Particle Swarm Optimization

- Inspired by biological and sociological motivations
  - Bird flocks
  - Fish schools
  - Swarms of insects

PSO: Characteristics

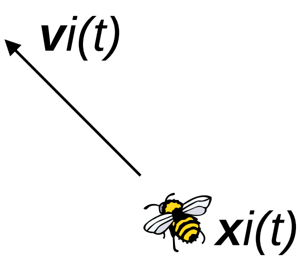
- Population-based optimization technique – originally designed for solving real-valued function optimizations
- Applicable for optimizations in rough, discontinuous and multimodal surfaces
- Does not require any gradient information of the function to be optimized
- Conceptually very simple

PSO: Characteristics

- Each **candidate solution** of continuous optimization problem is described (encoded) by a real vector N-dimensional search space: $\boldsymbol{x} = x_1, \ldots, x_n$
- Each candidate solution is called **PARTICLE** and represents one individual of a population called **SWARM**.
- The **particles** change their components and **FLY** through the multi-dimensional search space.
- Particles calculate their **FITNESS** function as the quality of their actual position in the search space using w.r.t. the function to be optimized.
- Particles also compare themselves to their neighbors and imitate the best of that neighbors.

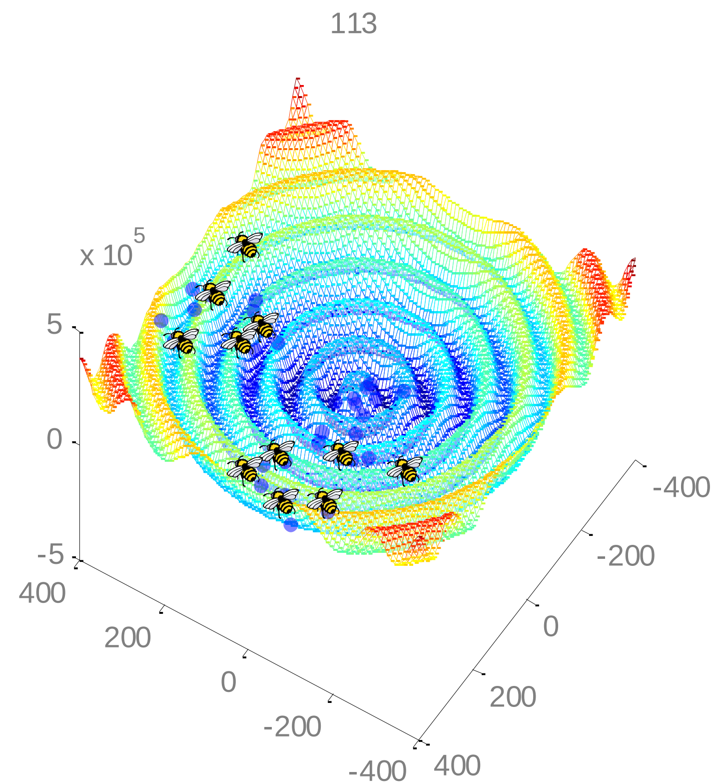
PSO: Fundamentals

- Swarm of particles is flying through the parameter space and searching for the optimum
- **Each particle** is characterized by
    - **Position vector…** $\boldsymbol{x_i(t)}$
    - **Velocity vector…** $\boldsymbol{v_i(t)}$



Particle i

swarm



PSO: Velocity Update

- Velocity update (i^{th} particle):

$$\boldsymbol{v_i(t+1) = \omega\, v_i(t) + C_1 \varphi_1 \,(pbest_i(t) - x_i(t)) + C_2 \varphi_2 \,(gbest(t) - x_i(t))}$$

$pbest_i(t)$ … personal best experience; the best value of the fitness function found by the i-th particle up to time t.

$gbest(t)$ … global best experience; the best value out of $pbest_i(t)$ values of all particles in the swarm found up to time t.

ω … inertial factor

φ_1 and φ_2 … uniformly distributed random numbers that determine the influence of $pbest_i(t)$ and $gbest(t)$.

C_1 … particle's self-confidence; controls the contribution towards the self-exploration.

C_2 … swarm confidence; controls the contribution towards the global direction.

