Robot Motion Planning II / Multi-Goal Planning

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Lecture 10

A4M36PAH - Planning and Games

Lecture Goals

- Provide an overview of the existing problem formulations in robotic planning
- Multi-Goal Path Planning a.k.a. robotic Traveling Salesman Problem (TSP)
- Inspection, exploration, and data collection missions
- Challenges in planning for non-holonomic vehicle (Dubins vehicle)
- Example of problem formulations suitable for **robotic data collection planning**

During the lecture, several problems formulation will be defined. Most of them are variants of the TSP. Each problem aims to address a specific issue related to a particular robotic application.

The main goal of the lecture is to make you familiar with the key challenges in the related problems and existing approaches.

The goal is not to memorize all the details and definitions!

Multi-Goal Planning

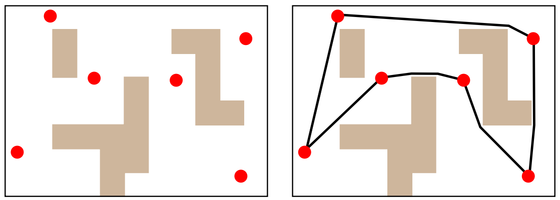
Multi-Goal Path Planning

Motivation

Having a set of locations (goals) to be visited, determine the cost efficient path to visit them and return to a starting location.

- Locations where a robotic arm performs some task
- Locations where a mobile robot has to be navigated

To perform measurements such as scan the environment or read data from sensors.



Alatartsev et al. (2015) – Robotic Task Sequencing Problem: A Survey

Traveling Salesman Problem (TSP)

Given a set of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city.

- The TSP can be formulated for a graph $\boldsymbol{G}(V, E)$, where V denotes a set of locations (cities) and E represents edges connecting two cities with the associated travel cost c (distance), i.e., for each $v_i, v_j \in V$ there is an edge $e_{ij} \in E$, $e_{ij} = (v_i, v_j)$ with the cost c_{ij}.
- If the associated cost of the edge (v_i, v_j) is the Euclidean distance $c_{ij} = |(v_i, v_j)|$, the problem is called the **Euclidean TSP** (ETSP).

  In our case, $v \in V$ represents a point in $\mathbb{R}^2$ and solution of the ETSP is a path in the plane.
- It is known, the TSP is NP-hard (its decision variant) and several algorithms can be found in literature.

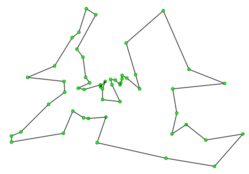
  William J. Cook (2012) – In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation

Solutions of the TSP

- Efficient heuristics from the Operational Research have been proposed
- LKH – K. Helsgaun efficient implementation of the Lin-Kernighan heuristic (1998)

  http://www.akira.ruc.dk/~keld/research/LKH/
- Concorde – Solver with several heuristic and also optimal solver

  http://www.math.uwaterloo.ca/tsp/concorde.html



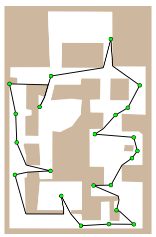
Problem Berlin52 from the TSPLIB

Beside the heuristic and approximations algorithms (such as Christofides 3/2-approximation algorithm), other („soft-computing") approaches have been proposed, e.g., based on genetic algorithms, and memetic approaches, ant colony optimization (ACO), and neural networks.

Multi-Goal Path Planning (MTP) Problem

Given a map of the environment $\mathcal{W}$, mobile robot $\mathcal{R}$, and a set of locations, what is the shortest possible **collision free path** that visits each location exactly once and returns to the origin location.

- MTP problem is de facto the TSP with the cost associated to the edges as the length of the *shortest* path connecting the locations
- For n locations, we need to compute up to n^2 shortest paths (solve n^2 motion planning problems)
- The paths can be found as the shortest path in a graph (roadmap), from which the $G(V, E)$ for the TSP can be constructed



Visibility graph as the roadmap for a point robot provides a straight forward solution, but such a shortest path may not be necessarily feasible for more complex robots

Multi-Goal Path Planning in Robotic Missions

Multi-goal path planning

- It builds on a simple path and trajectory planning
- It is a combinatorial optimization problem to determine the sequence to visit the given locations
- It allows to solve (or improve performance of) more complex problems such as
  - **Inspection planning** - Find the shortest tour to see (inspect) the whole environment
  - **Data collection planning** – Determine a cost efficient path to collect data from the sensor stations (locations)
  - *Robotic exploration* - Create a map of unknown environment as quickly as possible

Inspection Planning

Motivations (examples)

- Periodically visit particular locations of the environment to check, e.g., for intruders, and return to the starting locations
- Based on available plans, provide a guideline how to search a building to find possible victims as quickly as possible (search and rescue scenario)

Inspection Planning – Decoupled Approach

1. Determine sensing locations such that the whole environment would be inspected (seen) by visiting them

   A solution of the Art Gallery Problem
2. Create a roadmap connecting the sensing location

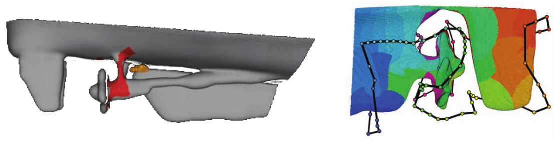
   E.g., using visibility graph or randomized sampling based approaches
3. Find the inspection path visiting all the sensing locations as a solution of the multi-goal path planning

   De facto solution of the TSP

Inspection planning is also called coverage path planning in literature.

