Complementary Filtering Approach to Orientation Estimation using Inertial Sensors Only

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Abstract— **Precise and reliable estimation of orientation plays crucial role for any mobile robot operating in unknown environment. The most common solution to determination of the three orientation angles: pitch, roll, and yaw, relies on the Attitude and Heading Reference System (AHRS) that exploits inertial data fusion (accelerations and angular rates) with magnetic measurements. However, in real world applications strong vibration and disturbances in magnetic field usually cause this approach to provide poor results. Therefore, we have devised a new approach to orientation estimation using inertial sensors only. It is based on modified complementary filtering and was proved by precise laboratory testing using rotational tilt platform as well as by robot field-testing. In the final, the algorithm well outperformed the commercial AHRS solution based on magnetometer aiding.**

I. INTRODUCTION

THE most crucial aspect that determines the successful performance of any mobile robot agent operating in unknown environment is the ability to estimate robot's orientation in space. Orientation is vital for path integration, planning and performance of other sensors, where measured data have to be transformed between different frames of reference, e.g. laser scanner, vision sensors. To this purpose a vast variety of sensors and signal processing approaches can be used, and most of them have one common modality – they rely on inertial sensors. The most common realization is often known as the Attitude and Heading Reference System (AHRS) [1], which nowadays usually consist of the combination of triaxial accelerometer, triaxial angular rate sensor, and triaxial magnetometer. This sensor suit can easily be exploited using well known estimation methods to provide robot's posture (see [2] [3] [4] [5]); determined by three angles: pitch, roll and yaw. It is also well known, that reliability of especially low-cost inertial sensors is given by means of how one can compensate for their errors: for the deterministic errors such as bias, sensing axis misalignment, and scale factor using calibration [6] [7], and for the random errors using estimation methods [8] [9]. Beside these errors appropriate placement of sensors plays even more crucial role for any strapdown inertial system due to environmental influences that may deteriorate the performance. This concerns mostly vibration and disturbances in magnetic field. Although there exist many approaches to data fusion and signal processing that provide reliable results under laboratory conditions for AHRS units, obtaining reliable results in field still proves to be rather challenging.

There exists a large variety of approaches to orientation estimation, but in general two major groups can be distinguished [10]. First, the Kalman filter based methods (e.g. see [11] [12]) that enable thorough error estimation and compensation based on noise statistics identified using for example the Allan variance method [13] [14]. Second, the complementary filtering approaches (e.g. see [15] [16] [17]) that offer more straightforward solution, which is not so computationally demanding and not susceptible to improper tuning, numerical instabilities, and divergence. There are also alternative approaches to tilt determination, such as addressed in [18].

In this article we introduce a new, computationally undemanding and effective algorithm for orientation determination based on inertial sensors only. In principle, it is based on complementary filtering approach to fusion of attitude angles determined from pre-filtered and pre-processed accelerometer signals and angles computed by integration of angular rates.

The motivation to developing this approach originated from the nature of target applications as well as from previous work [9] [19] [20]. The applications are two: first, the NIFTi project [21], which aims to develop a semi-autonomous search and rescue mobile robot (UGV) (see Fig. 1 left); second, the Bellanca Super Decathlon XXL aircraft model (UAV) (see Fig. 1 right), which requires attitude determination for the autopilot that is being developed. No matter that one platform is a ground vehicle and the other airborne, both are negatively influenced by strong vibration and magnetic field disturbances, making it difficult to process the inertial data and even impossible to use the magnetometer. Therefore, the traditional AHRS approach cannot be exploited and alternative solution was sought. Another challenge concerned the limited computational power. Although the NIFTi robot is equipped with a quad-core processor, vast majority of the computational power is planned to be consumed by computer vision algorithms processing images from the embedded Point Grey Ladybug 3 omnicamera, algorithms for the SICK LMS-151 laser scanner data processing, and trajectory planning algorithms that are currently being developed. Initially, we considered a full-state EKF estimation approach that we have already

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developed, see [20], but modified and accommodated accordingly to the sensors available. However, the computational load proved to be too high.

Fig. 1. (Left) The mobile robot developed for the NIFTi (www.nifti.eu) project; hardware design by Blue Botics (www.bluebotics.com), sensors equipped: Point Grey Ladybug 3 camera, rotating 2D laser scanner SICK LMS-151, Xsens MTi-G unit; (Right) The Bellanca Super Decathlon XXL aircraft model: 3m wing span, 13-15 kg weight, produced by Hacker Model Production Inc (www.hacker-model.com).

