# MULTI-GOAL MOTION PLANNING

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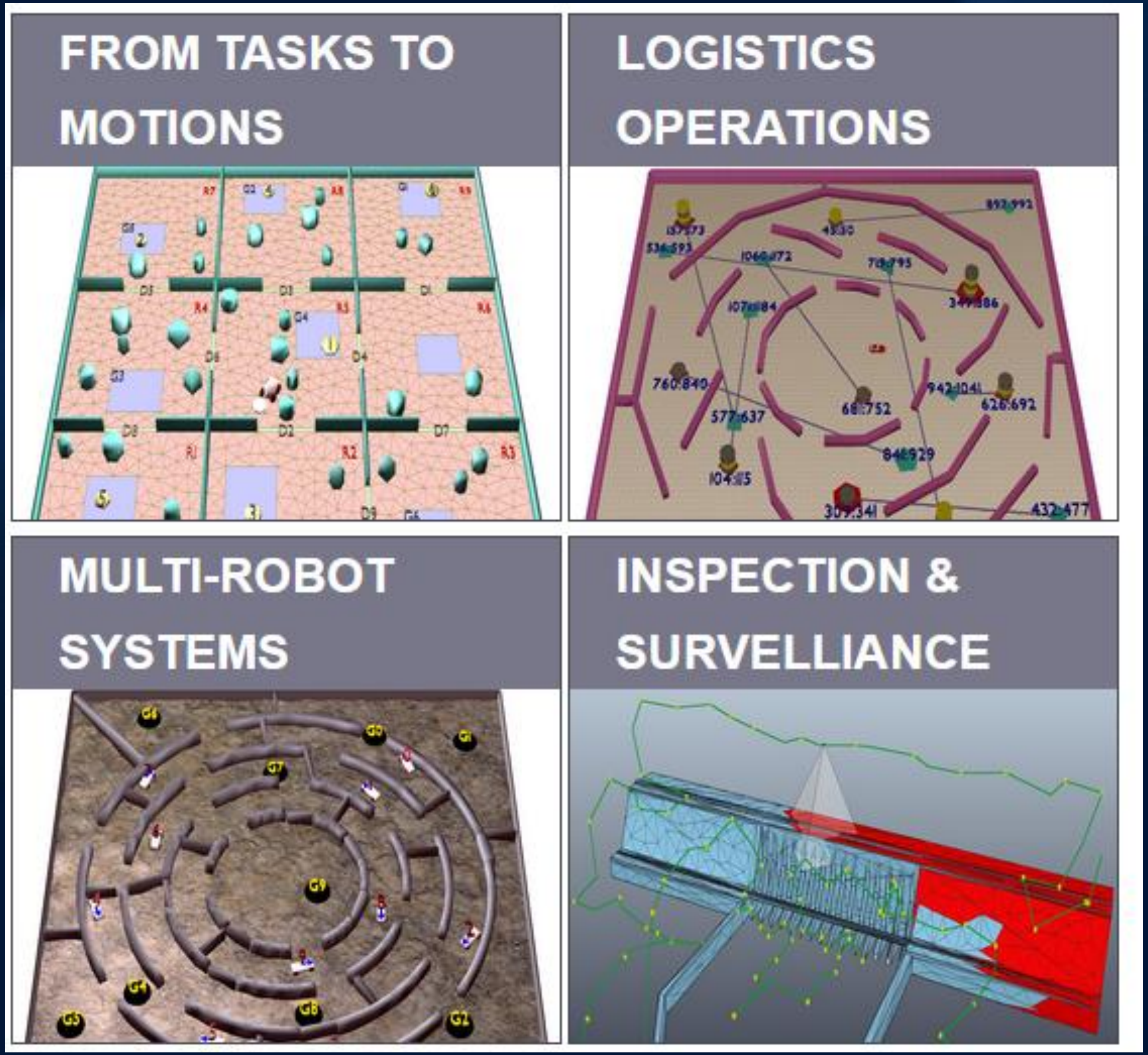
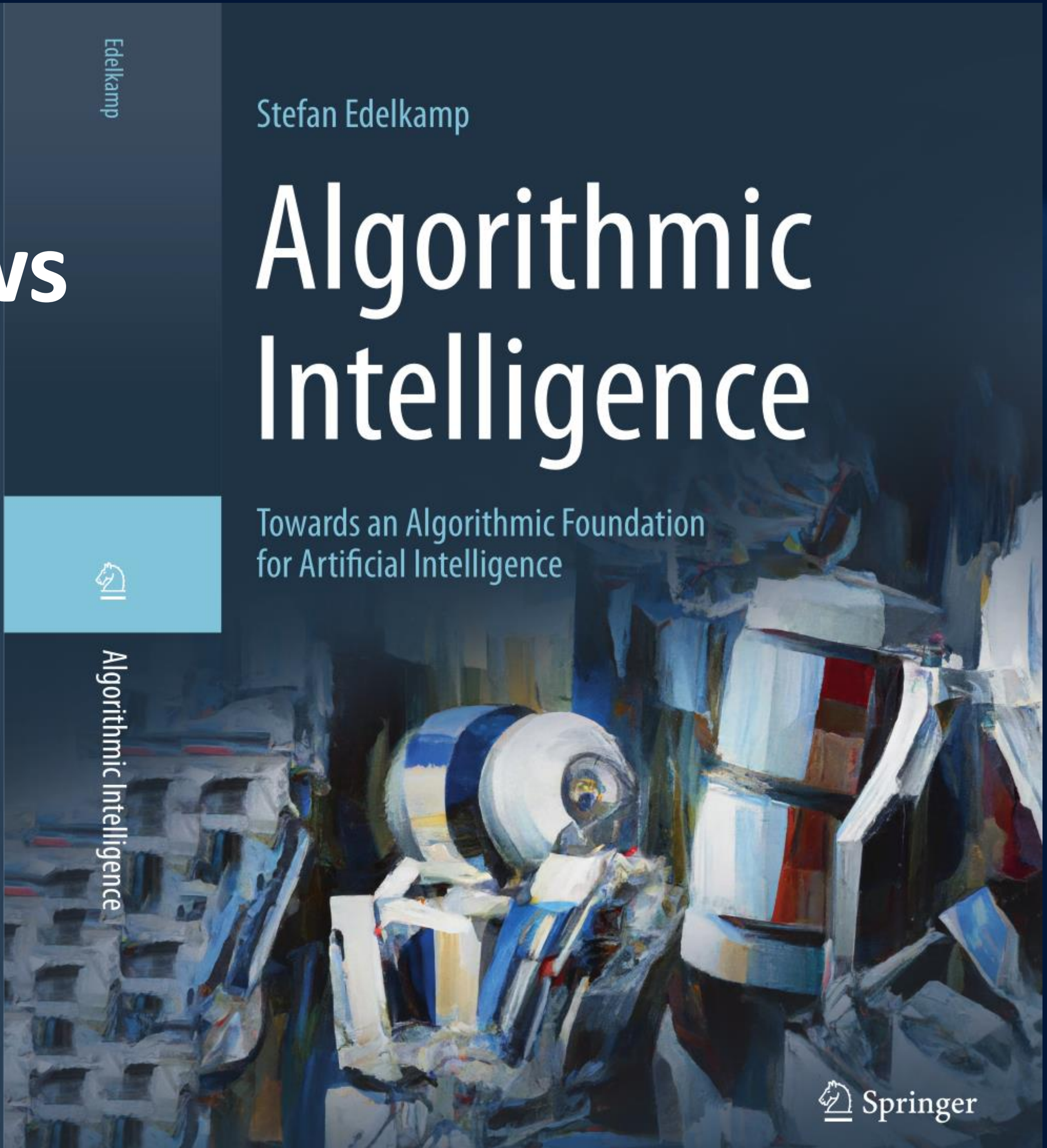
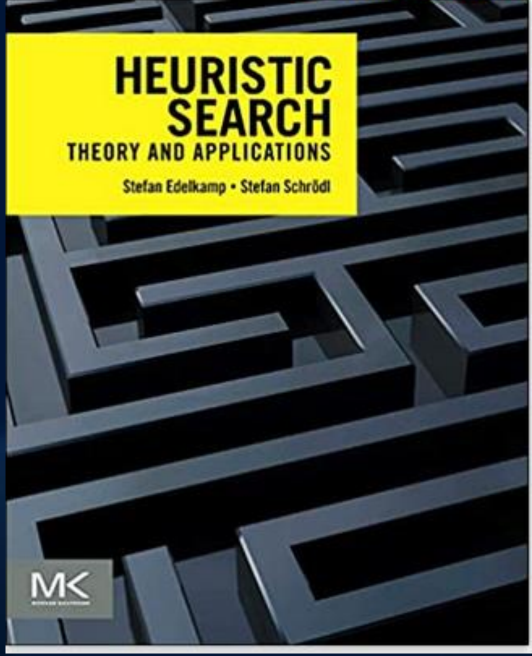


# News




- Demonstrates the algorithmic foundations of computer intelligence
- Integrates programming, theoretical computer science, optimization, machine learning, data mining, data analytics
- Covers searching, sorting and deep learning with applications to big data, games, biology, robotics, IT security

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Stefan Edelkamp

# Algorithmic Intelligence

## Towards an Algorithmic Foundation for Artificial Intelligence

In this book the author argues that the basis of what we consider computer intelligence has algorithmic roots, and he presents this with a holistic view, showing examples and explaining approaches that encompass theoretical computer science and machine learning via engineered algorithmic solutions.

Part I of the book introduces the basics. The author starts with a hands-on programming primer for solving combinatorial problems, with an emphasis on recursive solutions. The other chapters in the first part of the book explain shortest paths, sorting, deep learning, and Monte Carlo search. A key function of computational tools is processing Big Data efficiently, and the chapters in Part II of the book examine traditional graph problems such as finding cliques, colorings, independent sets, vertex covers, and hitting sets, and the subsequent chapters cover multimedia, network, image, and navigation data. The highly topical research areas detailed in Part III are machine learning, problem solving, action planning, general game playing, multiagent systems, and recommendation and configuration. Finally, in Part IV the author uses application areas such as model checking, computational biology, logistics, additive manufacturing, robot motion planning, and industrial production to explain how the techniques described may be exploited in modern settings.

The book is supported with a comprehensive index and references, and it will be of value to researchers, practitioners, and students in the areas of artificial intelligence and computational intelligence.

# Two Meanings of Planning

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## Long-Standing Challenge: Bridge the Gap

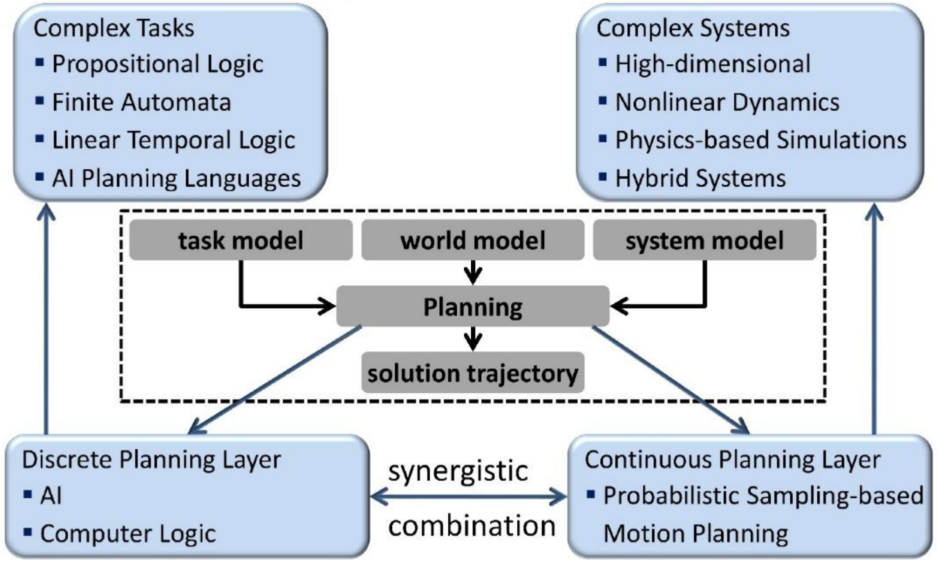
given a high-level task description compute the motions to enable the robot accomplish the task

**AI Planning**

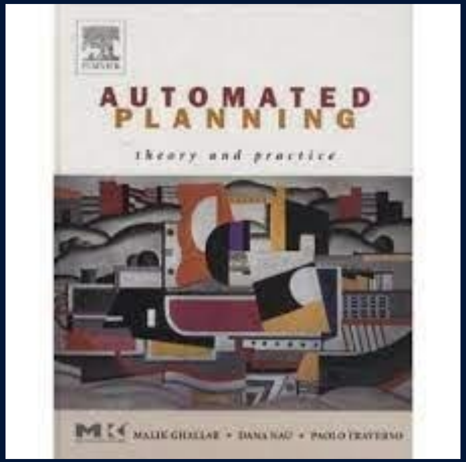
- ▷ Discrete world
- ▷ Discrete actions
- ▷ Complex task specifications

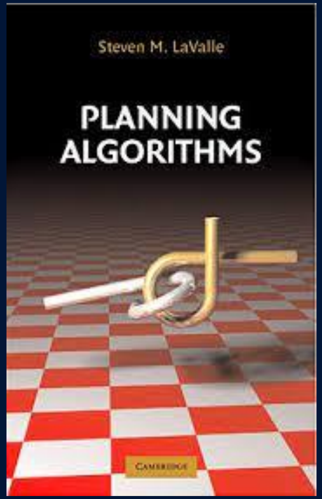
**Motion Planning**

- ▷ Continuous world
- ▷ Continuous controls/dynamics
- ▷ Simple task specifications




**Synergistic Combination of Discrete and Continuous Layers of Planning**



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# Driving Research Questions


# Steer Robots of the 21st Century

- How can we improve motion planning for complex systems?
- How can we develop motion planners that are generally applicable?
- How can we achieve planning efficiency even with nonlinear dynamics?
- How far back can we push the “curse of dimensionality”?
- How to integrate domain-independent planners for more flexible task planning?
- How to solve the inspection problem?
- How to combine efficient discrete solving with searching the continuous space of feasible motions?
- Is there Pareto optimality between efficiency and solution quality?
- What formal guarantees can we provide?
- How can we take resource and energy constraints into account?

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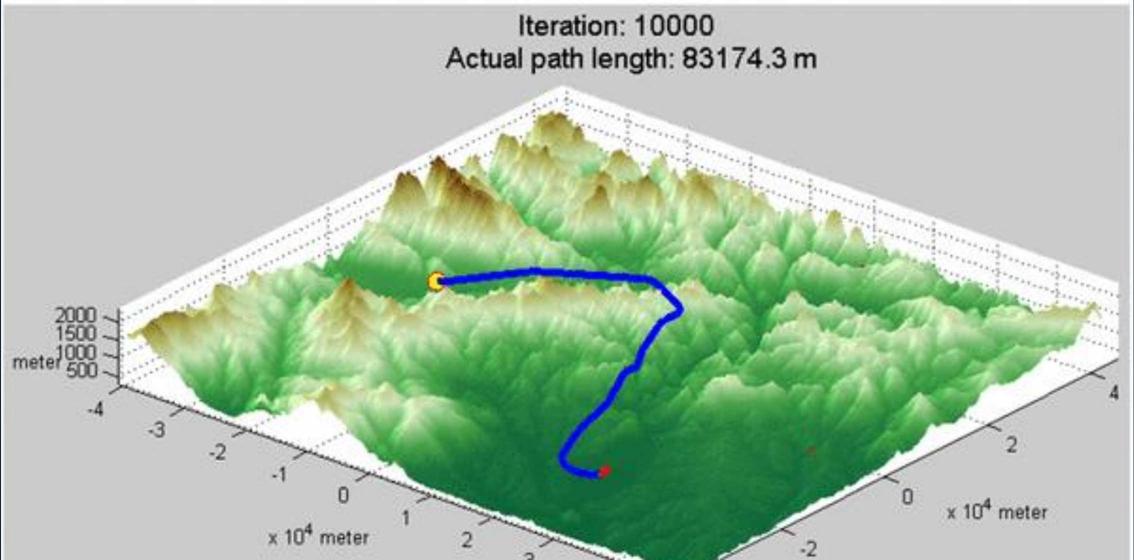
# Ongoing Work

SubT Challenge, Complex Resources, DeepRRT*, Emergency Planning, Location Routing, …



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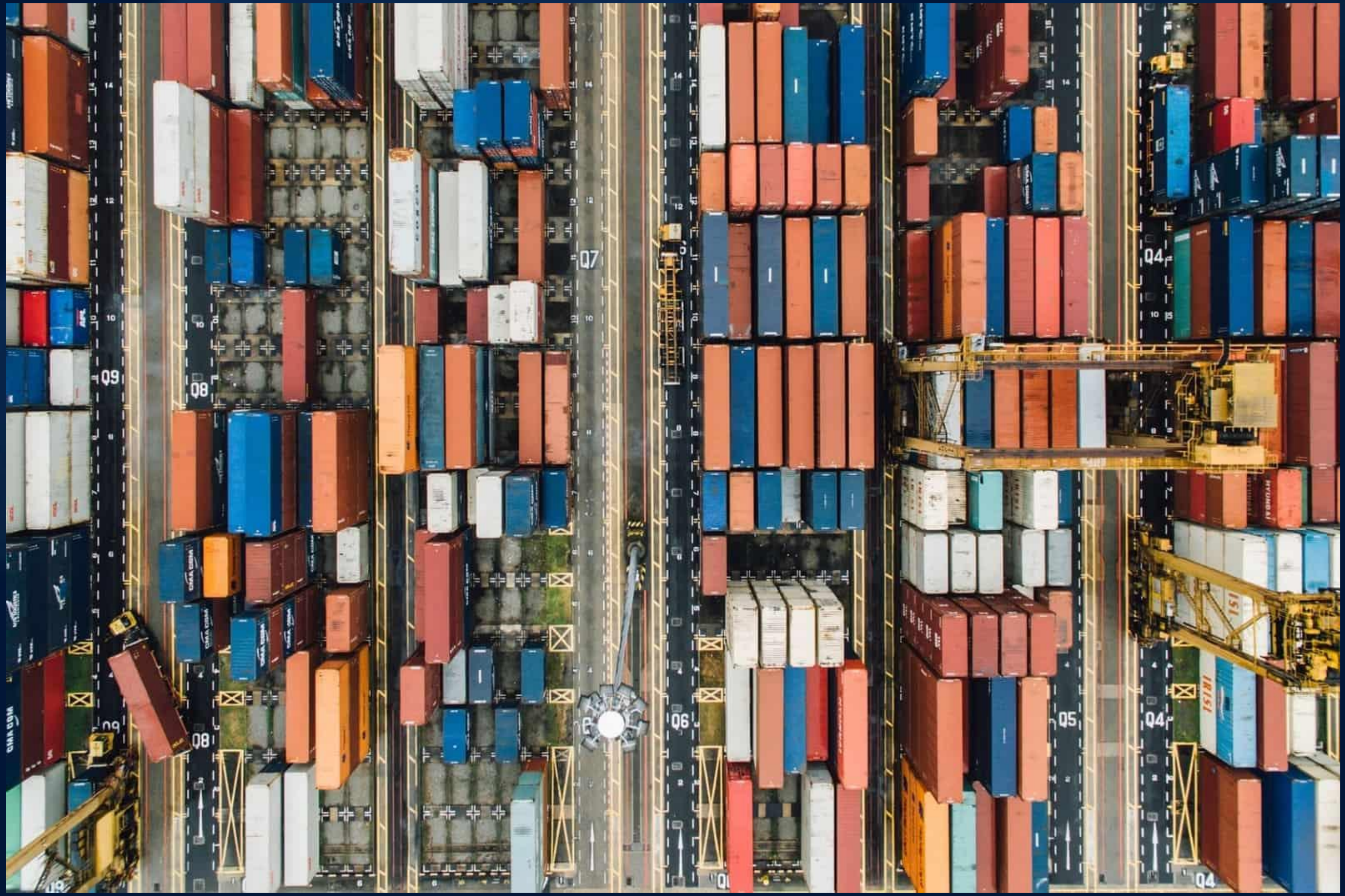
# Prelude: Planning Tours

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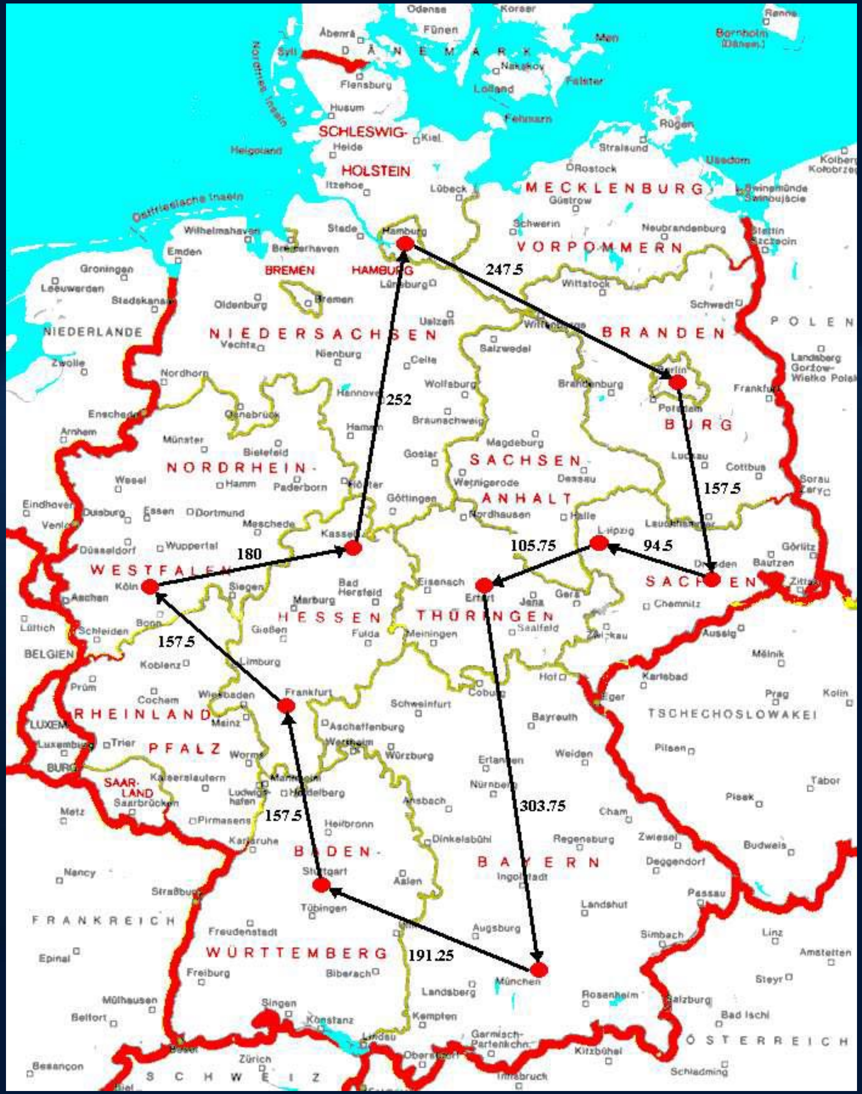
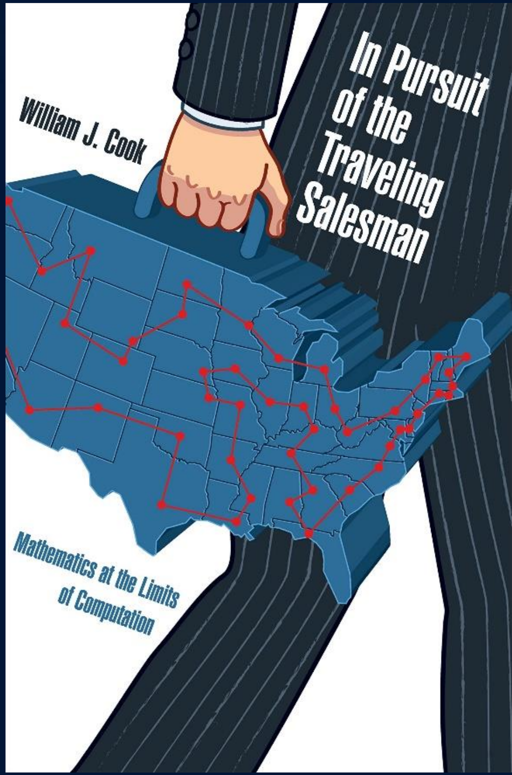
# Traveling Salesman Problem

Task

**Given a map, compute a minimum-cost round trip visiting certain cities**
**Shortest paths graph reduction**

precompute all-pairs-shortest-paths
with Dijkstra's SSSP algorithm



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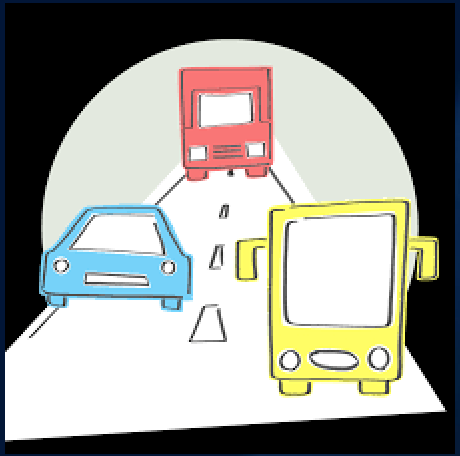
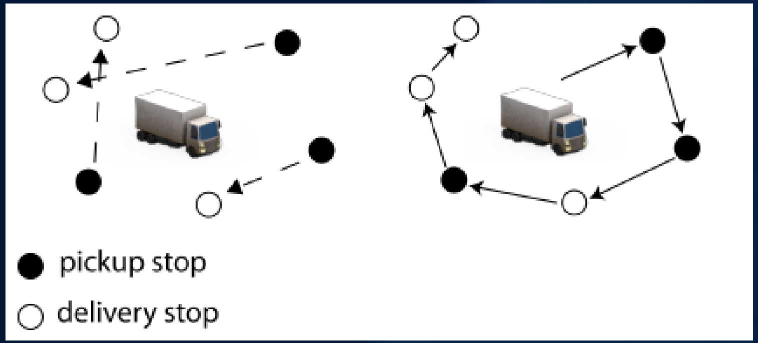
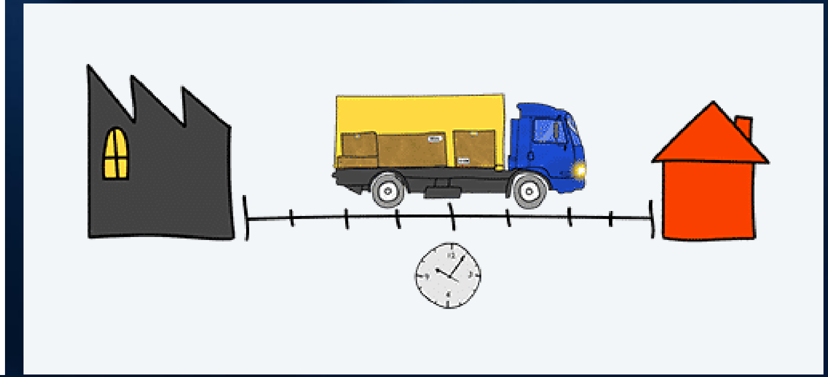
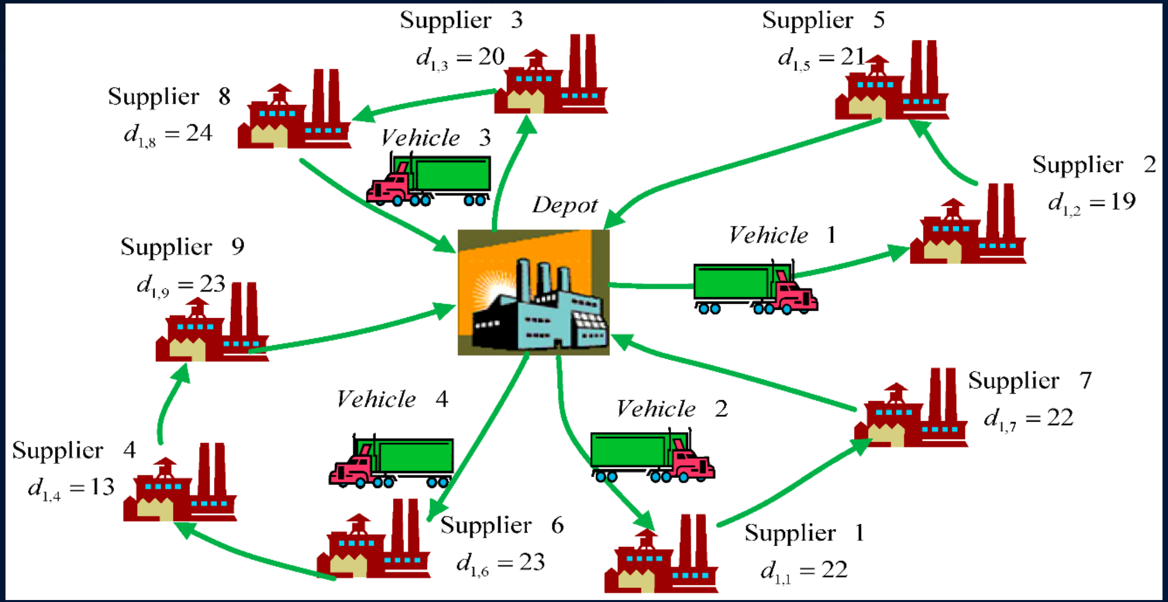
# TSP Variants








**Time Windows, Capacities, Premium Services, Pickup and Deliveries**

TSP+TW: Restricted time intervals / service times
C+TSP: Limited vehicle load
TSP+PD: Pickup and deliveries (PDP)
TSP*: Premium service – same-day delivery preferred
VRP: Vehicle routing – several vehicles
…

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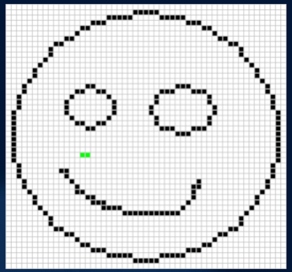
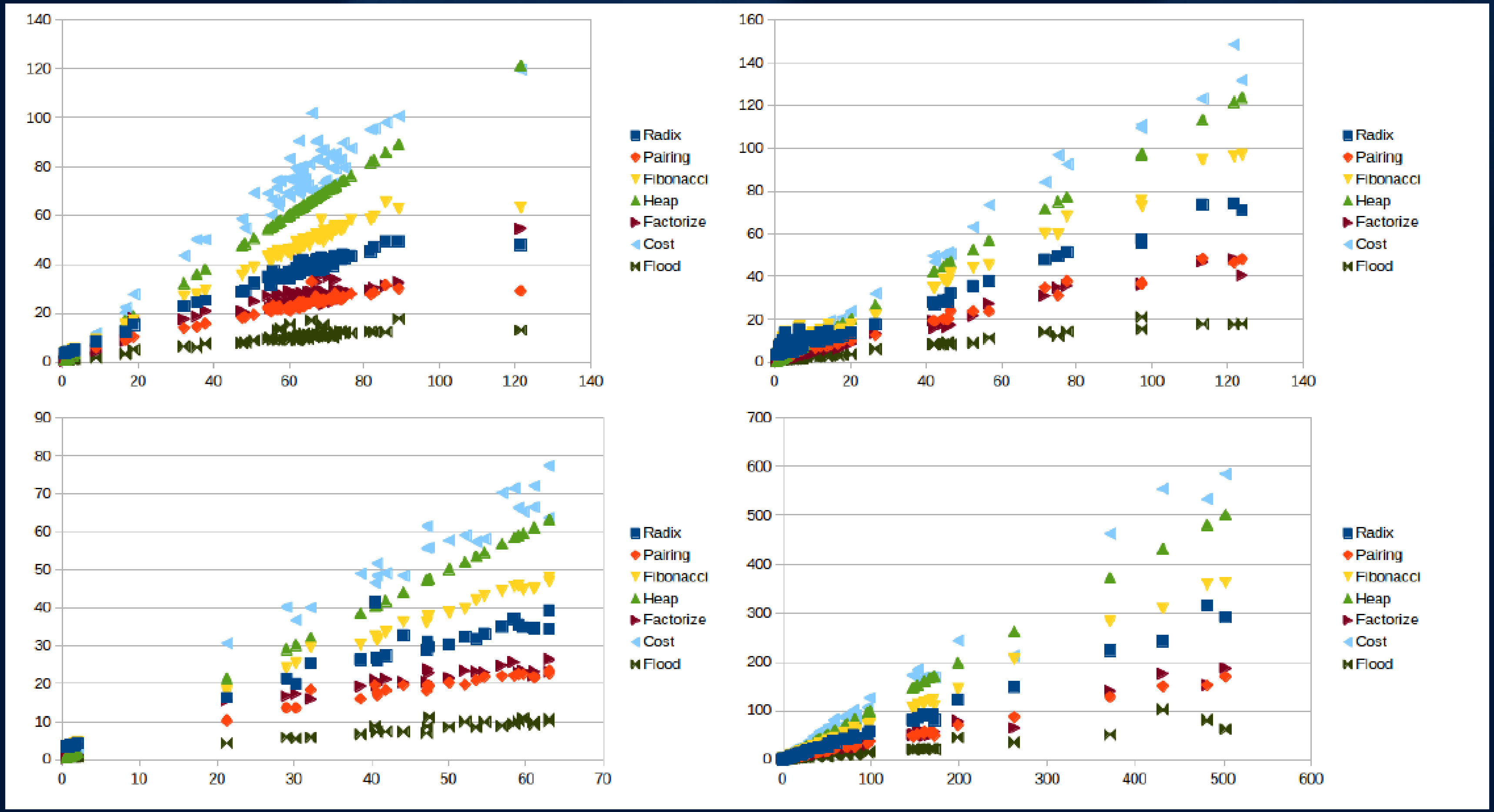
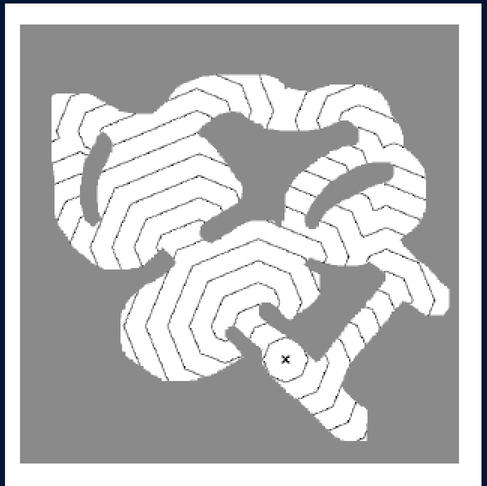


# Shortest Paths




## Cache-efficient Implementation of Dijkstra‘s algorithm



Time in milliseconds for shortest paths search in the game maps of Baldur’s Gate II, Starcraft, Warcraft III,and Dragon Age using scatter plots wrt the performance of shortest paths search with heaps.

