Alternatives to Explicit State Space Search
Symbolic Search

Álvaro Torralba

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About me

Dr. Álvaro Torralba
Saarland University, Saarbrücken, Germany
torralba@uni-saarland.de

Research Area: Search techniques for AI Planning

- Symbolic search
- Pruning techniques
- Applications

Agenda

1. Classical Planning: Models, Approaches
2. Symbolic Representation of Planning Tasks
3. Binary Decision Diagrams
4. Symbolic Search
5. Heuristic Search
6. Symbolic Abstraction Heuristics
7. Conclusion

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Classical Planning

Definition. *A planning task is a 4-tuple* $\Pi = (V, A, I, G)$ *where:*

- V *is a set of state variables, each* $v \in V$ *with a finite domain* D_v.
- A *is a set of actions; each* $a \in A$ *is a triple* (pre_a, eff_a, c_a), *of precondition and effect (partial assignments), and the action's cost* $c_a \in \mathbb{R}^{0+}$.
- *Initial state* I *(complete assignment), goal* G *(partial assignment).*

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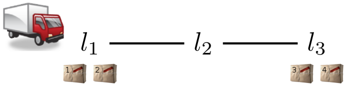
Running Example:



- $V = \{t, p_1, p_2, p_3, p_4\}$
  with $D_t = \{l_1, l_2, l_3\}$ and $D_{p_i} = \{t, l_1, l_2, l_3\}$.
- $A = \{load(p_i, x), unload(p_i, x), drive(x, x')\}$

Semantics – The State Space of a Planning Task

Definition. *Let $\Pi = (V, A, I, G)$ be an FDR planning task. The state space of Π is the labeled transition system $\Theta_\Pi = (S, L, c, T, I, S^G)$ where:*

- *The states S are the complete variable assignments.*
- *The labels $L = A$ are Π's actions; the cost function c is that of Π.*
- *The transitions are $T = \{s \xrightarrow{a} s' \mid pre_a \subseteq s, s' = s[\![a]\!]\}$.*

  If $pre_a \subseteq s$, then a is applicable in s and, for all $v \in V$, $s[\![a]\!][v] := \mathit{eff}_a[v]$ if $\mathit{eff}_a[v]$ is defined and $s[\![a]\!][v] := s[v]$ otherwise.

  If $pre_a \not\subseteq s$, then $s[\![a]\!]$ is undefined.
- *The initial state I is identical to that of Π.*
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→ Solution ("Plan"): Action sequence mapping I into $s \in S^G$.
Optimal plan: Minimum summed-up cost.

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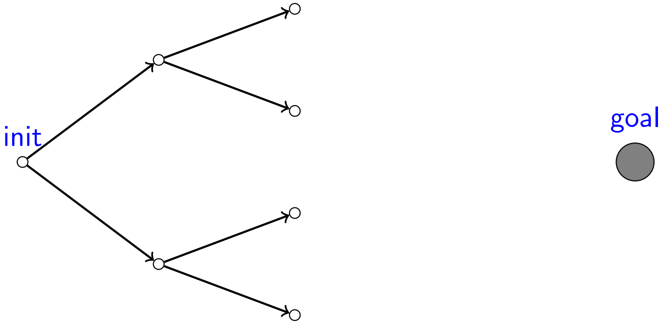

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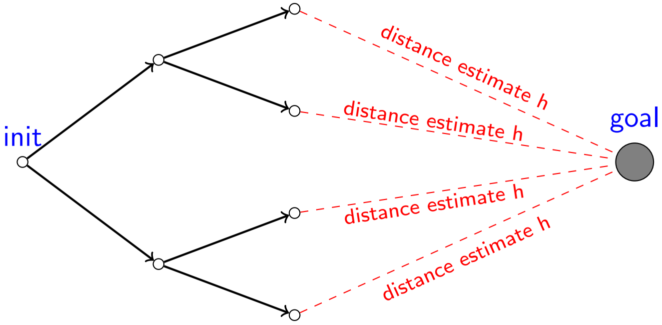
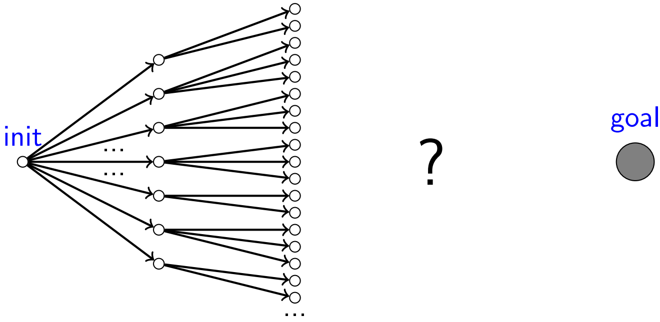


→ Forward state space search. Heuristic function h maps states s to an estimate $h(s)$ of goal distance.

