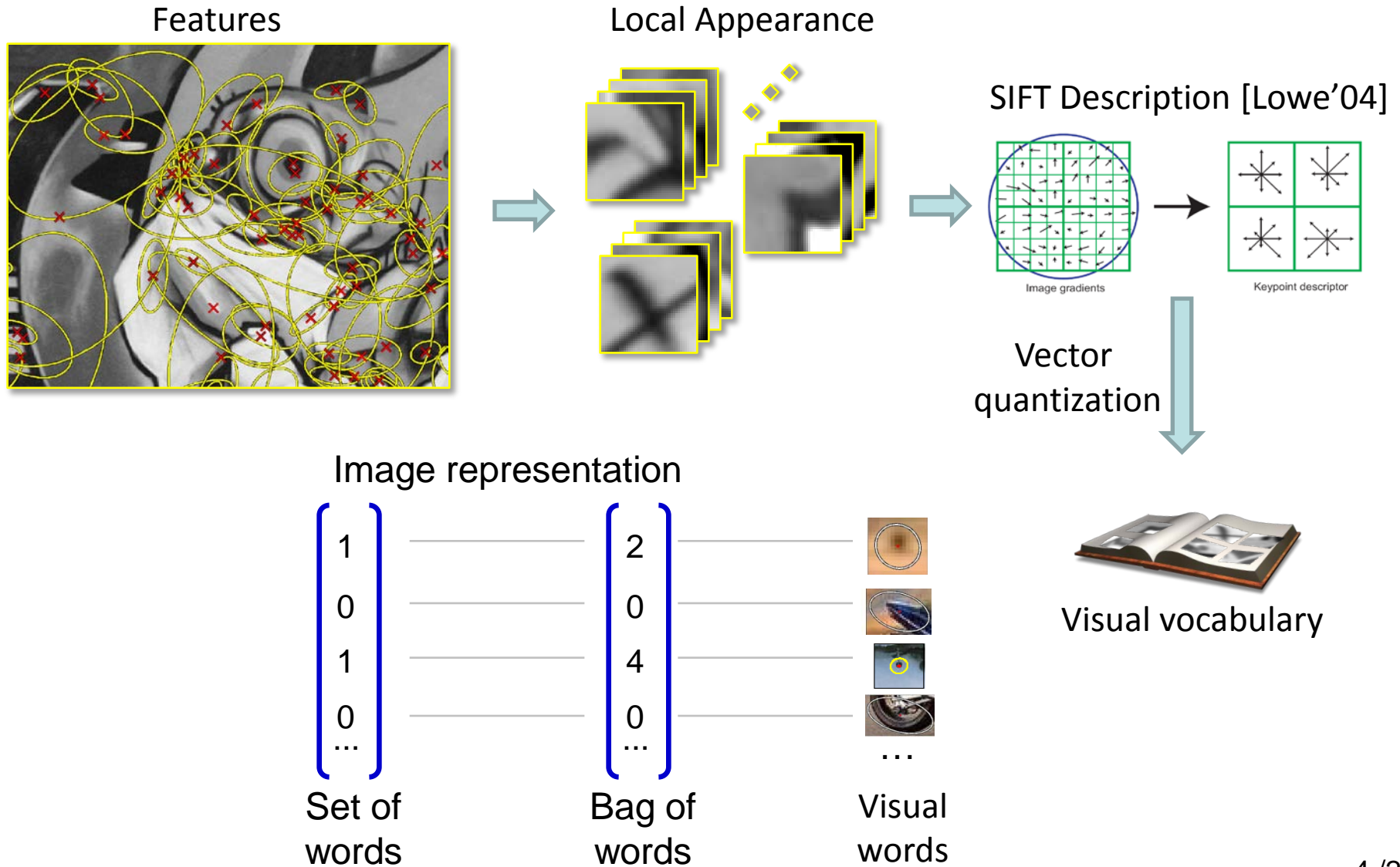
# Outline

1. Looking for representation of images that:
  - is compact (useful for very large datasets)
  - is fast (linear in the size of the output)
  - supports local (image and object) recognition/retrieval
  - is accurate (very low false positive rate)
2. min-Hash is a powerful representation, but we show that Geometric min-Hash is significantly more powerful
3. Applications:
  - Large database clustering
  - Discovery of (even small) objects

# Introduction to min-Hash for Images

min-Hash originates from the text retrieval community, originally used for detection of near duplicate documents

# Image Representation: a Set of Words



# min-Hash

min-Hash is a locality sensitive hashing (LSH) function  $m$  that selects an element (visual word)  $m(\mathcal{I}_i)$  from each set  $\mathcal{I}_i$  of visual words detected in image  $i$  so that

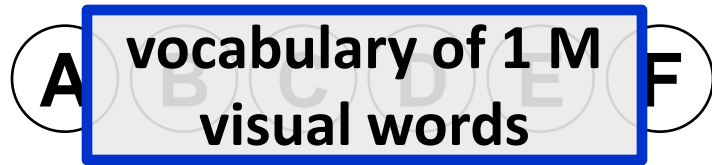
$$P\{m(\mathcal{I}_1) == m(\mathcal{I}_2)\} = \frac{|\mathcal{I}_1 \cap \mathcal{I}_2|}{|\mathcal{I}_1 \cup \mathcal{I}_2|}$$

This probability will be called “image similarity” and denoted

$$\text{sim}(\mathcal{I}_1, \mathcal{I}_2) = \frac{|\mathcal{I}_1 \cap \mathcal{I}_2|}{|\mathcal{I}_1 \cup \mathcal{I}_2|}$$