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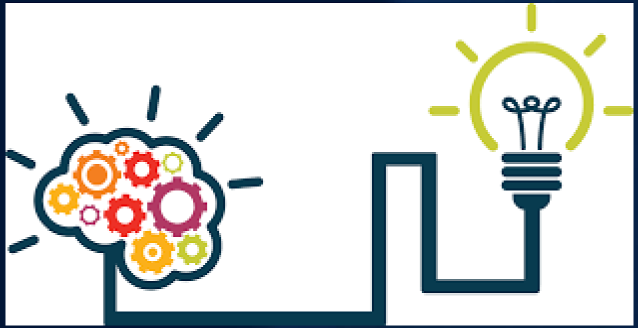
# Solving TSPs



Given a distance matrix, compute a minimum-cost trip

## Traditional

Model problem as an IP and call solver (CPLEX, IPSolve,. . . )

Neighborhood search (xOPT: SA; GA; AA; PSO; LNS,. . . )

**Depth-First Branch and Bound** with

**DFBnB**0 No Heuristic – incremental O(1) time

**DFBnB**1 Column/Row Minima – incremental O(1) time

**DFBnB**2 Assignment Problem – incremental $O(n^2)$ time

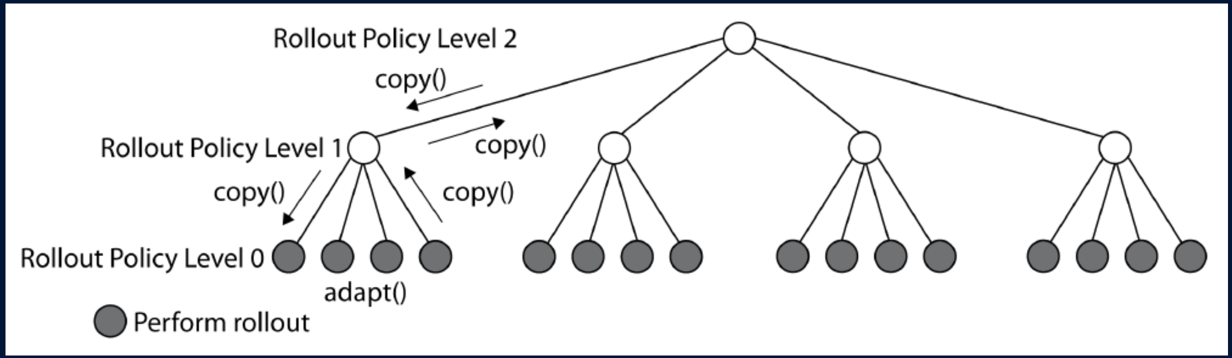
…

## NRPA (Reinforcement Learning)

**Nested-Monte-Carlo Tree Search (with Policy Adaptation)**

**Input:** Iteration **width** (exploitation), **nestedness** (exploration)

**Policy:** (city-to-city) Mapping IN x IN -> IR to be learnt



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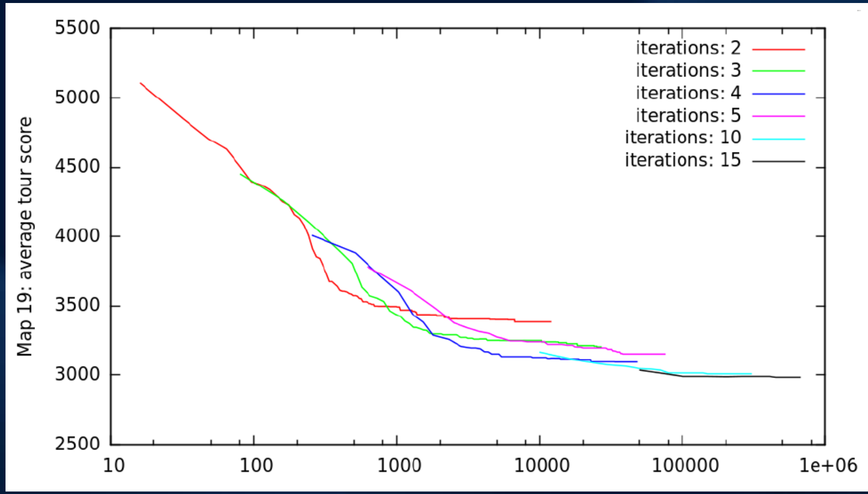
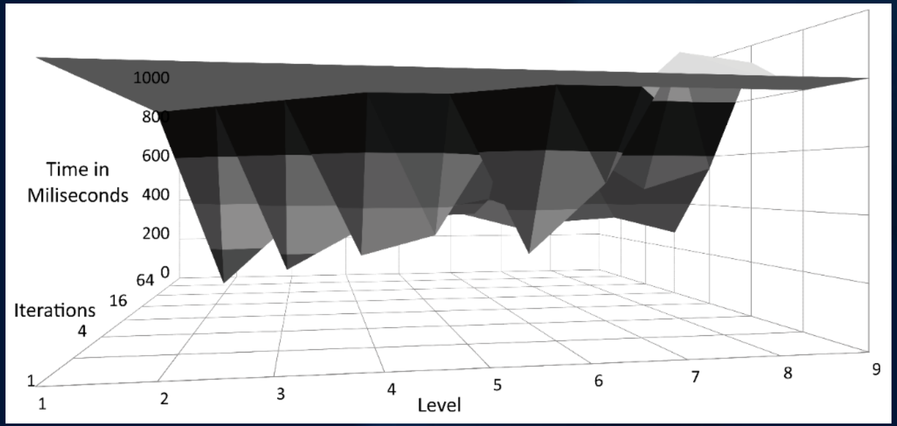
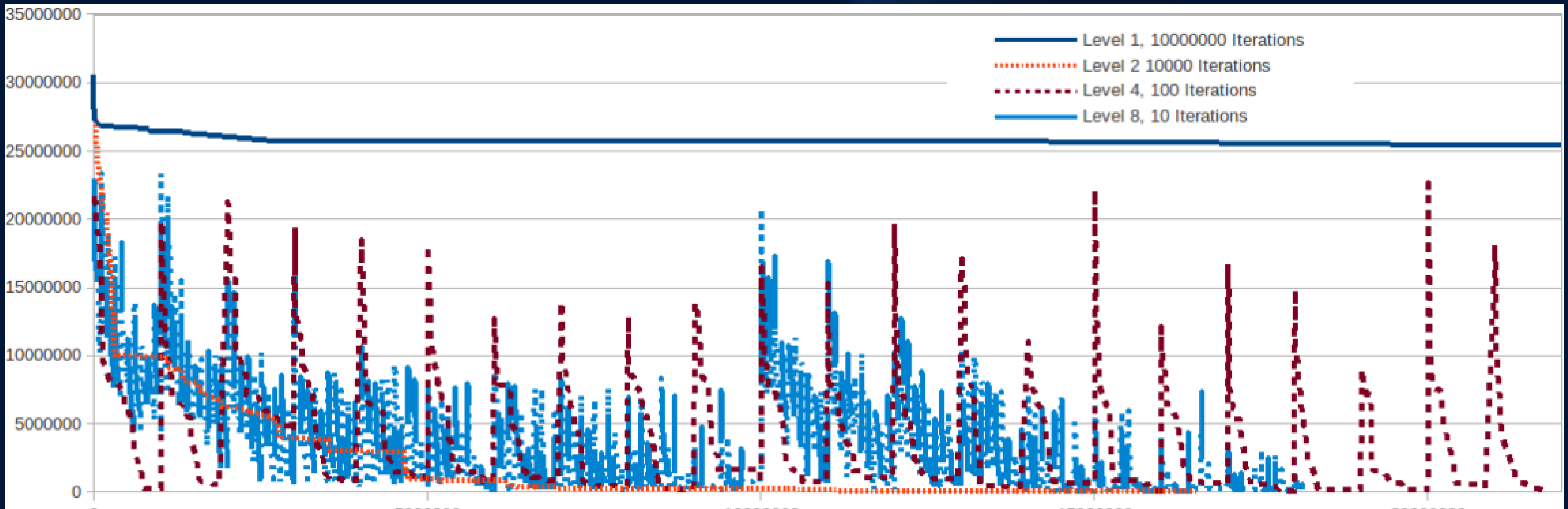
# Planning & Optimization with NRPA

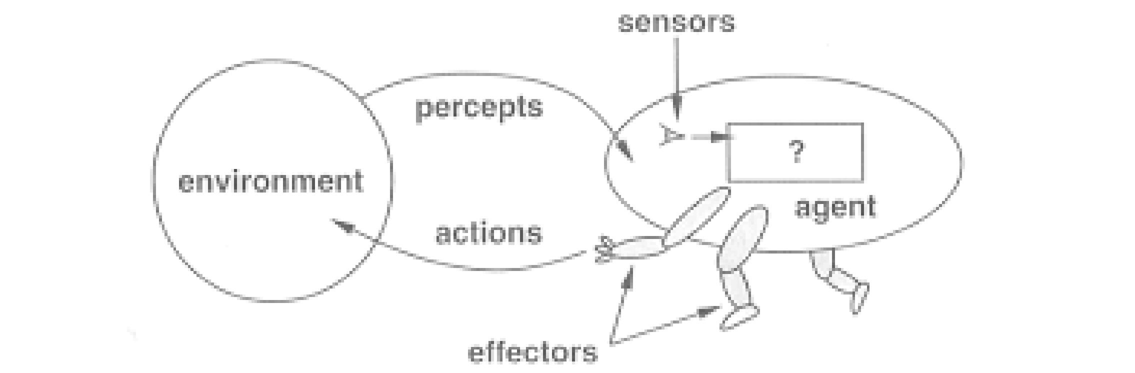


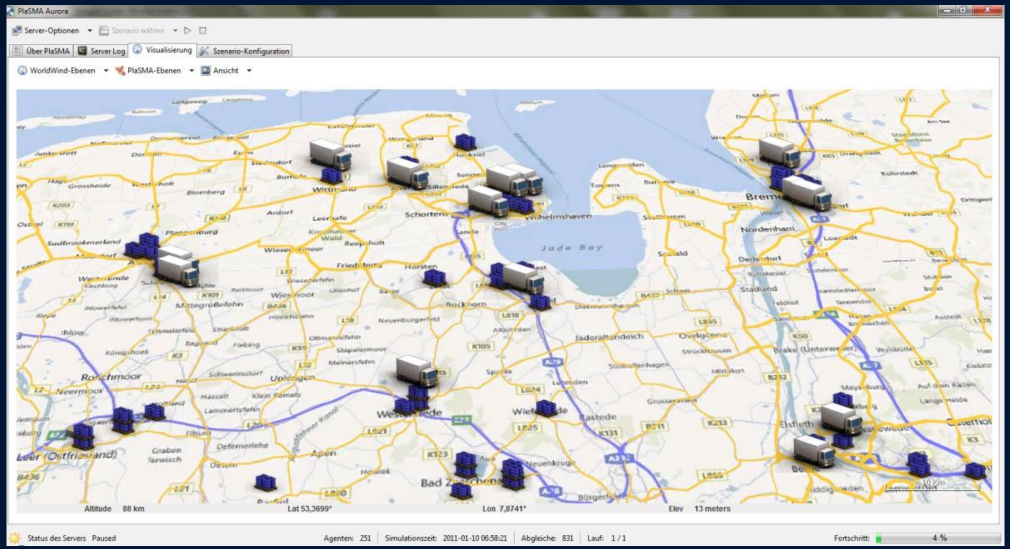




```
Tour search(level)
  best = new Tour();
  best.score = MAXVALUE;
  if (level == 0)
    best.score = rollout();
    best.tour = tour;
  for (int i = 0; i < ITERATIONS; i++)
    Tour r = search(level - 1);
    if (r.score < best.score)
      best.score = r.score;
      best.tour = r.tour;
      adapt(best.tour,level);
  return best;
```



# Integration in Multiagent System

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# In Motion

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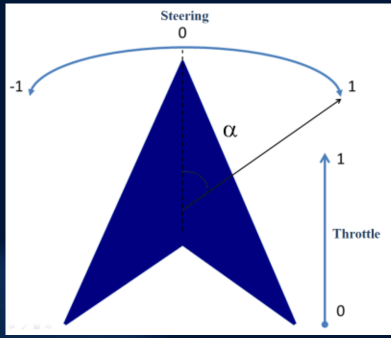
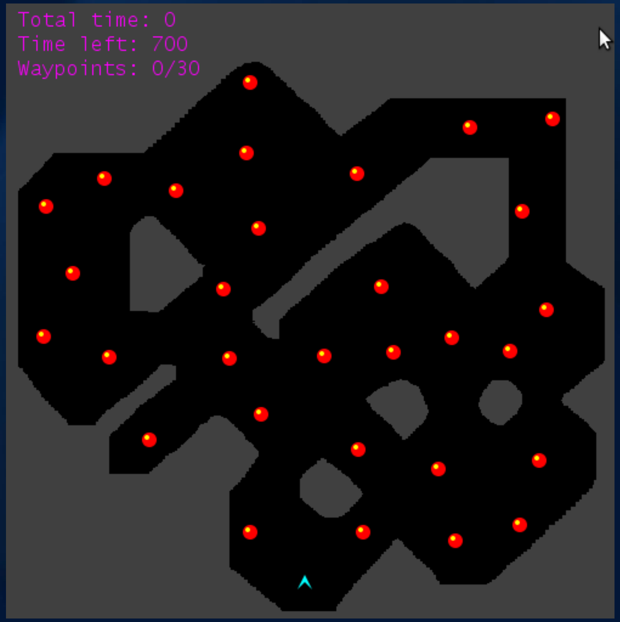
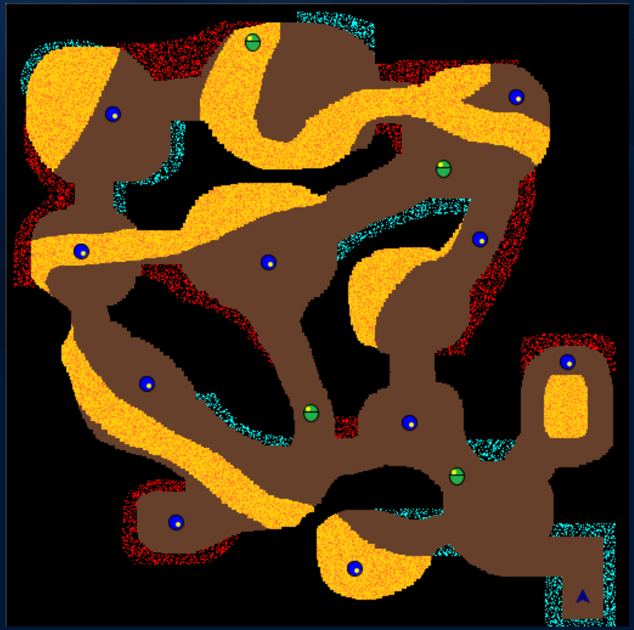



| Action ID | Steering | Throttle |
|---|---|---|
| 0 (*Neutral* action) | *Straight* (0) | *Off* (0) |
| 1 | *Left* (-1) | *Off* (0) |
| 2 | *Right* (1) | *Off* (0) |
| 3 | *Straight* (0) | *On* (1) |
| 4 | *Left* (-1) | *On* (1) |
| 5 | *Right* (1) | *On* (1) |

# Physical Traveling Salesman Problem



The main purpose of the PTSP is to provide a benchmark for combined task and motion planning in interactive computer games.

*To reach waypoint locations, the robot often has to avoid numerous obstacles. Planning such motions requires taking into account the robot geometry as well as constraints imposed by its dynamics.*

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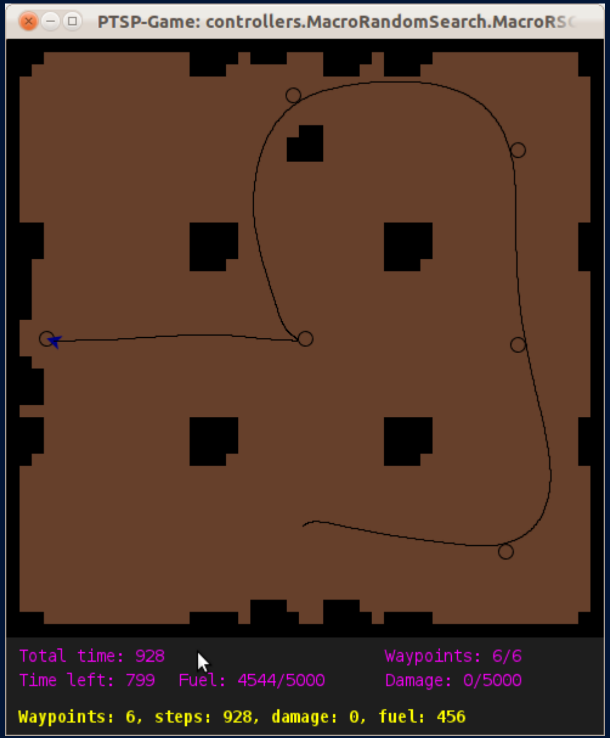
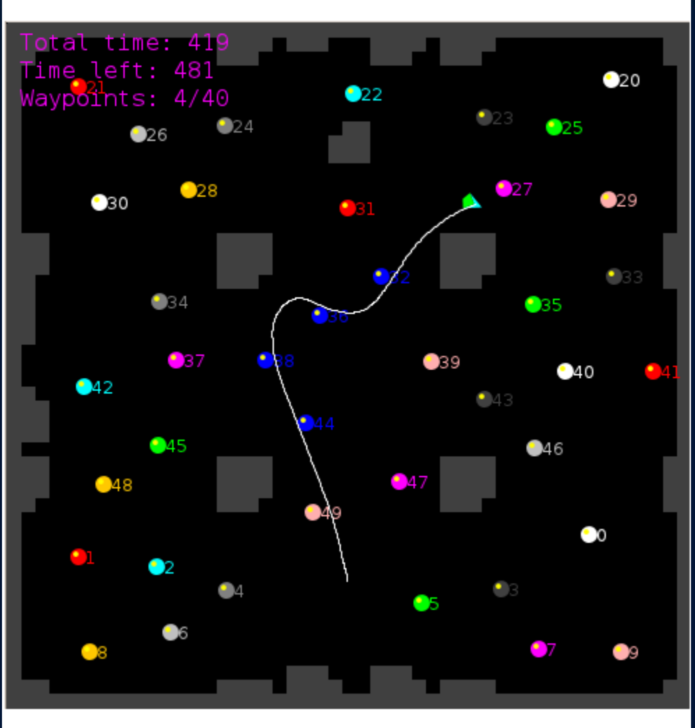
# Introducing Colors

## Clustered TSPs and **Generalized TSPs**

Either only one of each color or all of one color needs to be visited in sequence, minimizing traveling time







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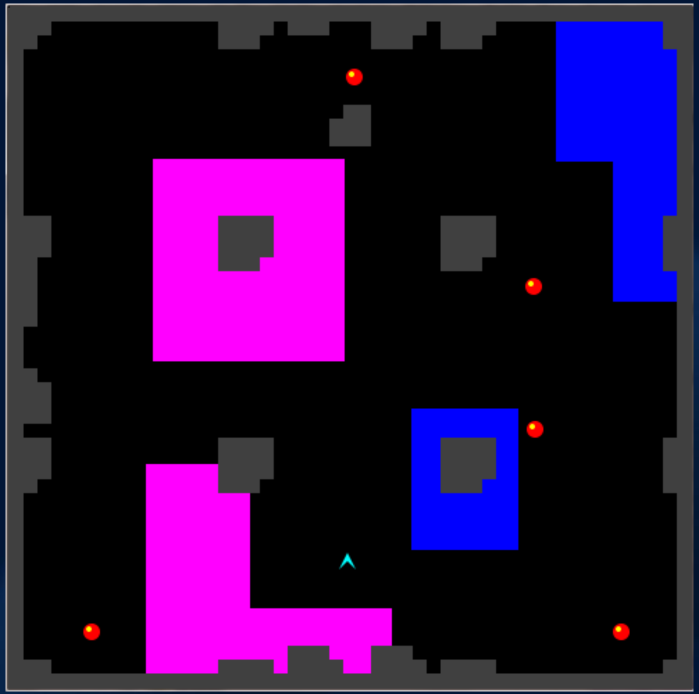
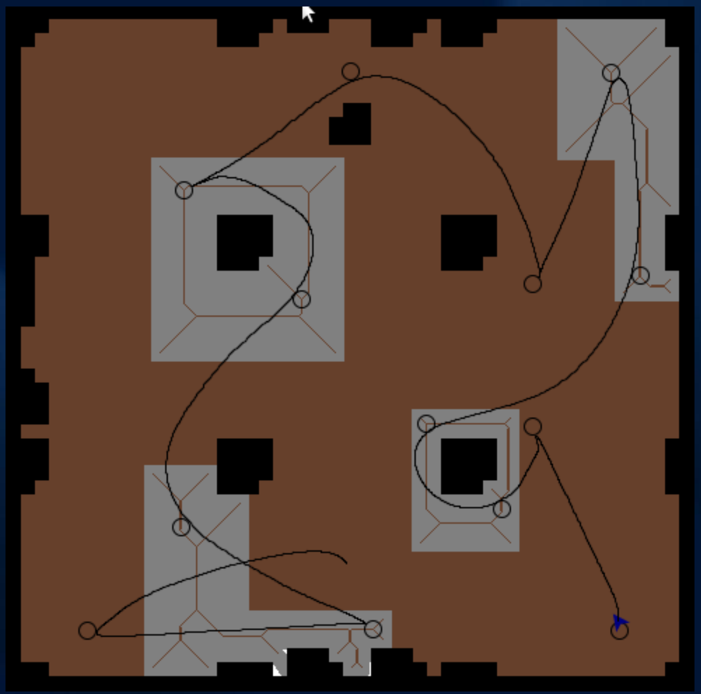
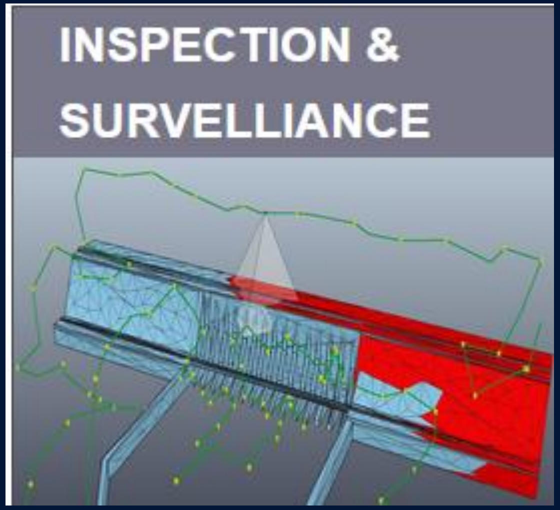
# Inspection








As defects to objects such as pipeline leakage can result in tremendous economical loss, the inspection problem is one of the most important problems in robotics.

Inspection poses significant challenges since the robot needs to determine and reach a set of locations whose combined visibility covers the inspection area.

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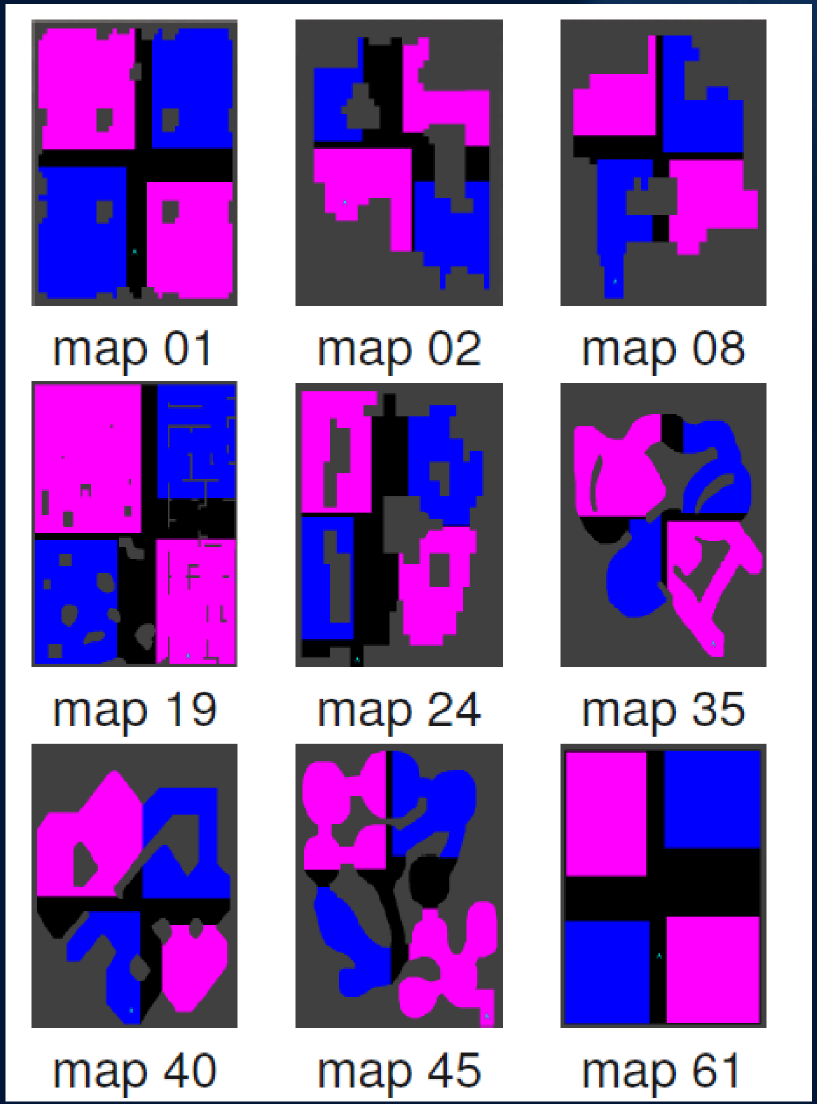
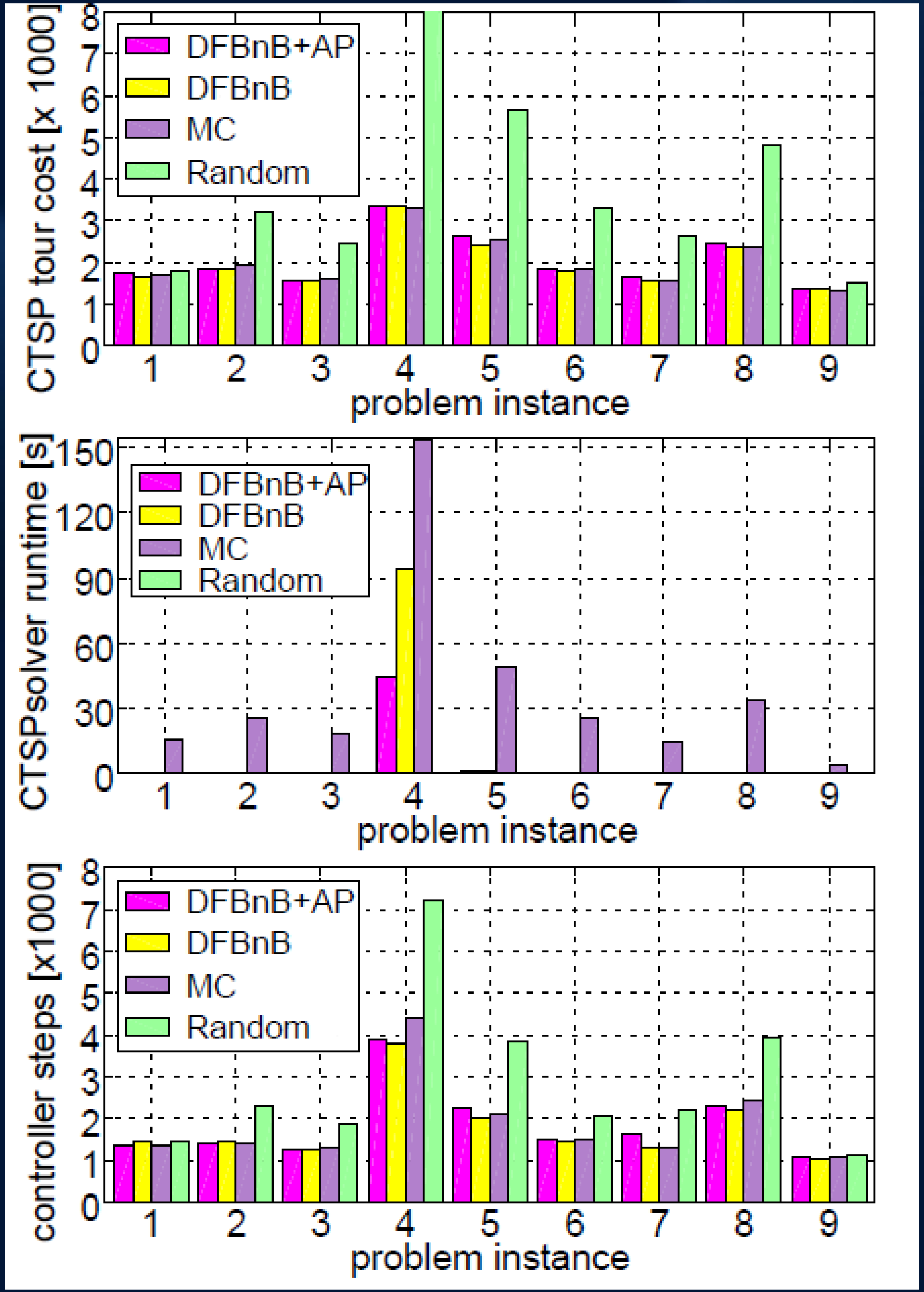
# Results



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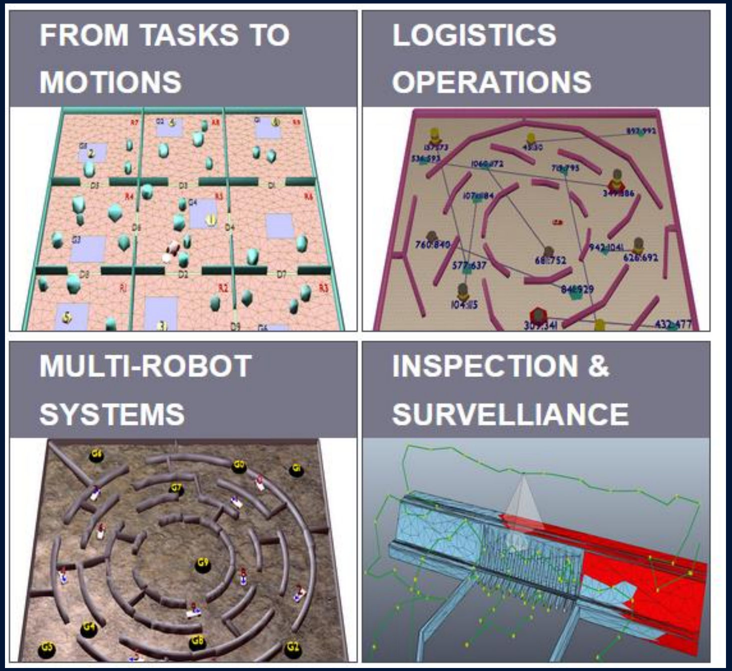
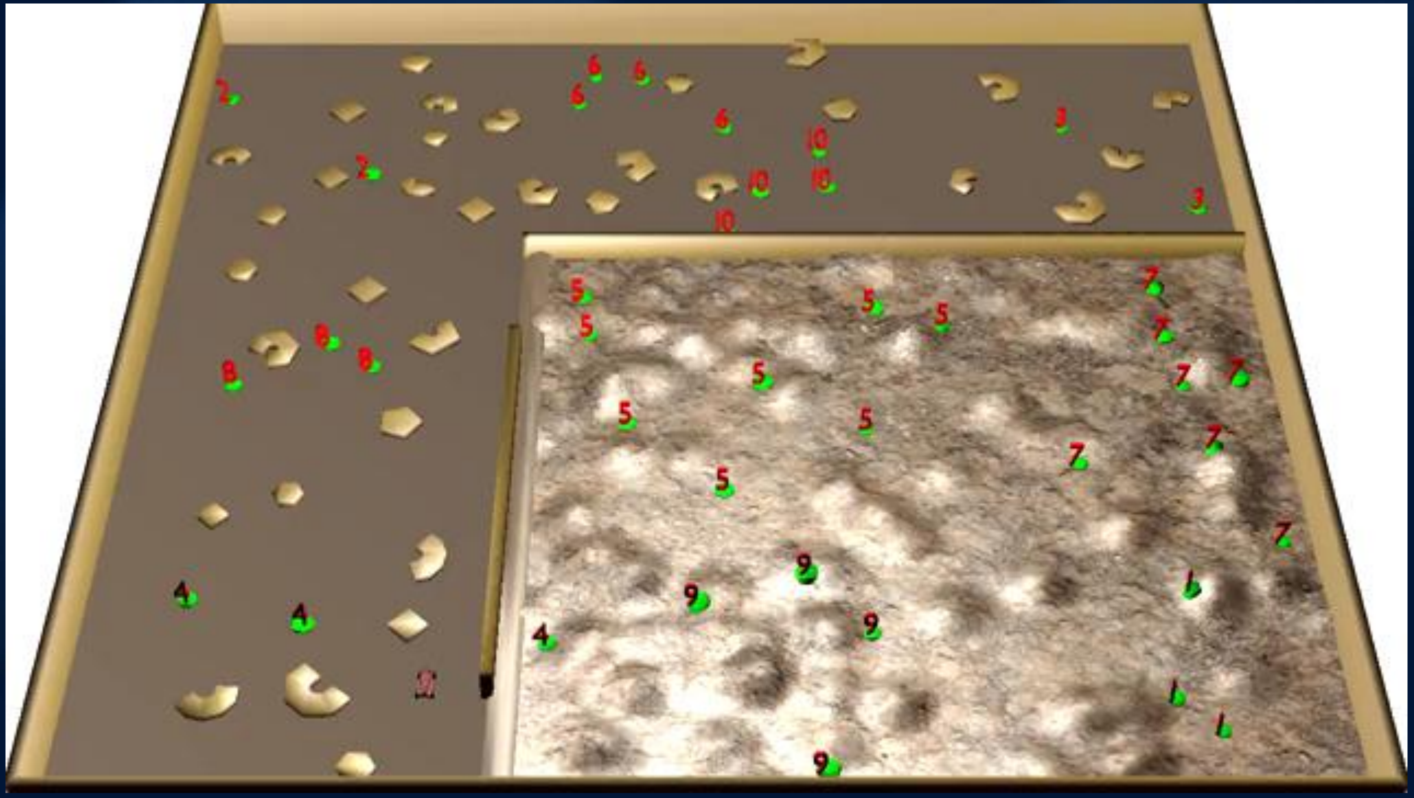
# Integration into the „Framework“