Alternatives to State Space Search (not covered here)

- **Planning as SAT**: Extensions use, e. g., heuristics, symmetry breaking. [KS92, KS96, EMW97, Rin98, Rin03, Rin12]
- **Property Directed Reachability** [Bra11, EMB11, Sud14]
- Planning via **Petri Net Unfolding** [GW91, McM92, ERV02, ELL04, HRTW07, BHHT08, BHK+14]
- **Partial-order Planning** [Sac75, KKY95, YS03, BGB13]
- **Factored Planning** [Kno94, AE03, BD06, KBHT07, BD08, BD13, FJHT10]
- ...

State Space Explosion



Huge branching factor $\rightarrow$ state space *explosion*

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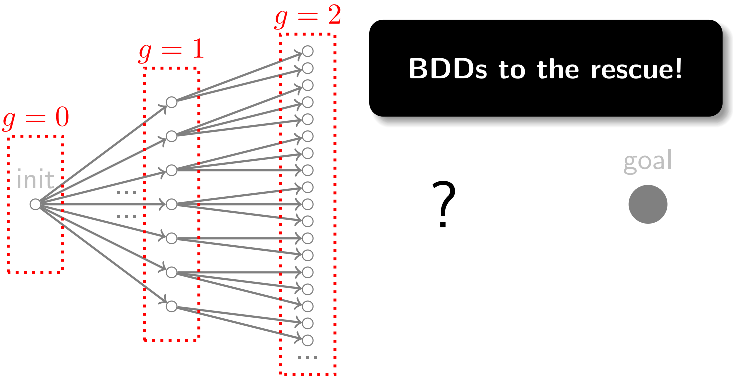

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Syntax of Propositional Logic

→ Atoms Σ in propositional logic = Boolean variables.

(Syntax of Propositional Logic). *Let Σ be a set of atomic propositions. Then:*

1. $\bot$ *and* $\top$ *are* *Σ-formulas.* ("False", "True")
2. *Each* $P \in \Sigma$ *is a* *Σ-formula.* ("Atom")

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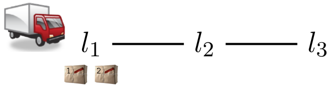
If φ and ψ are Σ-formulas, then so are:

4. $\varphi \wedge \psi$ ("Conjunction")
5. $\varphi \vee \psi$ ("Disjunction")
6. $\varphi \rightarrow \psi$ ("Implication")
7. $\varphi \leftrightarrow \psi$ ("Equivalence")

States as Logical Formulas

- Propositions (atoms in STRIPS): $\langle v, d\rangle$ Does var v have value d?
- How to represent a state as a logical formula?



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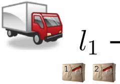
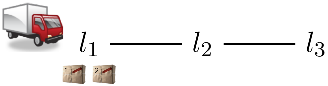


l_1 —— l_2 —— l_3 $\quad \langle t, l_1 \rangle \wedge \langle p_1, l_1 \rangle \wedge \langle p_2, l_1 \rangle$

Disclaimer: In propositional logic there is no closed-world assumption. In our examples, we ignore state invariants: $\langle t, l_1 \rangle \leftrightarrow (\neg \langle t, l_2 \rangle \wedge \neg \langle t, l_3 \rangle)$. Otherwise the state above would be written as:
$\langle t, l_1 \rangle \wedge \neg \langle t, l_2 \rangle \wedge \neg \langle t, l_3 \rangle \wedge \langle p_1, l_1 \rangle \wedge \neg \langle p_1, l_2 \rangle \wedge \neg \langle p_1, l_3 \rangle \wedge$
$\langle p_2, l_1 \rangle \wedge \neg \langle p_2, l_2 \rangle \wedge \neg \langle p_2, l_3 \rangle$