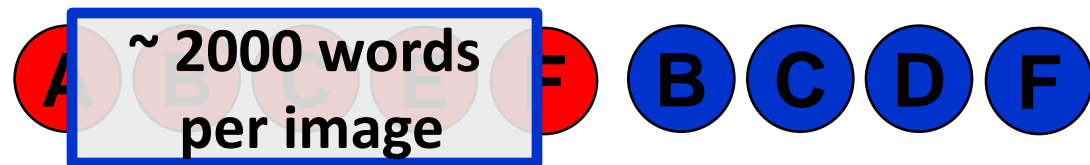
# min-Hash

Vocabulary



Set  $\mathcal{I}_1$

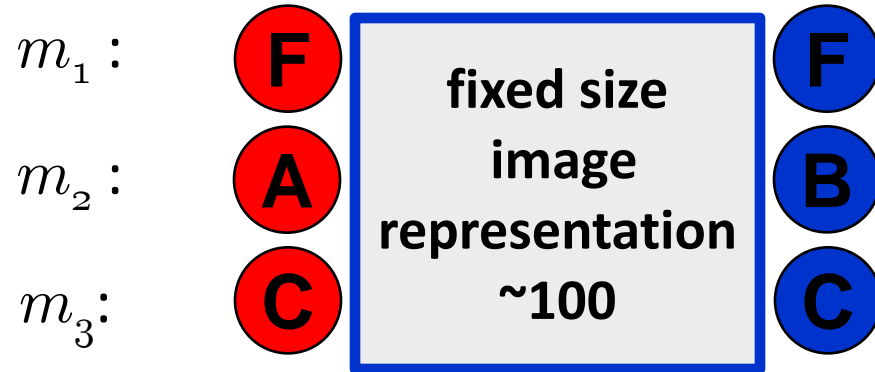
Set  $\mathcal{I}_2$



Random orderings

3	6	2	5	4	<u>1</u>
<u>1</u>	<u>2</u>	6	3	5	4
3	2	<u>1</u>	6	4	5

min-Hash

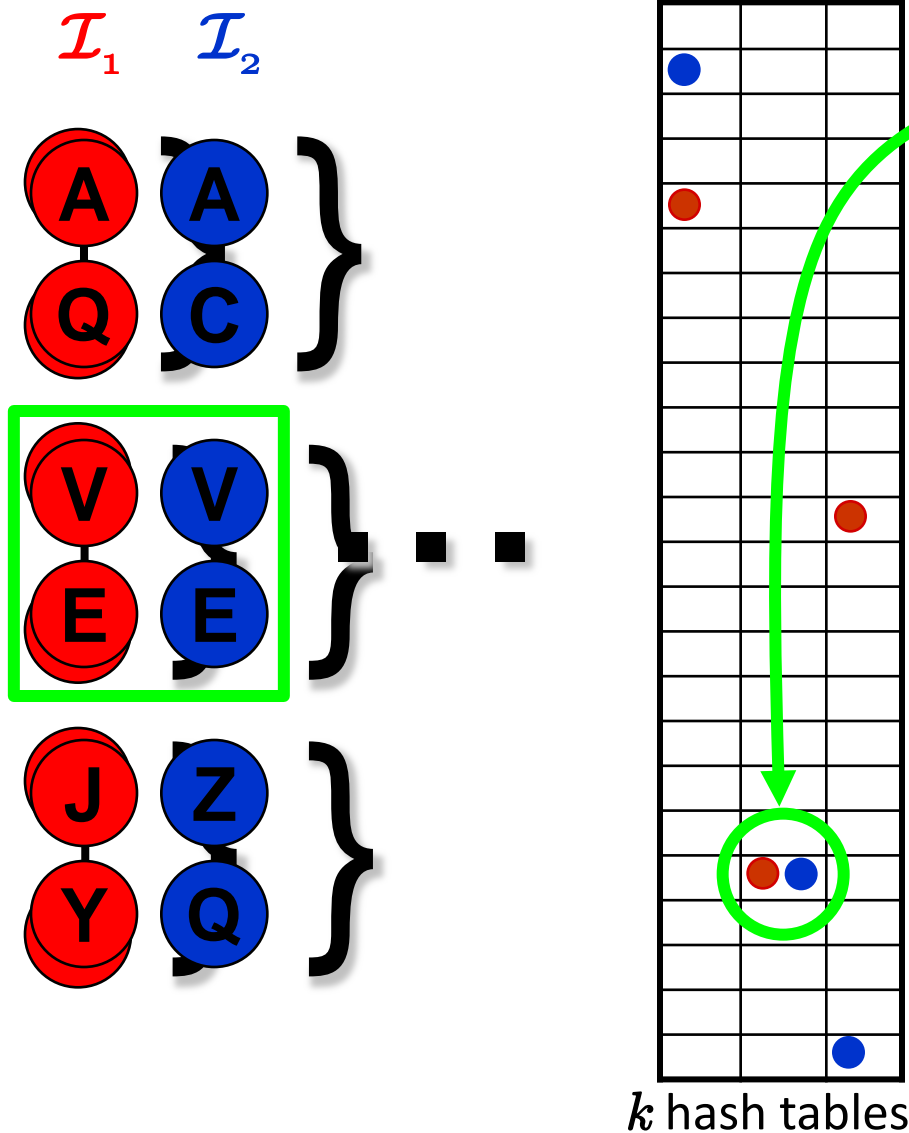


$$\text{sim}(\mathcal{I}_1, \mathcal{I}_2) = 1/2$$

Estimated similarity of  $\mathcal{I}_1$  and  $\mathcal{I}_2$  from 3 min-Hashes = 2/3

# min-Hash Retrieval

a sketch =  $s$ -tuple of min-Hashes



**Sketch collision**

**collision:**

all  $s$  min-Hashes must agree

$$P\{\text{collision}\} = \text{sim}(\mathcal{I}_1, \mathcal{I}_2)^s$$

**retrieval:**

1. generate  $k$  sketches
2. at least one of  $k$  sketches must collide

$$P\{\text{retrieval}\} = 1 - (1 - \text{sim}(\mathcal{I}_1, \mathcal{I}_2)^s)^k$$