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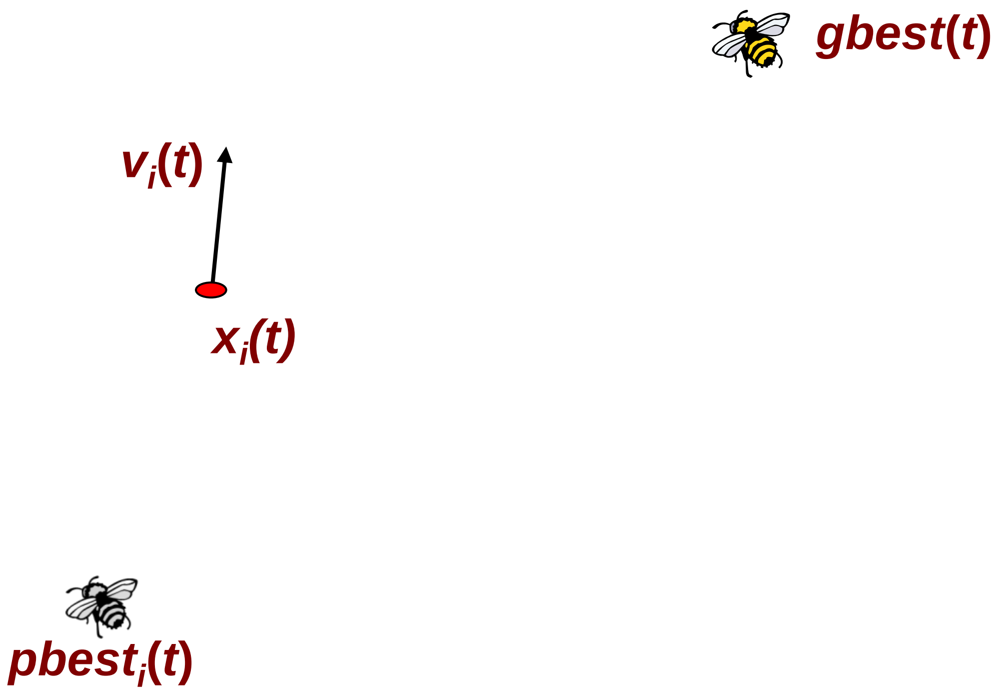

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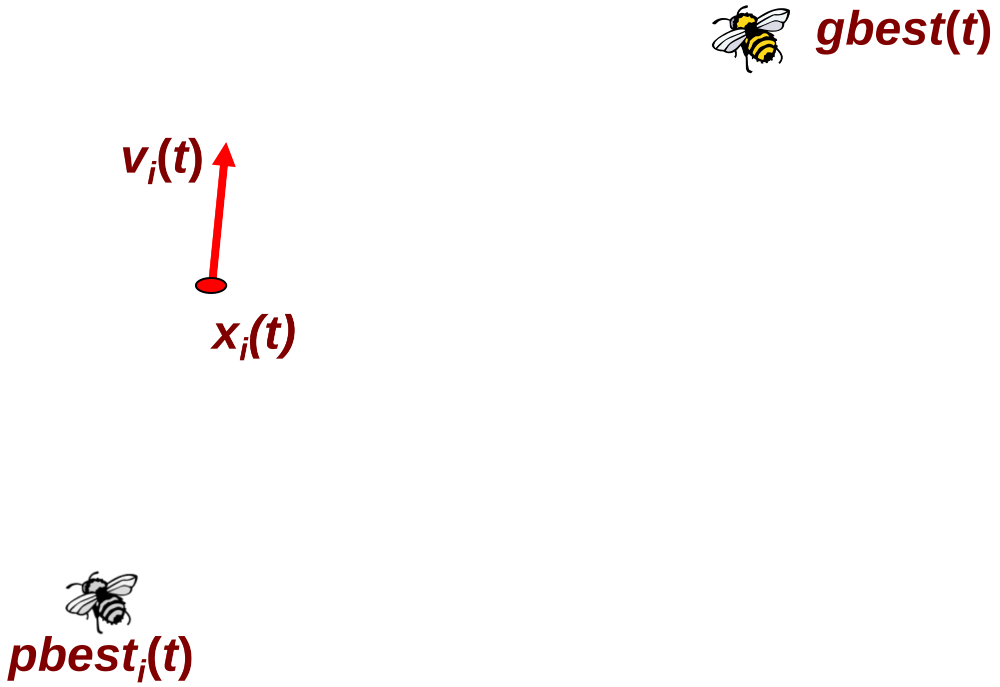