Example – Inspection Planning with AUV

- Determine shortest inspection path for Autonomous Underwater Vehicle (AUV) to inspect a propeller of the vessel

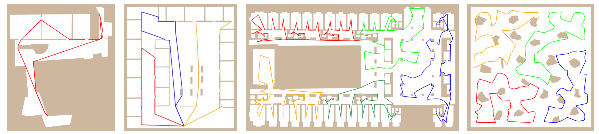


Three-dimensional coverage planning for an underwater inspection robot
Brendan Englot and Franz S. Hover
International Journal of Robotic Research, 32(9-10):1048–1073, 2013.

Inspection Planning – "*Continuous Sensing*"

- If we do not prescribe a discrete set of sensing locations, we can formulate the problem as the **Watchman route problem**

Given a map of the environment $\mathcal{W}$ determine the shortest, closed, and collision free path, from which the whole environment is covered by an omnidirectional sensors with the radius ρ.



Approximate Solution of the Multiple Watchman Routes Problem with Restricted Visibility Range
Jan Faigl
IEEE Transactions on Neural Networks, 21(10):1668–1679, 2010.

Self-Organizing Maps based Solution of the TSP

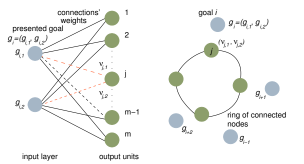
Kohonen's type of unsupervised two-layered neural network

- Neurons' weights represent nodes $\mathcal{N} = \{\nu_1, \ldots, \nu_m\}$ in a plane.
- Nodes are organized into a ring.
- Sensing locations $\boldsymbol{S} = \{s_1, \ldots s_n\}$ are presented to the network in a random order.
- Nodes compete to be winner according to their distance to the presented goal s

$$\nu^* = \mathrm{argmin}_{\nu \in \mathcal{N}} |\mathcal{D}(\nu, s)|$$

- The winner and its neighbouring nodes are adapted (moved) towards the city according to the neighbouring function

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{\sigma^2}} & \text{for } d < m/n_f, \\ 0 & \text{otherwise,} \end{cases}$$

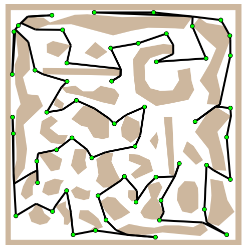


- Best matching unit ν to the presented prototype s is determined according to distance function $|\mathcal{D}(\nu, s)|$
- For the Euclidean TSP, $\mathcal{D}$ is the Euclidean distance
- However, for problems with obstacles, the multi-goal path planning, $\mathcal{D}$ should correspond to the length of the shortest, collision free path.

SOM for the Multi-Goal Path Planning

Unsupervised learning procedure

$\mathcal{N} \leftarrow$ initialization$(\nu_1, \ldots, \nu_m)$
repeat
  error $\leftarrow 0$
  foreach $g \in \Pi(\boldsymbol{S})$ **do**
    $\nu^* \leftarrow$ **selectWinner** $\mathrm{argmin}_{\nu \in \mathcal{N}} |S(g, \nu)|$
    adapt$(S(g, \nu), \mu f(\sigma, l)|S(g, \nu)|)$
    error $\leftarrow \max\{error, |S(g, \nu^*)|\}$
  $\sigma \leftarrow (1 - \alpha) \cdot \sigma$
until *error* $\leq \delta$



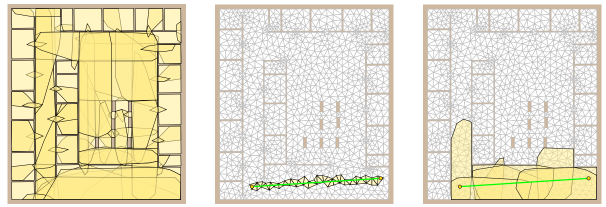
- For multi-goal path planning – the **selectWinner** and **adapt** procedures are based on the solution of the path planning problem

An Application of Self-Organizing Map in the non-Euclidean Traveling Salesman Problem
Jan Faigl, Miroslav Kulich, Vojtěch Vonásek and Libor Přeučil
Neurocomputing, 74(5):671–679, 2011.

SOM for the TSP in the Watchman Route Problem

During the unsupervised learning, we can compute coverage of $\mathcal{W}$ from the current ring (solution represented by the neurons) and adapt the network towards uncovered parts of $\mathcal{W}$

- Convex cover set of $\mathcal{W}$ created on top of a triangular mesh
- Incident convex polygons with a straight line segment are found by walking in a triangular mesh technique

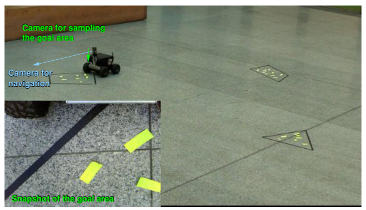


Jan Faigl (2010), TNN

Multi-Goal Path Planning with Goal Areas

- It may be sufficient to visit a goal region instead of the particular point location

  E.g., to take a sample measurement at each goal



Not only a sequence of goals visit has to be determined, but also an appropriate sensing location for each goal need to be found.

The problem with goal regions can be considered as a variant of the **Traveling Salesman Problem with Neighborhoods (TSPN)**.