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# Approach

**Sampling-based motion planning**

* generality: dynamics as black-box function snew ← MOTION(s,u,dt)
* continuous state/control spaces: probabilistic sampling to make it feasible
* high-dimensionality: search to find solution

**coupled with discrete abstractions**

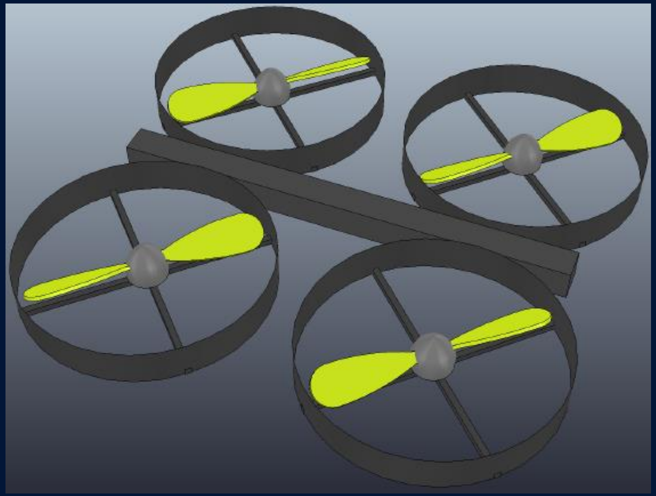
* provide simplified planning layer
* guide search in the continuous state/control spaces

**and motion controllers**

* open up the black-box MOTION function
* facilitate search expansion

**Formal guarantees**

* probabilistic completeness



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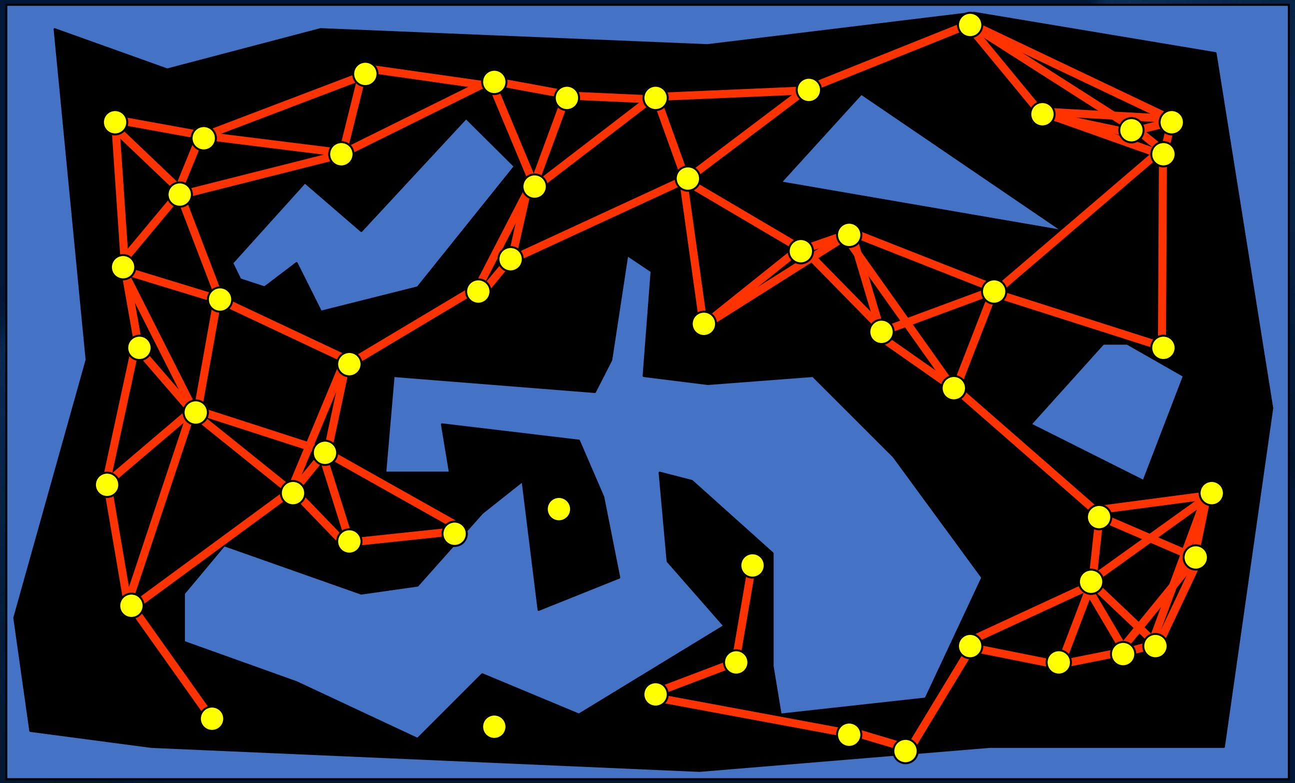
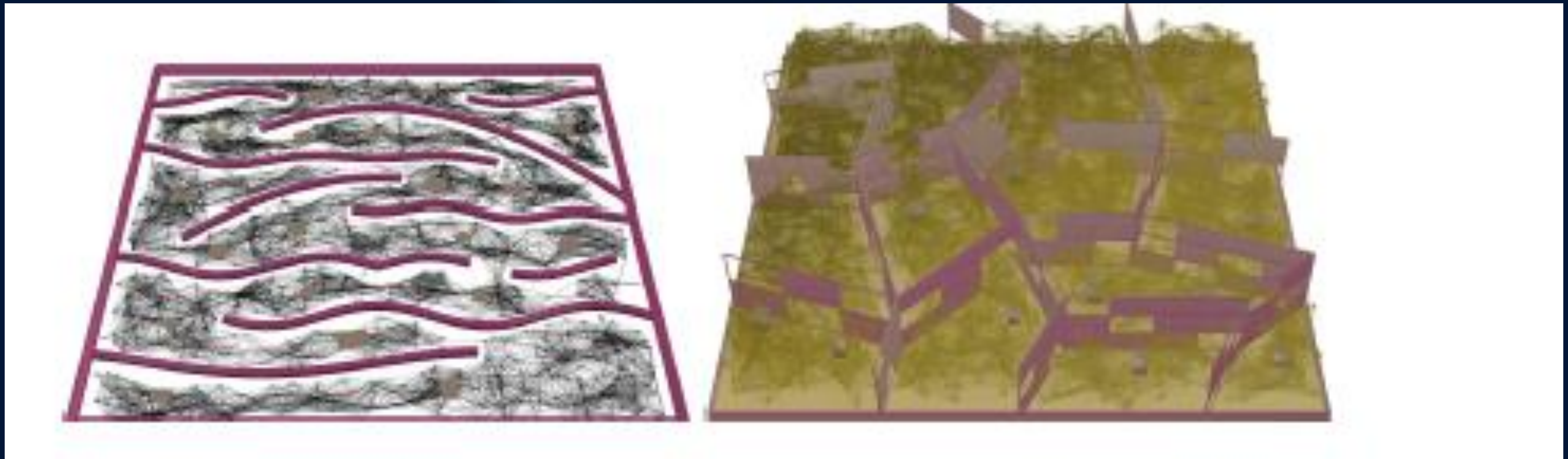
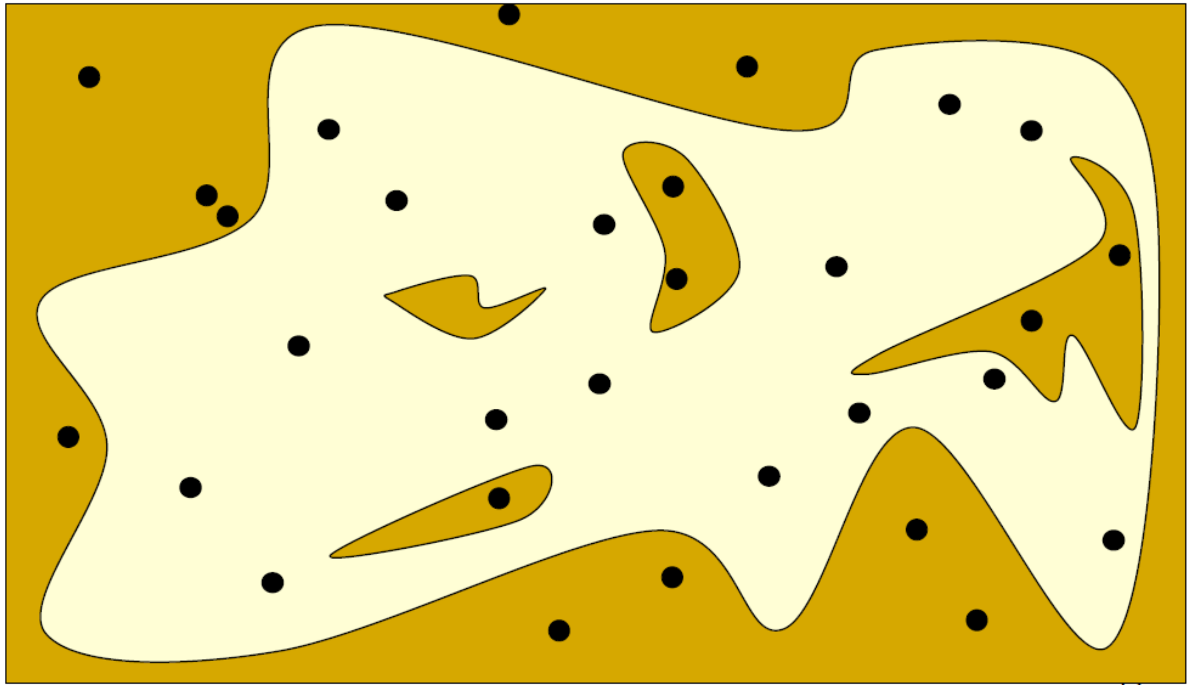
# Probabilistic Roadmap

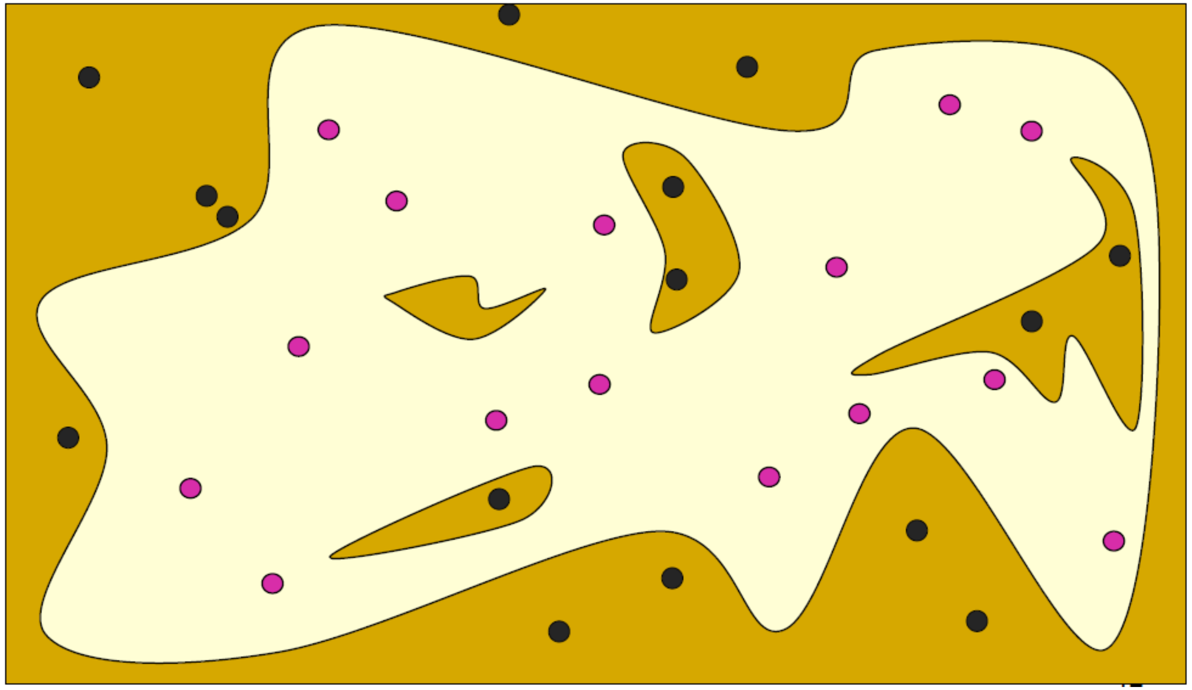




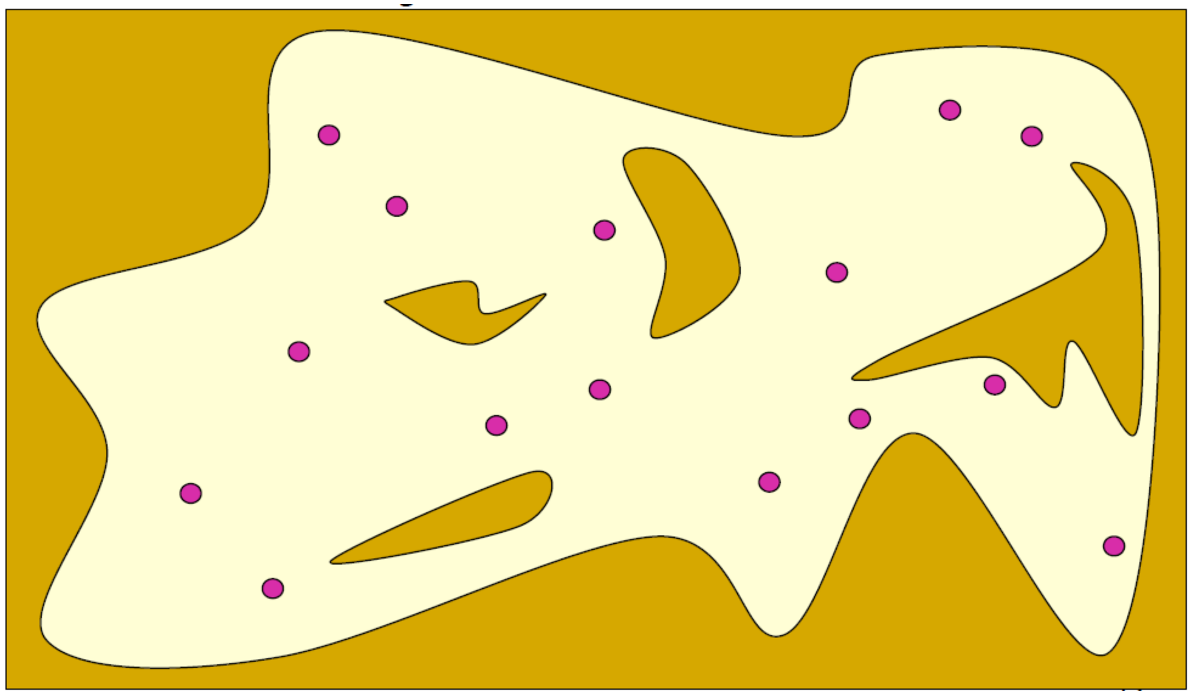
# PRM: (Uniform) Sampling



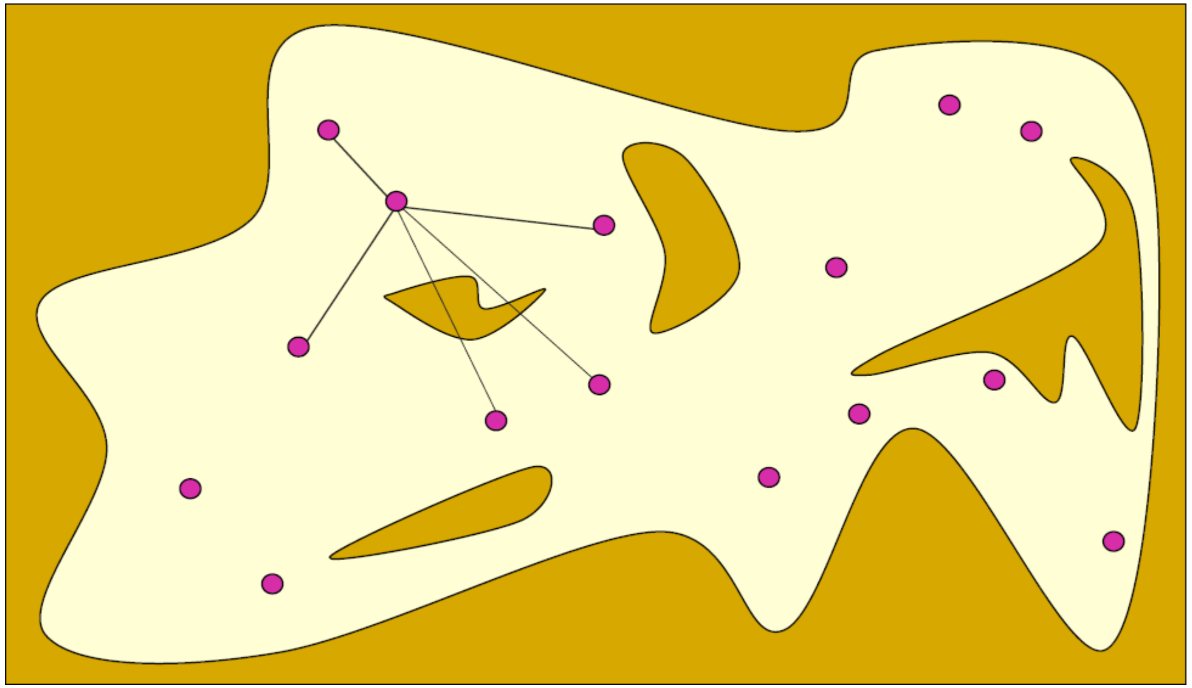
# PRM: Valid Nodes



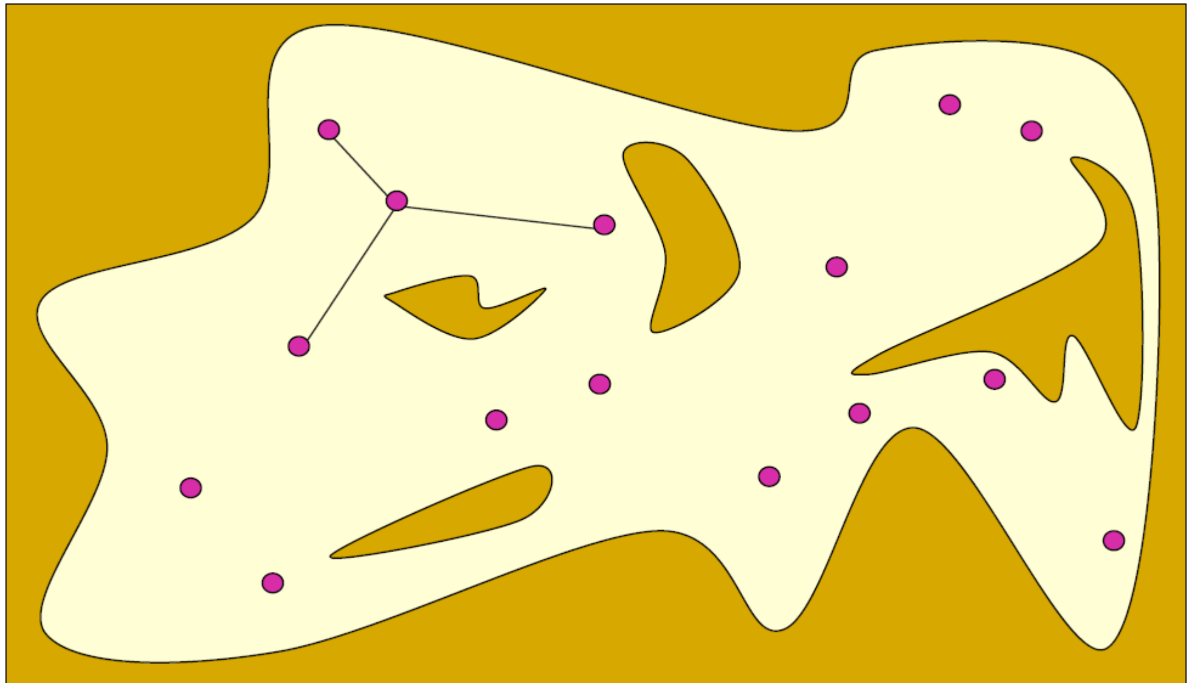
# PRM: Filtering Edges



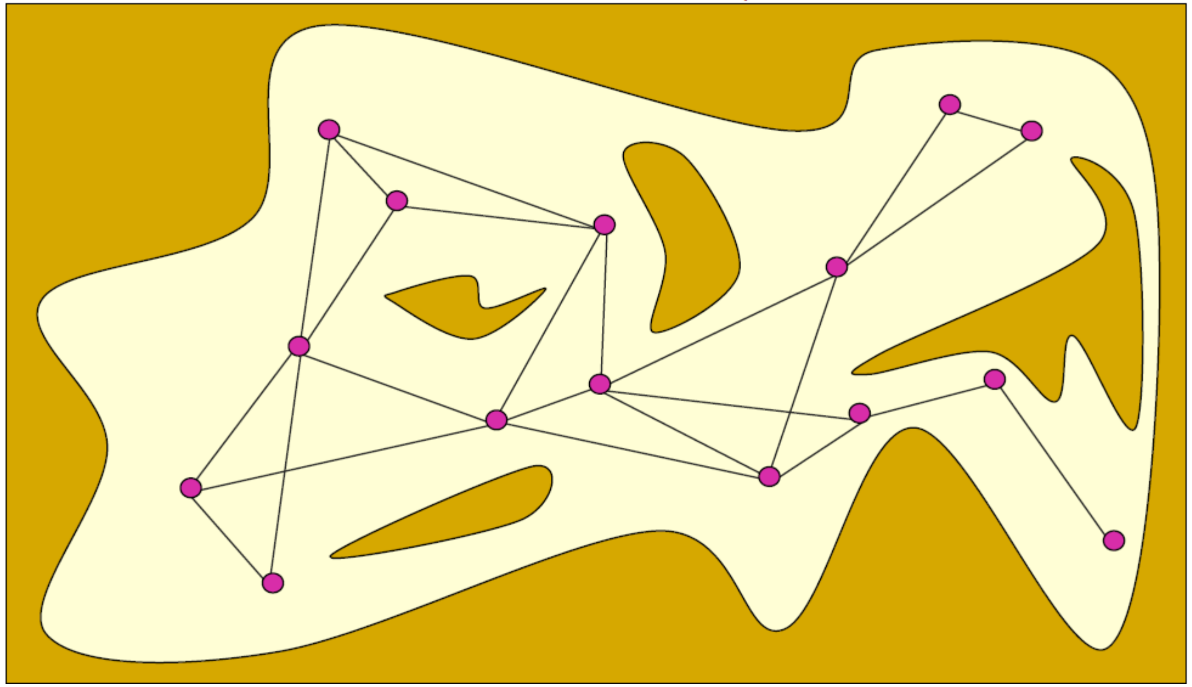
# PRM: Connect Nearest Neighbors



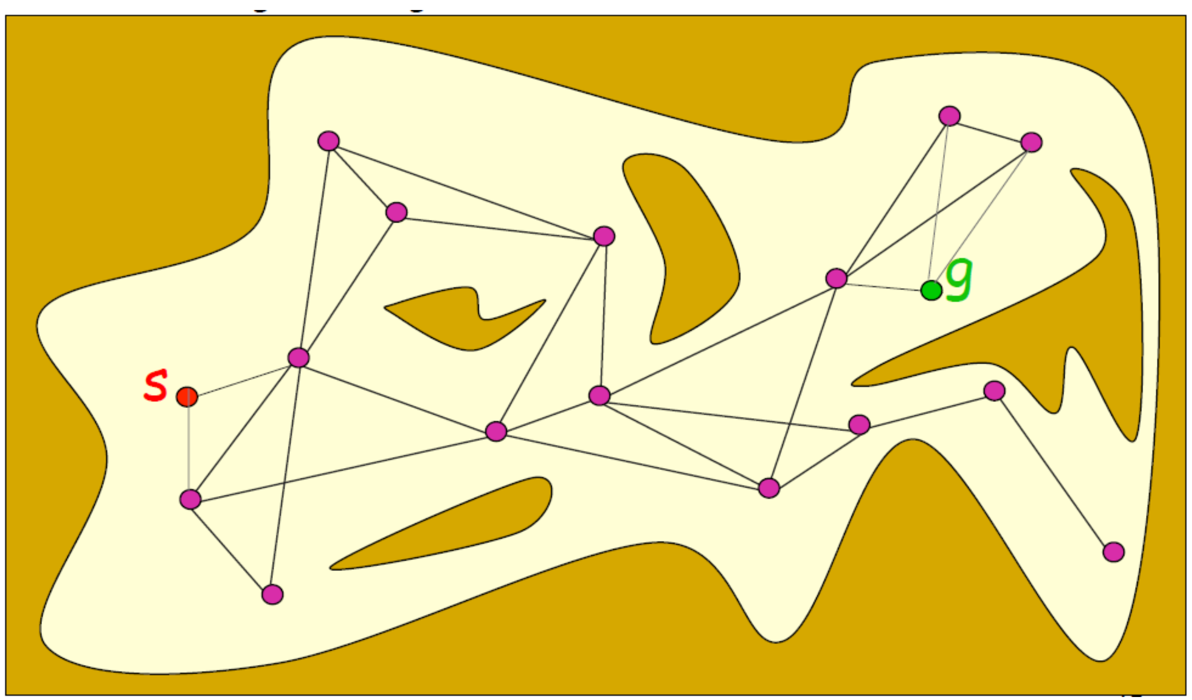
# PRM: Filtering Edges



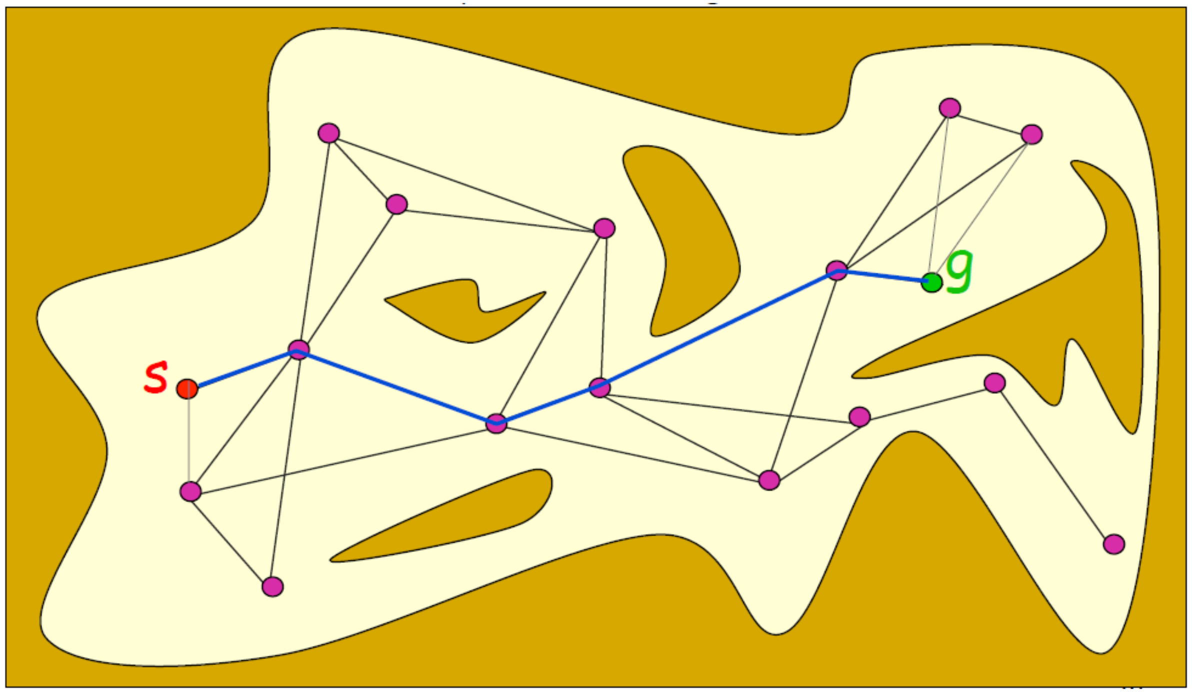
# PRM: Entire Graph



# PRM: Integrate Start and Goal



# PRM: Path Search



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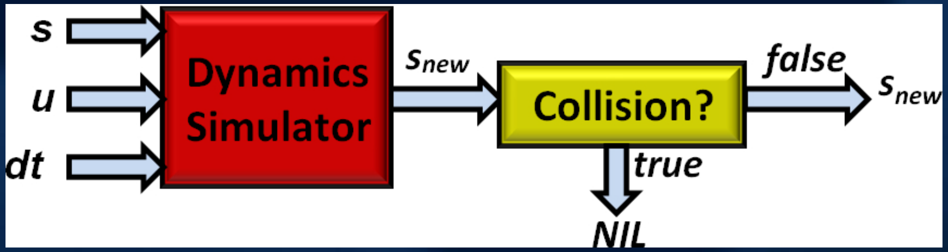



# Dynamics

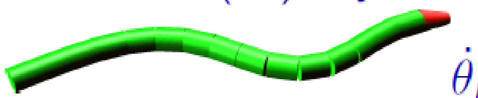
**Express relation between input controls and resulting motions**

**Modeled via physics-based engines**

$$\dot{s} = f(s, u)$$

$$s = (x, y, \theta_0, v, \psi, \theta_1, \ldots, \theta_n) \qquad u = (a, \omega)$$

$$\dot{x} = v\cos(\theta_0) \quad \dot{y} = v\sin(\theta_0) \quad \dot{\theta}_0 = v\tan(\psi) \quad \dot{v} = a \quad \dot{\psi} = \omega$$



$$\dot{\theta}_i = \frac{v}{d}\left(\prod_{j=1}^{i-1}\cos(\theta_{j-1} - \theta_j)\right)(\sin(\theta_{i-1}) - \sin(\theta))$$

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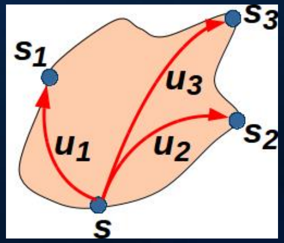
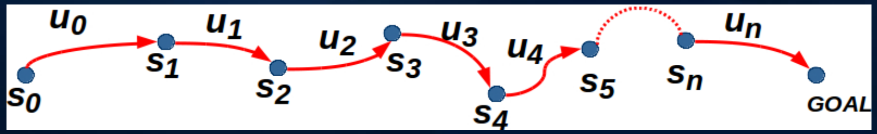
# Dynamics

**Necessary to plan motions that can be executed**
**Impose significant challenges**




- ➢ Constrain the feasible motions
- ➢ Often are nonlinear and high-dimensional
- ➢ Give rise to nonholonomic systems
- ➢ State and control spaces are continuous
- ➢ Solution trajectories are often long

**Computational complexity of motion planning with dynamics**

Point with Newtonian dynamics NP-Hard [DXCR 1993]

Polygon Dubin's car Decidable [CPK 2008]

General nonlinear dynamics Undecidable [Branicky 1995]

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# Introduce Discrete Layer to Guide the Search

Workspace decomposition provides discrete layer as adjacency graph $G = (R,E)$

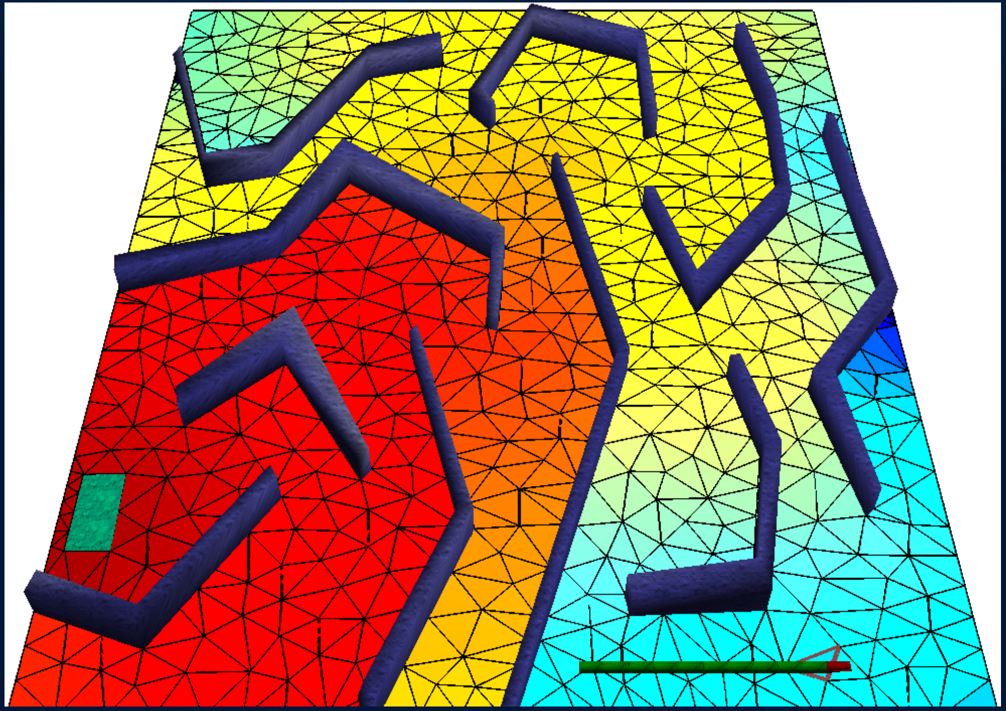
R denotes the regions of the decomposition

$E = \{(r_i,r_j) \mid r_i, r_j \text{ in } R \text{ are physically adjacent}\}$

hcost(r) estimates the difficulty of reaching the goal region from r defined as length of shortest path in $G = (R,E)$ from r to goal

[hcost(r1), hcost(r2),..., hcost(rn)]

computed via BFS/A* on G backwards from goal



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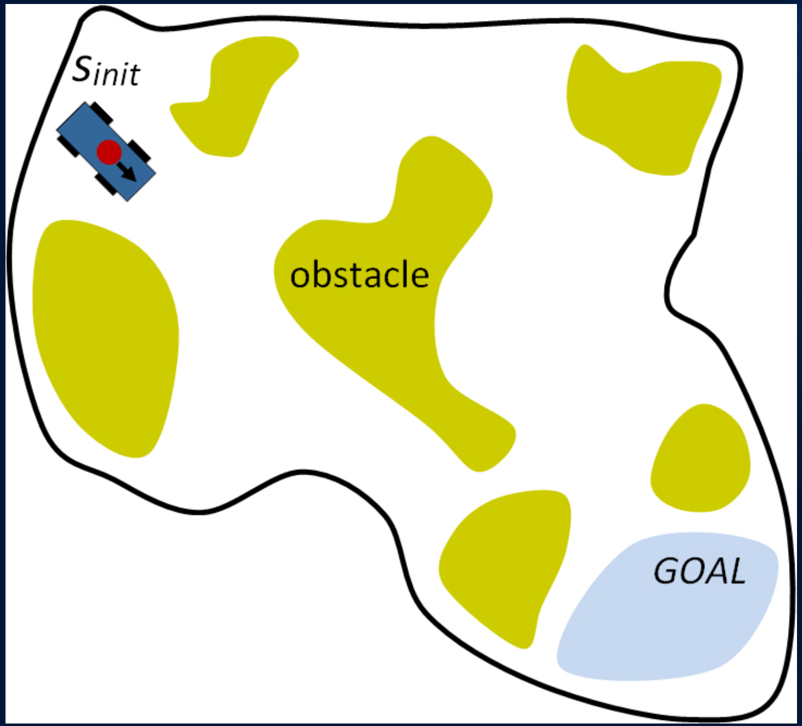
# Sampling Based Motion Planning