# Probability of Retrieving an Image Pair

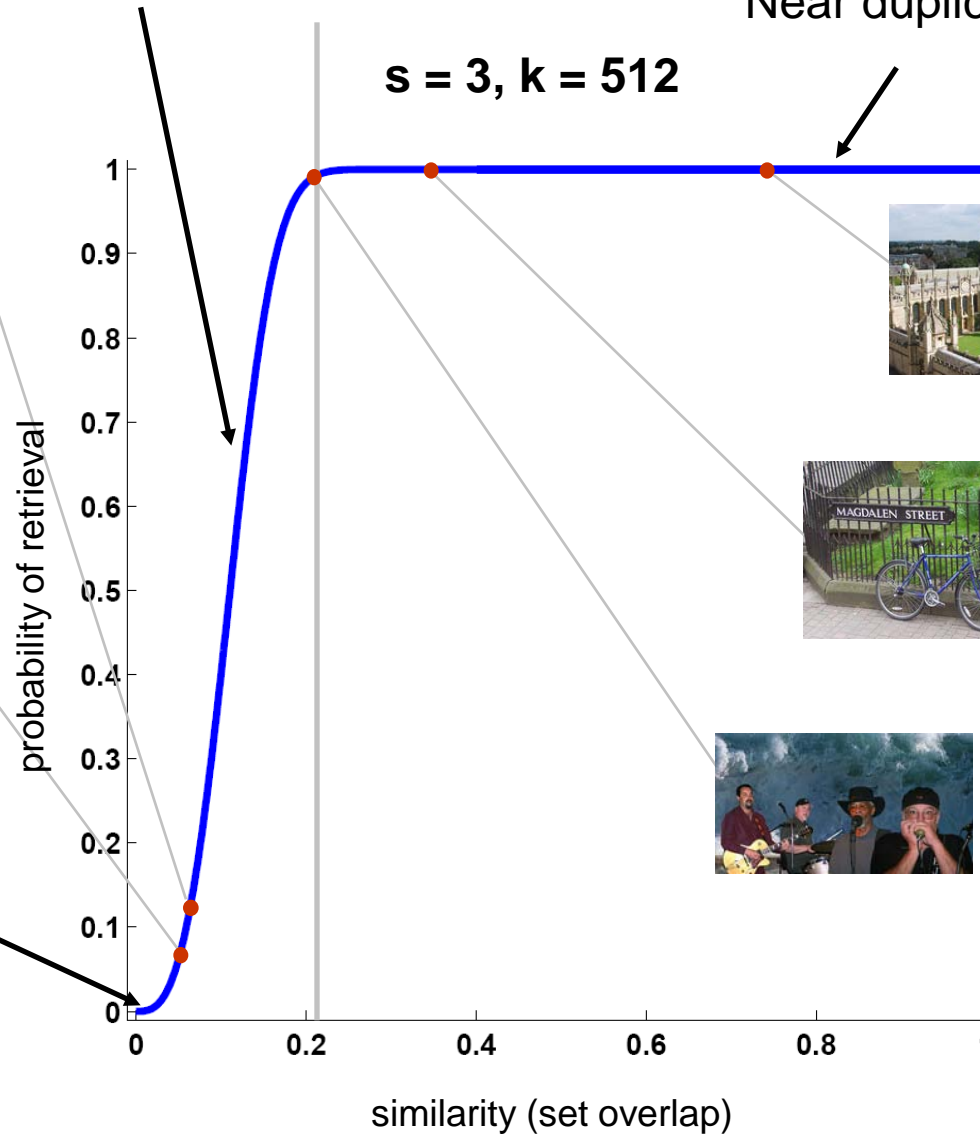
Images of the same object

Near duplicate images



Unrelated images

$s = 3, k = 512$



# Near Duplicate Images



# Near Duplicate Images



# Near Duplicate Images



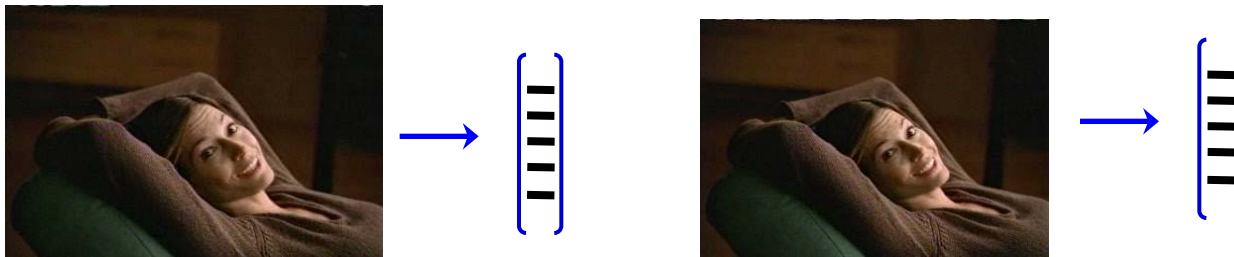
# Scalable Near Duplicate Image Detection

- Images perceptually (almost) identical but not identical (noise, compression level, small motion, small occlusion)
- Similar images of the same object / scene
- Large databases
- Fast – linear in the number of duplicates
- Store small constant amount of data per image



# Representation, Distance and Search

- Naïve method (space and time infeasible):
  - Representation: the whole image
  - Similarity: sophisticated visual similarity measure
  - Search:  $O(N^2)$  on  $N$  images
- Goal:
  - Compact representation – small constant number of bytes (that captures visual content addressed by near duplicate definition)
  - Similarity that allows for fast retrieval of similar images (that measures similarity according to near duplicate definition)
  - Search: linear in the number of near duplicates – needs to be indexing



# Word Weighting for min-Hash

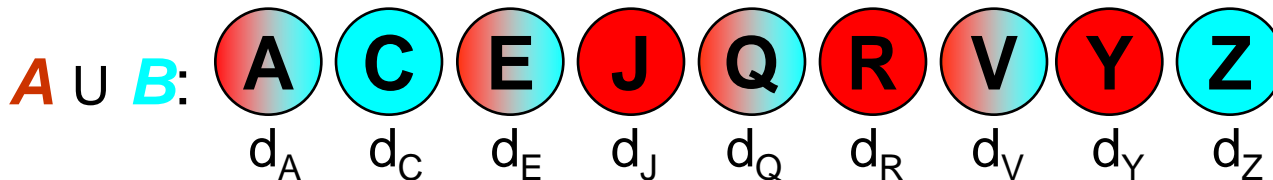
For hash function (set overlap similarity)  $f_j(X_w) = x \quad x \sim \text{Un}(1, 0)$

all words  $X_w$  have the same chance to be a min-Hash

For hash function

$$f_j(X_w) = \frac{-\log x}{d_w} \quad x \sim \text{Un}(1, 0)$$

the probability of  $X_w$  being a min-Hash is proportional to  $d_w$



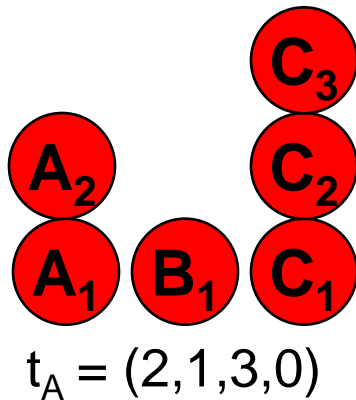
$$P(m(\mathcal{A}) = m(\mathcal{B})) = \frac{\sum_{X_w \in \mathcal{A} \cap \mathcal{B}} d_w}{\sum_{X_w \in \mathcal{A} \cup \mathcal{B}} d_w}$$

# Histogram Intersection Using min-Hash

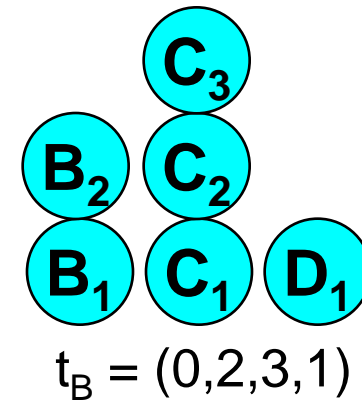
Idea: represent a histogram as a set, use min-Hash set machinery

Visual words: A B C D

Bag of words **A** / set **A'**



Bag of words **B** / set **B'**



min-Hash vocabulary: A<sub>1</sub> A<sub>2</sub> B<sub>1</sub> B<sub>2</sub> C<sub>1</sub> C<sub>2</sub> C<sub>3</sub> D<sub>1</sub>

**A' U B'**: A<sub>1</sub> A<sub>2</sub> B<sub>1</sub> B<sub>2</sub> C<sub>1</sub> C<sub>2</sub> C<sub>3</sub> D<sub>1</sub>

Set overlap of **A'** of **B'** is a histogram intersection of **A** and **B**



# Similarity Measures for min-Hash

Set representation

Bag of words representation

Equal weights

$$\text{sim}_s(\mathcal{A}_1, \mathcal{A}_2) = \frac{|\mathcal{A}_1 \cap \mathcal{A}_2|}{|\mathcal{A}_1 \cup \mathcal{A}_2|}$$

$$\text{sim}_{h_0}(\mathcal{A}_1, \mathcal{A}_2) = \frac{\sum_w \min(t_1^w, t_2^w)}{\sum_w \max(t_1^w, t_2^w)}$$

Weighted

$$\text{sim}_w(\mathcal{A}_1, \mathcal{A}_2) = \frac{\sum_{X_w \in \mathcal{A}_1 \cap \mathcal{A}_2} d_w}{\sum_{X_w \in \mathcal{A}_1 \cup \mathcal{A}_2} d_w}$$

$$\text{sim}_h(\mathcal{A}_1, \mathcal{A}_2) = \frac{\sum_w d_w \min(t_1^w, t_2^w)}{\sum_w d_w \max(t_1^w, t_2^w)}$$

# Min-Hash on TrecVid

- DoG features
- vocabulary of 64,635 visual words
- 192 min-Hashes, 3 min-Hashes per a sketch, 64 sketches
- similarity threshold 35%

Set overlap similarity measure



Query

Retrieved images with decreasing similarity

# Query Examples

Query image:



Results

Set overlap, weighted set overlap, weighted histogram intersection

# Geometric min-Hash (GmH)

*Can geometry help us in finding sketches of min-Hashes with much higher repeatability than random  $s$ -tuples?*

# The Idea

- For a sketch collision all  $s$  min-Hashes in the sketch must agree
- In the construction of a sketch, we can assume that the first min-Hash is matching
- If the assumption is violated, no harm is done, the sketch would not collide

We show how to exploit the assumption of matching  $m_1(\mathcal{I})$  to design the rest of the sketch (**not independent min-Hashes** anymore). The new procedure – Geometric min-Hash - is superior to the standard min-Hash.