Sets of States as Logical Formulas



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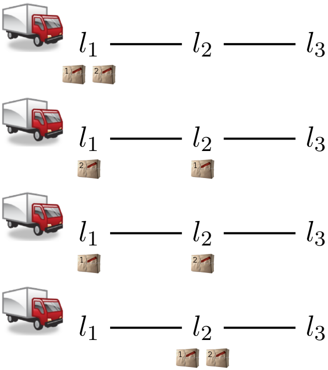

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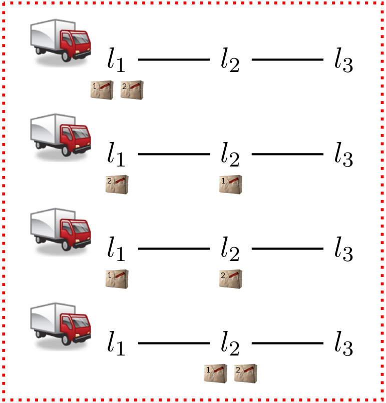

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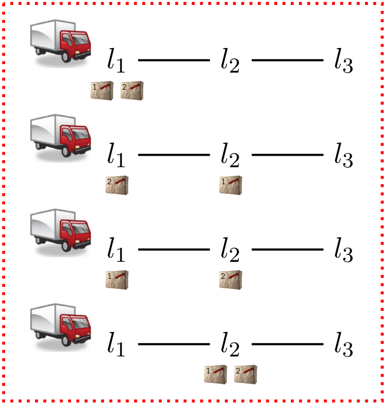

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$$\boldsymbol{\langle t, l_1\rangle \wedge (\langle p_1, l_1\rangle \vee \langle p_1, l_2\rangle) \wedge (\langle p_2, l_1\rangle \vee \langle p_2, l_2\rangle)}$$

Sets of States as Logical Formulas

- Formulas represent sets of states.
- Given a formula ψ, and a state s, is $s \in S_\psi$?.
- Example: $\psi = (\langle p_1, l_1\rangle \vee (\langle t, l_2\rangle \wedge \langle p_2, l_2\rangle)) \wedge (\langle t, l_1\rangle \vee \langle p_2, l_2\rangle)$
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  - Set of all states where the truck is at l_1: $\langle t, l_1 \rangle$

$\rightarrow$ Note that a very simple formula may represent exponentially many states!

Operating with Sets of States as Logical Formulas

- Once we represent sets of states as logical formulas, we can also operate on sets of states via performing operations on their corresponding logical formulas.
- Given two sets of states S_φ and S_ψ, represented by formulas φ and ψ. How to compute their:

Union $S_\varphi \cup S_\psi$:

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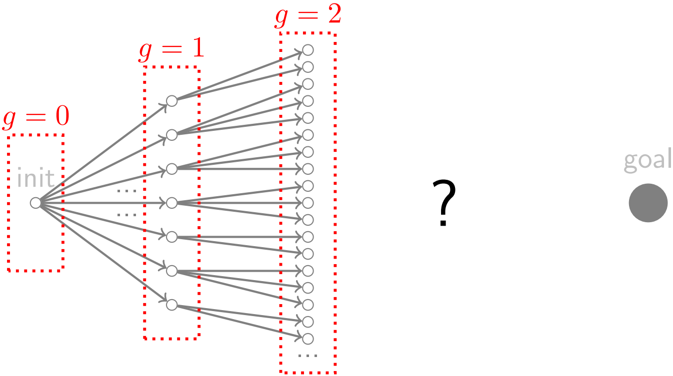
Operations on Set of States: Search

So, what is missing for doing search via operations on logical formulas?