Expand a tree T of collision-free and dynamically-feasible motions

- select a state s from which to expand the tree
- sample control input u
- generate new trajectory by
- applying u to s

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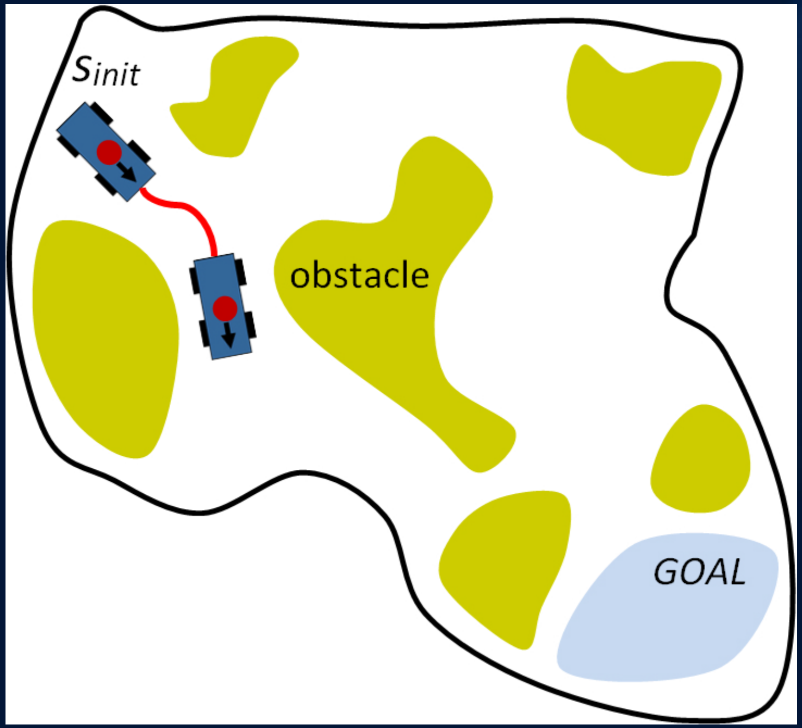
FEE CTU

# Sampling Based Motion Planning




Expand a tree T of collision-free and dynamically-feasible motions

- select a state s from which to expand the tree
- sample control input u
- generate new trajectory by
- applying u to s

delkamp

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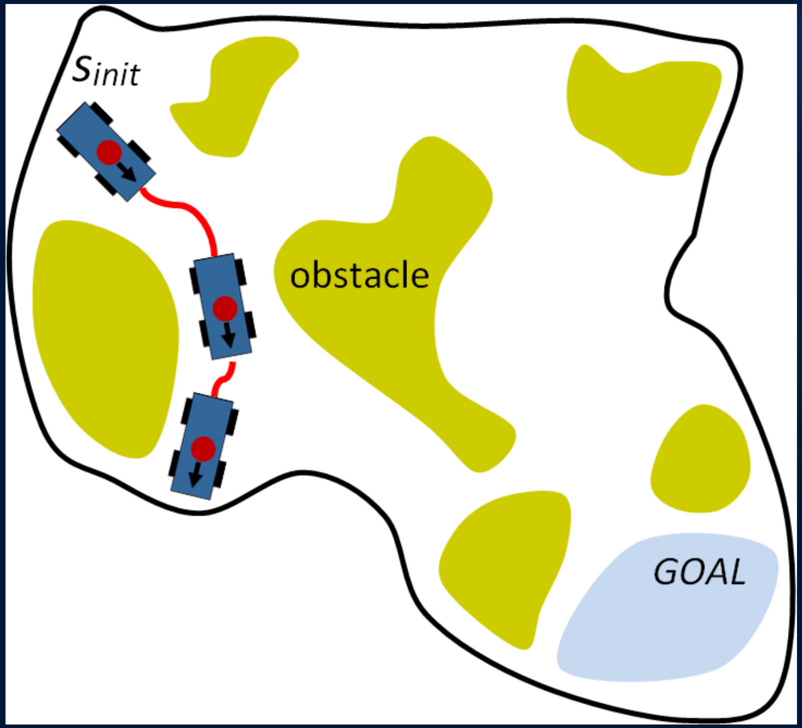
# Sampling Based Motion Planning




Expand a tree T of collision-free and dynamically-feasible motions

- select a state s from which to expand the tree
- sample control input u
- generate new trajectory by
- applying u to s

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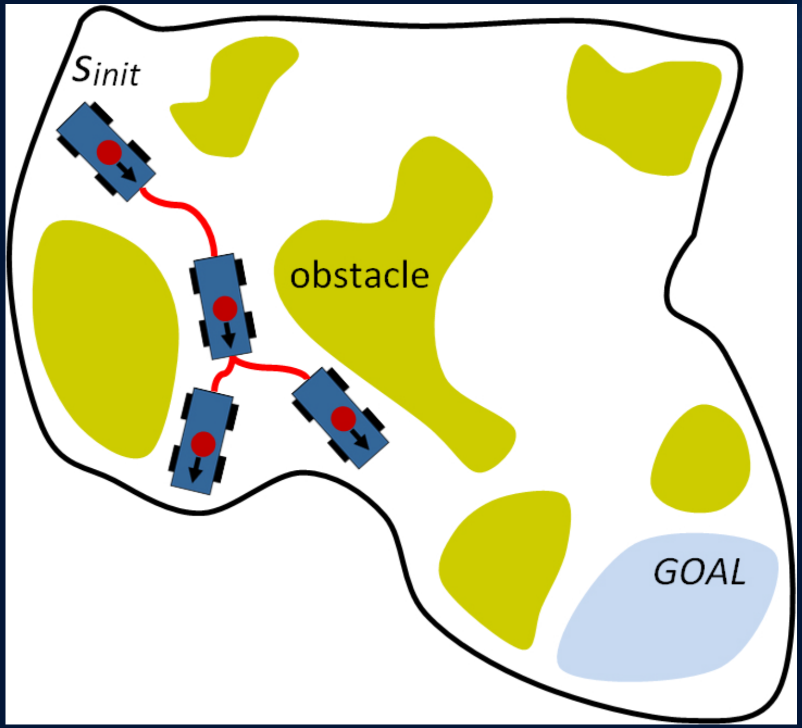
FEE CTU

# Sampling Based Motion Planning




Expand a tree T of collision-free and dynamically-feasible motions

- select a state s from which to expand the tree
- sample control input u
- generate new trajectory by
- applying u to s

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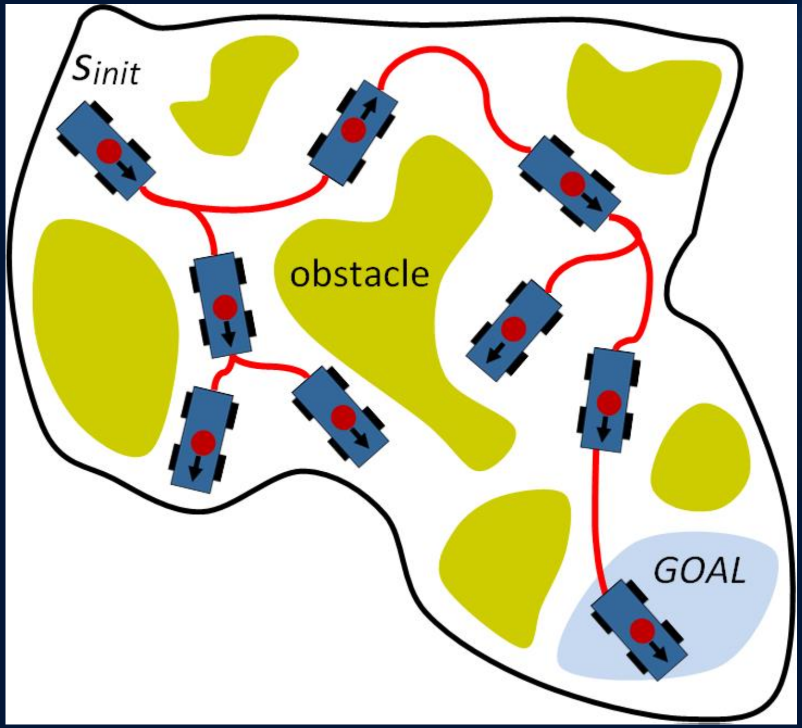
# Sampling Based Motion Planning




Expand a tree T of collision-free and dynamically-feasible motions

- select a state s from which to expand the tree
- sample control input u
- generate new trajectory by
- applying u to s

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# Guided Expansion of Motion Tree

selecting an equivalence class
from which to expand motion tree T

sampling-based motion planning to expand T

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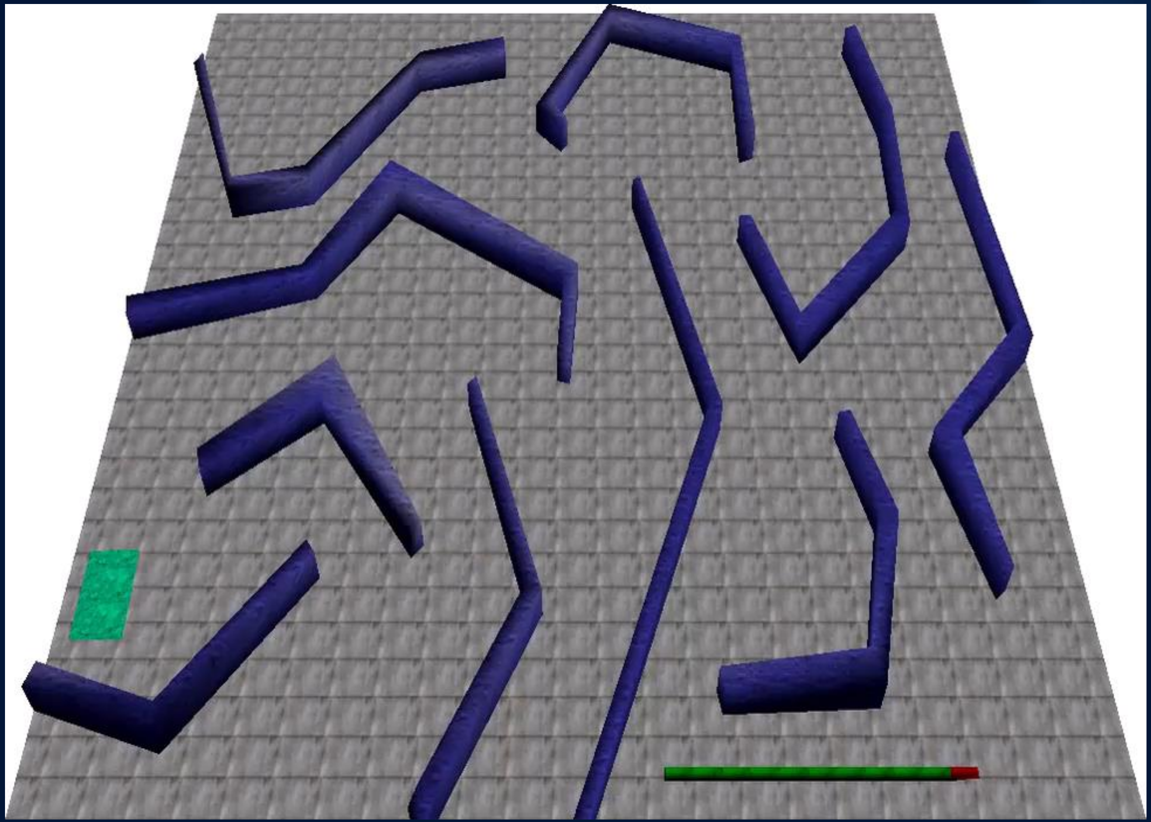
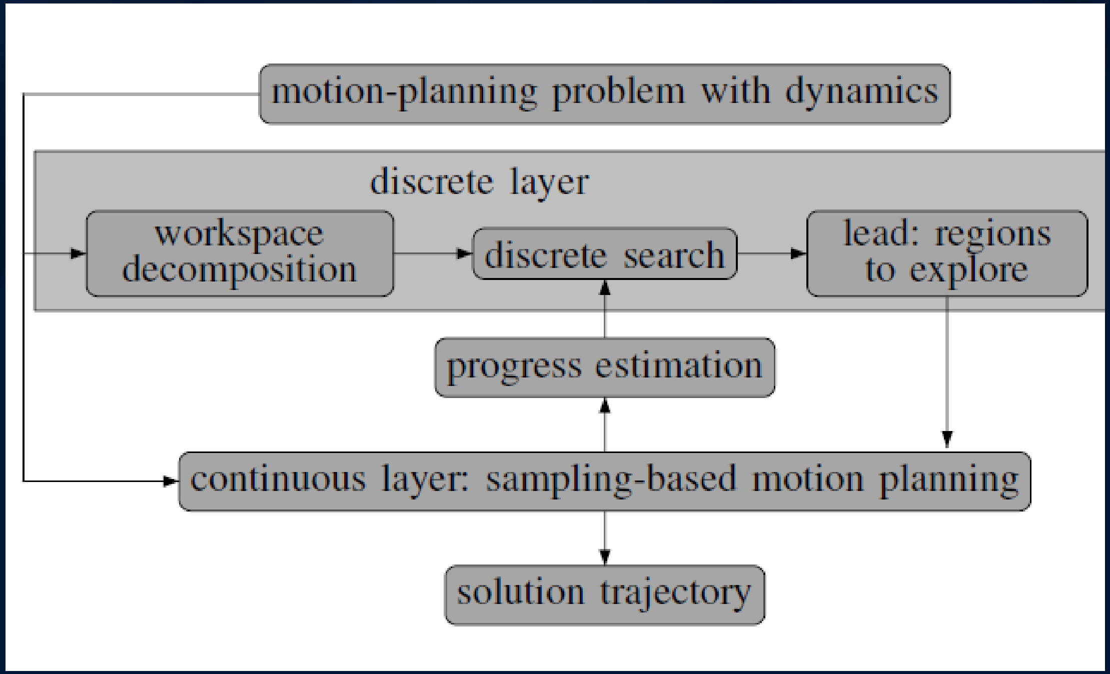
# Architecture





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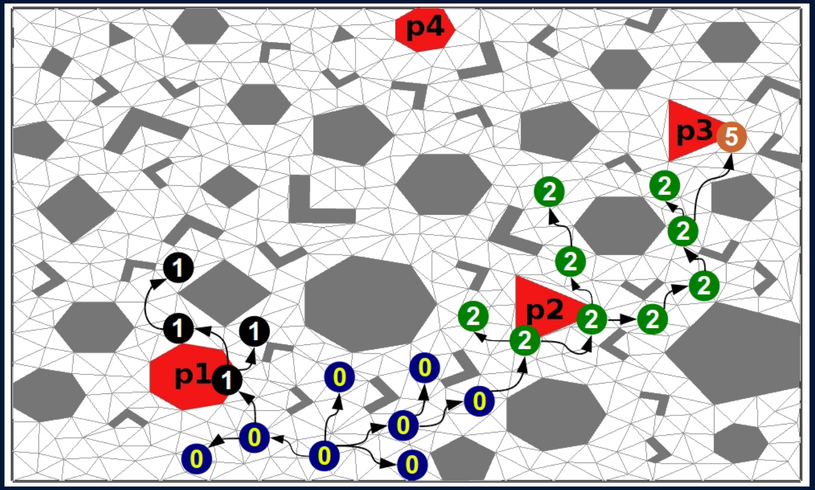
# Abstraction




**Used to induce partition of motion tree into equivalence classes**

vi = vj
iff
TRAJ(T,vi) provides same abstract information as TRAJ (T,vj)
iff
region(vi) = region(vj)

➔ equivalence class corresponding to abstract state <r>
Γ<r> = {v | v in T and region(v) = r}
➔ partition of motion tree T into equivalence classes
Γ = {Γ <r> : Γ <r> > 0}

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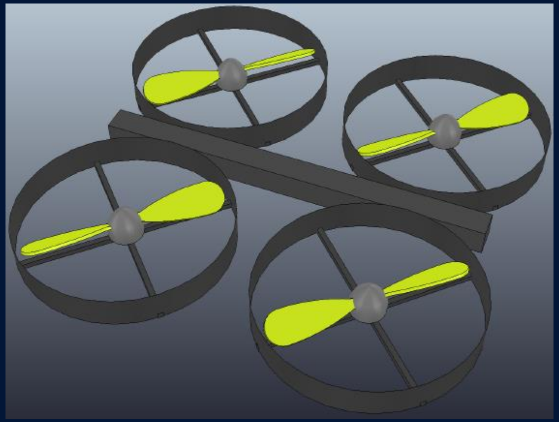
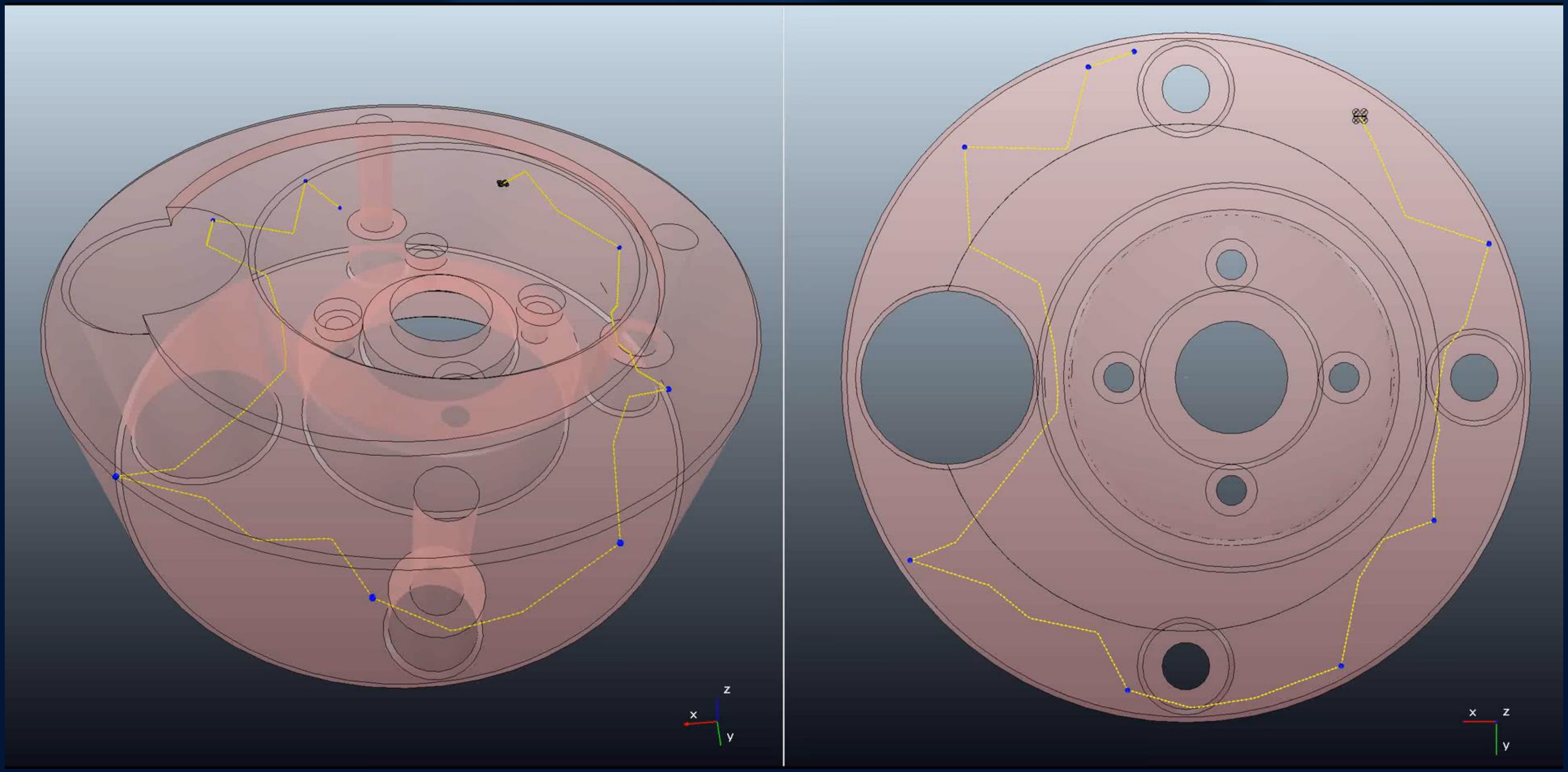
# Inspection



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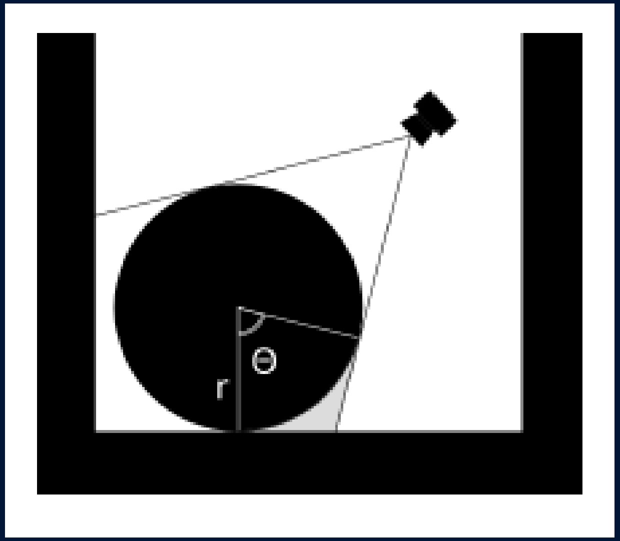
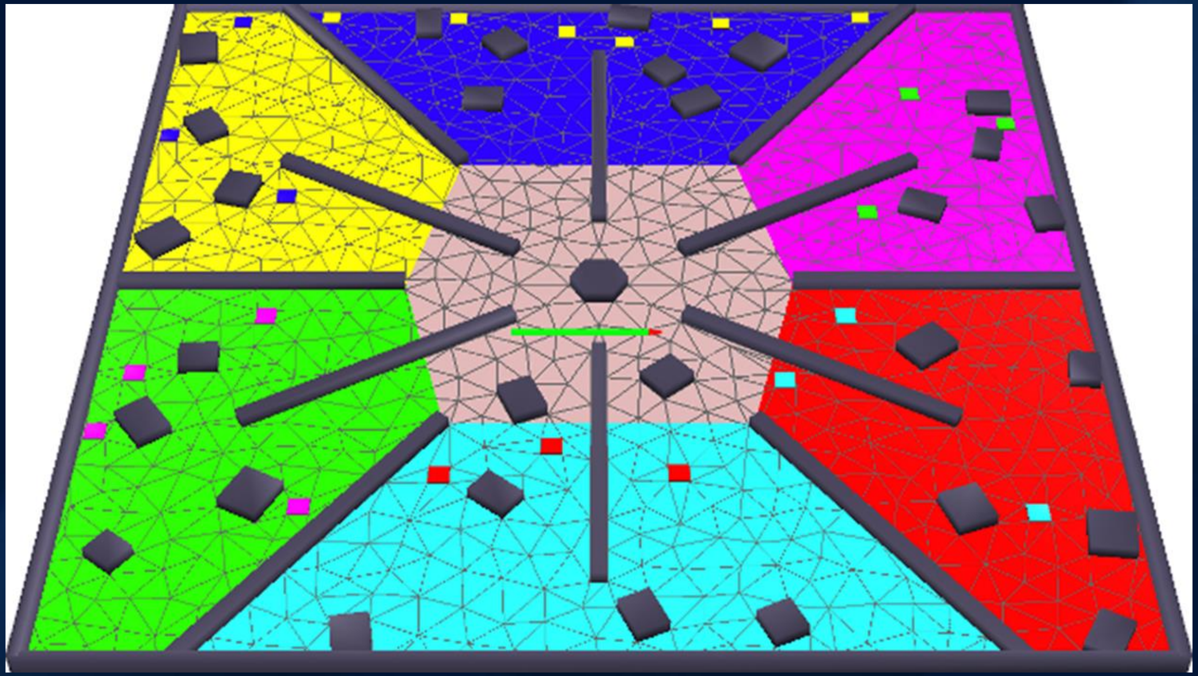
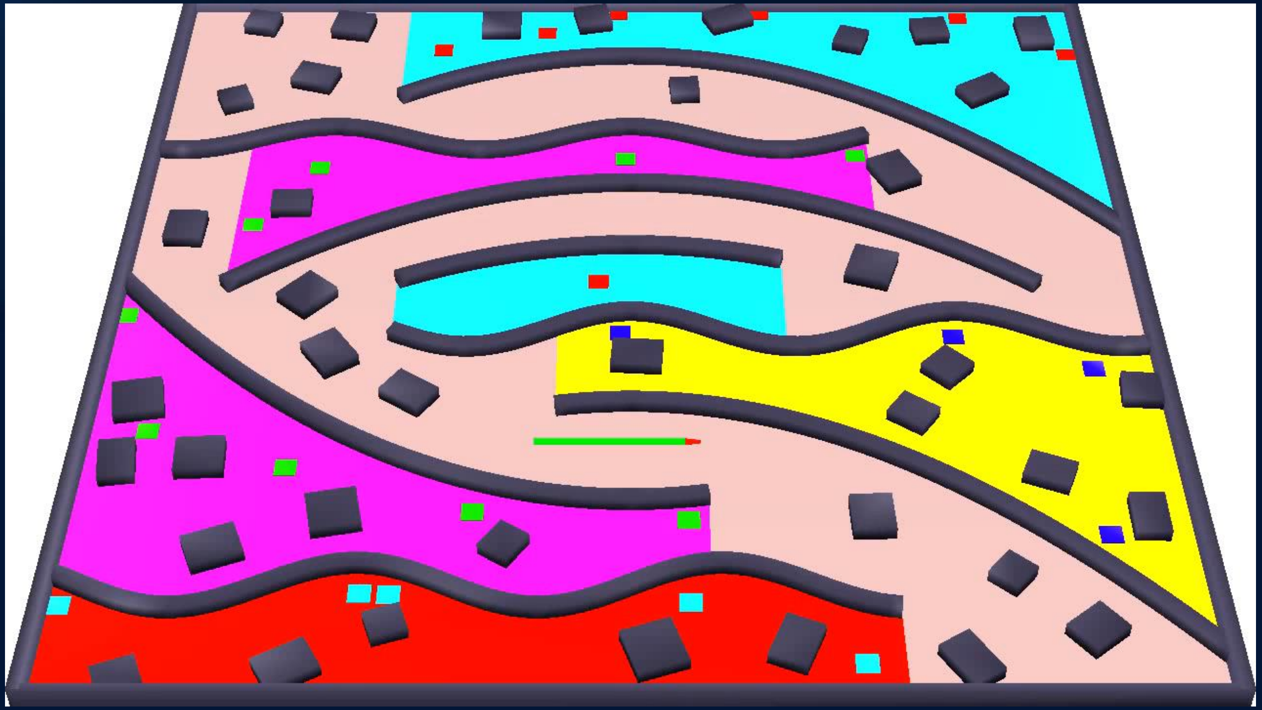
# In 2D





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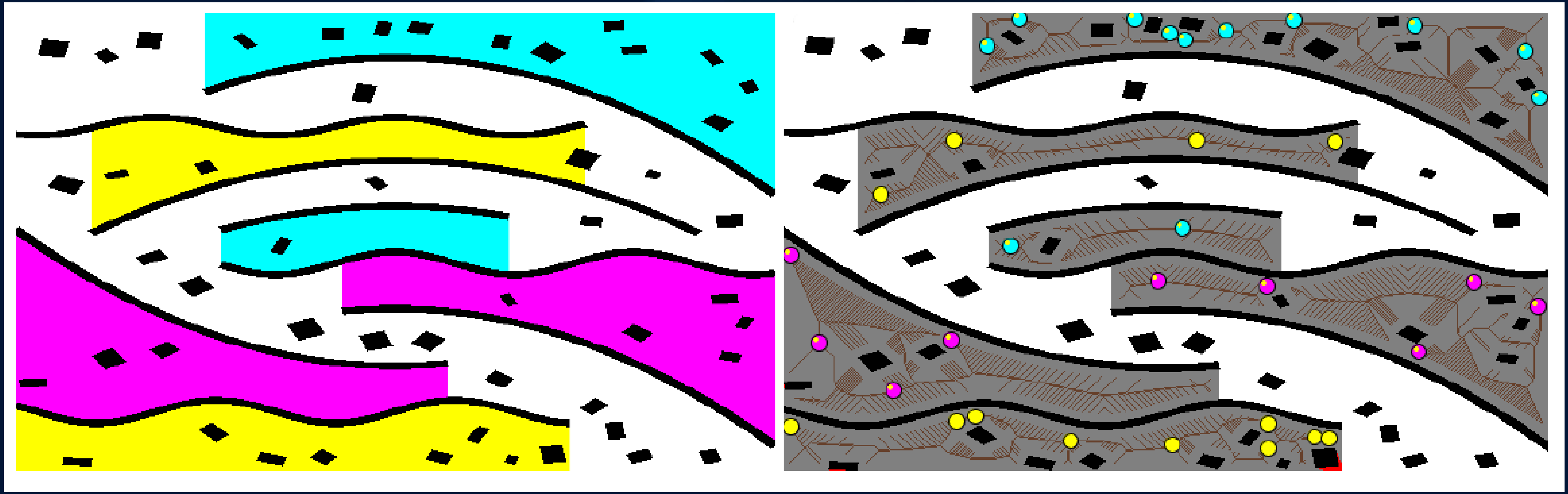
# Skeletonization via Grassfiring (Medial Axis) and Filtering



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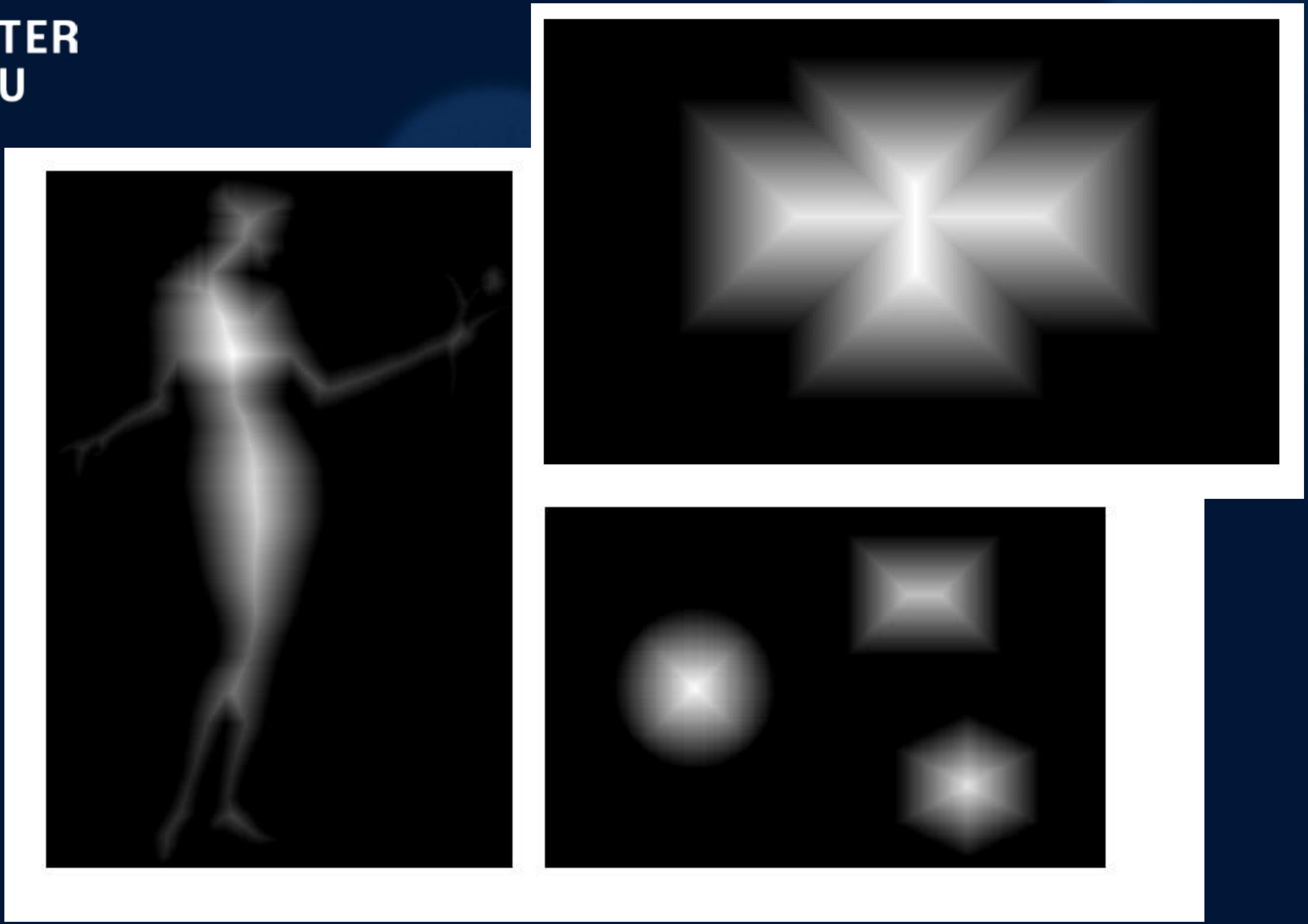
# Grassfiring



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# Generating Inspection Waypoints (1)


**Input:** $\mathcal{I}$: **bitmap image;** $\alpha$: **desired inspection quality,** $0 < \alpha \leq 1$

**Output: a set of inspection points**

---

$h \leftarrow \mathrm{height}(\mathcal{I})$; $w \leftarrow \mathrm{width}(\mathcal{I})$; $\mathcal{B} \leftarrow \mathrm{zeros}(h, w)$