The performance was evaluated both under laboratory conditions using precise Rotational Tilt Platform (ROTIP) [22] as well as using the NIFTi robot for both indoor and outdoor field-testing. The inertial units used for evaluation were the ADIS 16405 (Analog Devices) [23], which will be embedded on the UAV, and the MTi-G (Xsens) [24] mounted inside the UGV. The actual algorithm was implemented and optimized for real time operation in the Robot Operating System (ROS) environment [25]. Although our algorithm does not bring any revolutionary innovation to the state of the art estimation methods, it still possesses engineering ingenuity proven by fast, effective and reliable performance in the field under limiting conditions such as strong vibration and magnetic field disturbances. It is a feasible alternative to the traditional AHRS solution that fails under these circumstances.

The paper is structured as follows. Section II covers theory regarding the proposed algorithm. Section III presents spectral analysis and experimental evaluation during field testing and under laboratory conditions. Results are compared to the standard output that the MTi-G Xsens unit offers and to the ground truth. In Section IV we discuss on implications of our work and possible future work.

II. Theory and Methodology

A. Principles of the Orientation Estimation

This section describes our approach to full orientation determination, i.e. estimation of pitch, roll, and yaw angles. The algorithm we propose is initialized during static conditions by local gravity approximation. If intended for performance in the North-East-Down (NED) navigation frame, initial heading due to north has to be supplied at the initialization stage. There are two measurement channels: one processing three orthogonal accelerations and second three orthogonal angular rates; all input signals are calibrated (for details see [9]). The data fusion process can be described as follows (for the block scheme see Fig. 2):

First, the calibrated accelerations and angular rates are pre-filtered – various filter types were investigated to deal best with the vibrations (for results see Section III).

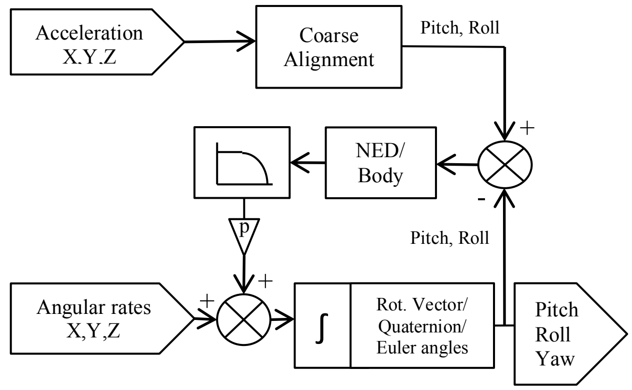

Fig. 2. Block scheme representing the principle of the proposed orientation determination algorithm: a complementary filtering approach that uses inertial data only.

Second, the *coarse alignment* algorithm (for details see next section *B*, or [26] [27]) is applied on the inertial data to obtain pitch and roll angles. Then, the angular rate data are numerically integrated; quaternions are used for attitude representation.

Third, angles obtained from the coarse alignment and by the integration are fused together using specially designed complementary filter (see next section *B* for details).

Fourth, results of the data fusion are fed back to the angular rates channel to ensure stable solution and to minimize the error due to noise integration and drift.

B. Algorithm for Orientation Estimation

The theoretical details to the computational procedures of the proposed algorithm are described in this section. The initial Euler angles are enumerated using the coarse alignment method, which provides coarse approximations of the pitch and roll angles. This method would be feasible also for yaw angle determination if high precision angular rate sensors able to sense the Earth rotation rate were available. Pitch, roll, and yaw angles are the three rotation angles that define the direct cosine matrix (DCM), which transforms inertial measurements from Body Frame (BF) to the NED frame, and can be obtained using the coarse alignment equation as follows:

$$\boldsymbol{C}_b^n = \begin{bmatrix} \frac{-\tan(\varphi)}{g} & \frac{1}{\omega_e \cos(\varphi)} & 0 \\ 0 & 0 & \frac{-1}{g\omega_e \cos(\varphi)} \\ \frac{-1}{g} & 0 & 0 \end{bmatrix} \begin{bmatrix} (\boldsymbol{a}^b)^T \\ (\boldsymbol{\omega}_{ib}^b)^T \\ (\boldsymbol{a}^b \times \boldsymbol{\omega}_{ib}^b)^T \end{bmatrix} \quad (1)$$

where $\boldsymbol{a}^b = [a_x \ a_y \ a_z]^T$ is the accelerometer measurements vector, $\boldsymbol{\omega}_{ib}^b$ is the angular rates measurement vector, g is the local gravity value, ω_e is the Earth rate and φ is latitude.