Traveling Salesman Problem with Neighborhoods

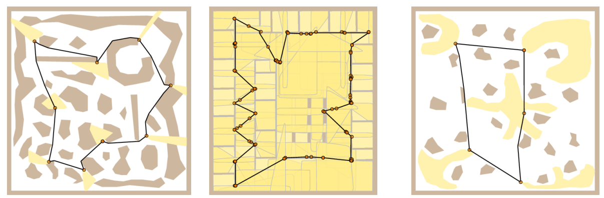
Given a set of n regions (neighbourhoods), what is the shortest closed path that visits each region.

- The problem is NP-hard and APX-hard, it cannot be approximated to within factor $2 - \epsilon$, where $\epsilon > 0$

  Safra and Schwartz (2006) – Computational Complexity
- Approximate algorithms exists for particular problem variants

  E.g., Disjoint unit disk neighbourhoods
- Flexibility of SOM for the TSP allows to generalize the unsupervised learning procedure to address the TSPN
- **TSPN provides a suitable problem formulation for planning various inspection and data collection missions**

SOM-based Solution of the Traveling Salesman Problem with Neighborhoods (TSPN)



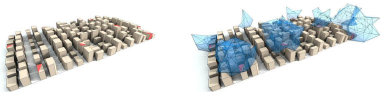
Polygonal Goals	Convex Cover Set	Non-Convex Goals
n=9, T= 0.32 s	n=106, T=5.1 s	n=5, T=0.1 s

Visiting Convex Regions in a Polygonal Map,
Jan Faigl, Vojěch Vonásek and Libor Přeučil
Robotics and Autonomous Systems, 61(10):1070–1083, 2013.

Example – TSPN for Inspection Planning with UAV

- Determine a cost efficient trajectory from which a given set of target regions is covered
- For each target region a subspace $S \subset \mathbb{R}^3$ from which the target can be covered is determined

  S represents the neighbourhood
- The PRM motion planning algorithm is utilized to construct a motion planning roadmap (a graph)
- SOM based solution of the TSP with a graph input is generalized to the TSPN



Janoušek and Faigl, (2013) – ICRA

Example – TSPN for Planning with Localization Uncertainty

- Selection of waypoints from the neighbourhood of each location
- P3AT ground mobile robot in an outdoor environment

TSP: L=184 m, E_{avg}=0.57 m

TSPN: L=202 m, E_{avg}=0.35 m

Real overall error at the goals decreased from 0.89 m → 0.58 m (about 35%)

- Decrease localization error at the target locations (indoor)

Error decreased from 16.6 cm → 12.8 cm

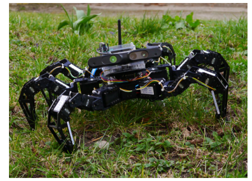
Improved success of the locations' visits 83%→95%

Faigl et al., (2012) – ICRA

Multi-Goal Motion Planning

- In the previous cases, we consider existing roadmap or relatively "simple" collision free (shortest) paths in the polygonal domain
- However, determination of the collision free path in a high dimensional configuration space ($\mathcal{C}$-space) can be a challenging problem itself
- Therefore, we can generalize the MTP to multi-goal **motion** planning (MGMP) considering motion (trajectory) planners in $\mathcal{C}$-space.
- An example of MGMP can be

> Plan a cost efficient trajectory for hexapod walking robot to visit a set of target locations.



Problem Statement – MGMP Problem

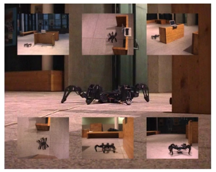
- The working environment $\mathcal{W} \subset \mathbb{R}^3$ is represented as a set of obstacles $\mathcal{O} \subset \mathcal{W}$ and the robot configuration space $\mathcal{C}$ describes all possible configurations of the robot in $\mathcal{W}$
- For $q \in \mathcal{C}$, the robot body $\mathcal{A}(q)$ at q is collision free if $\mathcal{A}(q) \cap \mathcal{O} = \emptyset$ and all collision free configurations are denoted as $\mathcal{C}_{free}$
- Set of n goal locations is $\mathcal{G} = (g_1, \ldots, g_n)$, $g_i \in \mathcal{C}_{free}$
- Collision free path from q_{start} to q_{goal} is $\kappa : [0,1] \to \mathcal{C}_{free}$ with $\kappa(0) = q_{start}$ and $d(\kappa(1), q_{end}) < \epsilon$, for an admissible distance ϵ
- Multi–goal path τ is **admissible** if $\tau : [0,1] \to \mathcal{C}_{free}$, $\tau(0) = \tau(1)$ and there are n points such that $0 \le t_1 \le t_2 \le \ldots \le t_n$, $d(\tau(t_i), v_i) < \epsilon$, and $\bigcup_{1<i\le n} v_i = \mathcal{G}$
- **The problem is to find path τ^* for a cost function c such that $c(\tau^*) = \min\{c(\tau) \mid \tau$ *is admissible multi–goal path*$\}$**

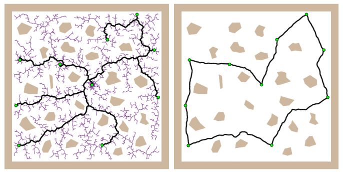
MGMP – Examples of Solutions

- We aim to avoid explicit determination of all paths connecting two locations $g_i, g_j \in \mathcal{G}$
- Various approaches can be found in literature, e.g.,
  - Considering Euclidean distance as approximation in solution of the TSP as the Minimum Spanning Tree (MST)