$\diamondsuit$ ***grassfire transformation***

**for** $(i, j) \in \{0, \ldots, h-1\} \times \{0, \ldots, w-1\}$ **do**

  **if** $\mathrm{color}(\mathcal{I}(i,j)) \notin \{\mathtt{black}, \mathtt{gray}\}$ **then**

    $\mathcal{B}(i,j) \leftarrow 1 + \min\{\mathcal{B}(i-1,j), \mathcal{B}(i,j-1)\}$

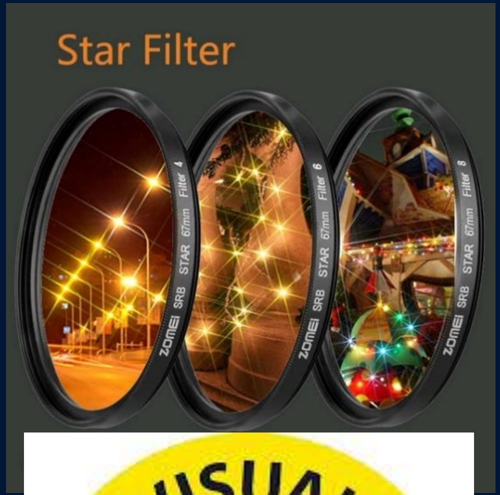
**for** $(i, j) \in \{h-1, \ldots, 0\} \times \{w-1, \ldots, 0\}$ **do**

  **if** $\mathrm{color}(\mathcal{I}(i,j)) \notin \{\mathtt{black}, \mathtt{gray}\}$ **then**

    $\mathcal{B}(i,j) \leftarrow 1 + \min\{\mathcal{B}(i+1,j), \mathcal{B}(i,j+1)\}$

$\mathrm{skeleton} \leftarrow$ **extract pixels making up the most intense lines in the brightness map** $\mathcal{B}$

# Generating Inspection Waypoints (2)





**Input:** $\mathcal{I}$: **bitmap image;** $\alpha$: **desired inspection quality,** $0 < \alpha \leq 1$
**Output: a set of inspection points**

---

$\diamondsuit$ ***select inspection points***

```
skeleton ← FILTER(skeleton)
inspectionPts ← skeleton
currScore ← VISSCORE(I, inspectionPts)
for p ∈ skeleton do
    newScore ← VISSCORE(I, inspectionPts \ {p})
    if newScore ≥ α ∨ currScore = newScore then
        inspectionPts ← inspectionPts \ {p}
        currScore ← newScore
return inspectionPts
```

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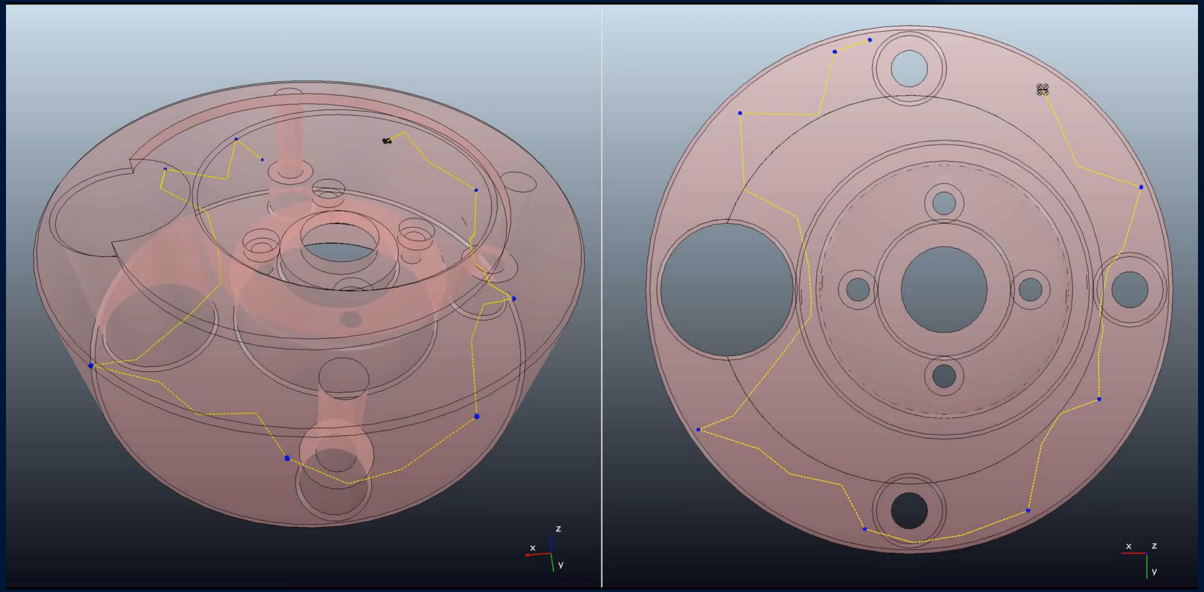
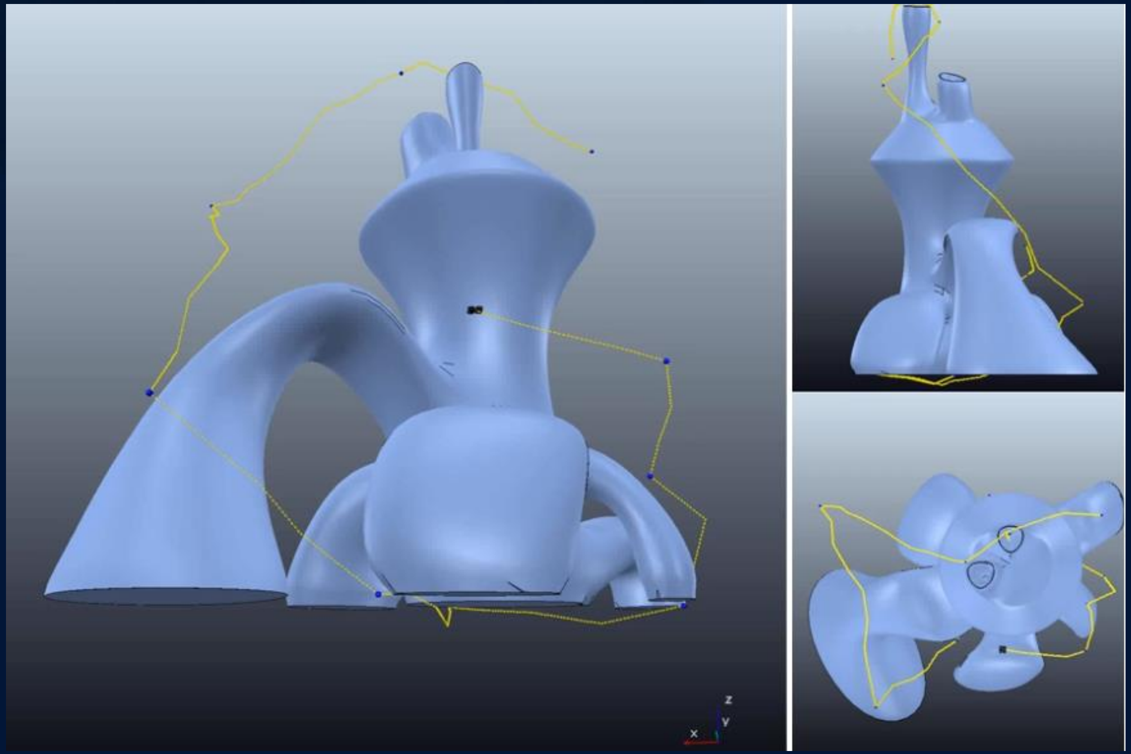
# In 3D

## a) Inside



## b) Outside



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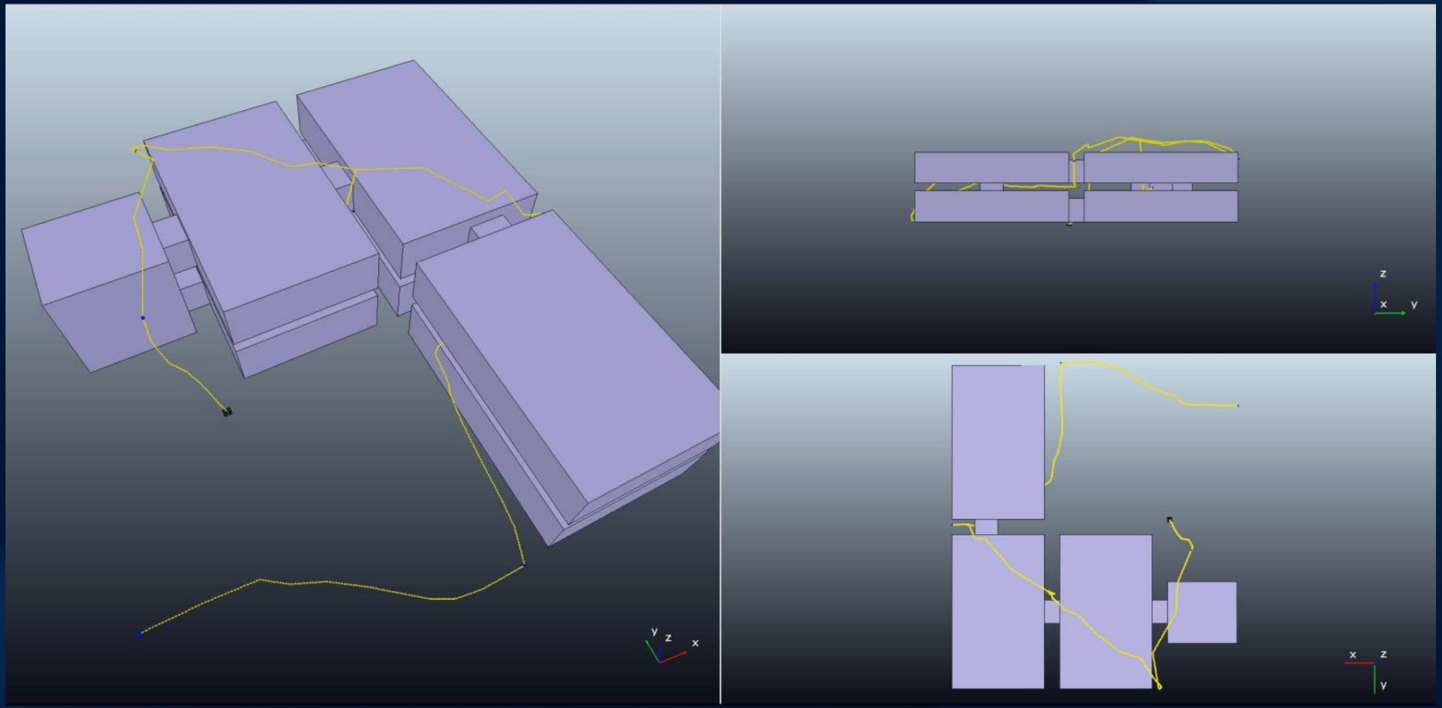
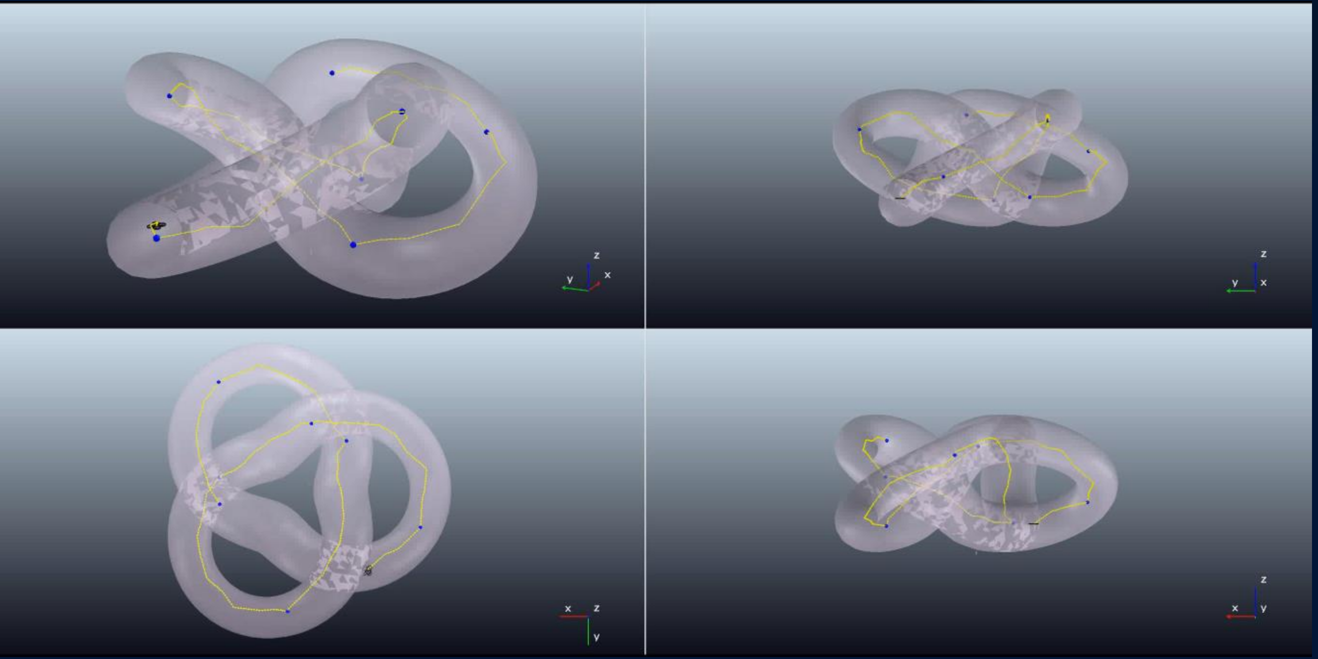
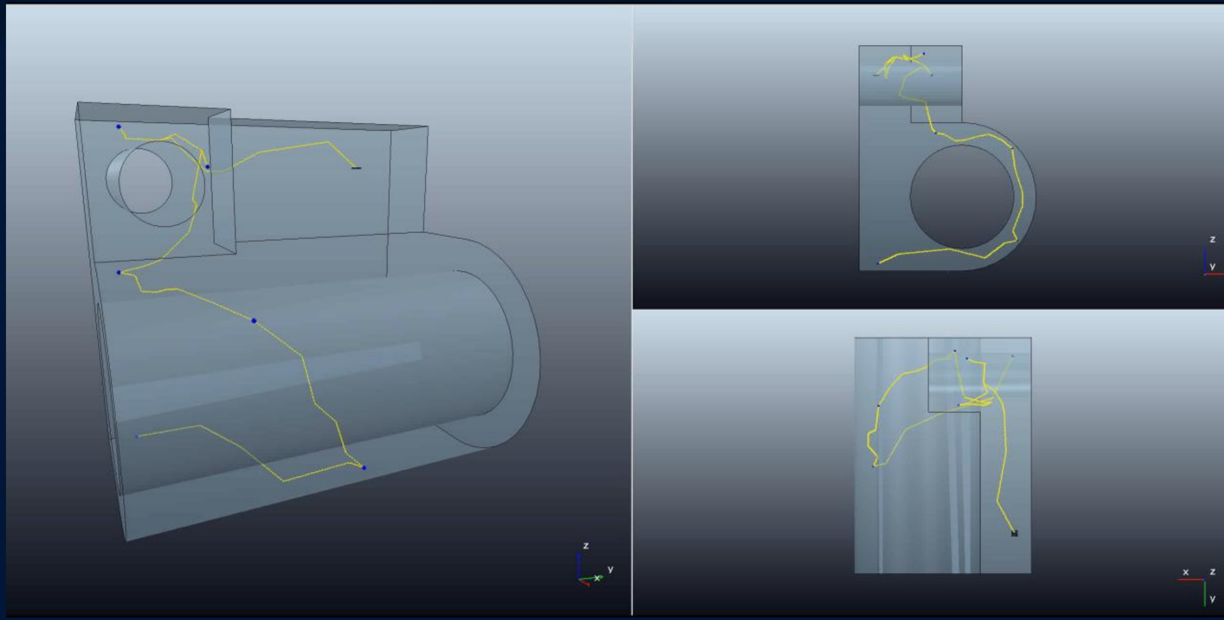
# Some more...







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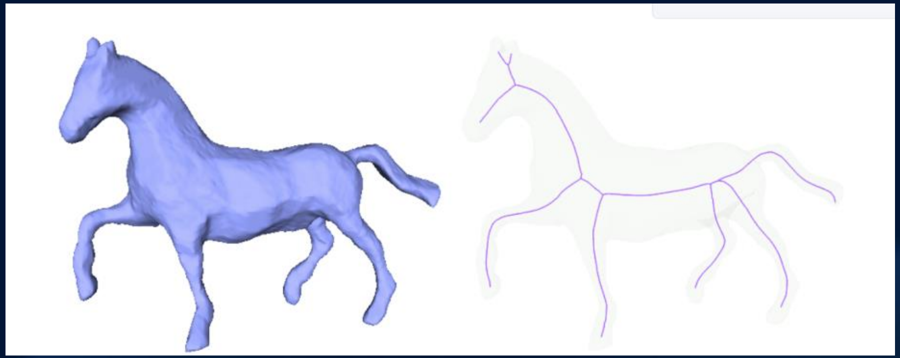
# Skeleton, Filtering Algorithm



$$\Sigma(\Omega) = \{x \in \Omega \mid \exists a, b \in \Delta \wedge \ a \neq b \wedge \ ||x - a||_2 = ||x - b||_2\}.$$

**Algorithm 1.** GenerateInspectionPts($\mathcal{W}, \mathcal{O}, \mathcal{R}, \alpha$)

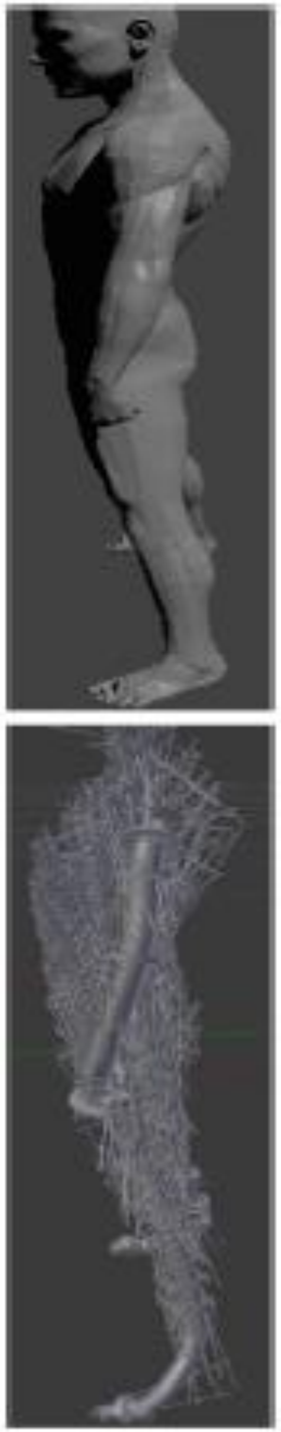
**Input:** $\mathcal{W}, \mathcal{O}, \mathcal{R}$ CAD objects
$\alpha$: desired inspection quality, $0 < \alpha \leq 1$
**Output:** a set of inspection waypoints

$skeleton \leftarrow$ SKELETONIZE($\mathcal{W}$)
$V_1 \leftarrow$ SAMPLE($skeleton$)
$V_2 \leftarrow$ SAMPLE($\mathcal{R}$)
$adjacent \leftarrow$ FilterInspection($V_1, V_2, \alpha$);
**return** HittingSet($adjacent, \alpha$)

**Algorithm 2.** FilterInspectionPts($V_1, V_2, \alpha$)

**Input:** $V_1, V_2$, $\alpha$: desired inspection quality, $0 < \alpha \leq 1$
**Output:** a set of inspection points

**for** $(i, j) \in \{0, \ldots, |V_1| - 1\} \times \{0, \ldots, |V_2| - 1\}$ **do**
    $adjacent(i, j) \leftarrow$ VISIBLE($v_i, v_j$)
**return** $adjacent$



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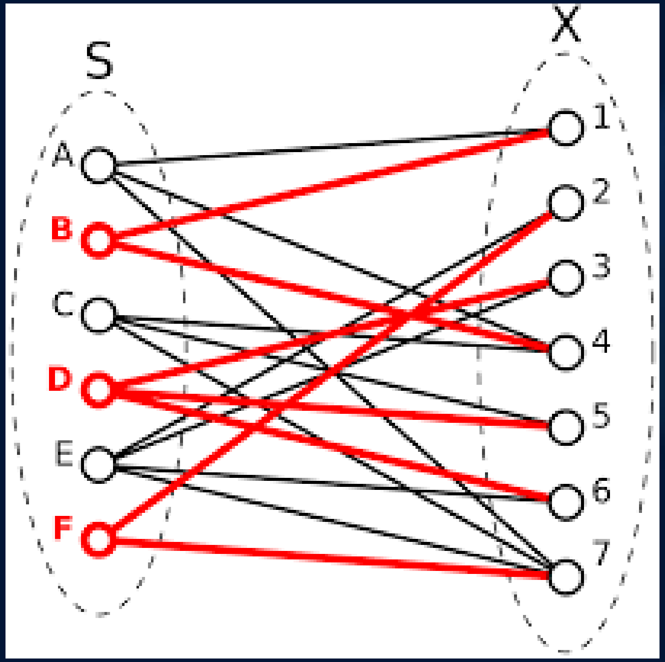
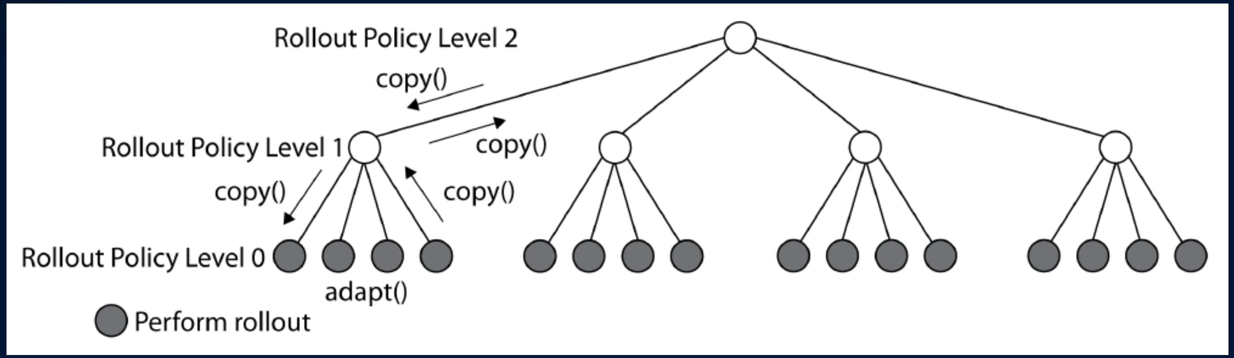
# Hitting Set (Exact Cover) Filtering




## Using NRPA again



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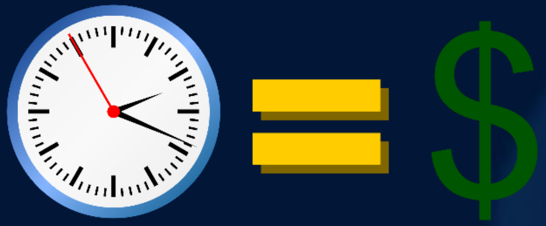
# Temporal Task-Motion Planning

Time is money
Real-world has and needs time constraints
Combining task with motion planning “holy grail”

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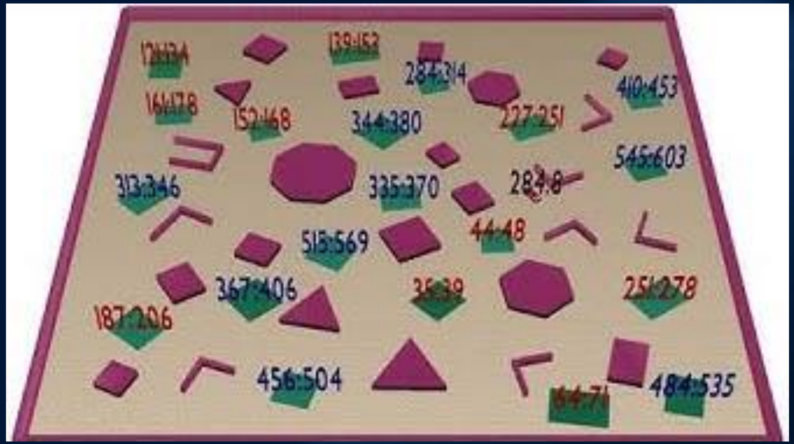
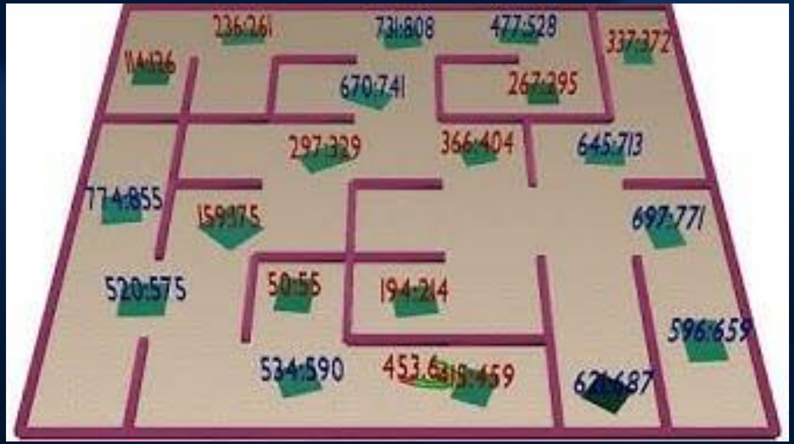
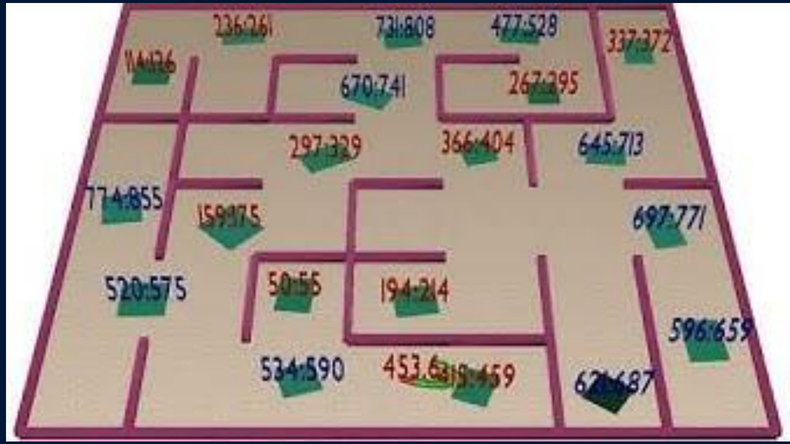
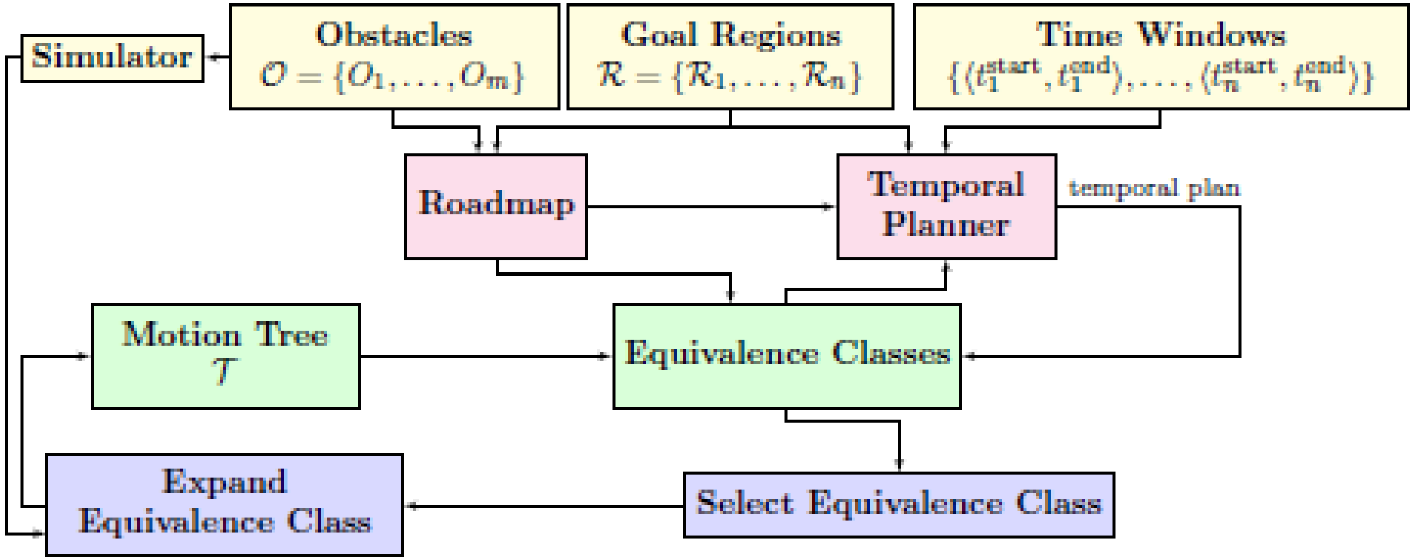
# Examples









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# Integration of Automated Planning

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# Interface with PDDL

TIL = Timed Initial Literal

(at timepoint (fact))
(at timepoint (not fact))

Specified in initial state

➔ actions time windows

```
v1  v3  v5  v4  v2
0.0   1.26   3.22   12.55   21.11
```

```
(at auv v0)
(connected v0 v1)
(connected v0 v2)
(connected v1 v2)
(= (traveltime v0 v1) 0.8)
(= (traveltime v0 v2) 1.5)
(= (traveltime v1 v2) 0.7)
(located task1 v1)
(located task2 v2)
(at 1.1 (tw_open task1))
(at 2.1 (not (tw_open task1)))
(at 2.3 (tw_open task2))
(at 3.3 (not (tw_open task2)))
```

coffee_errors.pddl

```
(define COFFEE

  (requirements
    :typing)

  (:types room - location
          robot human _ agent
          furniture door - (at ?l - location)
          kettle ?coffee cup water - movable
          location agent movable - object)

  (:predicates (at ?l - location ??o - object)
               (have ?m - movable ?a - agent)
               (hot ?m - movable) = true
               (on ?f - furniture ?m - movable))

  (:action boil
    :parameters (?m - movable $k - kettle ?a - agent)
    :preconditions (have ?m ?a)
    :effect (hot ?m))

```

Line 20, Column 22    Spaces: 2    PDDL

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# PDDL 3 Planning



**Monte-Carlo Search for Prize-Collecting Robot Motion Planning with Time Windows, Capacities, Pickups, and Deliveries**

```
(:durative-action execute_task_pickup
 :parameters (?v - vehicle ?wp - waypoint ?t - task)
 :duration ( = ?duration (taskduration ?t))
 :condition (and
   (at start (at ?v ?wp)) (at start (located ?t ?wp)
   (at start (todo ?t))
   (at start (<= (+ (customer ?wp) (cap ?v)) (max_cap ?v)))
   (at start (is-pickup ?wp)) (at start(tw_open ?t)))
 :effect (and
   (at start (not (todo ?t))) (at end (visited ?wp))
   (at end (increase (cap ?v) (customer ?wp)))
   (at end (decrease (profit ?v) (customer ?wp)))
   (at end (completed ?t)))
(:durative-action execute_task_delivery
 :parameters (?v - vehicle ?wp1 ?wp2 - waypoint ?t - task)
 :duration ( = ?duration (taskduration ?t))ov
 :condition (and
   (at start (at ?v ?wp1)) (at start (located ?t ?wp1))
   (at start (todo ?t)) (at start (is-delivery ?wp1))
   (at start (and (visited ?wp2) (link ?wp2 ?wp1)))
   (at start (tw_open ?t)))
 :effect (and
   (at start (not (todo ?t))) (at end (visited ?wp1))
   (at end (increase (cap ?v) (customer ?wp1)))
   (at end (decrease (profit ?v) (customer ?wp1)))
   (at end (completed ?t))))
```

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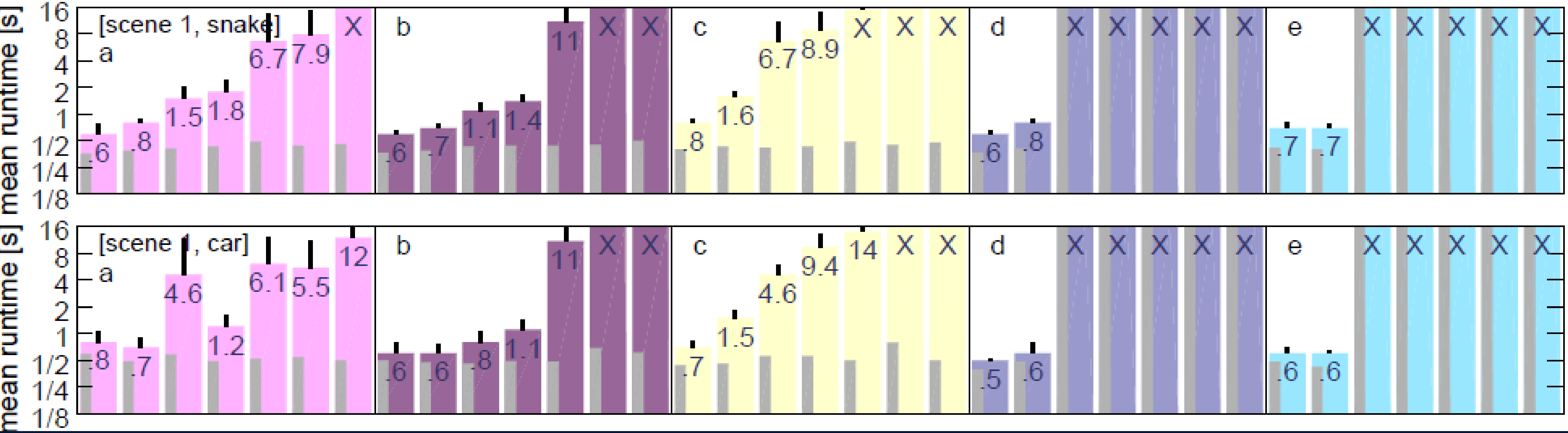
# NRPA BnB Optic OPTIMAL Random



