1 Prerequisites

You should have a basic understanding of probability theory, combinatorics and formal logic – propositional and first order. Here are a few questions you should be able answer quickly and without much thinking.

- What is a possible computer representation for a propositional interpretation (a.k.a. truth valuation)?
- What are the conjunctive and disjunctive normal forms?
- If I is an interpretation and ϕ is a formula, what does $I \vDash \phi$ mean?
- Prove that conjunction is associative in propositional logic.
- Rewrite the formula $(a \wedge b) \implies c$ to an equivalent clausal form.
- What does it mean when we say we take samples i.i.d.?

2 Exercises

Motivation The homework that will follow is essentially a computer implementation of the following exercises. If you manage to solve them, the homework should be easy for you.

Definitions A monotone conjunction (resp. disjunction) is a conjunction (resp. disjunction) of a number of propositional variables. In other words, it's a term (resp. caluse) with positive literals only. An s-clause is a clause containing at most s literals. An s-CNF is a conjunction of s-clauses.

Exercise - combinatorics Assume a propositional logic with n variables. Compute the following combinatoric problems:

- What is the number of monotone conjunctions? (No duplicate literals)
- What is the number of non-equivalent conjunctions?
- What is the number of s-CNFs? (No duplicate clauses) Break down the calculation to the following steps:
  - What is the number of caluses of length exactly s?
  - What is the number of s-caluses?
  - Apply an earlier result.

Theoretical exercise - generalizing algorithm Study the generalizing algorithm. Find answers to the following:

- The basic algorithm learns monotone conjunctive concepts from a set of propositional interpretations. How can you reduce the learning of non-monotone conjunctive concepts (i.e. terms) to the simpler monotone case?
- Using a similar idea, how could you reduce the learning of $s-$CNFs to the learning of monotone conjunctions?
  - Hint 1: Terms are actually 1-CNFs.
  - Hint 2: This is similar to the technique of polynomial expansion of features used in classical machine learning.
- Using De Morgan's laws, how can you alter the algorithm to learn monotone disjunctions instead?

Exercise - generalizing algorithm example Imagine a zoologist provides you with a dataset of animals and some of their observed features. Each animal is labeled whether it is or is not a mammal. You are to learn a mammal concept from the features, using the generalization algorithm. Emulate the generalization algorithm. First learn a conjunctive concept (non-monotone). Then learn a disjunctive concepts. Answers are on the next page.

	Bat	Dolphin	Earthworm	Bee	Carp	T-Rex	Penguin	Parrot	Goat	Platypus	Elephant	Frog	Hippo
Flies	✓			✓				✓					
Hair	✓								✓	✓			
Fins		✓			✓					✓		✓	
Feathers							✓	✓					
Scales					✓	✓							
Breathes air	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
Bones	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓
Funky nose		✓									✓		
Mammal	✓	✓							✓	✓	✓		✓

The correct answers are: $C_1 = \neg\text{Feathers} \wedge \neg\text{Scales} \wedge \text{Bones} \wedge \text{Breathes air}$ (conjunctive) and $C_2 = \text{Hair} \vee \text{Funky nose}$ (disjunctive). Knowing this, explain the following:

- Only one of the following is true. Determine which one and find a counterexample to the other.
  - $\text{Mammal} \models C_1$
  - $C_1 \models \text{Mammal}$
- Only one of the following is true. Determine which one and find a counterexample to the other.
  - $\text{Mammal} \models C_2$
  - $C_2 \models \text{Mammal}$