# Vocabulary Size and Set Representation



Small vocabulary (1k visual words)

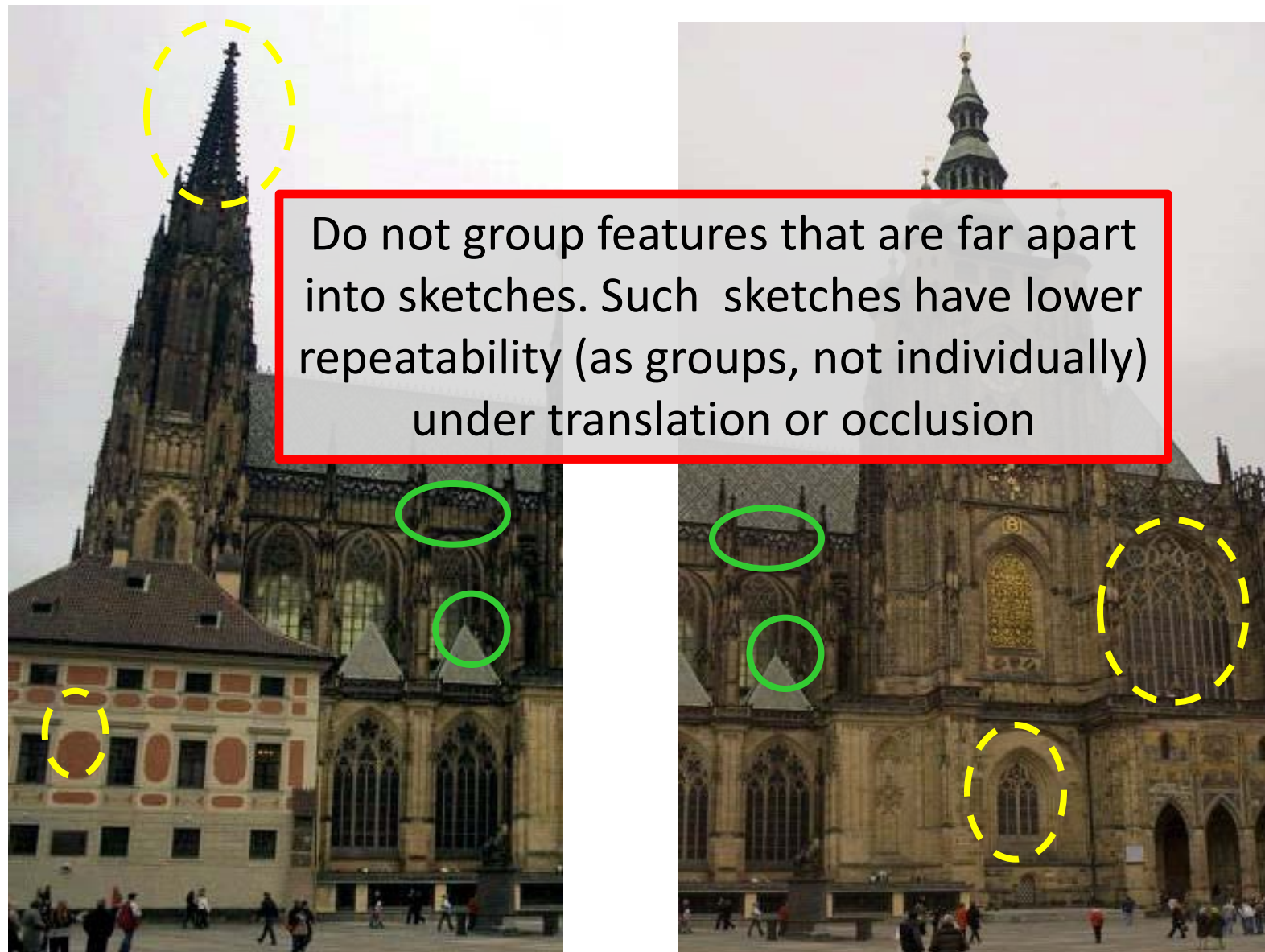


Large vocabulary (1M visual words)

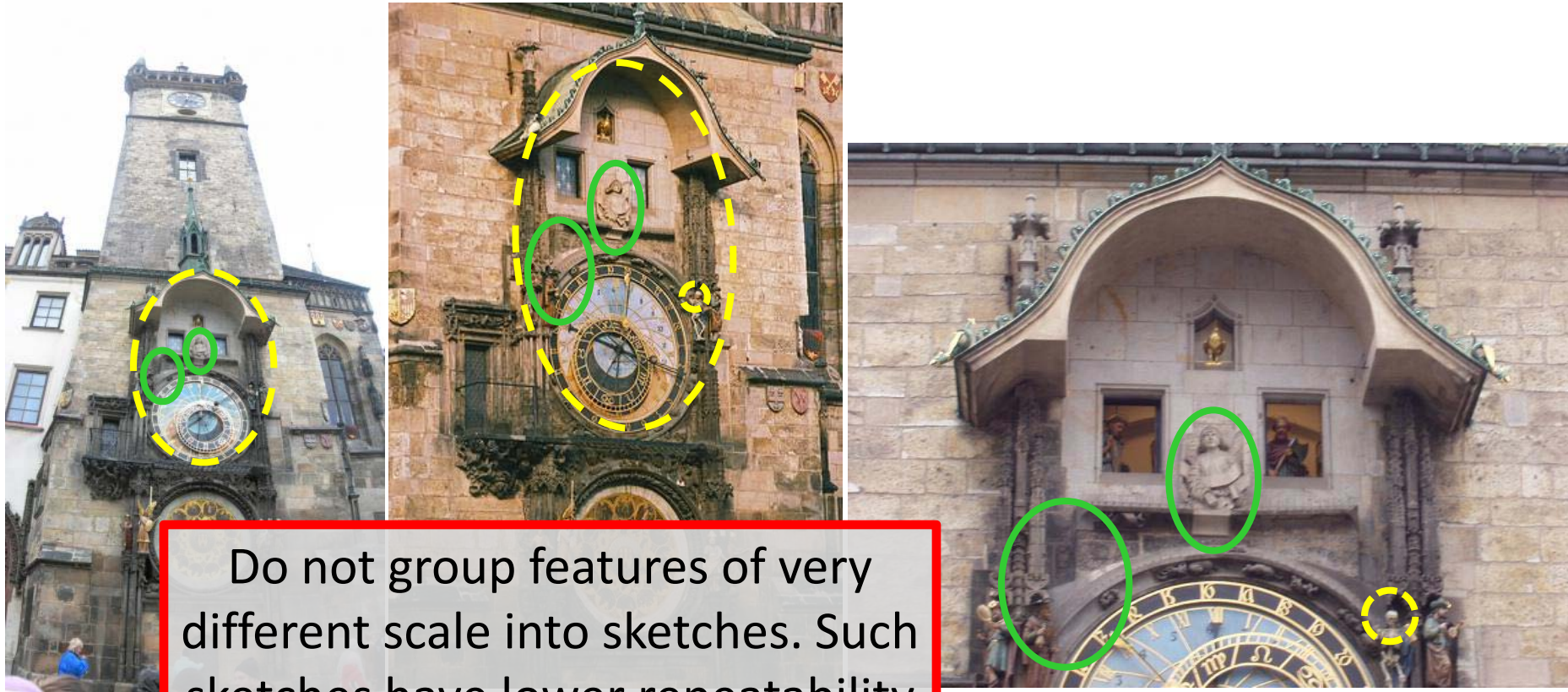
For large vocabularies selecting a visual word from an image is (almost) equivalent to selecting a feature (with location and scale)