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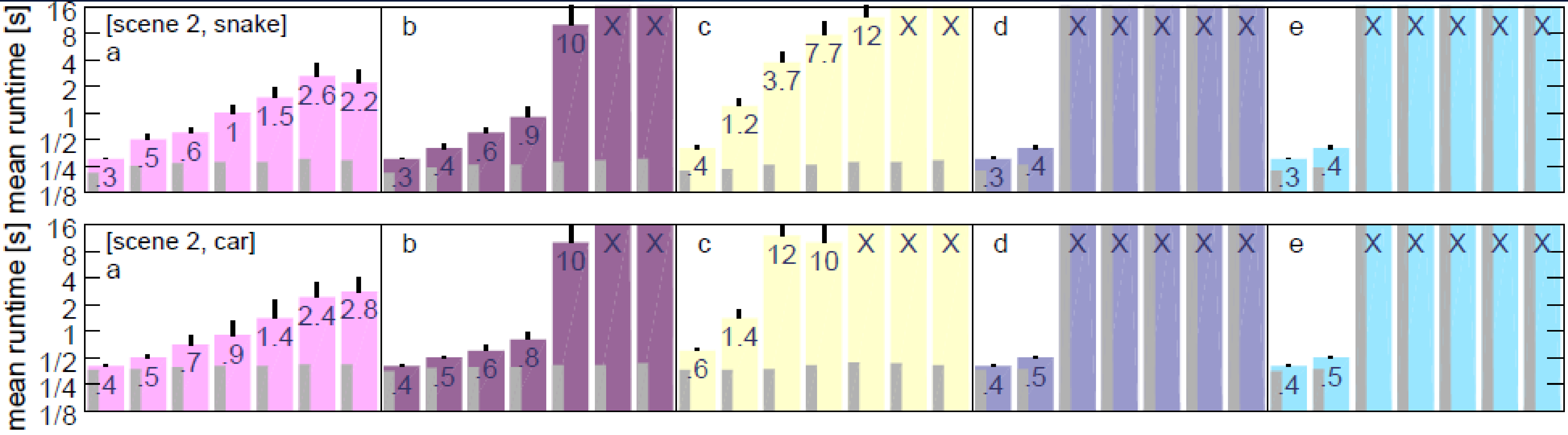
**NRPA** **BnB** **Optic** **OPTIMAL** **Random**



delkamp

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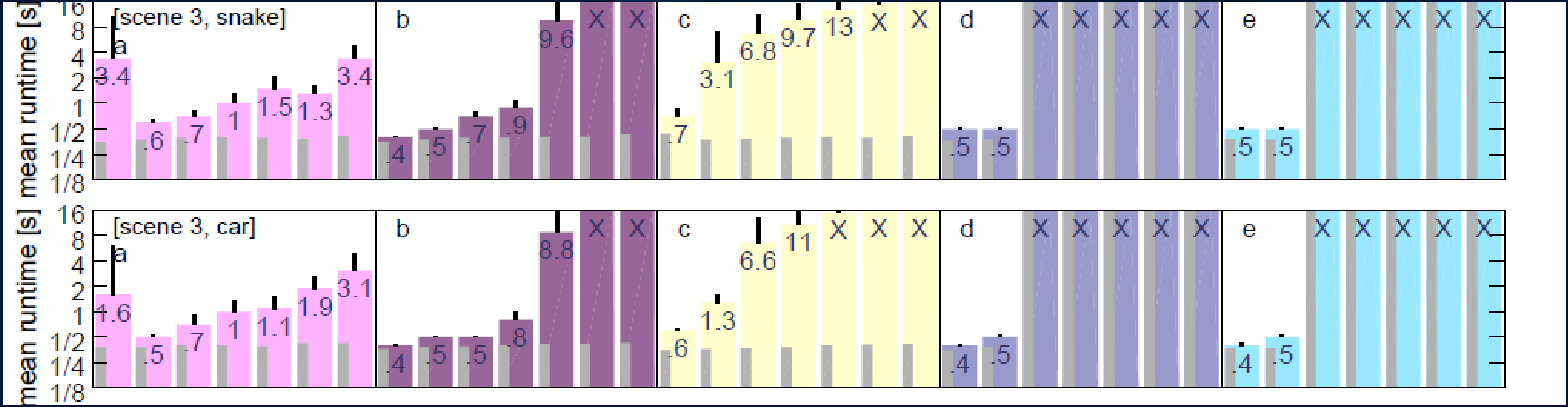
NRPA BnB Optic OPTIMAL Random



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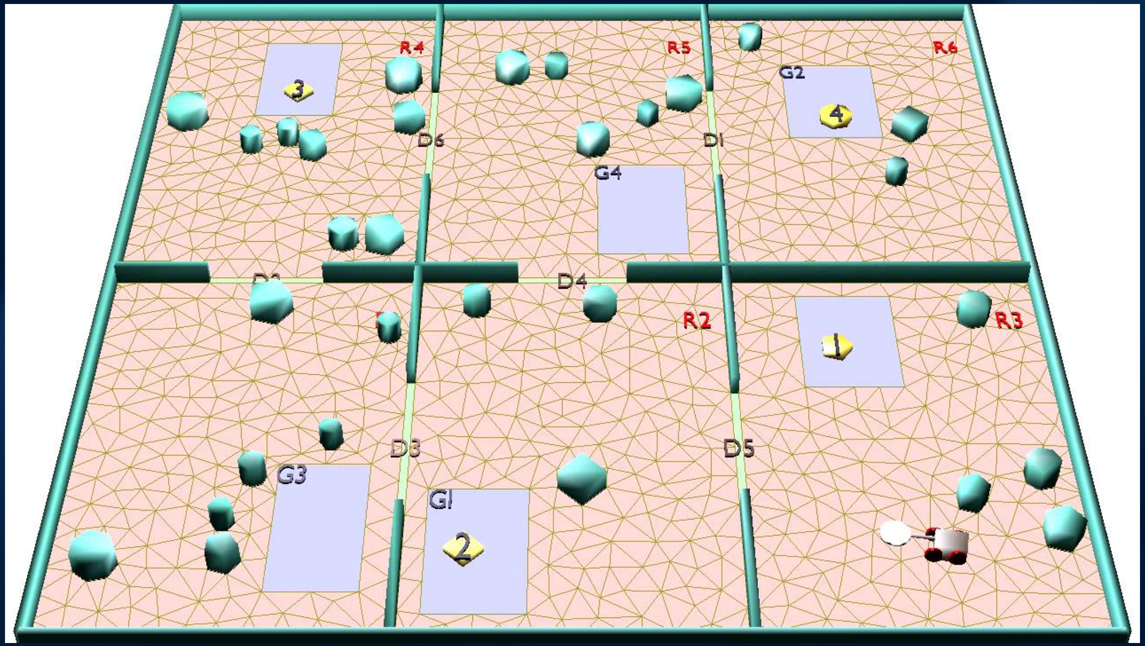
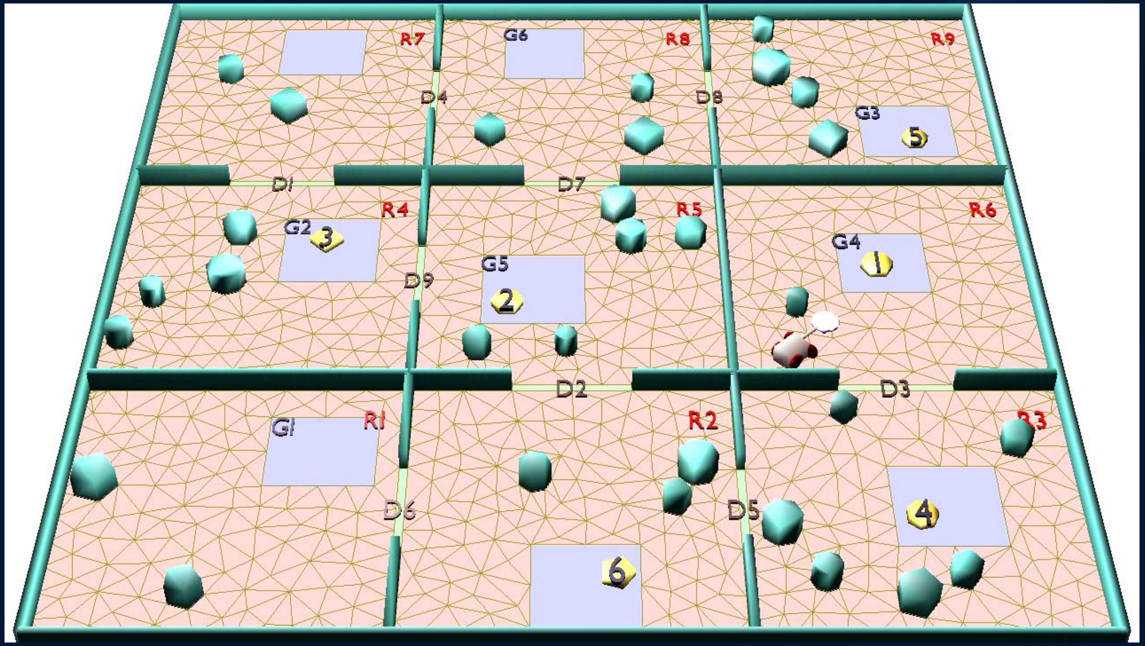
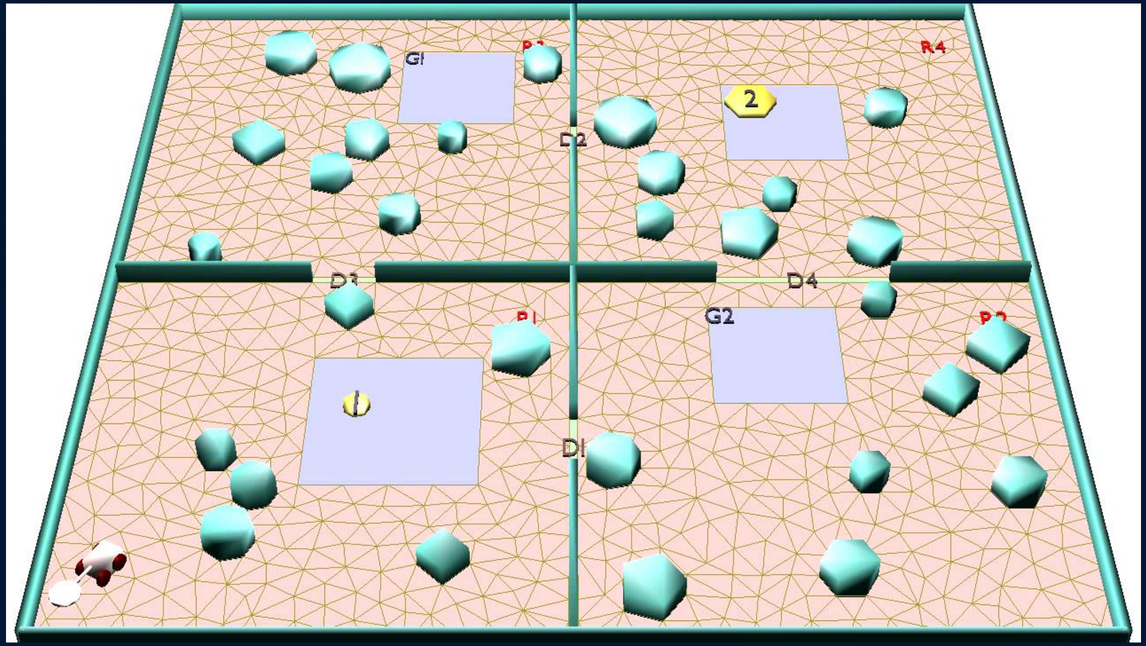
# Combined Task and Motion



0

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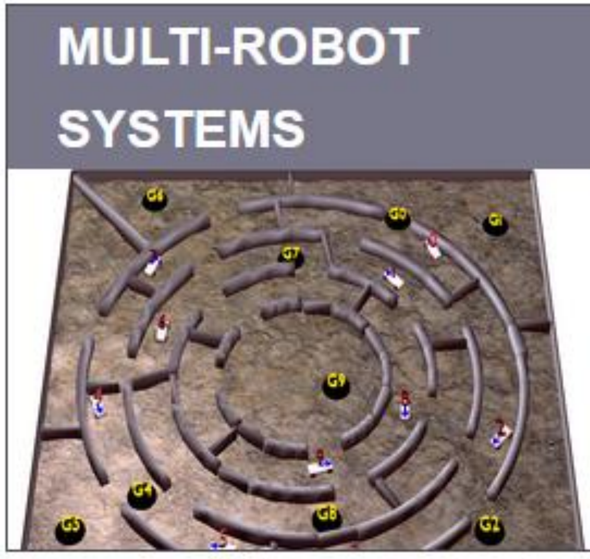
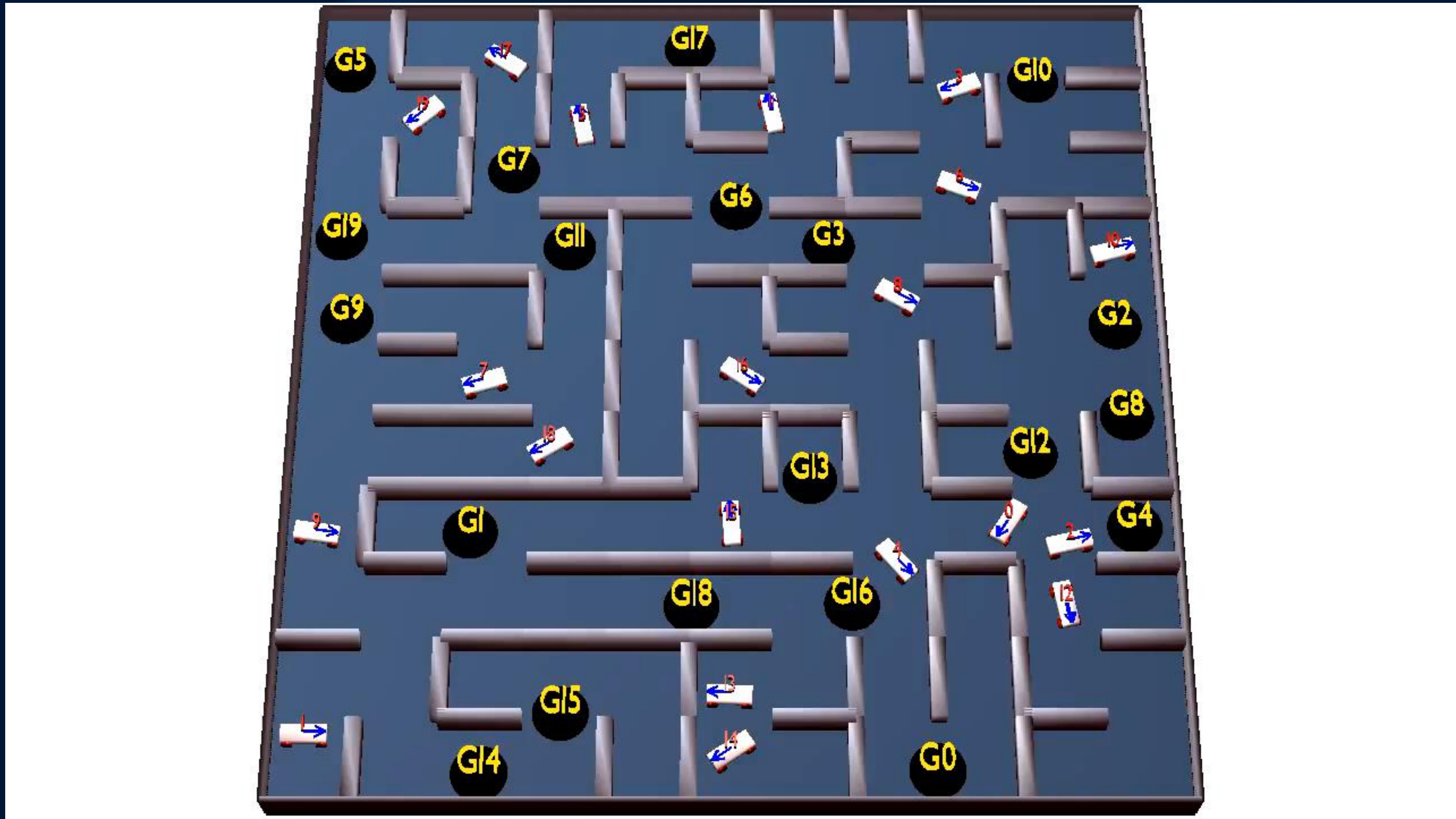



# More Robots

→ MAPF as
discrete problem

Multigoal Multirobot
Planning?



1

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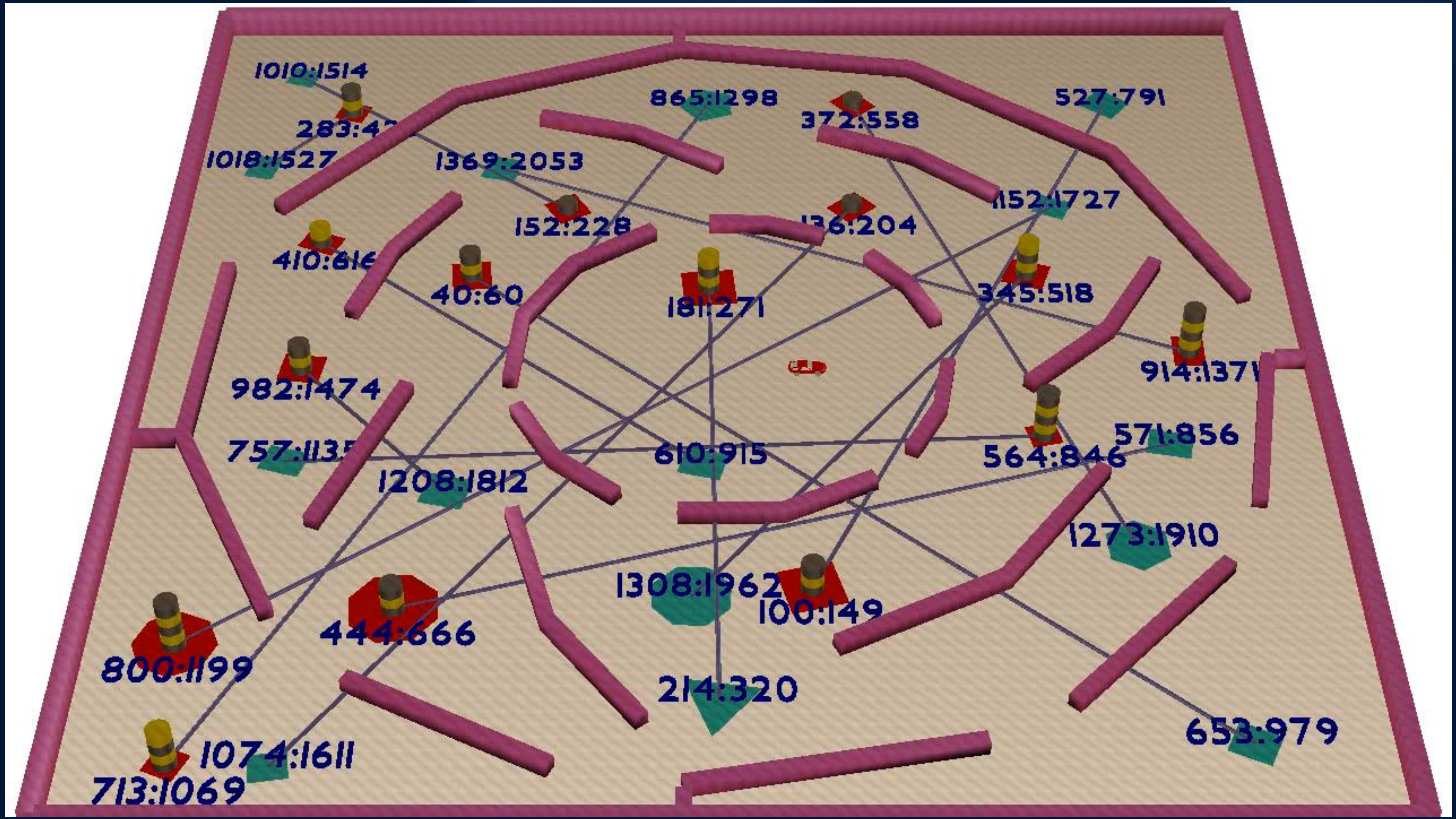
# Goods

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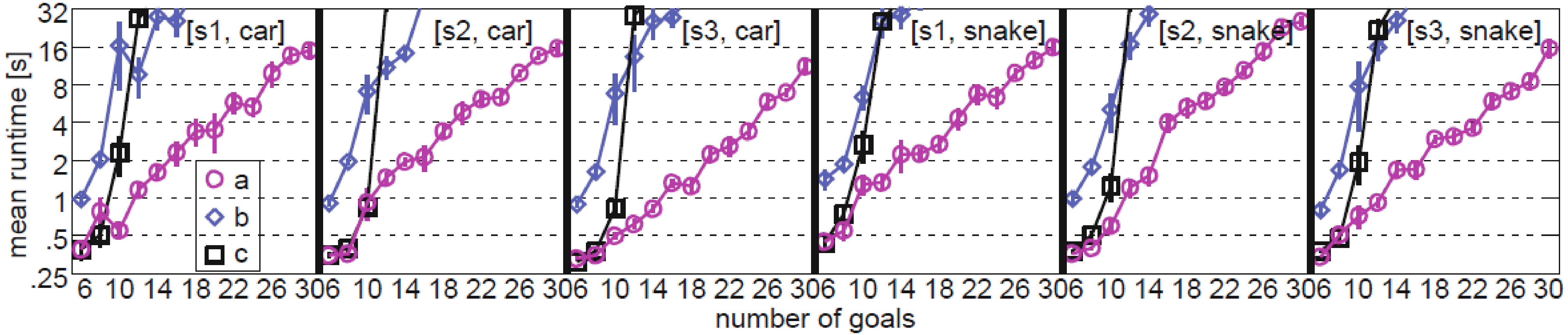
# (a) MC (B) OPTIC (C) Random



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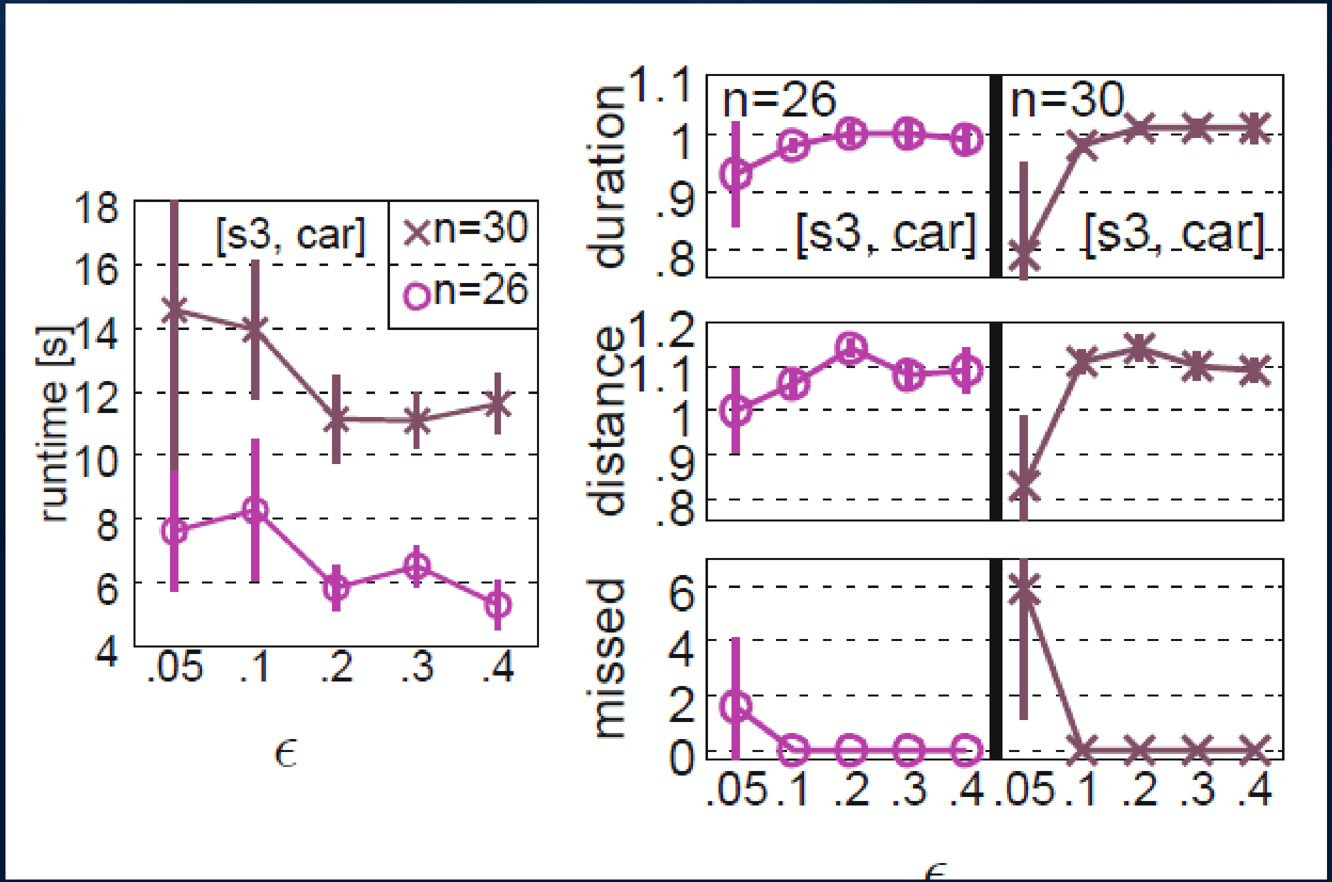
# Adjusting Time Windows



$$[(1-\epsilon)t_i, (1+\epsilon)t_i]$$



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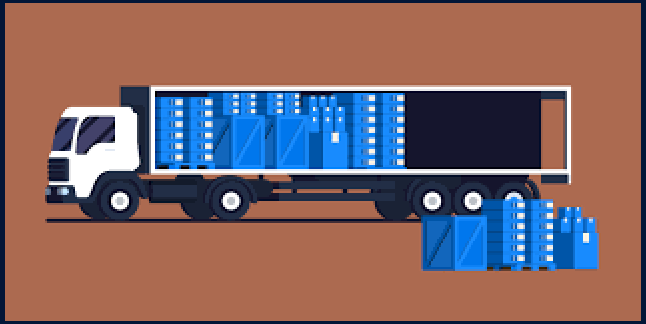
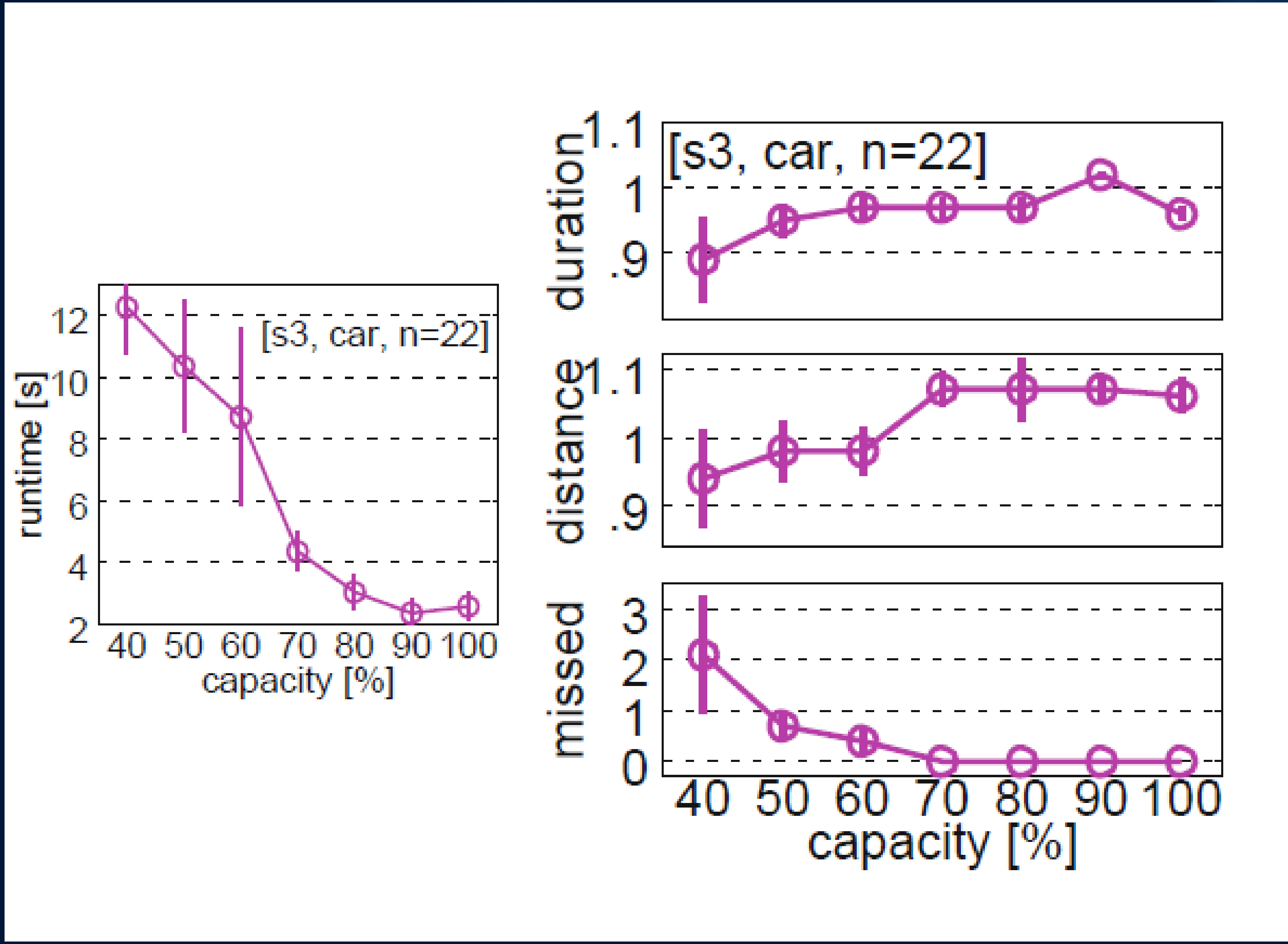
# Adjusting Vehicle Capacity





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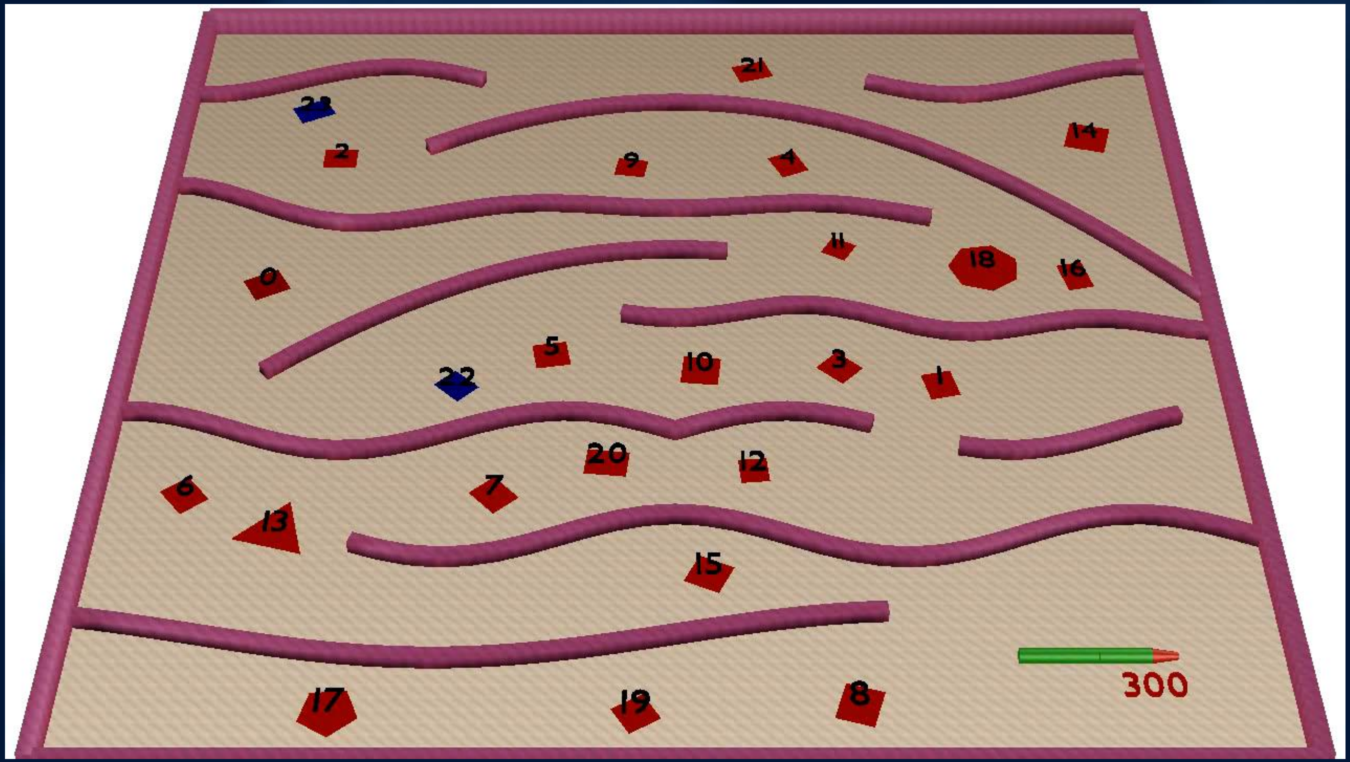
# Energy



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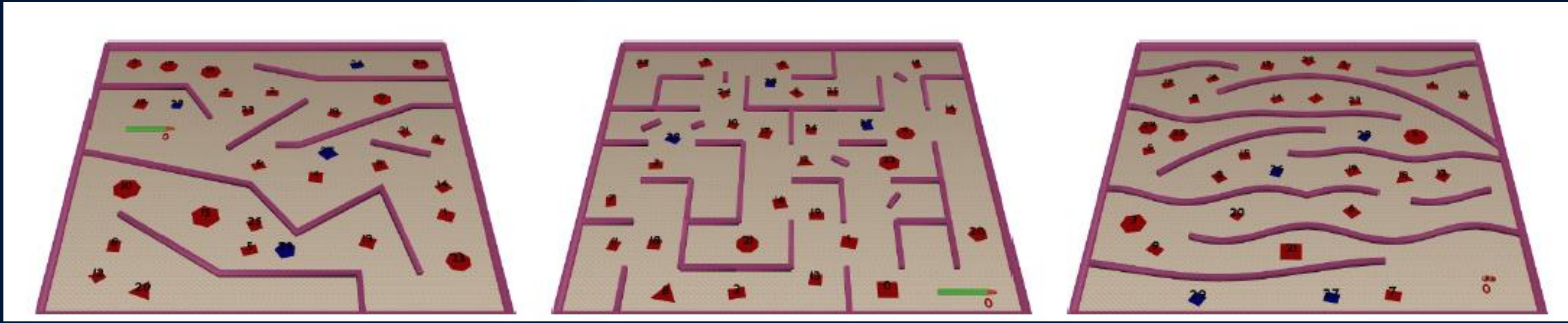
# Recharging

Multi-goal motion planning with multiple recharging stations. Goal regions, recharging stations, and obstacles are shown in red, blue, and magenta, respectively. The robot is required to visit each goal while reducing the distance traveled and the number of recharges