Because a low-pass filter is used in the algorithm, constants for its digital approximation are evaluated. The transfer function of the low-pass filter is as follows:

$$H(z) = \frac{1-e^{-aT_s}}{z-e^{-aT_s}} \quad (2)$$

where a $(rad \cdot s^{-1})$ is the cutoff frequency of the filter, z is a Z-transformation operator, and T_s is sampling period of the inertial data.

Let $v = e^{-aT_s}$ and $l = 1 - u$. We can rewrite state space equation of the filter using (2) as:

$$\boldsymbol{y}(k+1) = \boldsymbol{y}(k)v + \boldsymbol{u}(k)l \tag{3}$$

where $\boldsymbol{y}$ and $\boldsymbol{u}$ are output and input of the filter, respectively, k is a discrete time-step. The Earth rotation rate expressed in NED coordinate system is evaluated according to [28]:

$$\boldsymbol{\omega}_{ie}^{n} = [\omega_e cos\varphi, 0, -\omega_e sin\varphi]^T \tag{4}$$

Corresponding quaternion in NED is [28] as follows:

$$\mathbf{q}_{n(k-1)}^{n(k)} = \begin{bmatrix} \cos\|0.5\, \boldsymbol{\zeta}_N\| \\ -\frac{\sin\|0.5\, \boldsymbol{\zeta}_N\|}{\|0.5\, \boldsymbol{\zeta}_N\|} 0.5\, \boldsymbol{\zeta}_N \end{bmatrix} \tag{5}$$

where $\boldsymbol{\zeta}_N = \boldsymbol{\omega}_{ie}^{n} T_s$ is the rotation vector and $\mathbf{q}_{n(k-1)}^{n(k)}$ is the quaternion representing rotation of the NED frame. Note that rotation vector evaluation is simplified assuming that change of latitude of the Body frame can be neglected.

There are several main steps in computing the orientation: First, the feedback $\boldsymbol{b}$ is evaluated, but only roll and pitch angles are considered since coarse alignment cannot provide a reliable yaw angle (in case of low-cost sensors):

$$\boldsymbol{b}_k = \begin{bmatrix} \cos\Theta_{a,k-1} & 0 & 0 \\ 0 & 1 & 0 \\ \sin\Theta_{a,k-1} & 0 & 0 \end{bmatrix} \left(\begin{bmatrix} \Phi_{a,k-1} \\ \Theta_{a,k-1} \\ 0 \end{bmatrix} - \begin{bmatrix} \Phi_{\omega,k-1} \\ \Theta_{\omega.k-1} \\ 0 \end{bmatrix} \right) \tag{6}$$

where Φ_a and Θ_a are pitch and roll angles provided by the coarse alignment, Φ_ω and Θ_ω are corresponding angles provided by the integration of angular rates. Transformation from NED to BF is performed in (6) and the difference $\boldsymbol{b}$ is then filtered by (3).

Second, let the output of the filter be $\boldsymbol{b}_f$, hence the increment in angle $\Delta\boldsymbol{\theta}$ is evaluated as

$$\Delta\boldsymbol{\theta}(k) = \left(\frac{\boldsymbol{\omega}_{ib}^{b}(k-1) + \boldsymbol{\omega}_{ib}^{b}(k)}{2} + p\boldsymbol{b}_f(k) \right) T_s \tag{7}$$

which approximates the integration of angular rate sensors signals $\boldsymbol{\omega}_{ib}^{b}$ by a 2nd Runge-Kutta method, incorporating the feedback multiplied by a constant p. Based on (7), rotation vector $\boldsymbol{\zeta}_B$ is computed:

$$\boldsymbol{\zeta}_B(k) = \Delta\boldsymbol{\theta}(k) + \frac{1}{12}\, \Delta\boldsymbol{\theta}(k-1) \times \Delta\boldsymbol{\theta}(k) \tag{8}$$

where the second term represents the coning correction. The corresponding quaternion $\mathbf{q}_{b(k)}^{b(k-1)}$ is computed as [28]:

$$\mathbf{q}_{b(k)}^{b(k-1)} = \begin{bmatrix} \cos\|0.5\, \boldsymbol{\zeta}_N\| \\ \frac{\sin\|0.5\, \boldsymbol{\zeta}_N\|}{\|0.5\, \boldsymbol{\zeta}_N\|} 0.5\, \boldsymbol{\zeta}_N \end{bmatrix} \tag{9}$$