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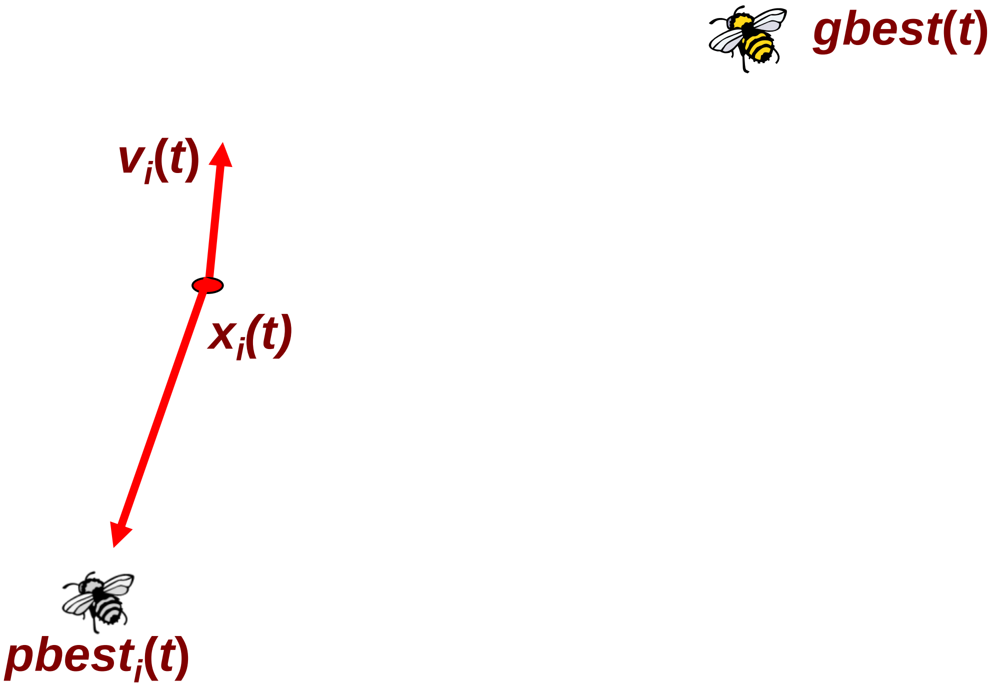

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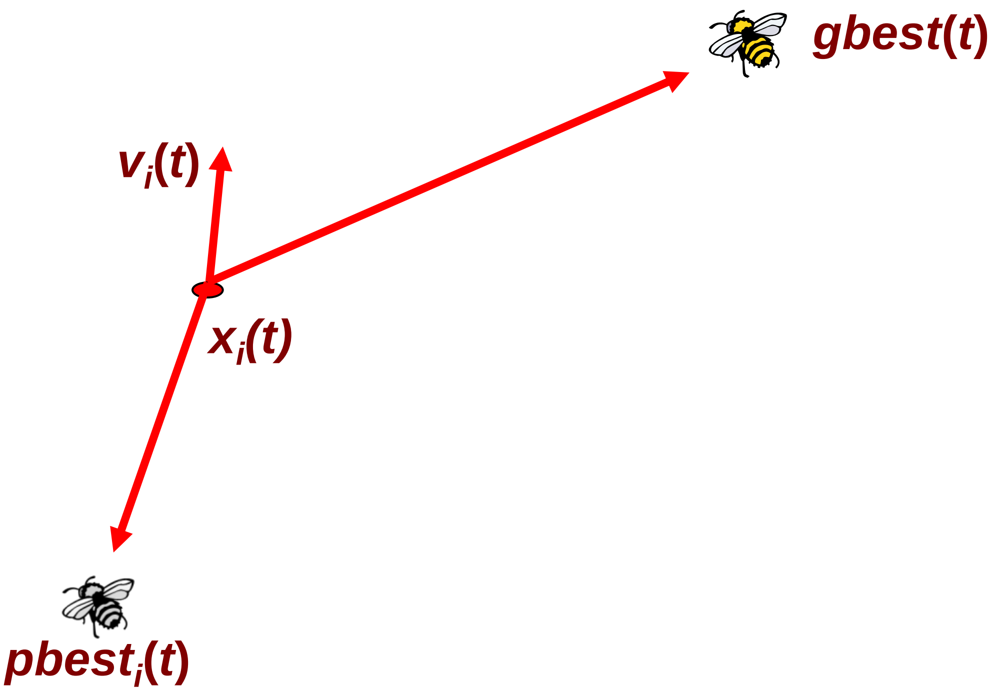