On a 100k dataset of images, 95% of features have a unique visual word in an image

# Repeatability of Feature Sets: Translation and Occlusion



# Repeatability of Feature Sets: Scale Change



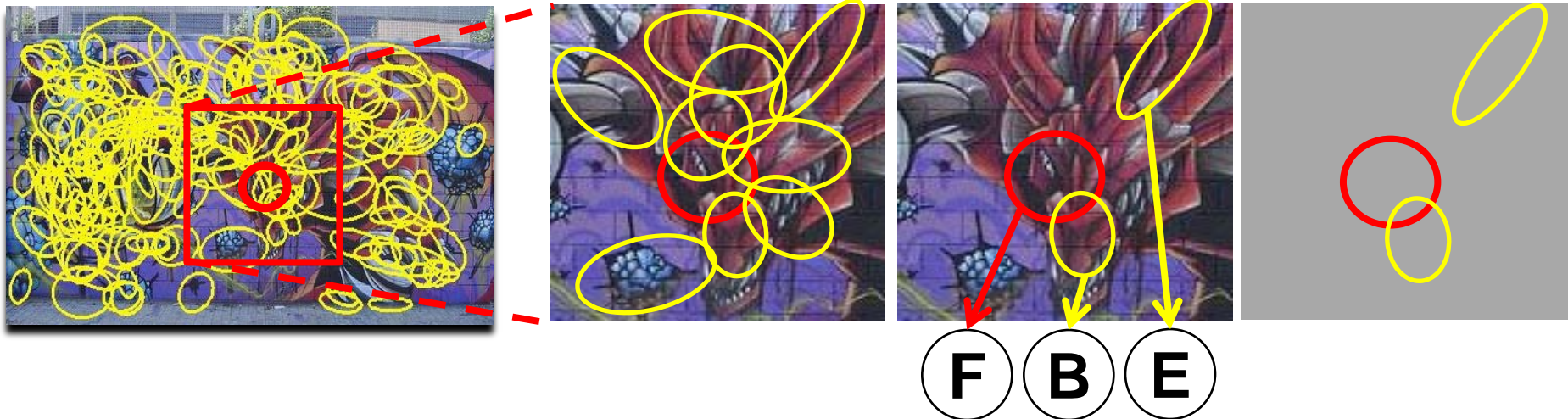
Do not group features of very different scale into sketches. Such sketches have lower repeatability under scale change.



# Geometric min-Hash algorithm

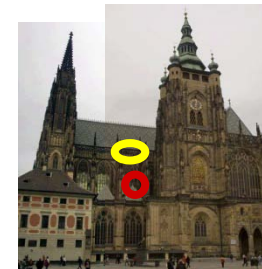
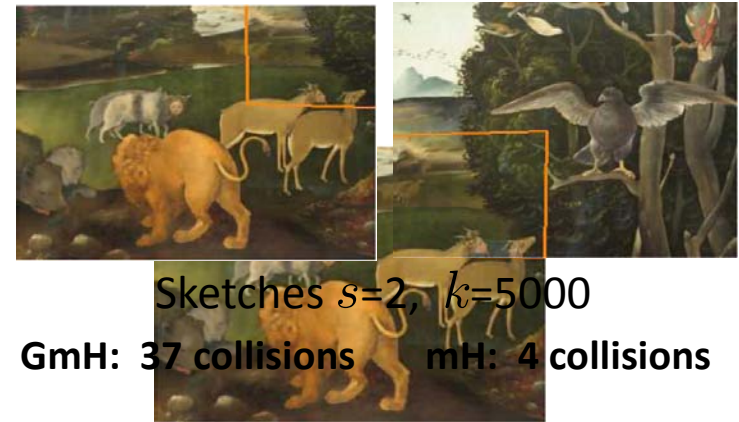
1. Keep features with unique visual word in the image
2. Obtain the “central feature” by min-Hash
3. Select scale and spatial neighbourhood of the central feature
4. Select secondary min-Hash(es) from the neighbourhood
5. Relative pose of the sketch features is a geometric invariant (as in geometric hashing)

Sketch of GmH: s-tuple of visual words + geometric invariant



# Geometric vs. Standard min-Hash

- Higher true positives
  - View point change
  - Severe occlusion
  - Scale change
  - Object on a different background
- Lower false positives
  - Additional geometric invariant (part of the hash key or verification)
  - Lower probability random sketch collisions (next slide)
- Faster spatial verification
  - Sketch collision defines geometric transformation



# Overlap of Random Sets

False positive = sketch collision of two random images

The probability of two random sets  $\mathcal{I}_1$  and  $\mathcal{I}_2$  having a common min-Hash (*i.e.* the average overlap of two random sets)

$$\frac{\min(|\mathcal{I}_1|, |\mathcal{I}_2|)}{2w} \leq \mathbb{E} \left( \frac{|\mathcal{I}_1 \cap \mathcal{I}_2|}{|\mathcal{I}_1 \cup \mathcal{I}_2|} \right) \leq \frac{\min(|\mathcal{I}_1|, |\mathcal{I}_2|)}{w}$$