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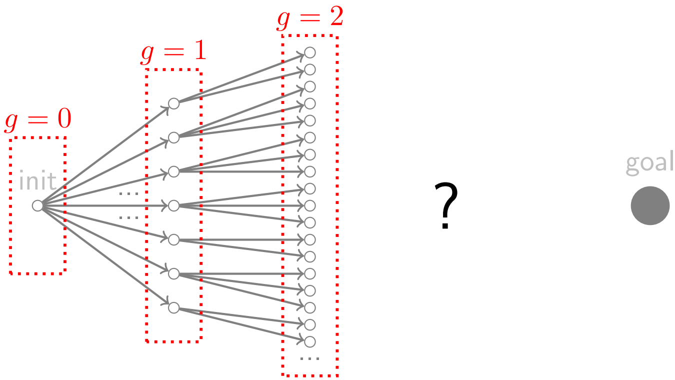

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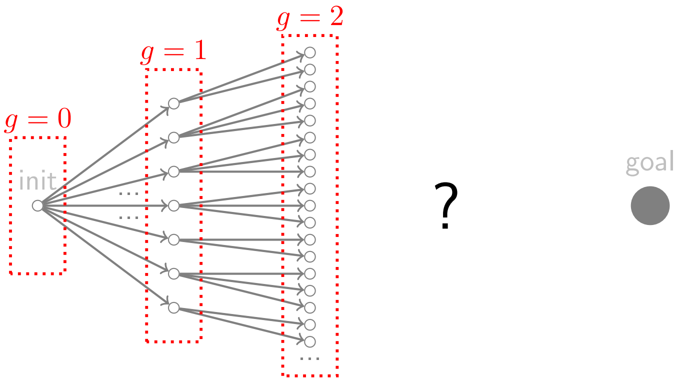

→Successor generation: given the set of states with $g = i$, what is the set of states with $g = (i + 1)$? depends on the set of actions!

Planning Actions as Logical Formulas

Action $a = \langle pre(a), \mathit{eff}(a) \rangle$

- Precondition: An action is only applicable in a state if $s \models pre(a)$
- Effect: changes the value of each variable v to $\mathit{eff}(a)[v]$ if $\mathit{eff}(a)[v]$ is specified

Transition Relation: represents an action a as the relation (set of pairs of states) containing (s, s') where a is applicable in s resulting in s'.

- We need to double all propositions: for s and s'
- The transition relation is

$$pre(a) \wedge \mathit{eff}'(a) \wedge \bigwedge_{v(x) \notin \mathit{eff}(a)} (x \leftrightarrow x')$$

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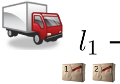
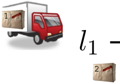


$l_1 - l_2 - l_3$



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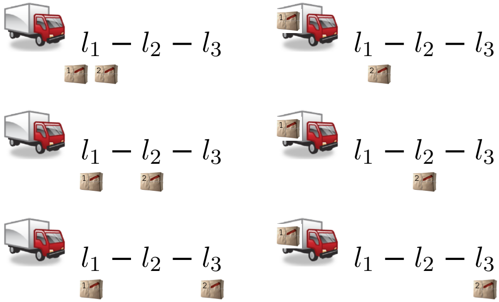

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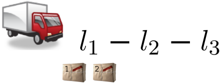

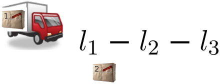

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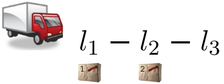

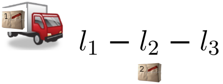

$\langle t, l_1\rangle \wedge \langle p_1, l_1\rangle \wedge \langle p_2, l_2\rangle \wedge$
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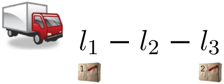

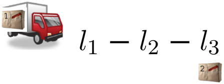

$\langle t, l_1\rangle \wedge \langle p_1, l_1\rangle \wedge \langle p_2, l_3\rangle \wedge$
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$\langle p_1, l_1\rangle \wedge \langle t, l_1\rangle \wedge \langle p_1, t\rangle' \wedge \langle t, l_1\rangle' \wedge (\langle p_2, l_1\rangle \leftrightarrow \langle p_2, l_1\rangle') \wedge (\langle p_2, l_2\rangle \leftrightarrow \langle p_2, l_2\rangle') \ldots$

Computing the Successors

Given a set of states $S(X)$ over propositions X and a TR $T(X, X')$ generate the successor states

Image operation:

$$\text{image}(S(X), T(X, X')) = \exists x \;.\; S(X) \wedge T(X, X')[X' \leftrightarrow X]$$

1. $S' = S(X) \wedge T(X, X')$: Select pairs $(s, s') \in T$ such that $s \in S$
2. $\exists x S'$: Get all states s' such that $(s, s') \in S'$ (forget about the predecessor states)
3. $[X' \leftrightarrow X]$: Replace X' by X so that the result is expressed by propositions in X