    Saha et al. (2006), IJJR
  - Steering RRG roadmap expansion by unsupervised learning of SOM for the TSP

    Faigl (2016), WSOM





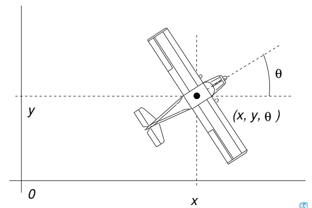
Dubins Vehicle

- Non-holonomic vehicle such as car-like or aircraft can be modeled as the Dubins vehicle
  - Constant forward velocity
  - Limited minimal turning radius ρ
  - Vehicle state is represented by a triplet $q = (x, y, \theta)$, where
  - $(x, y) \in \mathbb{R}^2$, $\theta \in \mathbb{S}^2$ and thus, $q \in SE(2)$

The vehicle motion can be described by the equation:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{u}{\rho} \end{bmatrix}, \quad |u| \le 1,$$

where u is the control input.



Optimal Maneuvers for Dubins Vehicle

- For two states $q_1 \in SE(2)$ and $q_2 \in SE(2)$ in the environment without obstacles $\mathcal{W} = \mathbb{R}^2$ the optimal paths can be characterized as one of two main types
  - **CCC** type: LRL, RLR;
  - **CSC** type: LSL, LSR, RSL, RSR;

  where S – straight line arc, C – circular arc oriented to left (L) or right (R)

  L. E. Dubins (1957) – American Journal of Mathematics
- The optimal paths are called **Dubins maneuvers**:
  - Constant velocity: $v(t) = v$ and turning radius ρ
  - **6** types of trajectories connecting any configuration in $\mathbb{R}^2 \times \mathbb{S}^1$

    without obstacles
  - The control u is according to C and S type one of the three possible values $u \in \{-1, 0, 1\}$

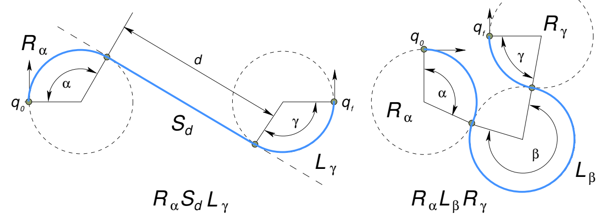
Parametrization of Dubins Maneuvers

- Parametrization of each trajectory phase:

$$\{L_\alpha R_\beta L_\gamma, R_\alpha L_\beta R_\gamma, L_\alpha S_d L_\gamma, L_\alpha S_d R_\gamma, R_\alpha S_d L_\gamma, R_\alpha S_d R_\gamma\}$$

for $\alpha \in [0, 2\pi)$, $\beta \in (\pi, 2\pi)$, $d \ge 0$

Notice the prescribed orientation at q_0 and q_f.



Planning with Dubins vehicle

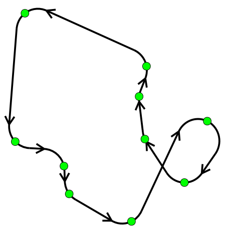
- The optimal path connecting two configurations can be found analytically

  E.g., for UAVs that usually operates in environment without obstacles
- The Dubins maneuvers can be used in randomized-sampling based motion planners, such as RRT, in the control based sampling
- We can consider the model of Dubins vehicle in the multi-goal path planning
  - Surveillance, inspection or monitoring missions to periodically visits given target locations (areas)
- **Dubins Traveling Salesman Problem DTSP**

> Given a set of locations, what is the shortest Dubins path that visits each location exactly once and returns to the origin location.

Dubins Traveling Salesman Problem (DTSP)

- Let have Dubins vehicle with minimal turning radius ρ
- Let the given set of n target locations be $G = \{g_1, \ldots, g_n\}$
- Let $\Sigma = (\sigma_1, \ldots, \sigma_n)$ be a permutation of $\{1, \ldots, n\}$
- Let $\mathcal{P}$ be projection form $SE(2)$ to $\mathbb{R}^2$ such that $\mathcal{P}(q_i) = (x_i, y_i)$, $q_i \in SE(2)$ and $g_i = (x_i, y_i)$.



- DTSP is a problem to determine the minimum length tour that visits every location $g_i \in G$ while satisfying motion constraints of the Dubins vehicle

DTSP – Optimization Criterion

- DTSP is an optimization problem over all permutations Σ and headings $\Theta = \{\theta_{\sigma_1}, \ldots, \theta_{\sigma_n}\}$ for the states $(q_{\sigma_1}, \ldots, q_{\sigma_n})$ such that $q_{\sigma_i} = (g_{\sigma_i}, \theta_{\sigma_i})$

$$\textit{minimize}_{\Sigma,\Theta} \quad \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \qquad (1)$$

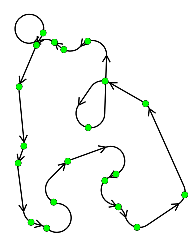
$$\textit{subject to} \quad q_i = (g_i, \theta_i) \; i = 1, \ldots, n \qquad (2)$$

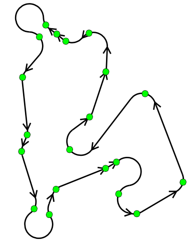
- $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$ is the length of the shortest possible Dubins maneuver connecting the states q_{σ_i} and q_{σ_j}.