where  $w$  is the size of the vocabulary

**The smaller the sets, the smaller probability of random collision**

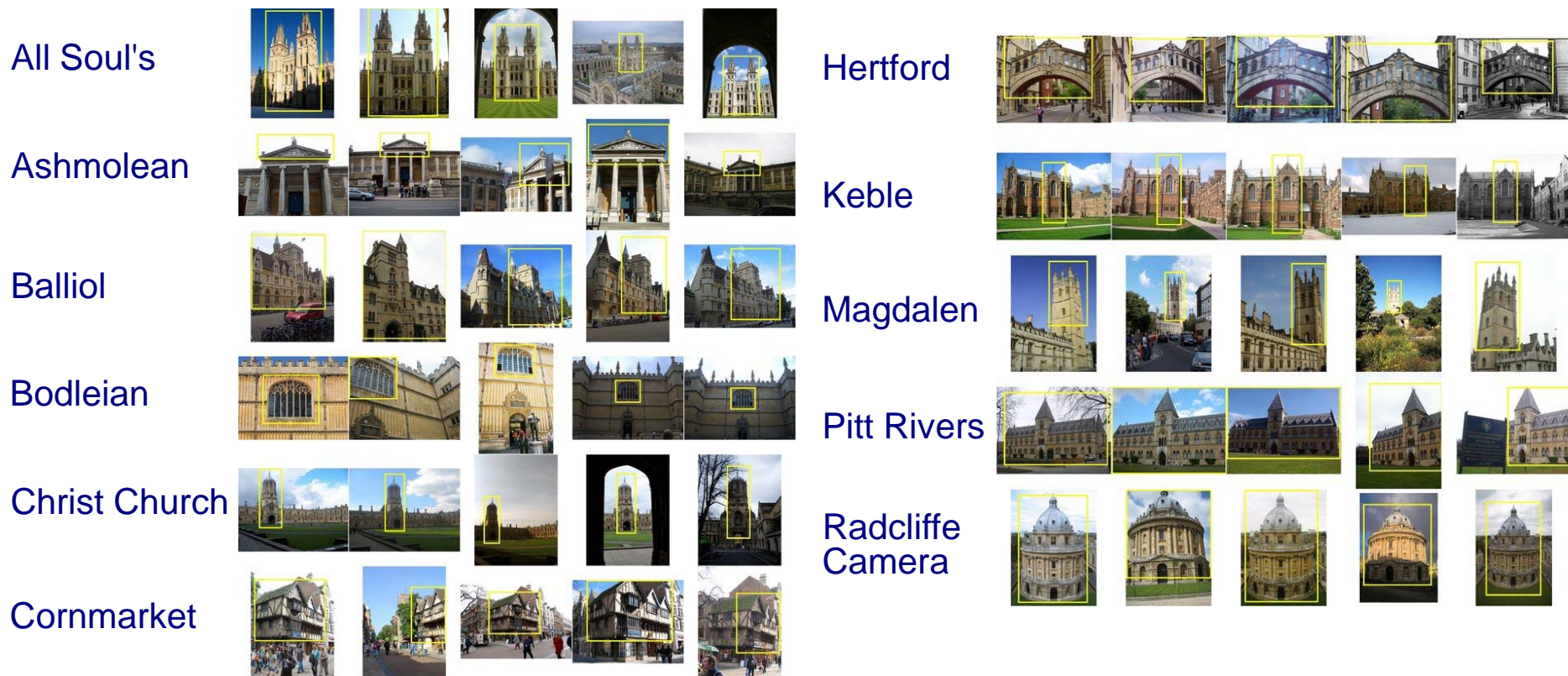
- Min-Hash: features in the whole image
- Geometric min-Hash: only small subset of the image

# Experiments

# Experiment 1: Clustering on Oxford 100k DB

## Randomized clustering

Cluster hypotheses by hashing, completion by retrieval



100 000 Images downloaded from FLICKR

Includes 11 Oxford Landmarks with manually labeled ground truth

# Clustering Results on Oxford 100k

	Geometric min-Hash		[Chum TR 2008]	
	Component Recall	fp	Component Recall	fp
all souls	<b>98.72</b>	0	97.44	0
ashmolean	<b>76.00</b>	0	68.00	0
balliol	<b>91.67</b>	0	33.33	0
bodleian	<b>100</b>	0	95.83	1
christ church	<b>97.44</b>	1	89.74	0
cornmarket	<b>77.78</b>	1	66.67	0
hertford	<b>100</b>	0	96.30	1
keble	<b>100</b>	0	85.71	0
magdalen	<b>38.89</b>	0	5.56	0
pitt rivers	<b>100</b>	0	100	0
radcliffe	<b>99.55</b>	0	98.64	0

**16 min\***

**$s = 2, k = 64$**

**~550 bytes per image**

**33 min\***

**$s = 3, k = 256$**

**~1600 bytes per image**



\* The time does not include feature detection, SIFT computation, vector quantization

# Experiment 2: Object Discovery



Geometric min-Hash  
 sketch collision  
 $s = 2, k = 256$

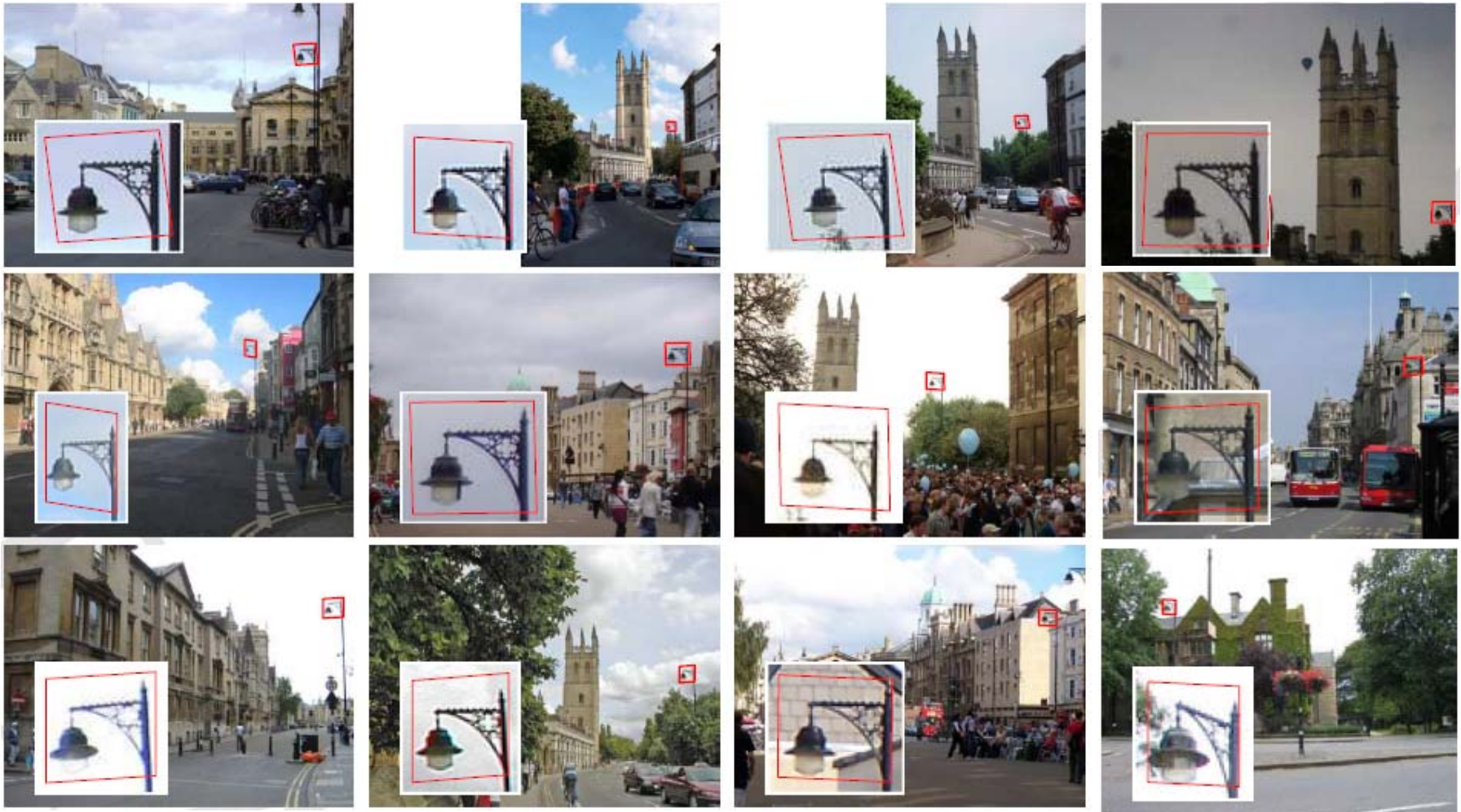
**Verification by co-segmentation  
 critical for small objects**



[Cech, Matas, Perdoch CVPR 08], code available on WWW  
 [Ferrari, Tuytelaars, Van Gool, ECCV 2004]