Example of Image Computation

$$\mathsf{image}(S(x), T(x,x')) = \exists x \;.\; S(x) \wedge T(x,x')[x' \leftrightarrow x]$$

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$S(x)$:

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l_1	l_1	l_1
l_1	l_1	l_2
l_2	l_1	l_3
l_1	l_3	l_1

$T(x, x')$: $(load(p_1, l_1))$

t	p_1	p_2	t'	p'_1	p'_2
l_1	l_1	l_1	l_1	t	l_1
l_1	l_1	l_2	l_1	t	l_2
l_1	l_1	l_3	l_1	t	l_3

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l_1	l_1	l_2	l_1	t	l_2
l_1	l_1	l_3	l_1	t	l_3

Result:

t	p_1	p_2	t'	p'_1	p'_2
l_1	l_1	l_1	l_1	t	l_1
l_1	l_1	l_2	l_1	t	l_2

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l_1	l_1	l_2	l_1	t	l_2
l_1	l_1	l_3	l_1	t	l_3

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			l_1	t	l_2

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l_1	l_1	l_3	l_1	t	l_3

Result:

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l_1	t	l_1			
l_1	t	l_2			

Computing the Predecessors (Pre-Image Computation)

Pre-image: Given a set of states $S(x)$ and a TR $T(x, x')$ generate the predecessor states

$$\text{pre-image}(S(x), T(x, x')) = \exists x' \ . \ S(x)[x' \leftrightarrow x] \wedge T(x, x')$$

- Corresponds to regression [Rin08]

Agenda

1. Classical Planning: Models, Approaches
2. Symbolic Representation of Planning Tasks
3. Binary Decision Diagrams
4. Symbolic Search
5. Heuristic Search
6. Symbolic Abstraction Heuristics
7. Conclusion

How to Represent Logical Formulas in Practice?

- To take advantage of the symbolic representations of sets of states, one needs an efficient representation
- The same set of states can be represented by very different formulas, e.g., the set of all states can be represented by $\top$ or by:

$$
\begin{gathered}
(\langle t, l_1\rangle \wedge \langle p_1, l_1\rangle \wedge \langle p_2, l_1\rangle) \vee (\langle t, l_1\rangle \wedge \langle p_1, l_1\rangle \wedge \langle p_2, l_2\rangle) \vee (\langle t, l_1\rangle \wedge \langle p_1, l_1\rangle \wedge \langle p_2, t\rangle) \vee \\
(\langle t, l_1\rangle \wedge \langle p_1, l_2\rangle \wedge \langle p_2, l_1\rangle) \vee (\langle t, l_1\rangle \wedge \langle p_1, l_2\rangle \wedge \langle p_2, l_2\rangle) \vee (\langle t, l_1\rangle \wedge \langle p_1, l_2\rangle \wedge \langle p_2, t\rangle) \vee \\
(\langle t, l_1\rangle \wedge \langle p_1, t\rangle \wedge \langle p_2, l_1\rangle) \vee (\langle t, l_1\rangle \wedge \langle p_1, t\rangle \wedge \langle p_2, l_2\rangle) \vee (\langle t, l_1\rangle \wedge \langle p_1, t\rangle \wedge \langle p_2, t\rangle) \vee \\
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\end{gathered}
$$

$\rightarrow$How to choose the "best" representation? $\rightarrow$As usual, it involves a trade-off between memory and time efficiency

How to Represent Logical Formulas in Practice?

Normal Form/Decision Diagram			
Negation NF (NNF)			
Disjunctive NF (DNF)			
Conjunctive NF (CNF)			
Binary DD (BDD)	[Bry86]		
Zero-sup DD (ZDD)	[Min93]		
Sentential DD (SDD)	[Dar11]		
Determ. DNNF (d-DNNF)	[Dar02]		
Decomp. NNF (DNNF)	[Dar01]		

How to Represent Logical Formulas in Practice?