Challenges of the Dubins Traveling Salesman Problem

- The key difficulty of the DTSP is that the path length mutually depends on
  - Order of the visits to the locations
  - Headings at the target locations

We need the sequence to determine headings, but headings may influence the sequence





Algorithms for the DTSP

Two fundamental approaches can be found in literature

- Considering a sequence of the visits is given

  E.g., found by a solution of the Euclidean TSP
- Sampling the headings at the locations into discrete sets of values and considering the problem as the variant of the Generalized TSP

  Sampling based approaches

Besides, further approaches are

- Approximation algorithms; optimal solutions for restricted variants
- Soft-computing technique such as genetic and memetic technique or neural networks

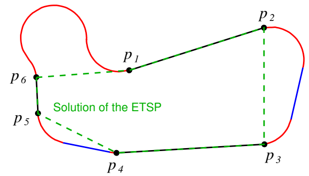
DTSP – Alternating Algorithm

`Alternating Algorithm` (AA) provides a solution of the DTSP for an even number of targets n

Savla et al. (2005)

1. Solve the related Euclidean TSP

   Relaxed motion constraints
2. Establish headings for even edges using straight line segments
3. Determine optimal maneuvers for odd edges



Courtesy of P. Váňa

AA is heuristic algorithm which solutions can be bounded by $L_{TSP}\kappa\lceil n/2\rceil\pi\rho$, where L_{TSP} is the length of the optimal solution of the ETSP and $\kappa < 2.658$.

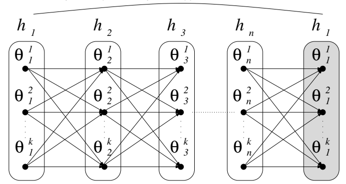
DTSP with the Given Sequence of the Visits to the Targets

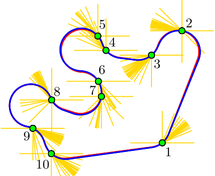
- If the sequence of the visits Σ to the target locations is given
- the problem is to determine the optimal heading at each location
- We call the problem as the **Dubins Touring Problem** (**DTP**)

  Váňa and Faigl (2016)
- Let for each location $g_i \in G$ sample possible heading to k values, i.e., for each g_i the set of headings be $h_i = \{\theta_1^1, \ldots, \theta_1^k\}$.
- Since Σ is given, we can construct a graph connecting two consecutive locations in the sequence by all possible headings
- For such a graph and particular headings $\{h_1, \ldots, h_n\}$, we can find an optimal headings and thus, the optimal solution of the DTP.

DTSP as a Solution of the DTP

The first layer is duplicated layer to support the forward search method





- The edge cost corresponds to the length of Dubins maneuver
- Better solution of the DTP can be found for more samples, which will also improve the DTSP but only for the given sequence

Two questions arise for a practical solution of the DTP

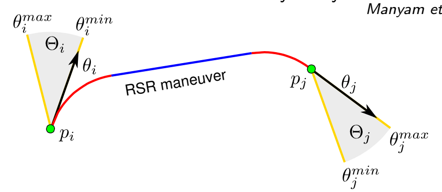
- How to sample the headings? Since more samples makes finding solution more demanding

  We need to sample the headings in a "smart" way.
- What is the solution quality? Is there a tight lower bound?

Dubins Interval Problem

- Dubins Interval Problem (DIP) is a generalization of Dubins maneuvers to the shortest path connecting two points p_i and p_j
- In the DIP, an leaving interval Θ_i at p_i and arrival interval Θ_j at p_j are allowed
- The optimal solution can be found analytically

Manyam et al. (2015)



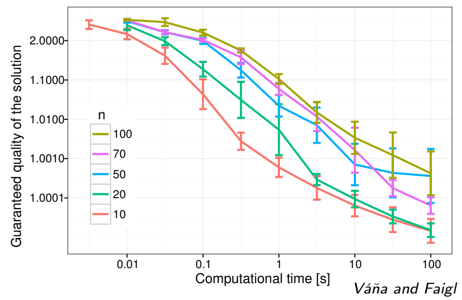
- Solution of the DIP is a tight lower bound for the DTP

  Manyam and Rathinam (2015)
- Solution of the DIP is not a feasible solution of the DTP

  Notice, for $\Theta_i = \Theta_j = \langle 0, 2\pi)$ the optimal maneuver for DIP is straight line segment

The DIP-based Sampling of Headings in the DTP

- A similar graph as for DTP can be used for heading intervals
- The solution of the DIP is a lower bound of the DTP
- It can be used to inform how to splitting heading intervals
- The ratio between the lower bound and feasible solution of the DTP provides estimation of the solution quality



Váňa and Faigl (2016)

DTSP – Sampling-based Approach

- Sampled heading values can be directly utilized to find the sequence as a solution of the Generalized Traveling Salesman Problem (GTSP)

Also called Set TSP or Covering Salesman Problem

Notice For Dubins vehicle, it is the Generalized Asymmetric TSP (GATSP)

> The problem is to determine a shortest tour in a graph that visits all specified subsets of the graph's vertices.

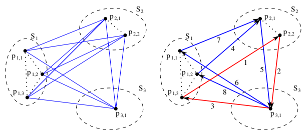
The TSP is a special case of the GTSP when each subset to be visited consists just a single vertex.