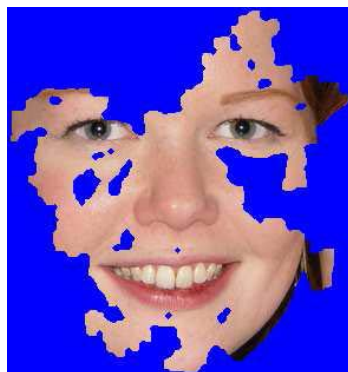
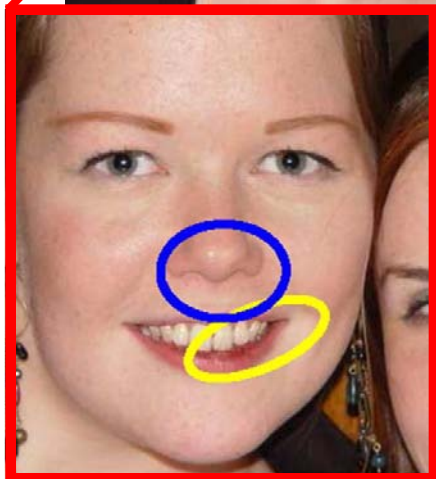
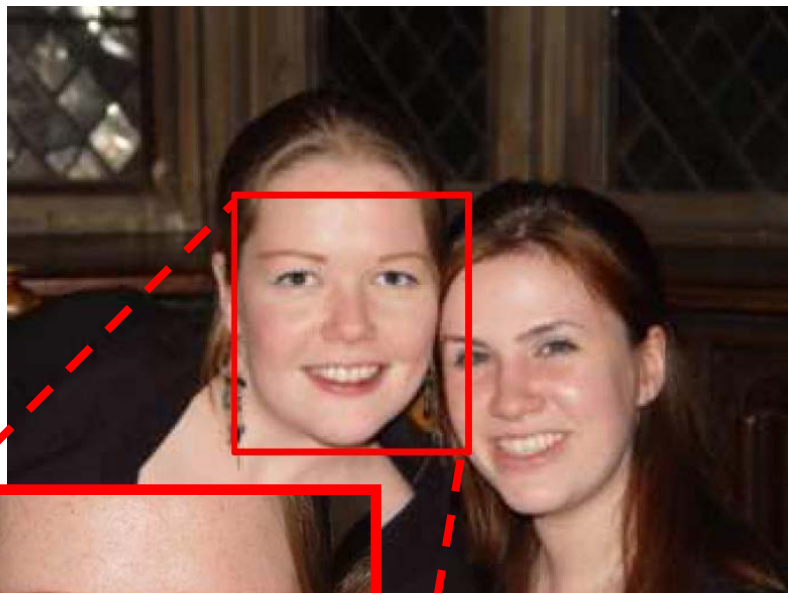
# Small Object Discovery

Other instances of the discovered object by (sub)image retrieval

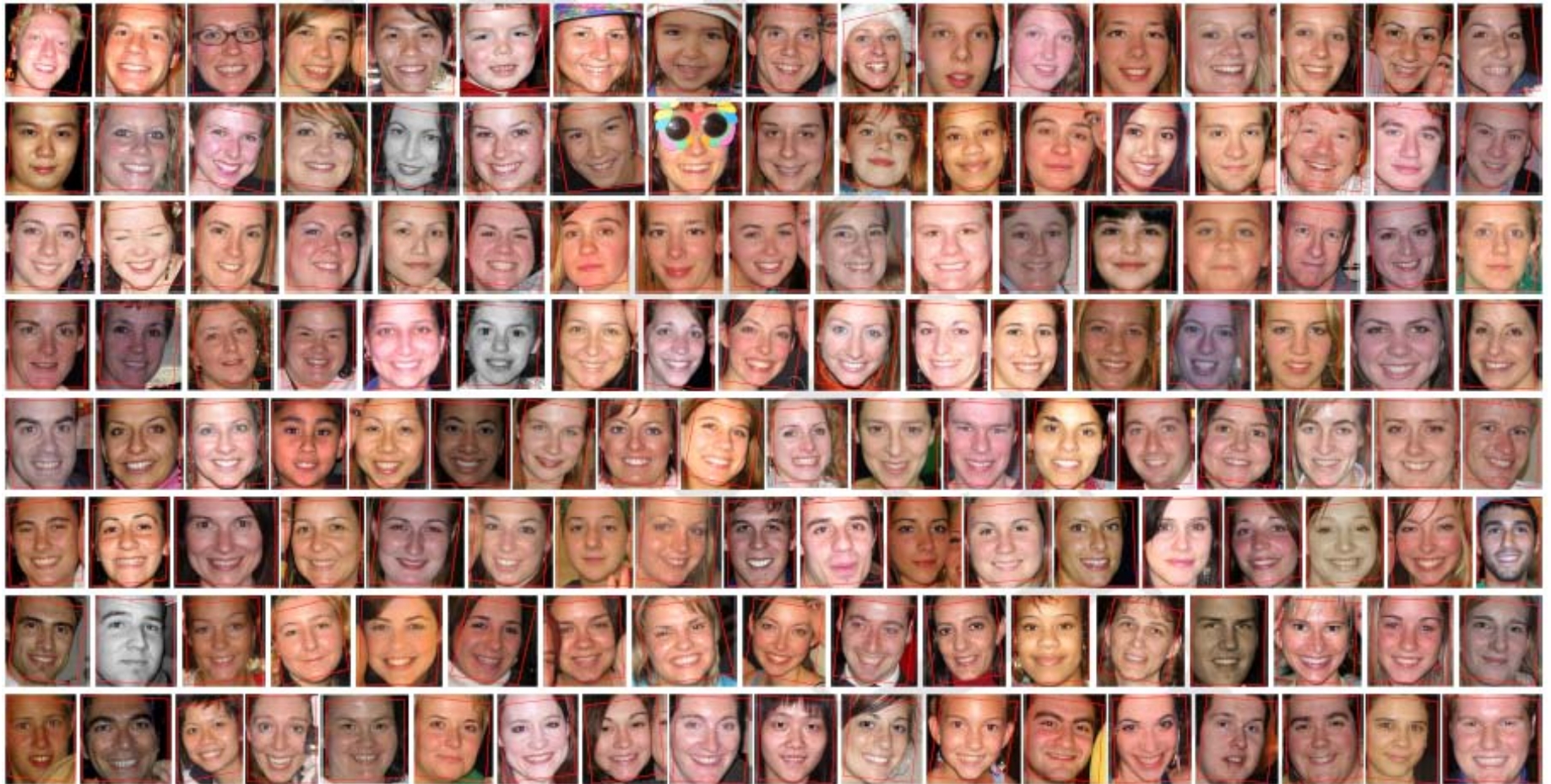




# Faces are Small Objects



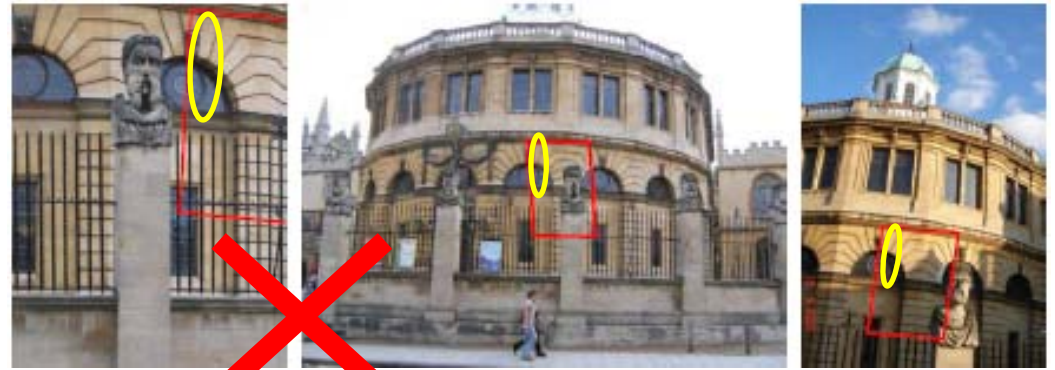
# Discovery of the Face Category: the Largest Cluster in Oxford 100k