Normal Form/Decision Diagram		$\vee$	$\wedge$	$\neg$	$\sigma \equiv \top$	$\sigma \equiv \bot$	$\sigma \equiv \sigma'$
Negation NF (NNF)		P	P	P	co-NP	NP	co-NP
Disjunctive NF (DNF)		P	E	E	co-NP	P	co-NP
Conjunctive NF (CNF)		E	P	E	P	NP	co-NP
Binary DD (BDD)	[Bry86]	E/P	E/P	P	P	P	P
Zero-sup DD (ZDD)	[Min93]	E/P	E/P	P	P	P	P
Sentential DD (SDD)	[Dar11]	E/P*	E/P*	P	P	P	P
Determ. DNNF (d-DNNF)	[Dar02]	P	E	E	co-NP	P	co-NP
Decomp. NNF (DNNF)	[Dar01]	P	E	E	co-NP	P	co-NP

$*$: In SDDs, $\vee$ and $\wedge$ with compression is not polynomial.

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Normal Form/Decision Diagram		$\vee$	$\wedge$	$\neg$	$\sigma \equiv \top$	$\sigma \equiv \bot$	$\sigma \equiv \sigma'$
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Binary DD (BDD)	[Bry86]	**E/P**	**E/P**	**P**	**P**	**P**	**P**
Zero-sup DD (ZDD)	[Min93]	**E/P**	**E/P**	**P**	**P**	**P**	**P**
Sentential DD (SDD)	[Dar11]	**E/P***	**E/P***	**P**	**P**	**P**	**P**
Determ. DNNF (d-DNNF)	[Dar02]	**P**	**E**	**E**	**co-NP**	**P**	**co-NP**
Decomp. NNF (DNNF)	[Dar01]	**P**	**E**	**E**	**co-NP**	**P**	**co-NP**

$*$: In SDDs, $\vee$ and $\wedge$ with compression is not polynomial.

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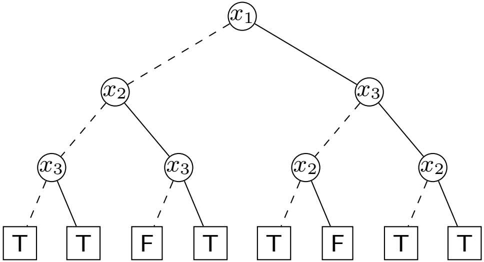
Binary Decision Diagrams (BDDs)

A function can be represented by a decision tree (e.g. $x_2 \to x_3$)



$\rightarrow$Each node decomposes the formula according to one variable:

$$\psi = (x_i \wedge \varphi) \vee (\neg x_i \wedge \varphi')$$

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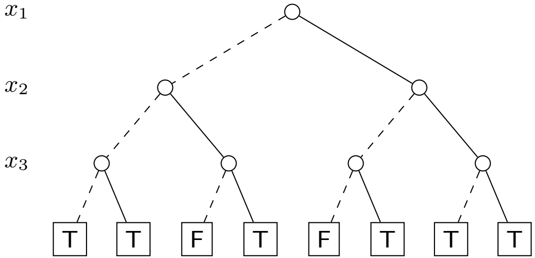

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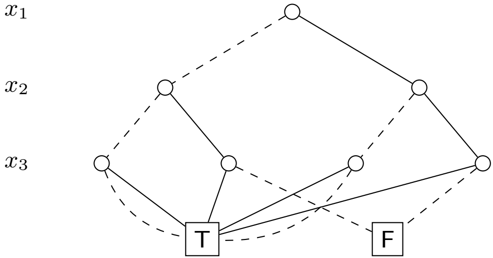

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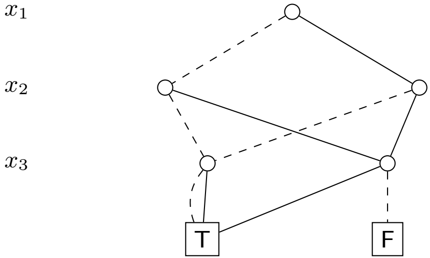

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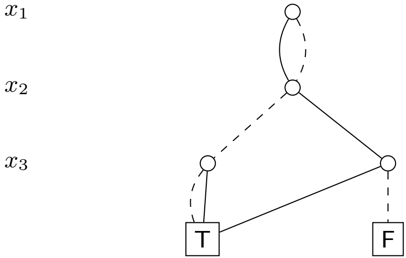

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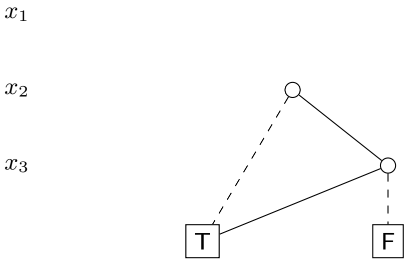