- GATSP → ATSP

Noon and Bean (1991)

- ATSP can be solved by LKH
- ATSP → TSP, which can be solved optimally

E.g., by Concorde



GTSP · GATSP

Dubins Traveling Salesman Problem with Neighborhoods

- In surveillance planning, it may be required to visit a set of target regions $\boldsymbol{G} = \{R_1, \ldots, R_n\}$ by the Dubins vehicle
- Then, for each target region R_i, we have to determine a particular point of the visit $p_i \in R_i$ and DTSP becomes the Dubins Traveling Salesman Problem with Neighborhoods (DTSPN)

In addition to Σ and headings Θ, locations P have to be determined.

DTSPN – Optimization Criterion

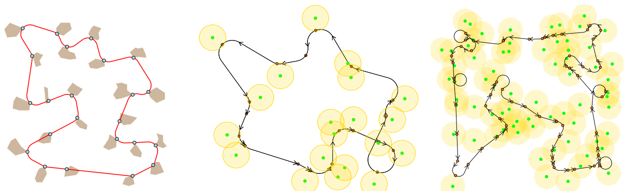
- DTSPN is an optimization problem over all permutations Σ, headings $\Theta = \{\theta_{\sigma_1}, \ldots, \theta_{\sigma_n}\}$ and points $P = (p_{\sigma_1}, \ldots, p_{\sigma_n})$ for the states $(q_{\sigma_1}, \ldots, q_{\sigma_n})$ such that $q_{\sigma_i} = (p_{\sigma_i}, \theta_{\sigma_i})$ and $p_{\sigma_i} \in R_{\sigma_i}$:

$$\textit{minimize}_{\Sigma,\Theta,P} \quad \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) + \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) \qquad (3)$$

$$\textit{subject to} \quad q_i = (p_i, \theta_i), p_i \in R_i \;\; i = 1, \ldots, n \qquad (4)$$

- $\mathcal{L}(q_{\sigma_i}, q_{\sigma_j})$ is the length of the shortest possible Dubins maneuver connecting the states q_{σ_i} and q_{σ_j}.

DTSPN – Examples of Solution



Váňa and Faigl (2015), (IROS)

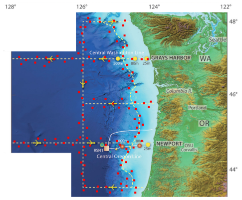
Faigl and Váňa (2016)

Autonomous Data Collection

- Having a set of sensors (sampling stations), we aim to determine a cost efficient path to retrieve data from the individual sensors

E.g., Sampling stations on the ocean floor

- The planning problem is a variant of the Traveling Salesman Problem



Two practical aspects of the data collection can be identified

1. Data from particular sensors may be of different importance
2. Data from the sensor can be retrieved using wireless communication

These two aspects can be considered in Prize-Collecting Traveling Salesman Problem (PC-TSP) and Orienteering Problem (OP) and their extensions with neighborhoods.

Prize-Collecting Traveling Salesman Problem with Neighborhoods (PC-TSPN)

- Let n sensors be located in $\mathbb{R}^2$ at the locations $S = \{s_1, \ldots, s_n\}$
- Each sensor has associated penalty $\zeta(s_i) \geq 0$ characterizing additional cost if the data are not retrieved from s_i
- Let the data collecting vehicle operates in $\mathbb{R}^2$ with the motion cost $c(p_1, p_2)$ for all pairs of points $p_1, p_2 \in \mathbb{R}^2$
- The data from s_i can be retrieved within δ distance from s_i

PC-TSPN – Optimization Criterion

The PC-TSPN is a problem to

- Determine a set of unique locations $G = \{g_1, \ldots, g_k\}$, $k \leq n$, $g_i \in \mathbb{R}^2$, at which data readings are performed
- Find a cost efficient tour T visiting G such that the total cost $C(T)$ of T is minimal

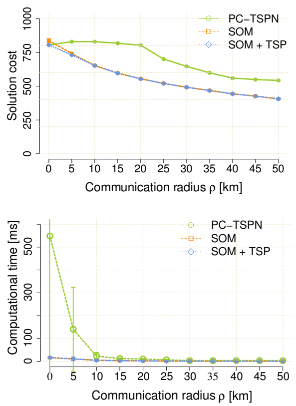
$$\mathcal{C}(T) = \sum_{(g_{l_i}, g_{l_{i+1}}) \in T} c(g_{l_i}, g_{l_{i+1}}) + \sum_{s \in S \setminus S_T} \zeta(s), \qquad (5)$$

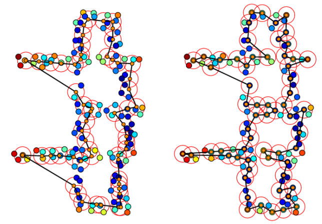
where $S_T \subseteq S$ are sensors such that for each $s_i \in S_T$ there is g_{l_j} on $T = (g_{l_1}, \ldots, g_{l_{k-1}}, g_{l_k})$ and $g_{l_j} \in G$ for which $|(s_i, g_{l_j})| \leq \delta$.