Third, knowing quaternions $\mathbf{q}_{n(k-1)}^{n(k)}$ and $\mathbf{q}_{b(k)}^{b(k-1)}$, we can compute a new set of Euler angles represented by quaternion $\mathbf{q}_{b(k)}^{n(k)}$ using the chain rule [28, p. 35] as follows:

$$\mathbf{q}_{b(k)}^{n(k)} = \mathbf{q}_{n(k-1)}^{n(k)} * \left(\mathbf{q}_{b(k-1)}^{n(k-1)} * \mathbf{q}_{b(k)}^{b(k-1)} \right) \tag{10}$$

Finally fourth, Euler angles are easily extracted from $\mathbf{q}_{b(k)}^{n(k)}$ and saved for the next computation cycle.

C. *Complementary Filter Tuning*

According to Fig. 2, the information about the actual orientation is sensed by both accelerometers and angular rate sensors. In order to perform the complementary data fusion, the feedback filter is proposed with the transfer function as follows:

$$T_a(s) = \frac{s^2 + sa}{s^2 + sa + pa} + \frac{pa}{s^2 + sa + pa} \tag{11}$$

where p is the constant from (7) and s a Laplace operator; the first term corresponds to transfer of the attitude obtained by the angular rates integration, the second term to the transfer of attitude from the coarse alignment. Resulting transfer function is similar to the proposed ones in [10], but the main difference lies in the order of the transfer functions; our solution leads to filters of the second order with slopes ± 40 dB/dec. Therefore, the feedback filter behaves as a low-pass filter for the integrated angular information and as a high-pass filter for the output of the coarse alignment, (see Fig. 3). This is especially beneficial because of the character of the noises present in the signals.

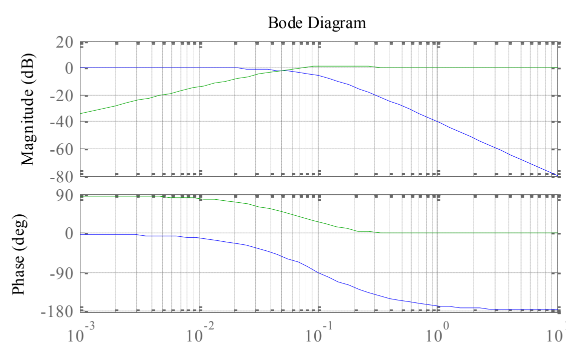

Fig. 3. Frequency characteristics of the transfer functions for the combination of attitude angles obtained using coarse alignment (green) and through integration of measured angular rates (blue); the cutoff frequency is set to 0.1 rad/s for demonstration purposes.

Constants of the filter can be adjusted arbitrarily under condition that the denominator of the transfer function has stable roots. In our approach, we define only the cutoff frequency ω_c (rad/s), the constants of the filters are then determined according to (11) as: $p = \frac{\omega_c}{2}$ and $a = 2\omega_c$. This setting ensures critical damping and thus the transfer functions show only minimal overshoot at the cutoff frequency. Otherwise, the algorithm would amplify noise present at the cutoff frequency.

III. Experimental Evaluation and Results

Experimental evaluation of the proposed algorithm consisted of two main phases: laboratory evaluation using precise *Rotational Tilt Platform* (ROTIP), see Fig. 4, and both indoor and outdoor field-testing in arena with concrete ramps of given slope.

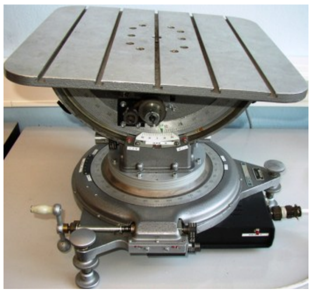

Fig. 4. Rotational Tilt Platform (ROTIP) for automatic positioning and precise attitude angle measurement; granted angular resolution below 2.6 arcsec [19, p. 243].