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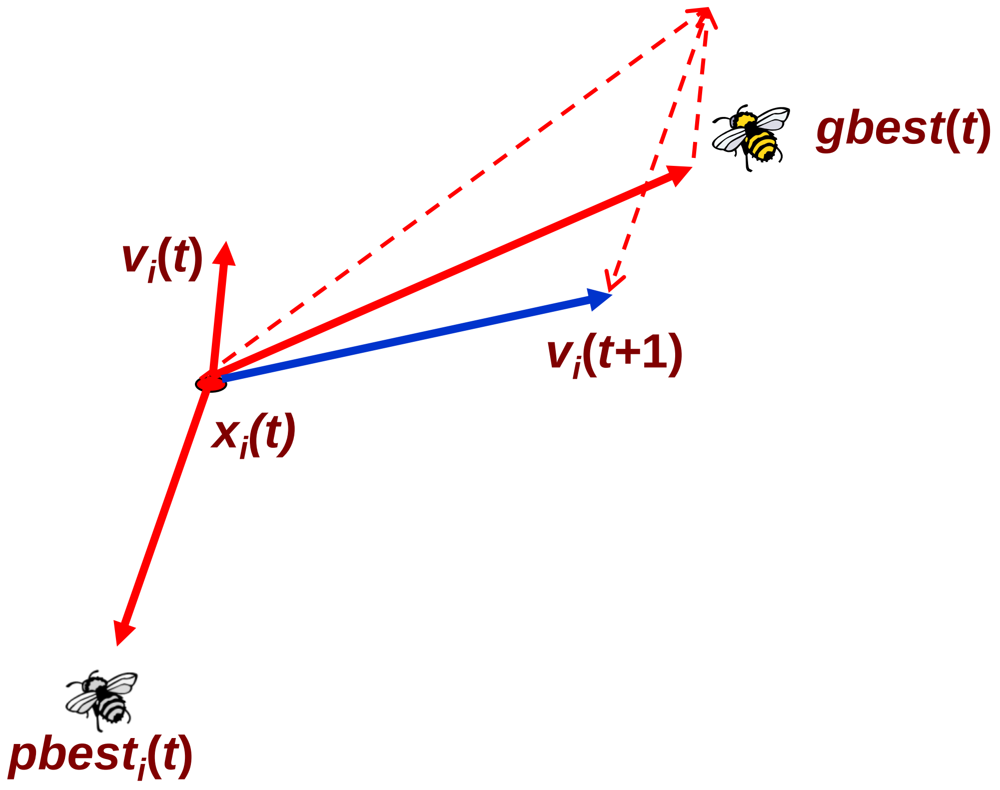
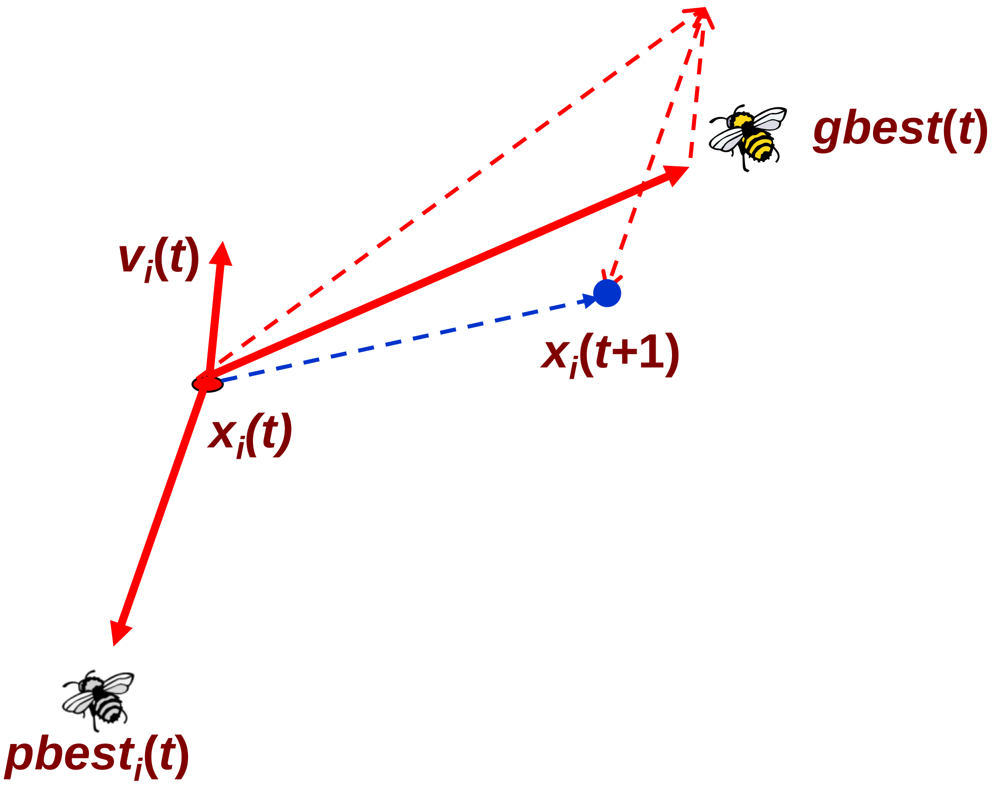