- PC-TSPN includes other variants of the TSP
  - for $\delta = 0$ it is the PC-TSP
  - for $\zeta(s_i) = 0$ and $\delta \geq 0$ it is the TSPN
  - for $\zeta(s_i) = 0$ and $\delta = 0$ it is the ordinary TSP

PC-TSPN – Example of Solution

Ocean Observatories Initiative (OOI) scenario





SOM PCTSPN · PCTSPN

Faigl and Hollinger (2014) – IROS

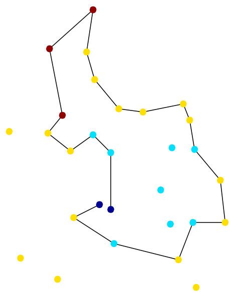
Orienteering Problem

- The Orienteering Problem (OP) originates from the orienteering outdoor sport
- The problem is to collect as many rewards as possible within the given travel budget
- It is similar to the PC-TSP, but the tour length must not exceed the prescribed maximize tour length T_{max}
- In OP, the starting and termination locations are prescribed, and they can be different

The solution may not be a closed tour as in the TSP

Orienteering Problem – Specification

- Let the given set of n sensors be located in $\mathbb{R}^2$ with the locations $S = \{s_1, \ldots, s_n\}$, $s_i \in \mathbb{R}^2$
- Each sensor s_i has an associated score ς_i characterizing the reward if data from s_i are collected
- The vehicle is operating in $\mathbb{R}^2$ and the travel cost is the Euclidean distance
- The starting and termination locations are prescribed



- We aim to determine a subset of k locations $S_k \subseteq S$ that maximizes the sum of the collected rewards while the travel cost to visit them is below T_{max}.

Orienteering Problem – Optimization Criterion

- Let $\Sigma = (\sigma_1, \ldots, \sigma_k)$ be a permutation of k sensor labels, $1 \le \sigma_i \le n$ and $\sigma_i \neq \sigma_j$ for $i \neq j$
- Σ defines a tour $T = (s_{\sigma_1}, \ldots, s_{\sigma_k})$ visiting the selected sensors S_k
- Let the start and end points of the tour be $\sigma_1 = 1$ and $\sigma_k = n$
- The **Orienteering problem** (**OP**) is to determine the number of sensors k, the subset of sensors S_k, and their sequence Σ such that

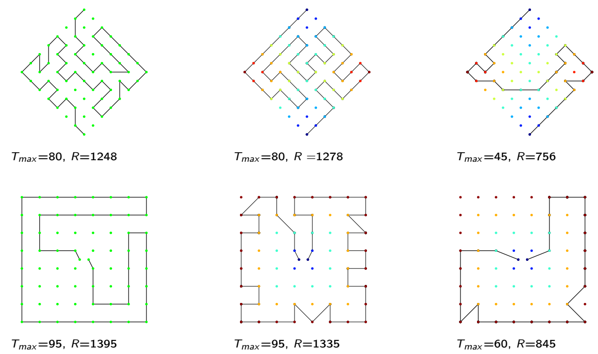
$$\begin{aligned} & \textit{maximize}_{k,S_k,\Sigma} \quad R = \sum_{i=1}^{k} \varsigma_{\sigma_i} \\ & \textit{subject to} \quad \sum_{i=2}^{k} |(s_{\sigma_{i-1}}, s_{\sigma_i})| \le T_{max} \ \textit{and} \\ & \qquad s_{\sigma_1} = s_1, s_{\sigma_k} = s_n. \end{aligned} \quad (6)$$

The OP combines the problem of determining the most valuable locations S_k with finding the shortest tour T visiting the locations S_k. It is NP-hard, since for $s_1 = s_n$ and particular S_k it becomes the TSP.

Orienteering Problem – Example of Solutions

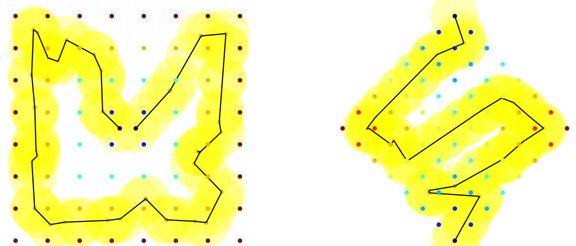
- Heuristic algorithms have been proposed

E.g., Ramesh et al. (1991), Chao et. al. (1996)



Orienteering Problem with Neighborhoods

- Similarly to the TSP with Neighborhoods and PC-TSPN we can formulate the **Orienteering Problem with Neighborhoods**.



T_{max}=60, δ=1.5, R=1600 T_{max}=45, δ=1.5, R=1344

Mobile Robot Exploration

- Create a map of the environment
- **Frontier**-based approach

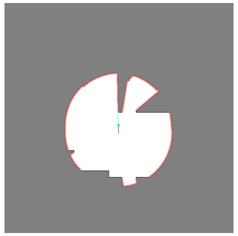
Yamauchi (1997)

- Occupancy grid

Moravec and Elfes (1985)

- Laser scanner sensor
- Next-best-view approach

Select the next robot goal



Performance metric:

Time to create the map of the whole environment

search and rescue mission

Distance Cost Variants

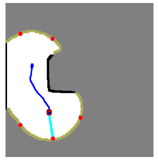
Simple robot–goal distance

- Evaluate all goals using the robot–goal distance

a length of the path from the robot position to the goal candidate

- Greedy goal selection

Select the closest goal candidate



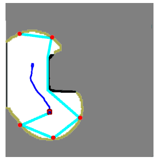
TSP distance cost

On Distance Utility in the Exploration Task
Miroslav Kulich, Jan Faigl and Libor Přeučil
ICRA, 2011, 4455-4460.

