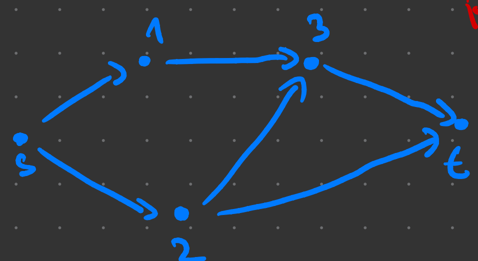
Assume max-flow in G with zero lower bound:

$$\max \sum_{p \in \bar{P}} x_p \qquad \xleftrightarrow{\text{LP duality}} \qquad \min \sum_{e \in E} u(e) \cdot y_e$$

$$\text{s.t.} \quad \sum_{p \in \bar{P} : e \in p} x_p \leq u(e) \quad \forall e \in E \qquad\qquad \text{s.t.} \quad \sum_{e \in p} y_e \geq 1 \quad \forall p \in \bar{P} \quad (*)$$

$$x_p \geq 0 \quad \forall p \in E \qquad\qquad y_e \geq 0 \quad \forall e \in E$$

→ $\bar{P}$ is set of all s-t paths in G.



$\bar{P} = \{(s-1-3-t),$
$(s-2-t),$
$(s-2-3-t)\}$

↓ solve by lazy separation of $(*)$

1) start with a single path (feasible solution)
2) find the most violated path p $(*)$ in current solution $\hat{y}_e$

constraints: $\sum_{e \in p} y_e - 1 \geq 0$ $(*)$

Most violated in current solution $\hat{y}_e$: $p^* := \arg\min_{p \in \bar{P}} \sum_{e \in p} \hat{y}_e - 1$

↳ but this is s-t shortest path problem in G with costs $\hat{y}_e$!

↳ non-negative $\hat{y}_e$, use e.g. Dijkstra

↳ if you cannot find shortest path lower than 1, then finish.