A. Evaluation under Laboratory Conditions

The laboratory testing aimed at verification of the precision, stability and reliability of attitude determination with precise ground truth available due to the ROTIP, which granted angular resolution of less than 2.6 arcsec in all three axes. Since investigation of influence of vibration on precision of attitude determination was the main aim of this test, frequency spectrum analysis of both the UAV and UGV was performed on data collected using ADIS 16405 (UAV) and MTi-G Xsens (UGV); for example see Fig. 5.

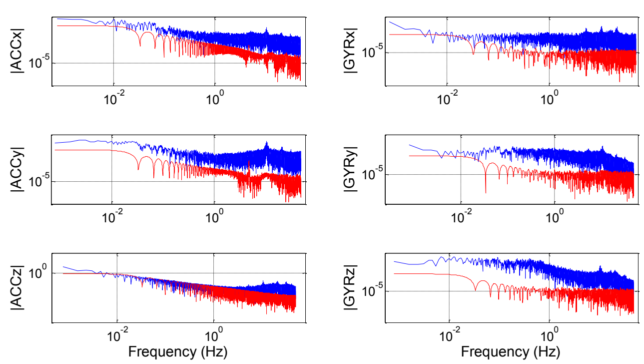

Fig. 5. Frequency characteristics showing frequency spectrum of all inertial sensors when stationary (red) and during field-testing (blue); blue frequency spectrum covers motion dynamics and vibration, measured at 100 Hz using MTi-G Xsens.

Based on the analysis, artificial source of vibration was created using an electric motor with deviated balance wheel, which was mounted on the ROTIP. The motor was tuned to provide vibration as close to the real measured spectrum as possible. This simulation using ROTIP positioning with precise ground truth enabled us to analyze the expected performance and to design appropriate filters for inertial data pre-filtering – first step in each measurement channel.

The effect of vibration on performance of our algorithm for different filters is concluded in Table I and Table II. Since the algorithm fuses two procedures of attitude computation, influence of pre-processing using different filters on each of the two procedures was evaluated for case with vibration (see Table I) and without (see Table II). From the results achieved it can be concluded: if no vibrations are present, pre-filtering should not proceed. If vibrations do affect the inertial data, moving average filter should be used in the coarse alignment channel and IIR low-pass filter with cut-off frequency according to expected motion dynamics should be used in the channel for angular rates integration.

TABLE I
RMS Error of Attitude Determination (without Vibrations)

Filter Type	Coarse Alignment RMS Error (°)		Angular Rates Integration RMS Error (°)		
	Pitch	Roll	Pitch	Roll	Yaw
No Filter	**1.1**	**1.3**	**0.9**	**1.9**	**2.3**
IIR Low-pass filter Transition frequency:					
4-10 Hz	1.1	1.9	1.6	2.3	2.4
16-24 Hz	1.1	1.5	1.0	2.0	2.4
36-44 Hz	1.1	1.4	0.9	1.9	2.3
Wavelet denoising: *Wavelet db8, decomposition level 6*	1.1	1.3	0.9	1.9	2.3
Moving Average Filter; order:					
10	1.1	1.3	0.9	1.9	2.3
100	1.1	1.8	0.8	2.5	2.9
300	1.6	3.1	1.3	3.5	3.9

TABLE II
RMS Error of Attitude Determination (with Vibrations)

Filter Type	Coarse Alignment RMS Error (°)		Angular Rates Integration RMS Error (°)		
	Pitch	Roll	Pitch	Roll	Yaw
No Filter	**3.1**	**2.1**	**1.2**	**1.4**	**1.6**
IIR Low-pass filter Transition frequency:					
4-10 Hz	0.8	1.0	1.1	1.6	0.7
16-24 Hz	1.2	1.3	0.5	1.5	1.2
36-44 Hz	1.6	1.4	0.9	1.4	1.5
Wavelet denoising: *Wavelet db8, decomposition level 6*	3.0	1.9	0.5	1.4	1.6
Moving Average Filter; order:					
10	1.4	1.4	3.9	1.5	1.1
100	0.5	0.6	7.5	1.8	0.5
300	0.3	0.4	8.3	2.0	0.6

To evaluate performance of the proposed complementary data fusion, vibrations were introduced to the measured data. RMS error with respect to the ground truth was evaluated for case with the proposed feedback filtering and without. An example of dynamic positioning test consisting of combined motions in all three axes at different rates is shown in Fig. 6. The results were convincing even after 15 min of dynamic motion and strong vibrations: RMS error in pitch and roll angles was *4.1 deg and 2.8 deg for the feedback filtering,* respectively, and 9.7 deg and 30.0 deg for case of angular rates integration without feedback, respectively. Therefore, it can be concluded, that the functionality of the proposed feedback filter was verified and for the data fusion it improves the performance significantly. For case without vibrations the precision corresponded to the values of standard AHRS solutions, in accordance of the used inertial sensors.