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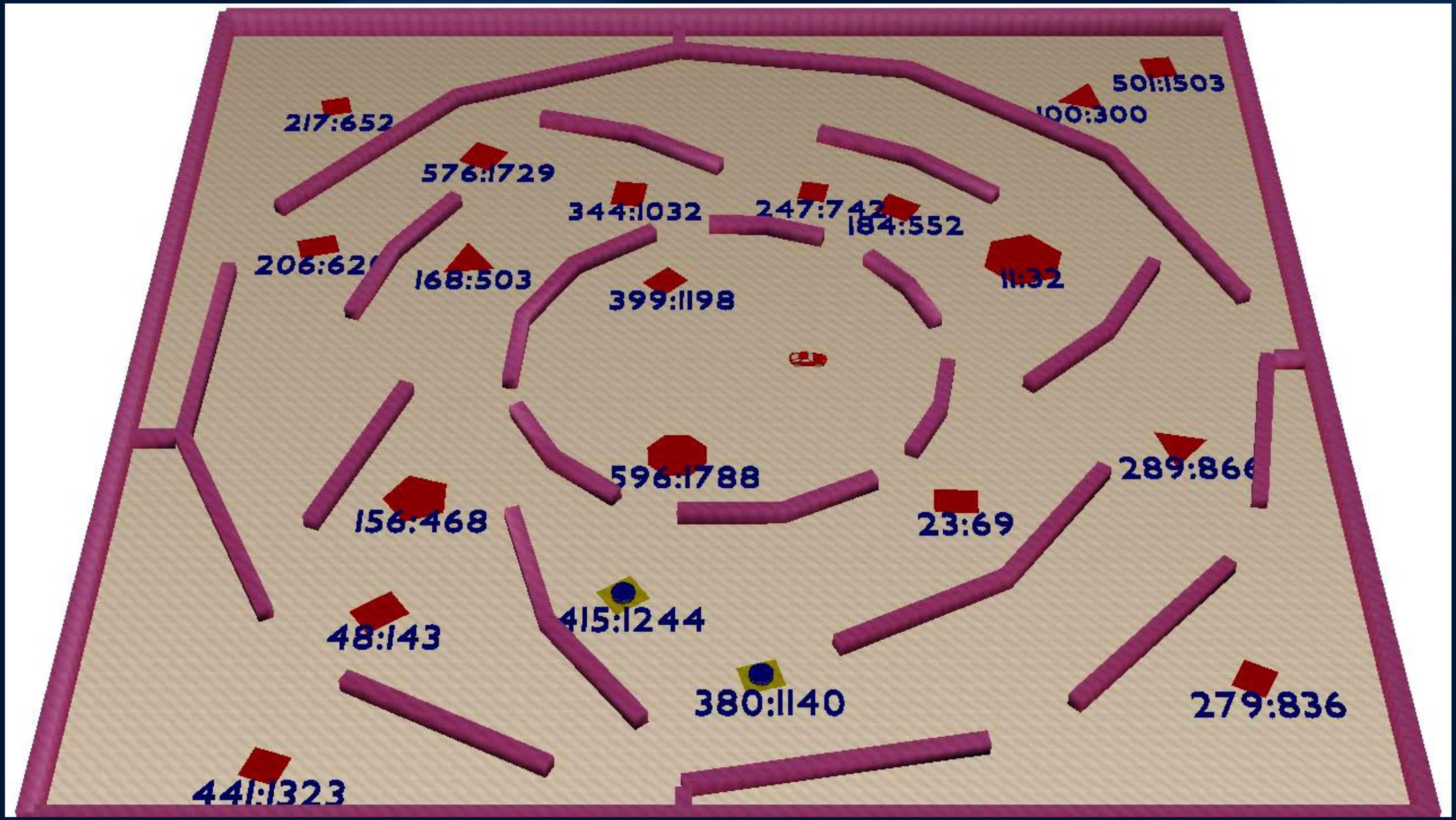
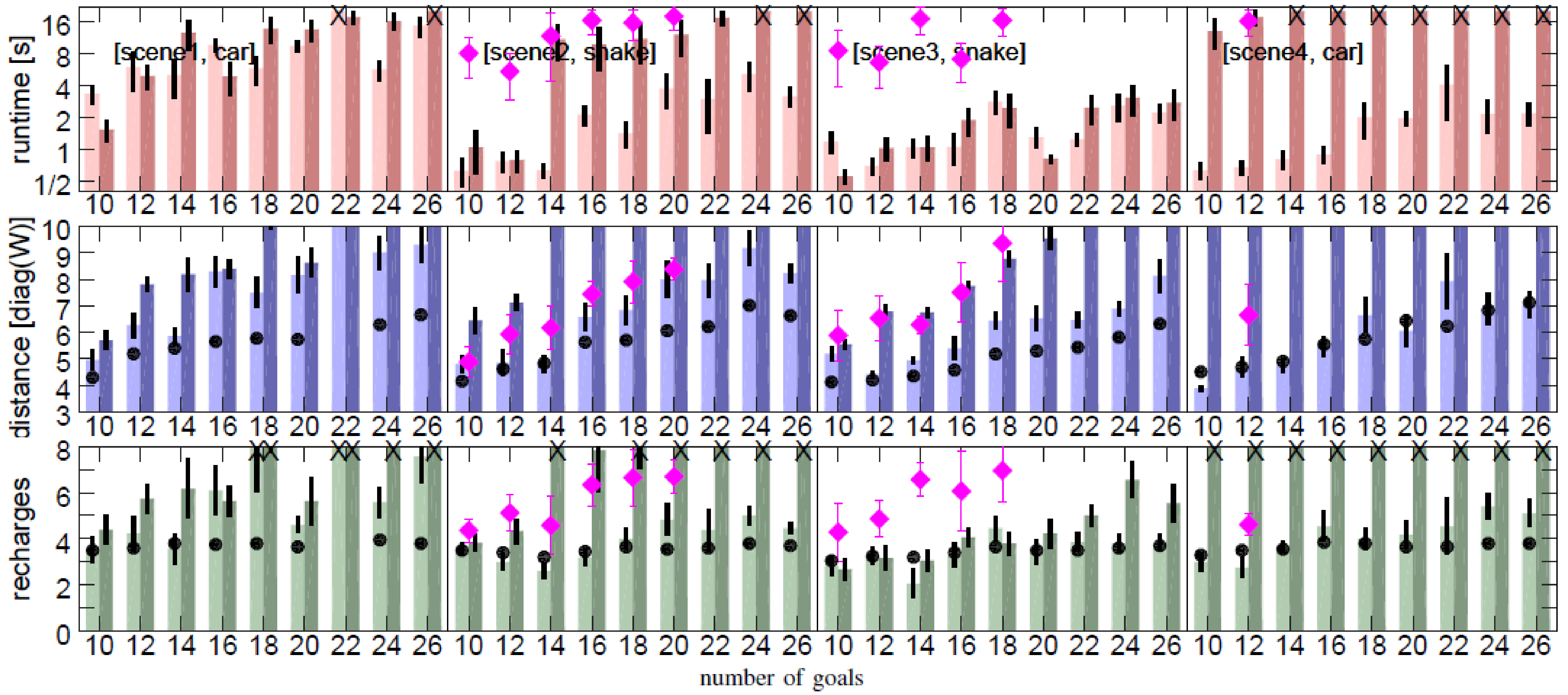
# Recharging with Temporal Constraints




# Results



runtime [s]
16
8
4
2
1
1/2
[scene1, car]
[scene2, snake]
[scene3, snake]
[scene4, car]
distance [diag(W)]
10
9
8
7
6
5
4
3
recharges
8
6
4
2
0
10 12 14 16 18 20 22 24 26
number of goals

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# Conclusion

**Full-Fledged Solution:**

- high-dimensional robotic systems with nonlinear dynamics and
- nonholonomic constraints
- visit all goal regions fast in suitable cost-minimizing order
- unstructured, complex environments

**and efficiently computes**

- collision-free, dynamically-feasible, low-cost trajectories that
- enable the robot to satisfy the task specification

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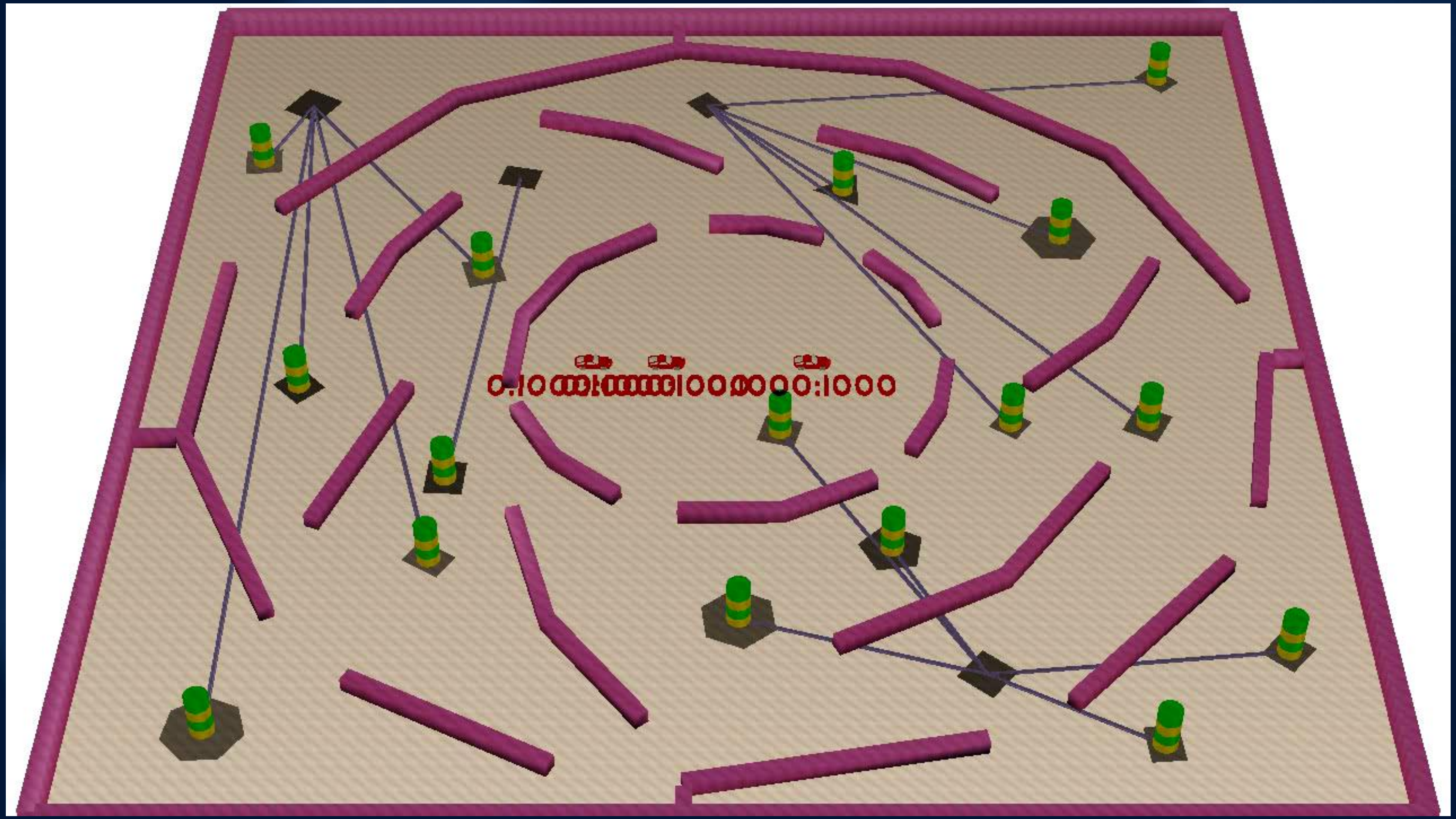
# Physical VRP



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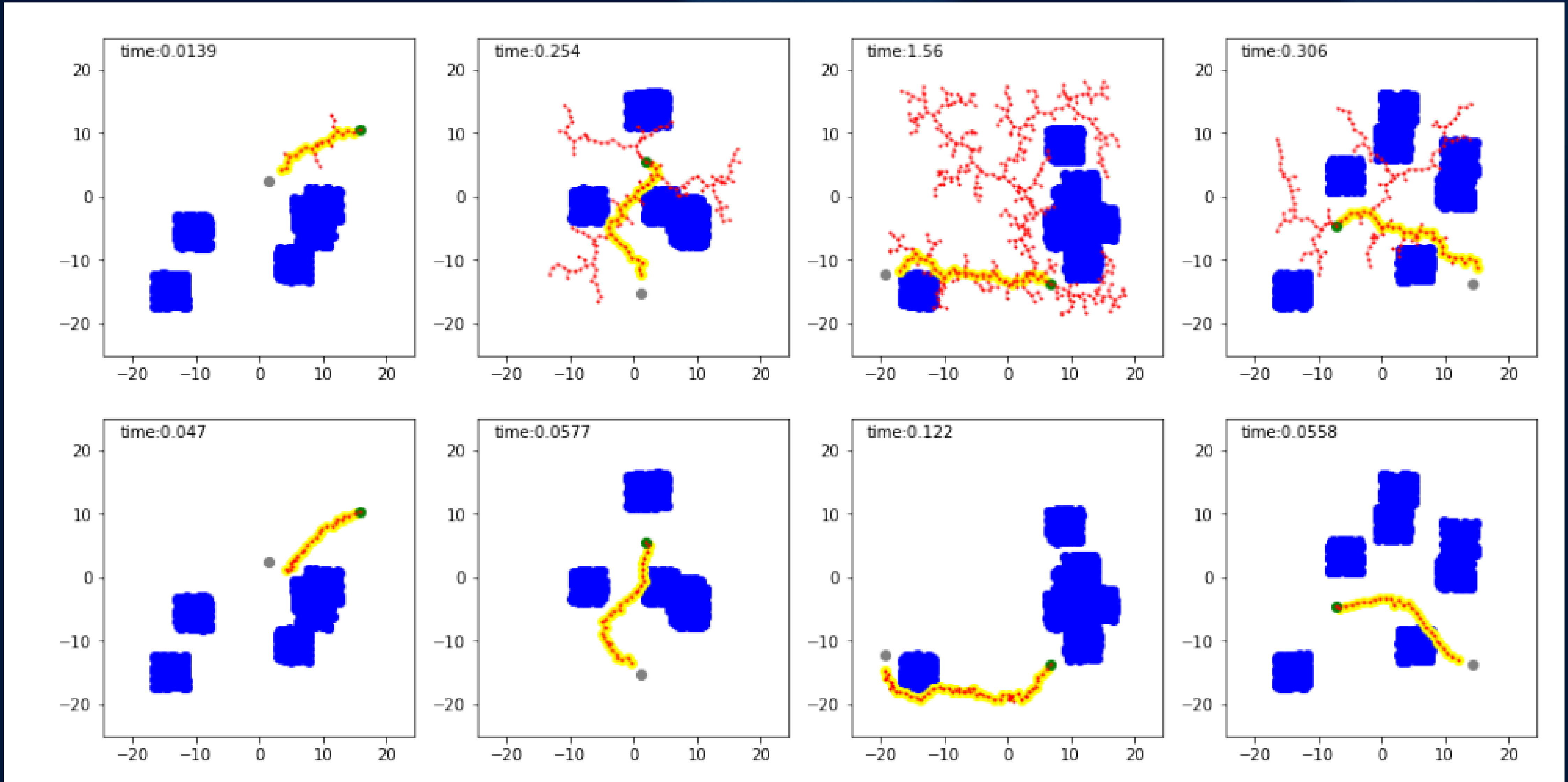
# Deep RRT*



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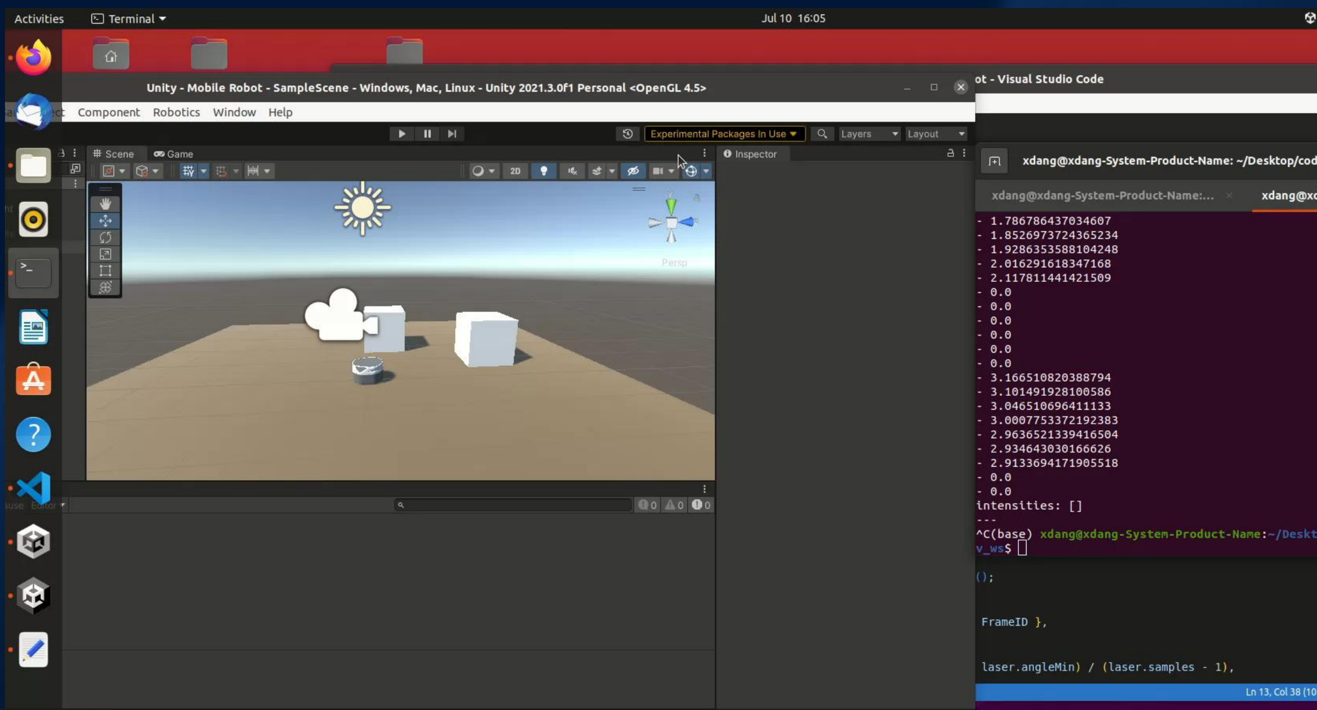
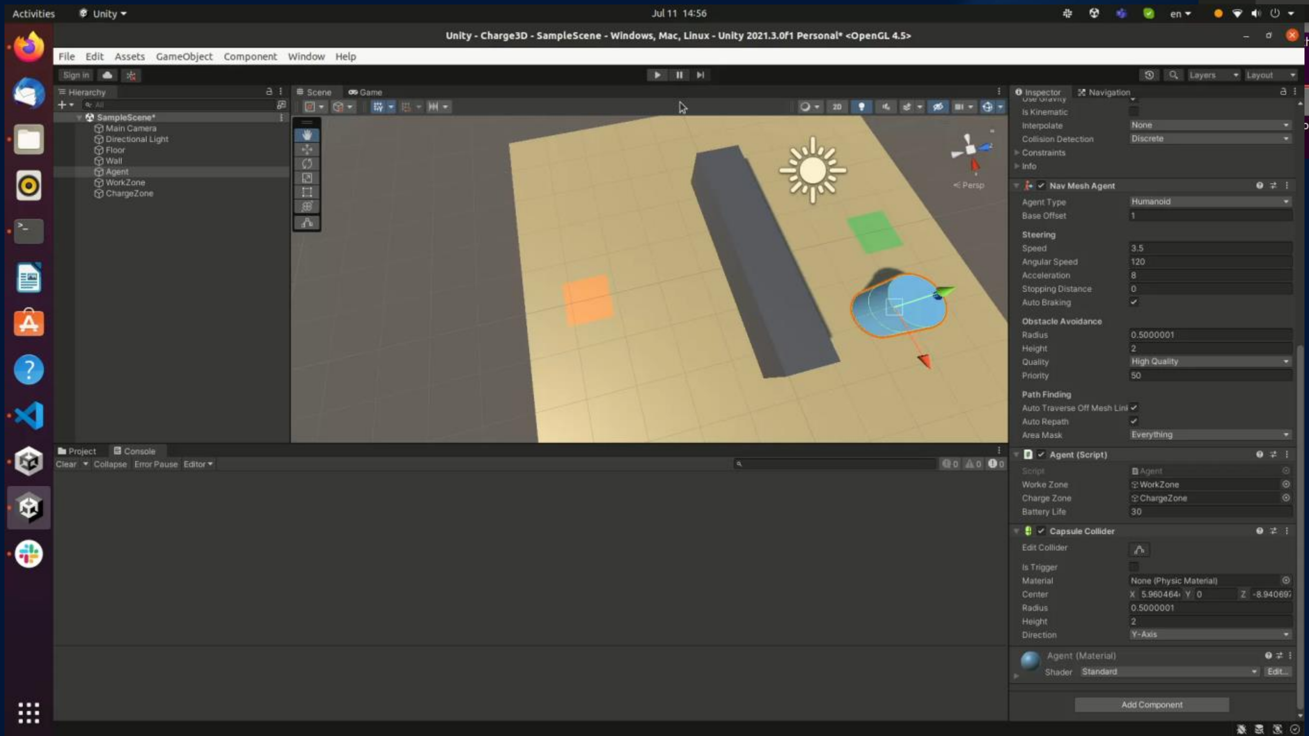
# Unity / ROS [many]



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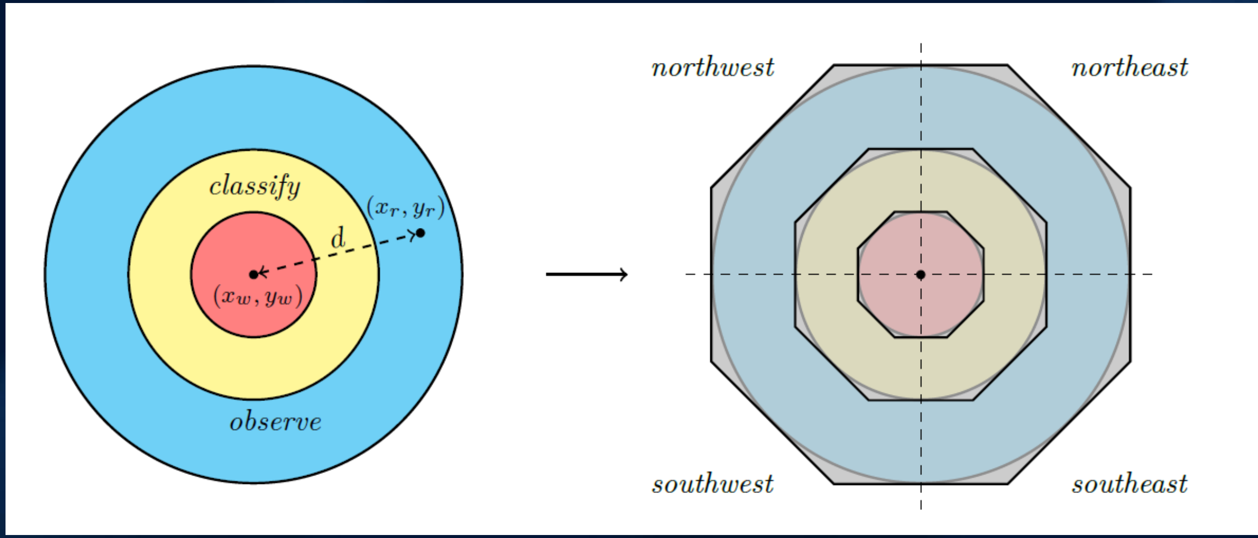
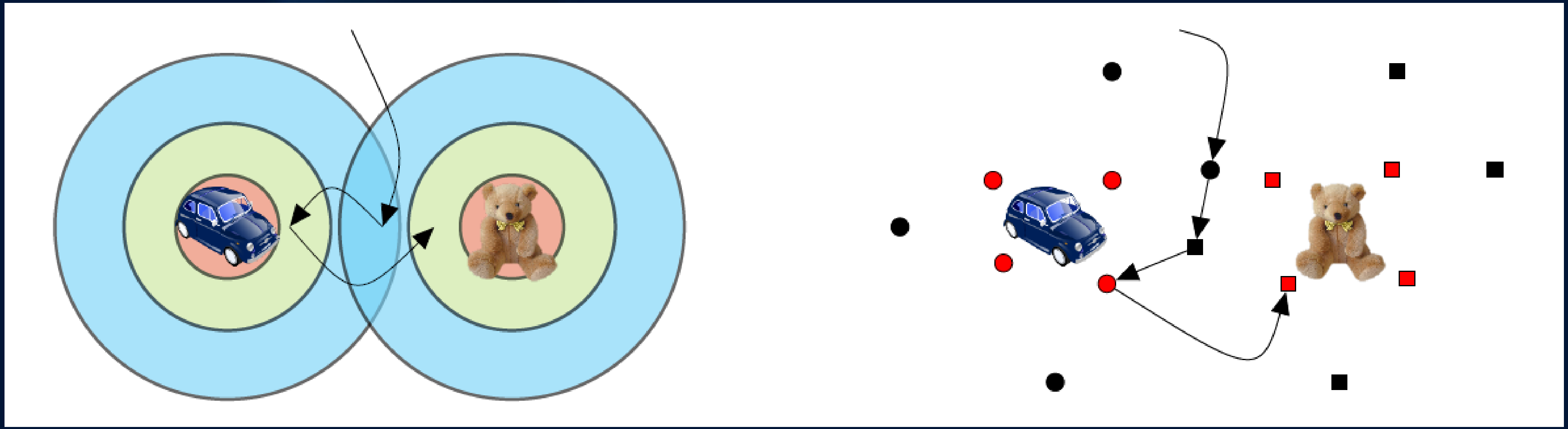
# Geometric Reasoning in AI Planning



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# PDDL can represent a lot

```
(:functions (x_location ?loc - location) (y_location ?loc - location)
    (x_robot) (y_robot) (maxV) (classify_min_distance)
    (observe_min_distance) (classify_min_distance_times_sqrt2)
    (observe_min_distance_times_sqrt2) (observe_max_distance)
    (classify_max_distance_times_sqrt2) (classify_max_distance)
    (observe_max_distance_times_sqrt2))

(:durative-action move_robot
:parameters()
:control (?dx ?dy - number)
:duration (and (>= ?duration (/ ?dx (maxV)))
    (>= ?duration (/ (- ?dx) (maxV)))
    (>= ?duration (/ ?dy (maxV)))
    (>= ?duration (/ (- ?dy) (maxV)))
    (>= ?duration (+ (/ ?dy (maxV)) (/ ?dx (maxV))))
    (>= ?duration (+ (/ (- ?dy) (maxV)) (/ (- ?dx) (maxV))))
    (>= ?duration (+ (/ (- ?dy) (maxV)) (/ ?dx (maxV))))
    (>= ?duration (+ (/ ?dy (maxV)) (/ (- ?dx) (maxV))))
    (<= ?duration 200))
:condition (and (at start (ready))
    ;; the displacement of robot is inside [-100,100], [-100,100] square
    (at start (>= ?dx -100)) (at start (<= ?dx 100))
    (at start (>= ?dy -100)) (at start (<= ?dy 100))
    (at end (>= ?dx -100)) (at end (>= ?dy -100))
    (at end (<= ?dx 100)) (at end (<= ?dy 100))
    ; the robot must stay within [0, 100] [0, 100] square
    (over all (>= (x_robot) 0)) (over all (<= (x_robot) 100))
    (over all (>= (y_robot) 0)) (over all (<= (y_robot) 100)))
:effect (and (at start (not (ready))) (at end (ready))
    (at start (increase (x_robot) ?dx))
    (at start (increase (y_robot) ?dy))))
```

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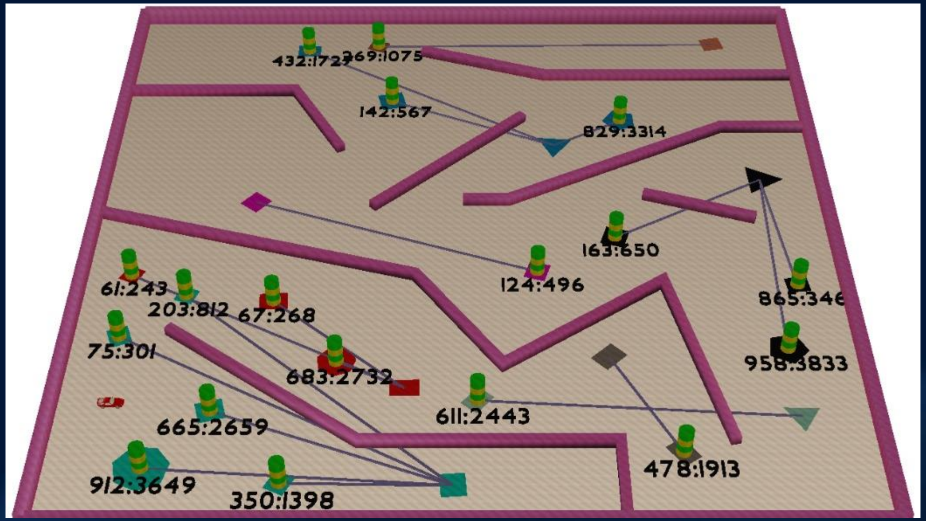
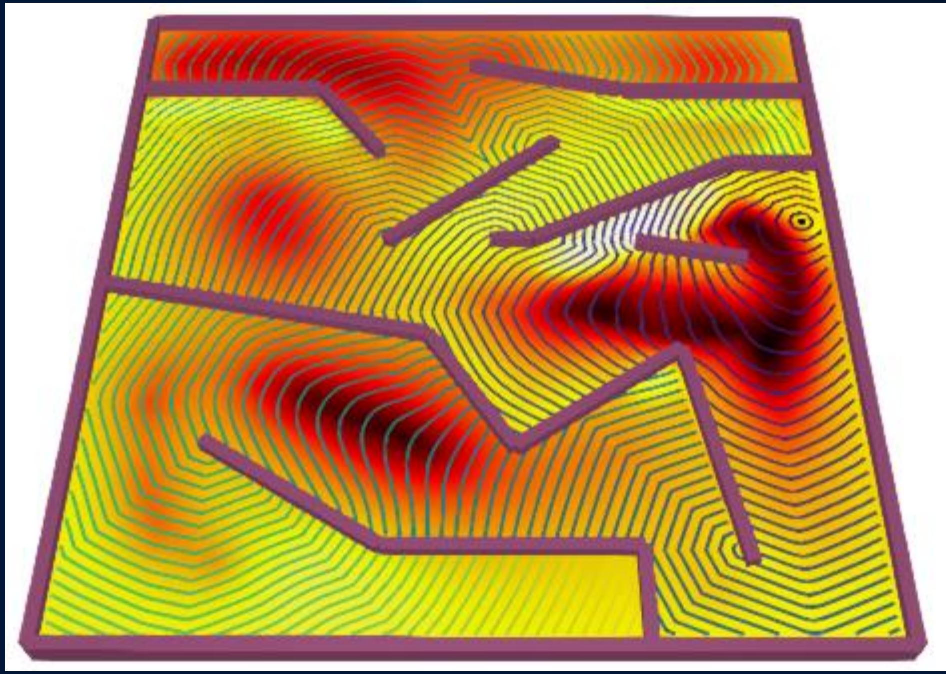
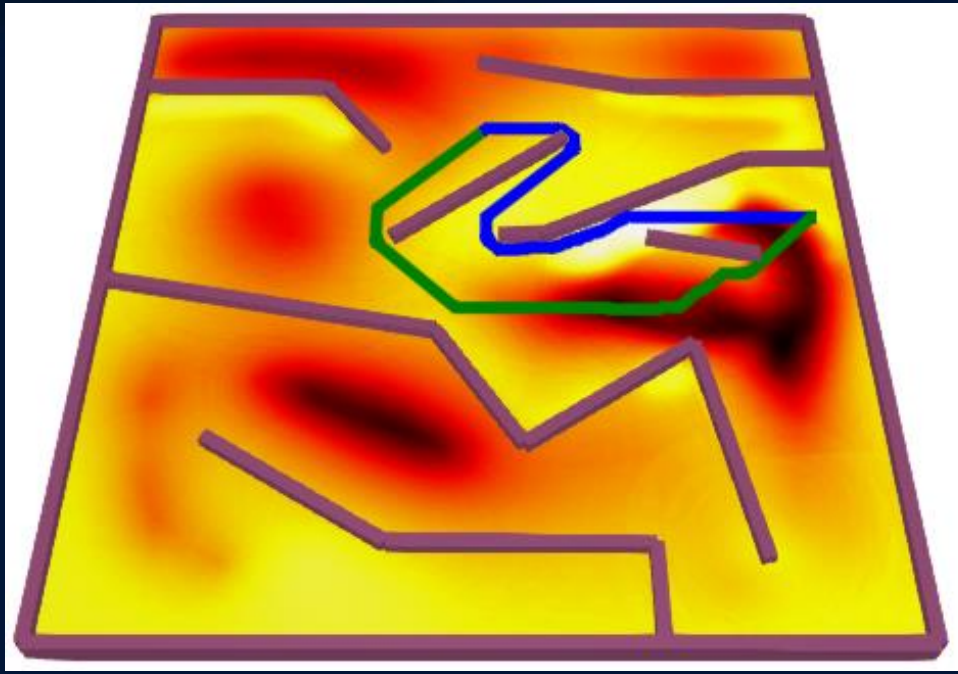
# Risk-Aware MGMP
# + Energy
# + Item Load
# + TW
# + PD





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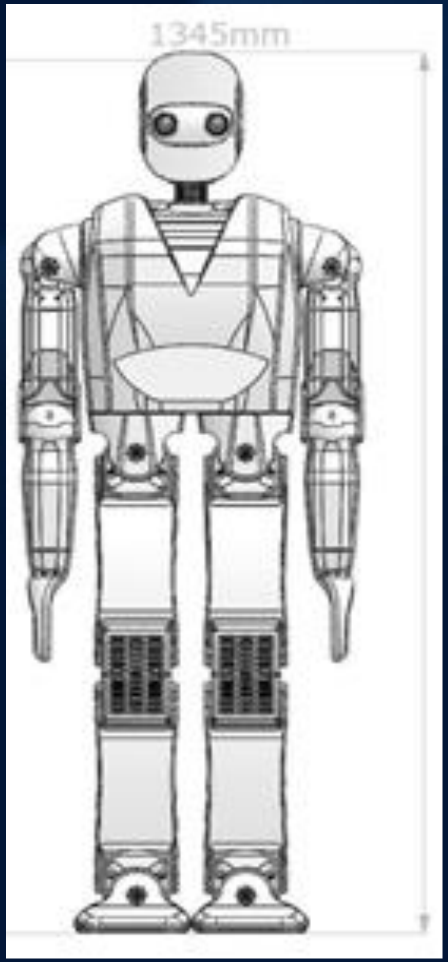
# www.aic.fel.cvut.cz