PSO: Position Update

- Position update (i^{th} particle):

  $$\boldsymbol{x_i(t+1) = x_i(t) + v_i(t+1)}$$



PSO: Algorithm

Input: Randomly initialized position and velocity of the particles: $\boldsymbol{X}_i(0)$ and $\boldsymbol{V}_i(0)$

Output: Position of the approximate global optima $\boldsymbol{X}^*$

```
begin
     while terminating condition is not reached do
     begin
             for i=1 to number_of_particles
                     calculate fitness f(X_i)
                     update p_i and g_i
                     adapt velocity of the particle
                     update position of the particle
             increase i
     end
end
```

PSO: Setting the Inertia Factor ω

- Static parameter setting
  - ω<<1 … only little momentum is preserved from the previous time-step.

    ω=0 … the particle moves in each step totally ignoring information about the past velocity.
  - ω>1 … particles can hardly change their direction which implies a reluctance against convergence towards optimum.

    ω>1 is always used with V_{max} to avoid *swarm explosion*.
- Dynamic parameter setting – annealing scheme; ω decreases linearly with time from ω=0.9 to ω=0.4.

  Globally explores the search space in the beginning of the run.

  Performs local search in the end.
- V_{max} can be set to the full search range of the particle's position in order to allow global search.

PSO: Swarm Size

- Swarm size has no significant effect on the performance of the PSO.

  Typical values are 20-60.

PSO: Acceleration Coefficients C_1 and C_2

- Static setting

  Usually C_1=C_2 and range from [0, 4], for example C_1=C_2=1.494.

- Dynamic setting - coefficients vary with time according to

$$C_1 = (C_{1f} - C_{1i})\frac{i}{MAXITER} + C_{1i}$$

$$C_2 = (C_{2f} - C_{2i})\frac{i}{MAXITER} + C_{2i}$$

  where C_{1f} and C_{2f} are final values for C_1 and C_2, C_{1i} and C_{2i} are current values at iteration i, and $MAXITER$ is the maximum number of iterations.

  Particular scheme: C_1 decreases from 2.5 to 0.5; C_2 increases from 0.5 to 2.5.

  Effect: Global search during the early phase of the optimization process; convergence to global optimum at the final stage of the optimization process.

References

Das S. et al.: Particle Swarm Optimization and Differential Evolution Algorithms: Technical Analysis, Applications and Hybridization Perspectives, 2008

http://www.softcomputing.net/aciis.pdf