- Consider visitations of all goals

Solve the associated traveling salesman problem (TSP)

- A length of the tour visiting all goals
- Goal representatives

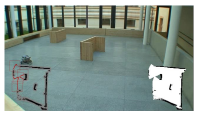


TSP distance cost improves performance about 10-30%

Multi-Robot Exploration Strategy

- A set of m robots at positions $\boldsymbol{R} = \{r_1, r_2, \ldots, r_m\}$
- At time t, let a set of n goal candidates be $\boldsymbol{G}(t) = \{g_1, \ldots, g_n\}$

e.g., frontiers



- The exploration strategy (at the planning step t):

Select a goal $g \in \boldsymbol{G}(t)$ for each robot $r \in \boldsymbol{R}$ that will minimize the required time to explore the environment.

The problem is formulated as the **task-allocation problem**

$$(\langle r_1, g_{r_1}\rangle, \ldots, \langle r_m, g_{r_m}\rangle) = \text{assign}(\boldsymbol{R}, \boldsymbol{G}(t), \mathcal{M}),$$

where $\mathcal{M}$ is the current map

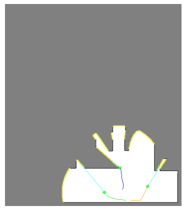
We consider only the distance cost for the assignment

Multi-Robot Exploration – Problem Definition

A problem of creating a grid map of the unknown environment by a set of m robots $\boldsymbol{R} = \{r_1, r_2, \ldots, r_m\}$.

Exploration is an iterative procedure:

1. Collect new sensor measurements
2. Determinate a set of goal candidates
   $\boldsymbol{G}(t) = \{g_1, g_2, \ldots, g_n\}$
   e.g., frontiers
3. At time step t, select next goal for each robot as the task-allocation problem
   $$(\langle r_1, g_{r_1}\rangle, \ldots, \langle r_m, g_{r_m}\rangle) = \text{assign}(\boldsymbol{R}, \boldsymbol{G}(t), \mathcal{M}(t))$$
   using the distance cost function
4. Navigate robots towards goal
5. If $|\boldsymbol{G}(t)| > 0$ go to Step 1; otherwise terminate



Proposed Multiple Traveling Salesman Approach

- Consider the task-allocation problem as the **Multiple Traveling Salesman Problem (MTSP)**
- MTSP heuristic ⟨*cluster–first, route–second* ⟩
  1. Cluster the goal candidates $\boldsymbol{G}$ to m clusters
     $\boldsymbol{C} = \{C_1, \ldots, C_m\}, C_i \subseteq \boldsymbol{G}$
     using K-means
  2. For each robot $r_i \in \boldsymbol{R}, i \in \{1, \ldots m\}$ select the next goal g_i from C_i using the TSP distance cost
     Kulich et at., ICRA (2011)
     - Solve the TSP on the set $C_i \cup \{r_i\}$
       the tour starts at r_i
     - The next robot goal g_i is the first goal of the found TSP tour

Goal Assignment using Distance Cost in Multi-Robot Exploration
Jan Faigl, Miroslav Kulich and Libor Přeučil
IROS, 2012, 3741–3741.

Comparison – Goal Assignment Strategies

1. **Greedy Assignment**
   Yamauchi B, Robotics and Autonomous Systems 29, 1999
   - Randomized greedy selection of the closest goal candidate
2. **Iterative Assignment**
   Werger B, Mataric M, Distributed Autonomous Robotic Systems 4, 2001
   - Centralized variant of the broadcast of local eligibility algorithm (BLE)
3. **Hungarian Assignment**
   - Optimal solution of the task-allocation problem for assignment of n goals and m robots in $O(n^3)$
   Stachniss C, C implementation of the Hungarian method, 2004
4. **MTSP Assignment**
   - $\langle$cluster–first, route–second$\rangle$, the TSP distance cost

In all strategies, we use the identical selection of the goal candidates from the frontiers.

Statistical Evaluation of the Exploration Strategies

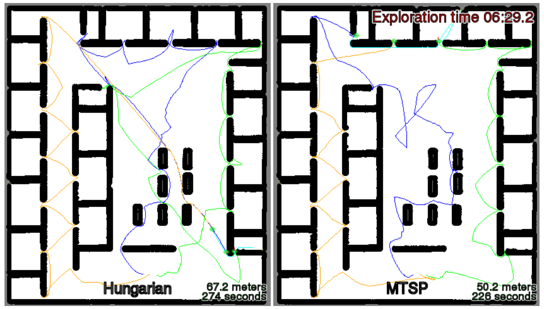
- Evaluation for the number of robots m and sensor range ρ

ρ	m	Iterative vs Greedy	Hungarian vs Iterative	MTSP vs Hungarian
3.0	3	+	=	+
3.0	5	+	=	+
3.0	7	+	=	+
3.0	10	+	+	−
4.0	3	+	=	+
4.0	5	+	=	=
4.0	7	+	=	+
4.0	10	+	+	−
5.0	3	+	=	+
5.0	5	+	=	+
5.0	7	+	=	+
5.0	10	+	+	−

Total number of trials 14 280

Performance of the MTSP vs Hungarian Algorithm

- Replanning as quickly as possible; $m = 3, \rho = 3$ m



The MTSP assignment provides better performance

Summary

- Introduction to multi-goal path planning
  Robotic TSP
- Overview of Dubins planning and DTSP
- Data collection planning
- Overview of multi-robot exploration based on the TSP