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[9] Stefan Edelkamp , Christoph Greulich, and Denis Golubev . *Solving the Physical Vehicle Routing Problem for Improved Multi-Robot Freespace Navigation.* Proc. of KI, 2016.
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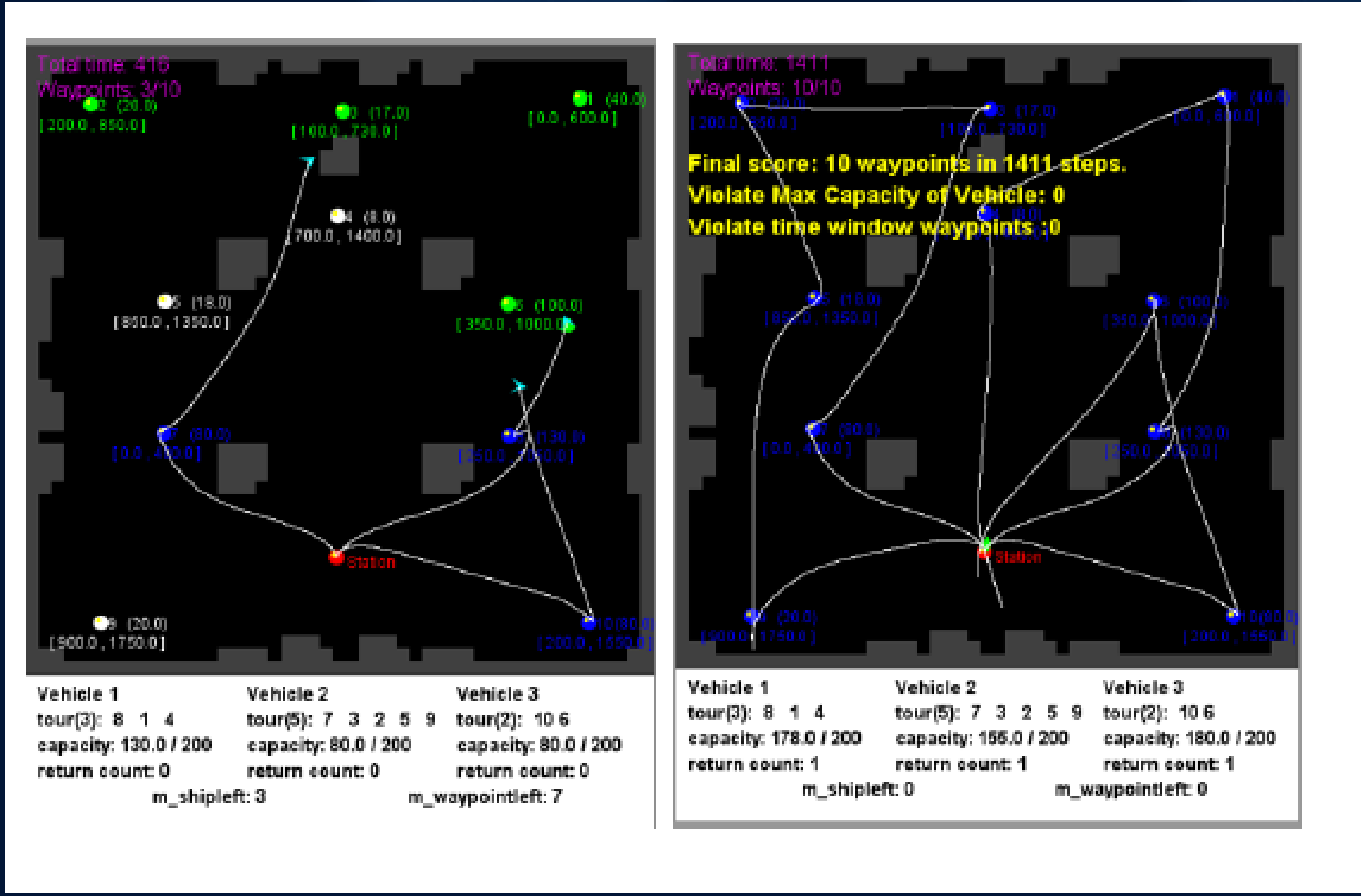
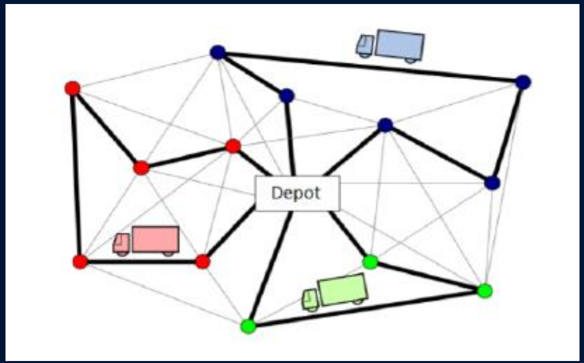
[11] Edelkamp S, Warsame Y, and Plaku E (2019): *Monte-Carlo Search for Prize-Collecting Robot Motion Planning with Time Windows, Capacities, Pickups, and Deliveries*. KI, vol. 11793, pp. 154-167
[12] Edelkamp S, Lahijanian M, Magazzeni D, and Plaku E (2018): *Integrating Temporal Reasoning and Sampling-Based Motion Planning for Multi-Goal Problems With Dynamics and Time Windows.* IEEE Robotics and Automation Letters, vol. 3, pp. 3473--3480

# Related Work: Physical Vehicle Routing Problem

Multi-Robot Multi-Goal Motion Planning with Time and Resources

Stefan Edelkamp Junho Lee




TAROS (1) 2019: 288-299

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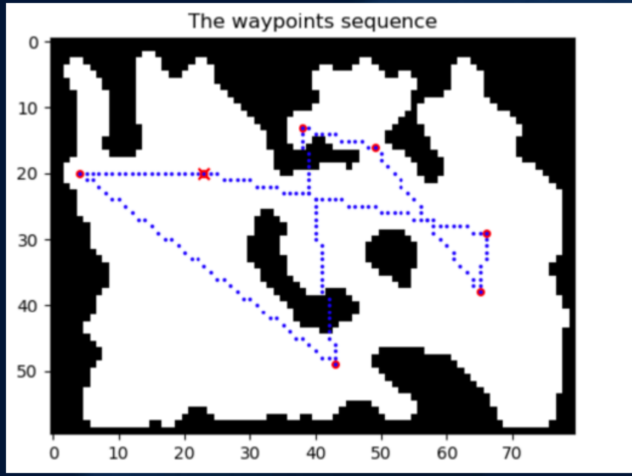
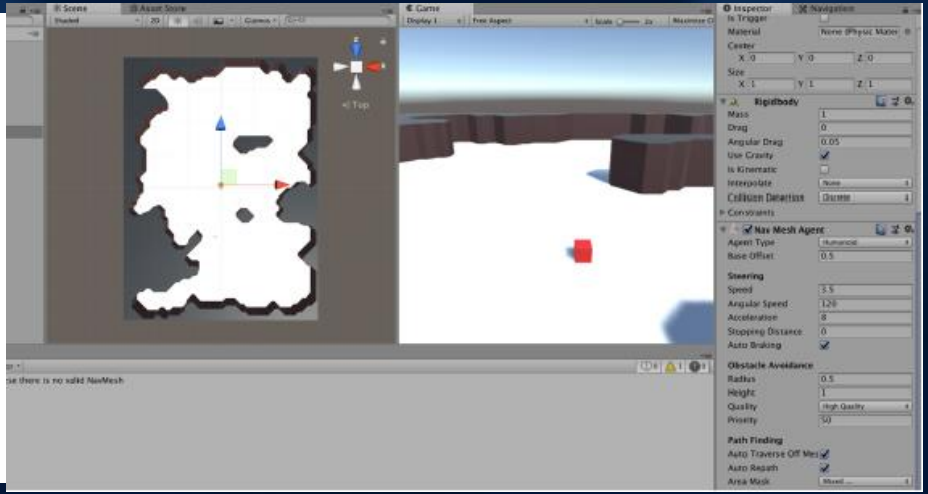
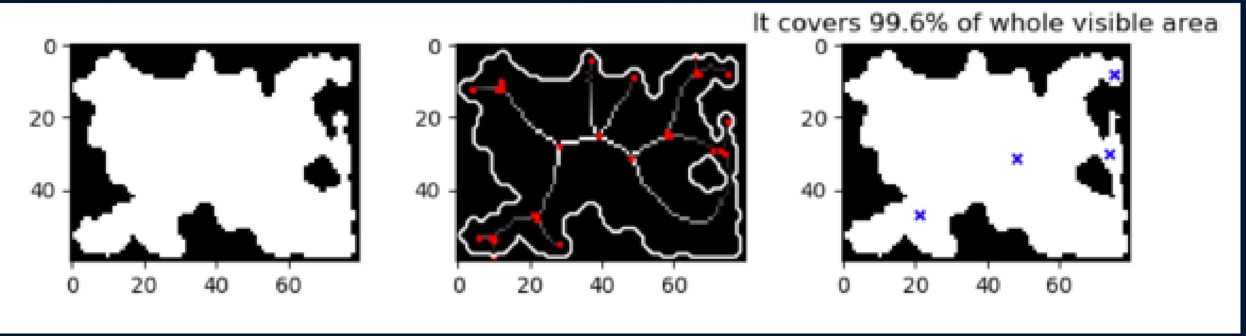
Watchman Routes for Robot Inspection

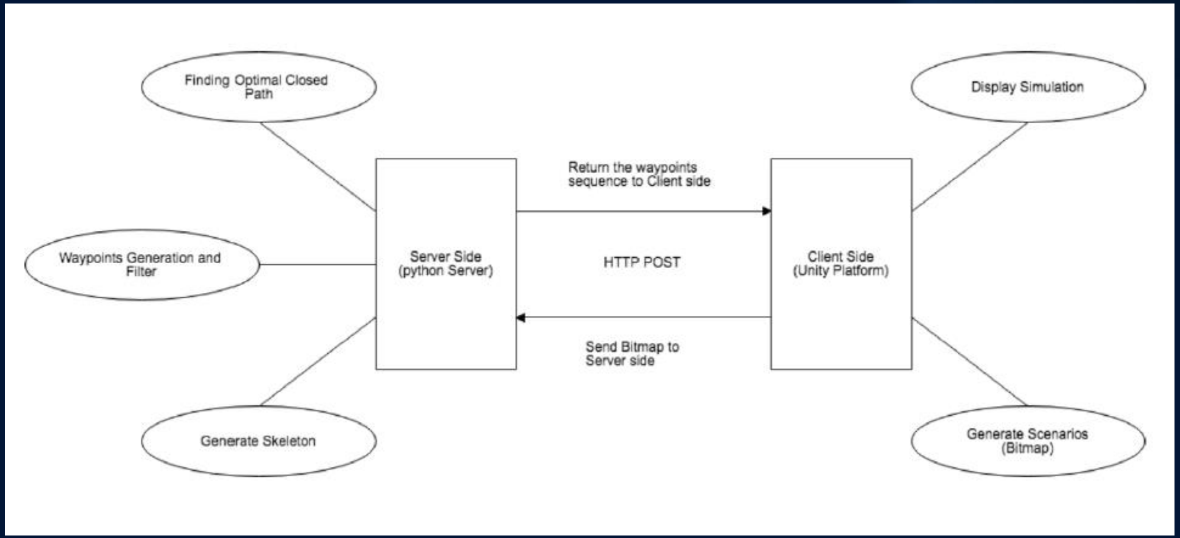
Stefan Edelkamp Zhuowei Yu

# Related Work: Robot Simulation in Unity






TAROS (2) 2019: 179-190

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# Related Work: Planning Flow Production

## A case study of planning for smart factories

Int. J. Softw. Tools Technol. Transf. 20(5): 515-528 (2018)

Stefan Edelkamp[1] · Christoph Greulich[2]

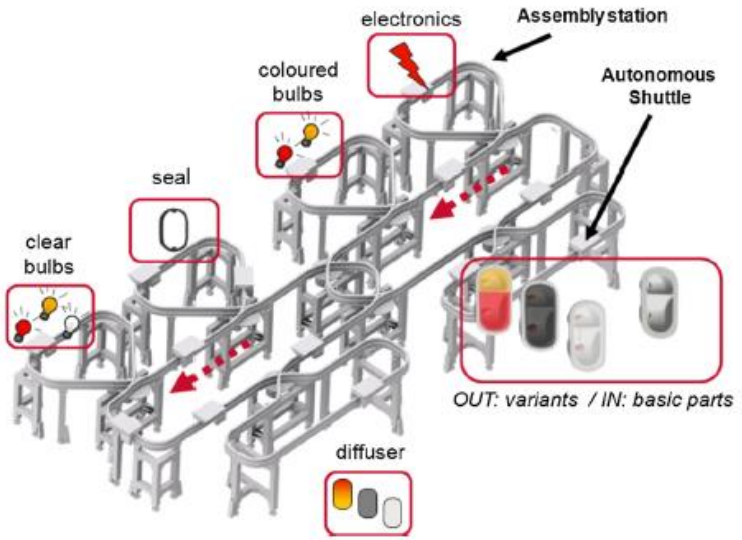


Fig. 1 Assembly scenario for taillights [40]

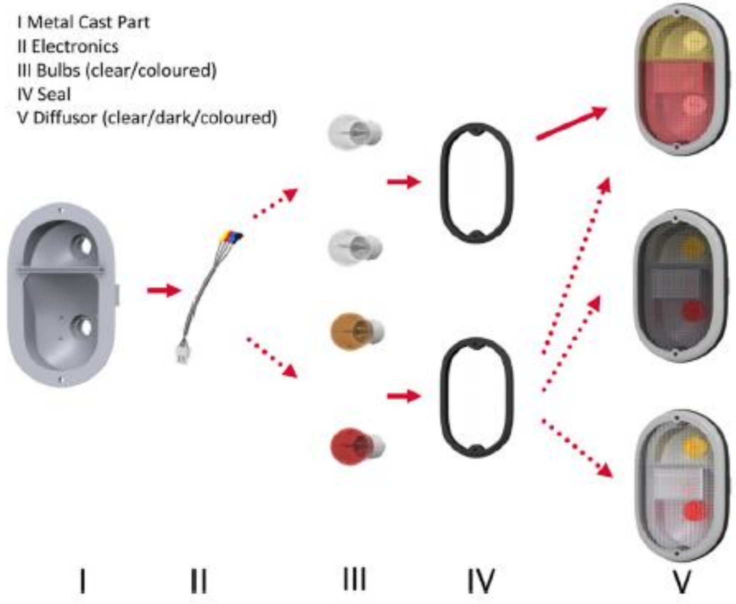


Fig. 2 Assembly states of taillights [21]

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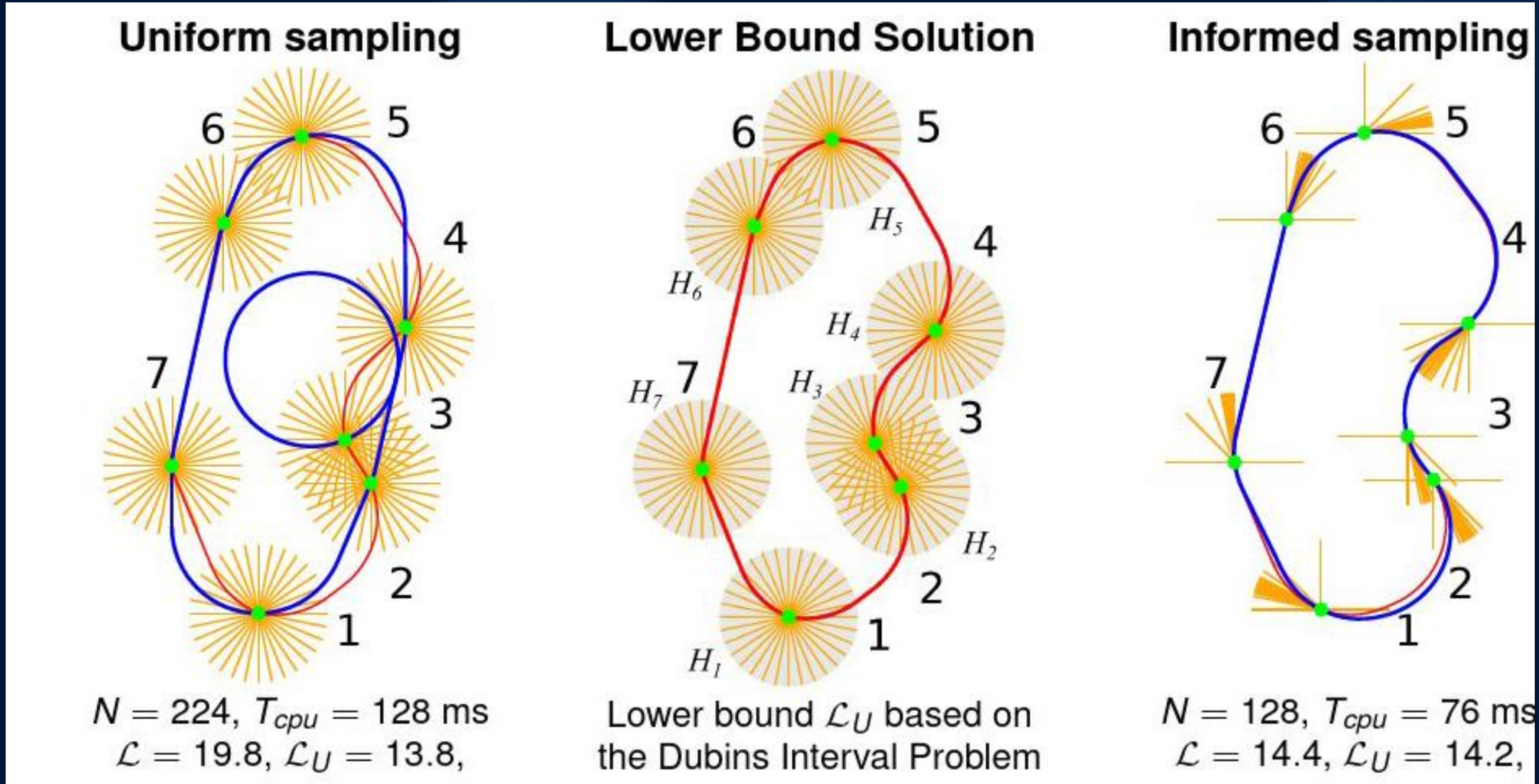
# Related Work: Dubins Car Touring Problem




- $N$ – the total number of samples (up to 32 samples per waypoint)
- $\mathcal{L}$ is the length of the tour (blue) and $\mathcal{L}_U$ is the lower bound (red) determined as a solution of the **Dubins Interval Problem** (**DIP**)

Jan Faigl , Petr Vana , Martin Saska, Tomás Báca, Vojtech Spurný:
On solution of the Dubins touring problem. ECMR 2017: 1-6

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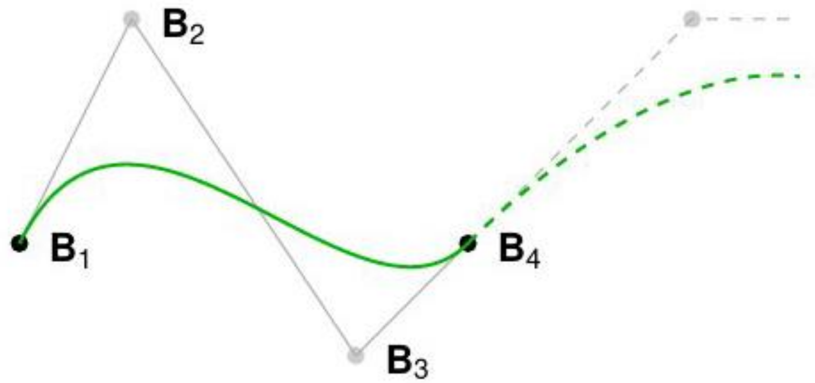
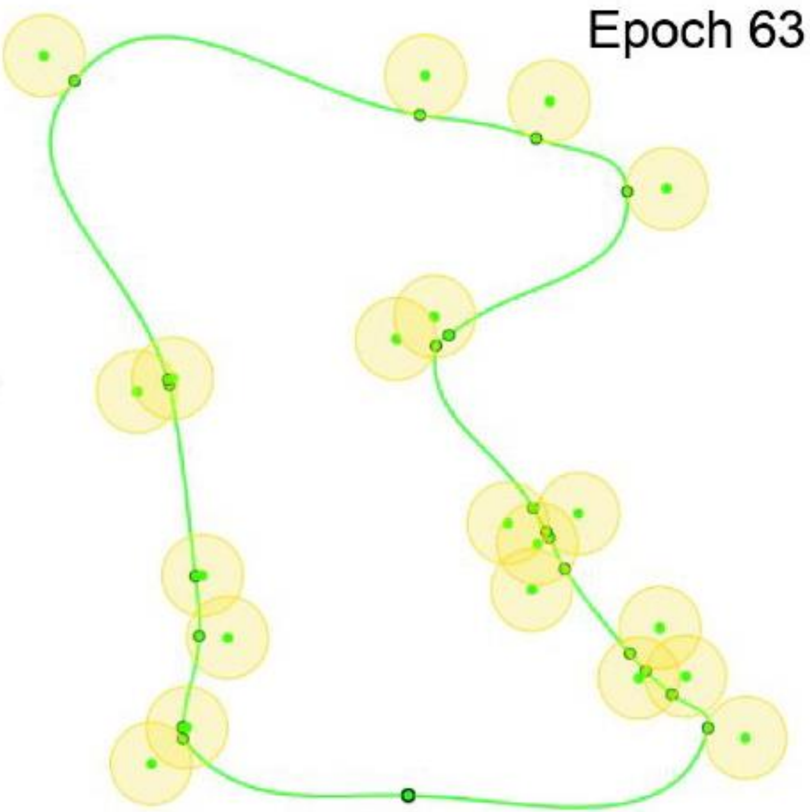
# Related Work: Bezier Curve Touring Problem

## Benefits of Bézier curves

- Flexible and easy to use
- Start/end direction is given by the first/last two control points




Example of a cubic Bézier curve




$$\mathbf{X}(\tau) = \mathbf{B}_0(1-\tau)^3 + 3\mathbf{B}_1\tau(1-\tau)^2 + 3\mathbf{B}_2\tau^2(1-\tau) + \mathbf{B}_3\tau^3$$


Jan Faigl, Petr Vána, Robert Penicka :
Multi-Vehicle Close Enough Orienteering Problem with Bézier Curves for Multi-Rotor Aerial Vehicles. ICRA 2019: 3039-3044

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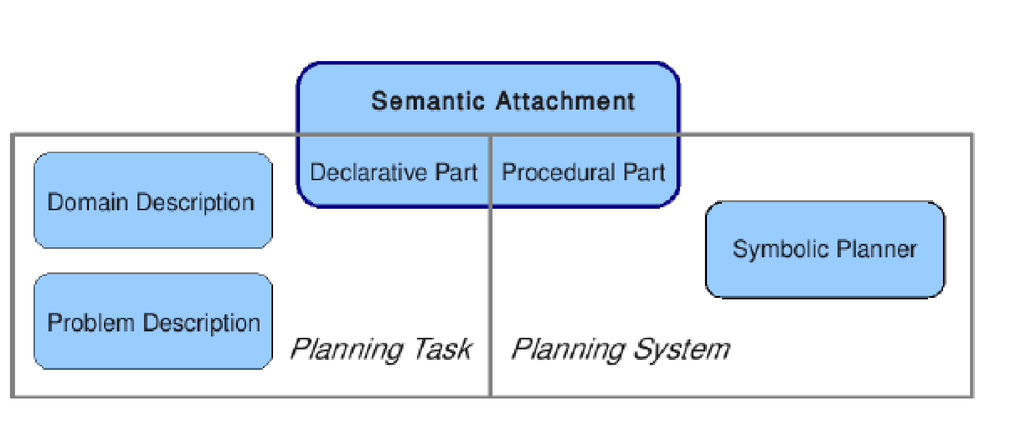
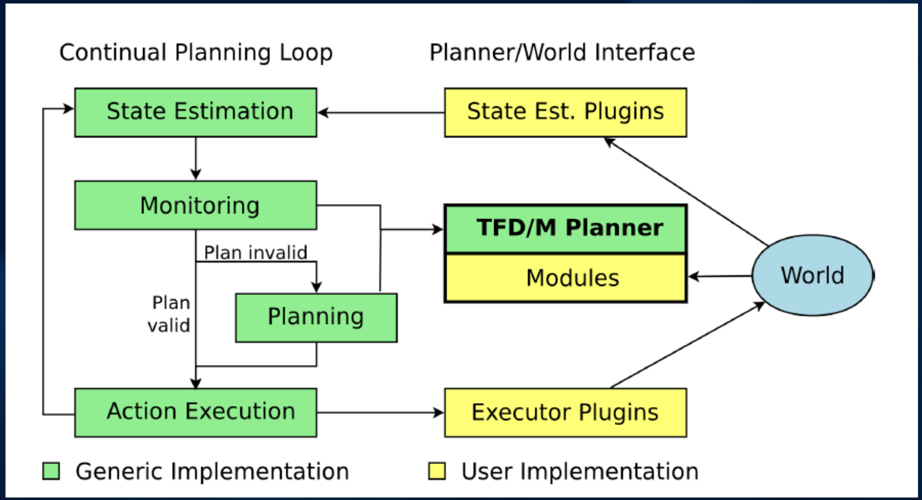
# Related Work: PDDL/M Planning with Semantic Attachments




Andreas Hertle, Christian Dornhege, Thomas Keller, Bernhard Nebel
Planning with Semantic Attachments:
An Object-Oriented View. ECAI 2012: 402-407

```
(:modules
   (canLoad ?v - vehicle ?p - package
    conditionchecker canLoad@libTrans.so) )
```

```
(:action put-down
 :params (?o - movable ?p - base ?g - grasp)
 :condition (... ([checkPutDown ?o ?p ?g]))
 :effect (and (on ?o ?p) (handempty)
  (not (holding ?o ?g)) ([putDown ?o ?p ?g])
```

```
(:modules
  (putDown ?o - movable ?p - base ?g - grasp
    (q0) (q1) (q2) (q3) (q4) (q5) (q6)
    (x ?o) (y ?o) (z ?o)
    (yaw ?o) (pitch ?o) (roll ?o)
   effect putDown@libTrajectory.so))
```

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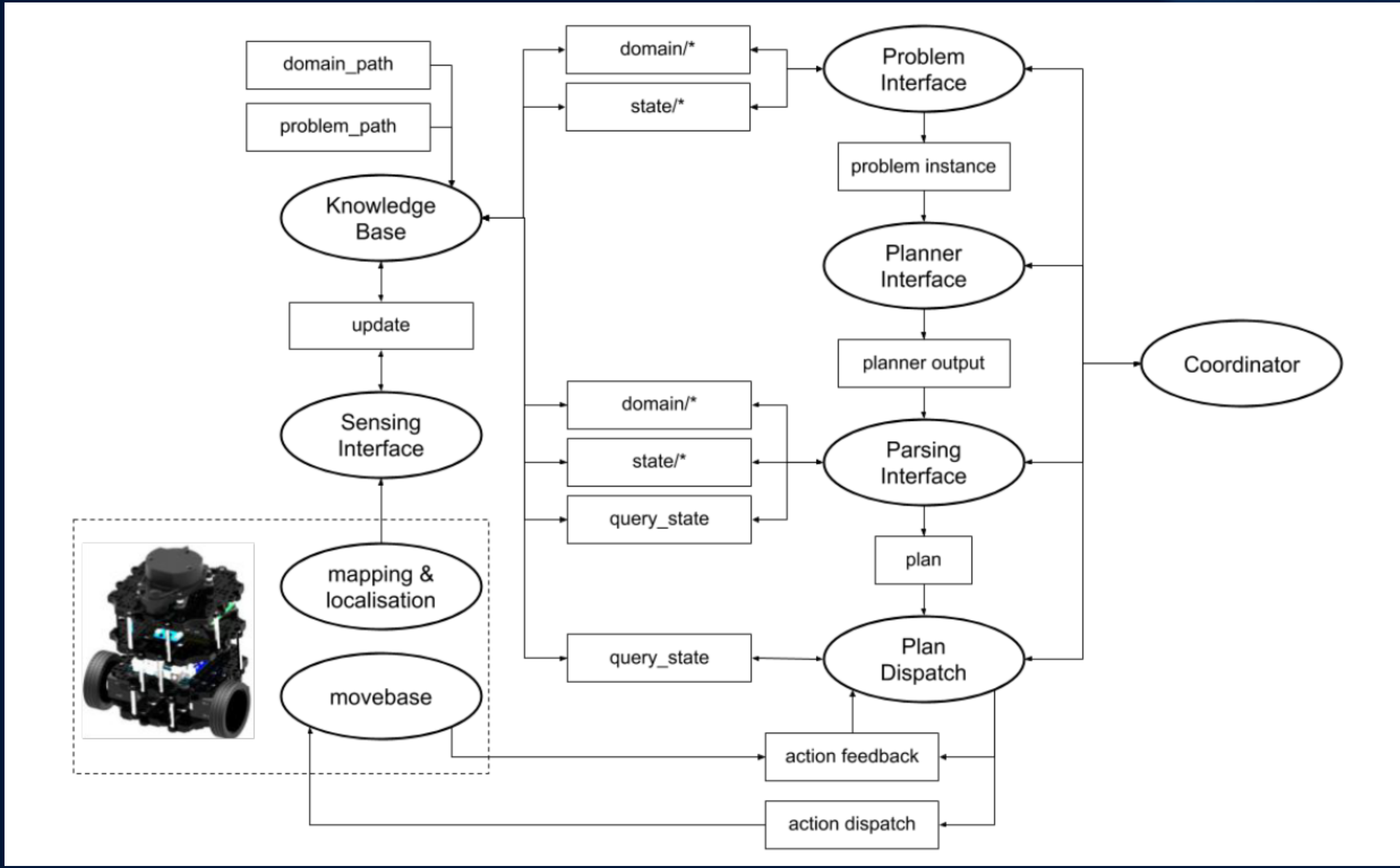
# Related Work: ROS Plan





Michael Cashmore, Maria Fox, Derek Long, Daniele Magazzeni, Bram Ridder, Arnau Carrera, Narcís Palomeras, Natàlia Hurtós, Marc Carreras: ROSPlan: Planning in the Robot Operating System. ICAPS 2015: 333-341

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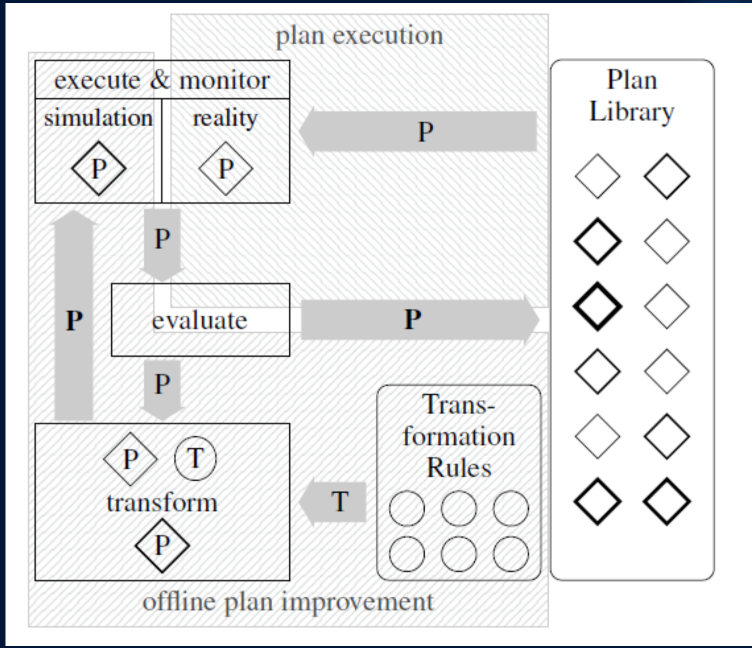
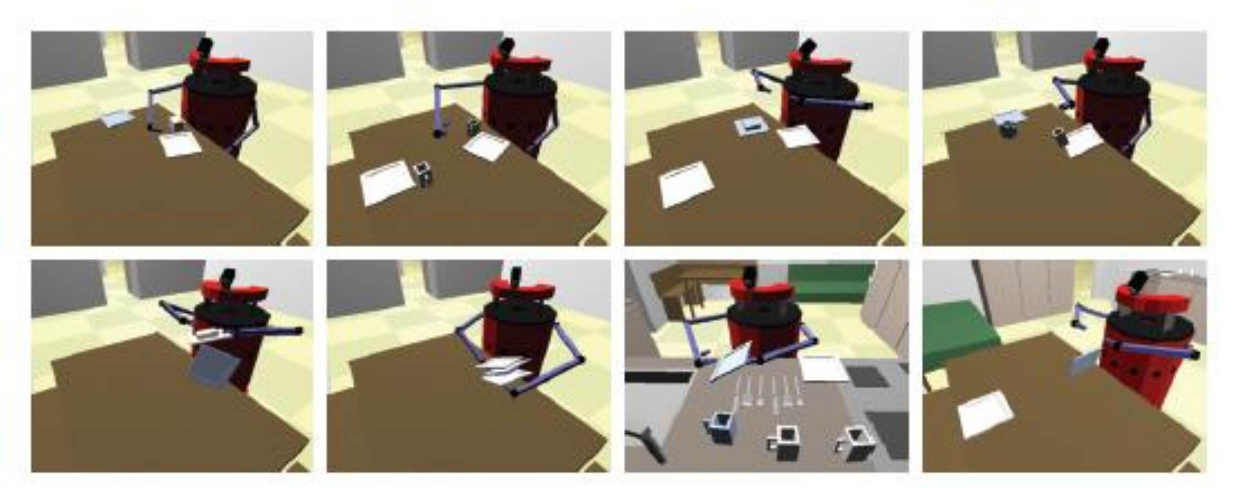
# Related Work: Transformational Planning

```
(define-plan (achieve (table-set ?persons))
  ( achieve-for-all
    (lambda (person)
      (with-designators
          ( (table '(the entity (type table)
                                (used-for meals)))
            (seating-location '(the location (at ,table)
                                             (preferred-by ,?person)))
            (plate '(an entity (type plate)
                               (status unused)))
            (cup '(an entity (type cup)
                             (status unused))) )
        (achieve (placed-on
                   plate
                   table
                   '(the location (on ,table)
                                  (matches (entity-location ,plate))
                                  (matches ,seating-location))))
        (achieve (placed-on
                   cup
                   table
                   '(the location (on ,table)
                                  (matches (entity-location ,cup))
                                  (matches ,seating-location))))))
    ?persons))
```



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Transformational Planning for Everyday Activity. ICAPS 2007: 248-255

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# Acknowledgements

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