# The Importance of the Co-segmentation in Object Discovery



Seed image pair

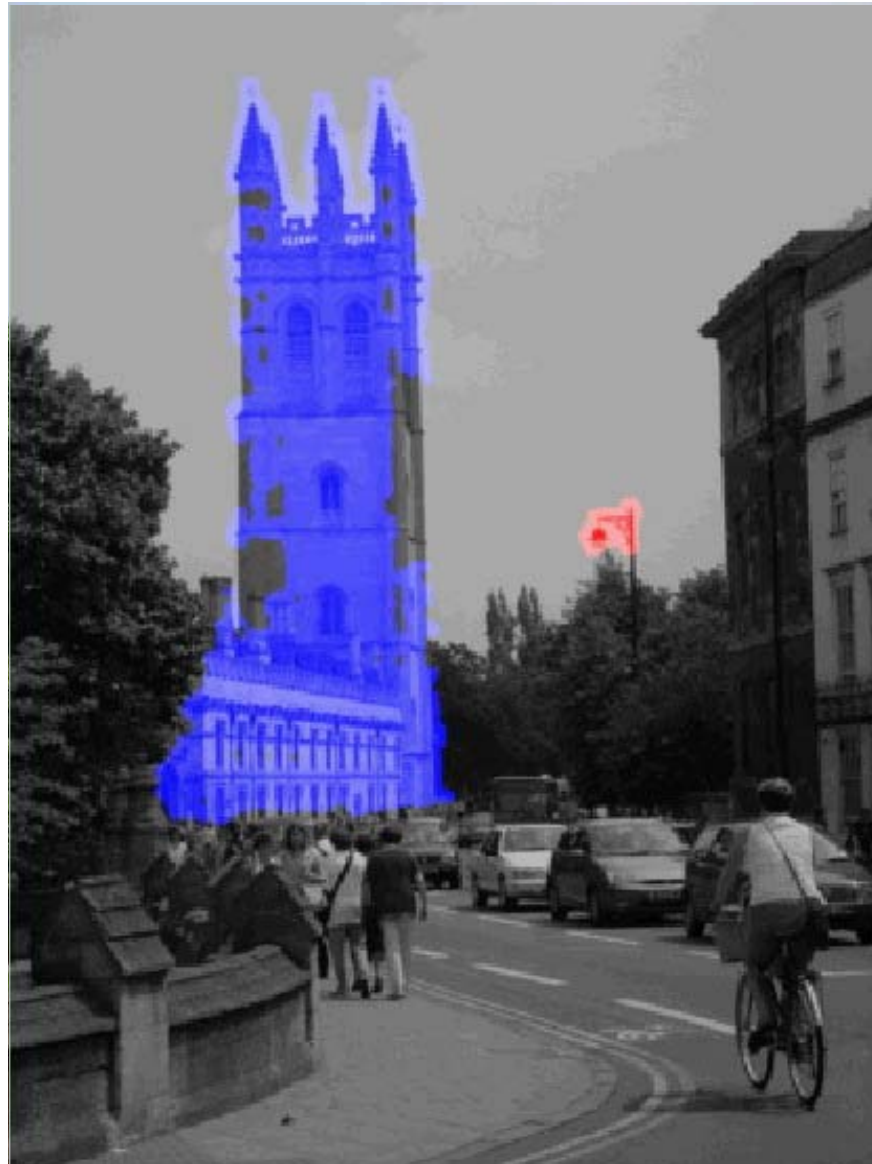
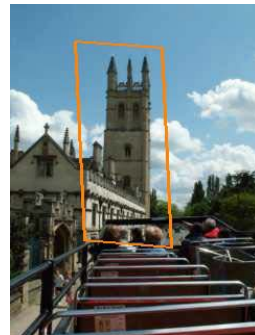
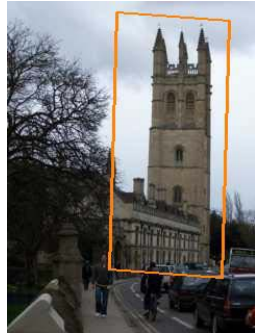


Using the whole bounding box



Result by query expansion using features inside the segmentation

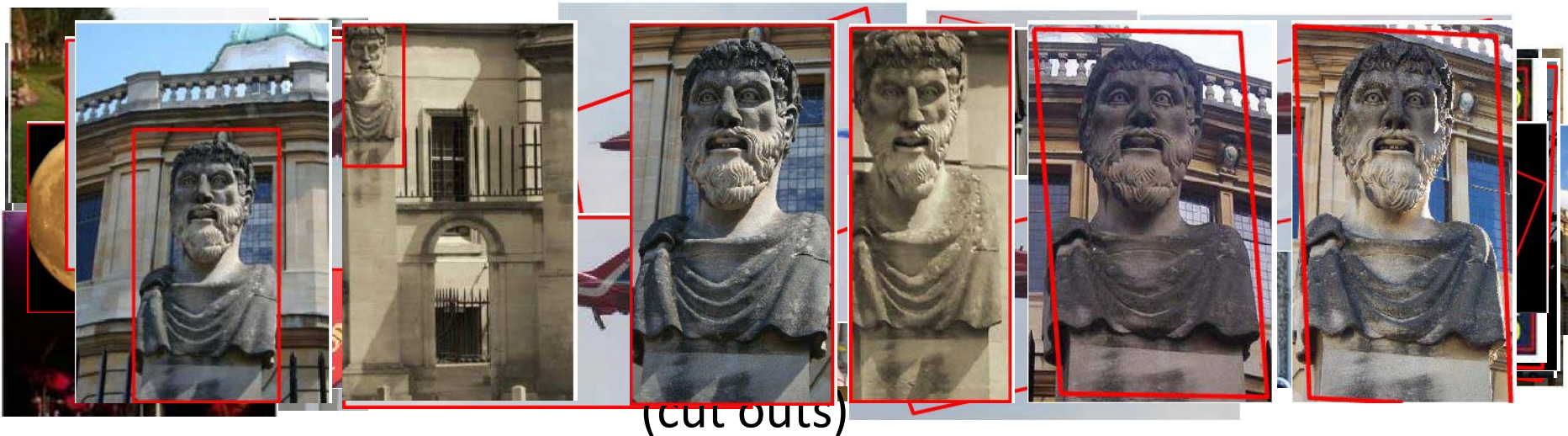
# Visual Content Hyperlinks



# Conclusions

- Novel representation for hashing was introduced
  - Significantly improves the recall (reduces false negatives)
  - Reduces the number of false positives
  - Reduces memory footprint of image representation
  - Connects min-Hash and geometric Hashing
  - The first efficient combination of appearance and geometry in large scale indexing
- Applications
  - Clustering of spatially related images
  - Discovery of small objects

Thank you!



Example of discovered object



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