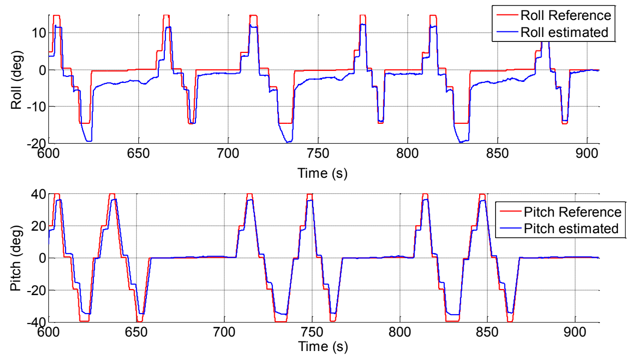

Fig. 6. A long-term dynamic positioning experiment performed using ROTIP with an electric motor used as source of vibration to test performance of the feedback filtering in all three axes of motion; the yaw axis is not shown since it was not influenced by the feedback. The overall RMS error of the pitch and roll angles was 4.1 deg and 2.8 deg, respectively, after 15 minutes of testing compared to 9.7 deg and 30.0 deg in pitch and roll obtained for case without the feedback.

B. *Evaluation by Field-testing*

As described in the previous section, we optimized and evaluated the proposed algorithm for performance under strong vibration conditions using ROTIP. Hence, the next step was to test it on the UGV platform (UAV systems still in development). On the contrary to the laboratory testing, field-testing unfortunately lacked such precise ground truth for the attitude angles.

To prove the benefit of our method, which does not rely on magnetic field measurements, in applications, where magnetic disturbances are eminent, we compared the results to the standard MTi-G Xsens output. The algorithm was running in ROS in real time as the robot was driving over concrete ramps of given slope of 11 deg over 15 minutes; see Fig. 7, Fig. 8, and Fig. 9 for typical example of results. These figures capture all three attitude angles during an experiment when the UGV was teleoperated as follows:

First, the UGV was placed on uneven surface and during the first 5s it proceeded with initial alignment while staying still (indicated by zero output values).

Second, at the time of 35s the UGV was driven over a doorstep, resulting in dynamic change in pitch angle, slight change in roll and yaw angles. Then it was teleoperated to the area with ramps (two 90 deg turns at 70s and 130s) over an uneven terrain (time interval 150s – 250s).

Third, during the time interval of 280s – 430s the UGV was driven up and down a ramp with 11 deg slope (forth and back in same heading). This was repeated 7 times in a row without turning and with short pauses between individual movements to show how the angles return to 0 deg on the flat ground.

Fourth, during 435s – 490s the UGV performed full 180 deg rotation while staying on the ramp such that the pitch and roll changed accordingly, on complementing the change in the other. At 500s the UGV returned from the ramp back to the flat ground.

Fifth, robot continued moving for one more minute in a straight line, accelerating and decelerating, and finally stopped. The final value of the yaw angle did correctly correspond to the initial one.

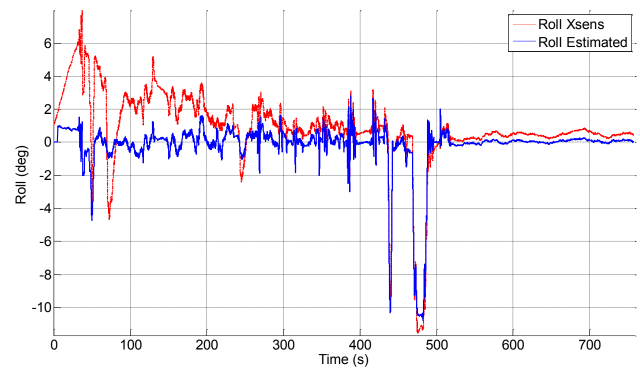

Fig. 7. Roll angle estimated during NIFTi robot field-testing on ramps of given slope: roll angle estimated using proposed complementary filtering (blue), standard roll angle output using MTi-G Xsens (red).

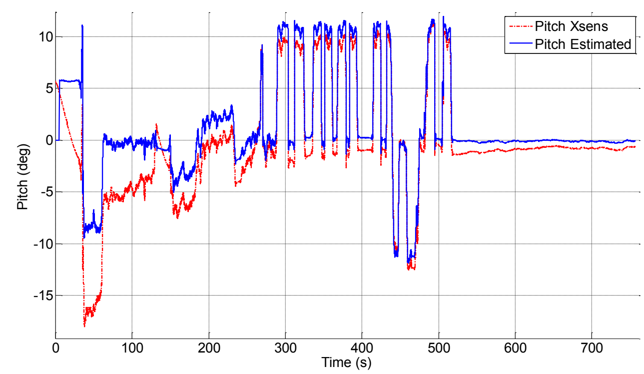

Fig. 8. Pitch angle estimated during NIFTi robot field-testing on ramps of given slope: pitch angle estimated using proposed complementary filtering (blue), standard pitch angle output using MTi-G Xsens (red)

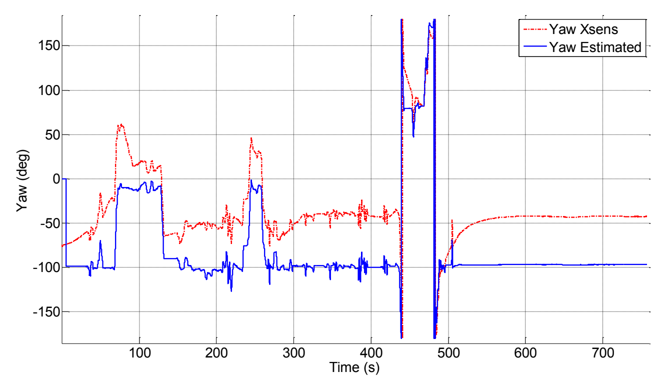

Fig. 9. Yaw angle estimated during NIFTi robot field-testing on ramps of given slope: yaw angle estimated using proposed complementary filtering (blue), standard yaw angle output using MTi-G Xsens (red).

From Fig. 7 to Fig. 9 it is clear by inspection, that the proposed algorithm outperformed the magnetometer based approach, which is common to most of the AHRS units, represented in this case by the MTi-G Xsens unit.

IV. Conclusion

In this article we comment on the usability and reliability of standard AHRS units and attitude estimation algorithms that exploit magnetometer measurements. There are applications where strong vibrations and magnetic field disturbances negatively affect such traditional approaches and commercial solutions. These applications were our main motivation to develop a complementary filtering algorithm for orientation determination using the inertial data only. We have evaluated this algorithm both under laboratory conditions with precise ground truth as well as on the UGV platform in field using the MTi-G Xsens unit. The algorithm was also developed and tested using the ADIS16405 inertial measurement unit for the UAV platform, which is still in development.

At this moment we consider it important to compare our solution to a similar complementary filtering approach proposed in [10], which also contributed to our inspiration when writing this paper:

First, comparing the RMS errors with similar solutions, it is important to consider all the dynamic movements ROTIP provided - experiments in [10] involved rotations parallel with the roll axis, perpendicular to the other two while we have excited all three sensing axes with high dynamics and artificial vibration with frequency spectrum corresponding to the frequency spectrum of the real platforms.

Second, in our approach we use Euler angles representation for the feedback implementation and quaternions for the rest of the computations. This allows us to exploit the benefits of quaternion computation as well as easy selection of the sensing axes to be included in the feedback.

Third, sensor requirements: as mentioned already no magnetometer is needed, 3 accelerometers and 3 angular rate sensors are the only source of information (except for determining the initial heading due to north if navigation in NED is requested). However, it has to be noted, that the performance of the proposed solution is application specific – in cases, where vibration and magnetic field disturbances are not an issues, standard AHRS solution with magnetometer, such as in [10], ensures better long term stability especially of the yaw angle. We assume that using navigation grade gyros might diminish this drawback of our approach.

Fourth, we have evaluated experiments in length of 15 minutes. We consider this time interval to be sufficient enough to prove challenge for any inertial sensors only low-cost solution.

Finally, regarding the future work, this algorithm will be further enhanced and become an integral part of a full 3D dead reckoning solution for the UGV as well as the core algorithm for the artificial horizon unit of the UAV.

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