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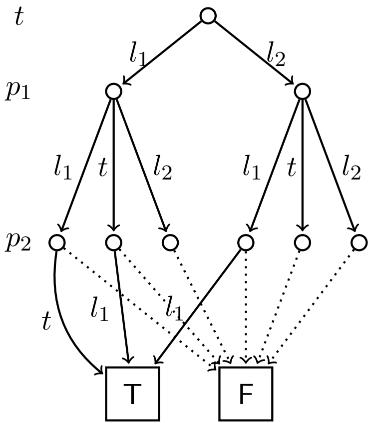
Binary Decision Diagrams (BDDs)

Multi-valued Decision Diagram (MDD)

DAG with a fixed variable ordering.

t	p_1	p_2
l_1	l_1	t
l_2	l_1	l_1
l_1	t	l_1



Binary Decision Diagrams (BDDs)

Multi-valued Decision Diagram (MDD)

DAG with a fixed variable ordering.

Reduction rules:

1. Each node represented only once
2. Nodes whose children are all the same are ommited

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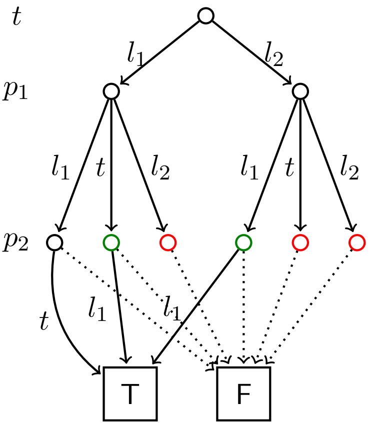

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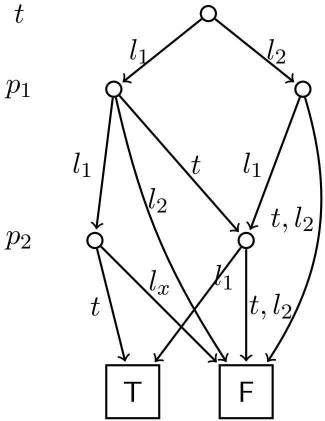

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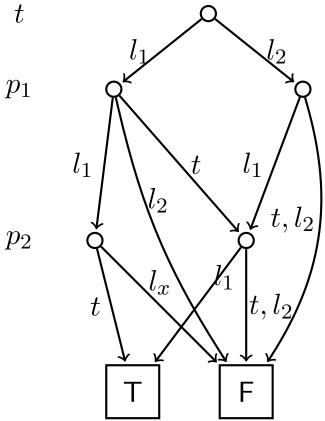

Binary Decision Diagrams: MDDs where variables are all binary
→Compilation that uses $log_2|D_v|$ binary variables per FDR variable v

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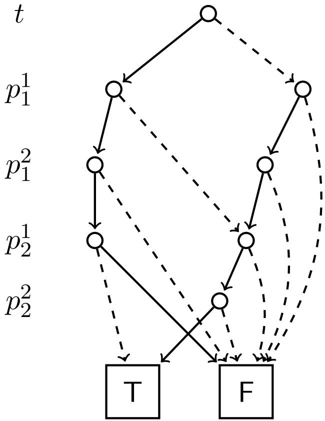
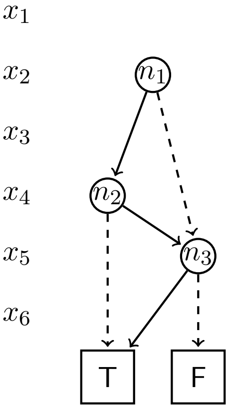


Binary Decision Diagrams: MDDs where variables are all binary
$\rightarrow$Compilation that uses $log_2|D_v|$ binary variables per FDR variable v

How to Count

Question!

How many states are in the following BDD?



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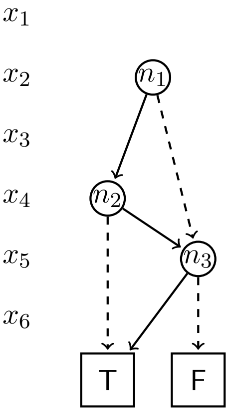

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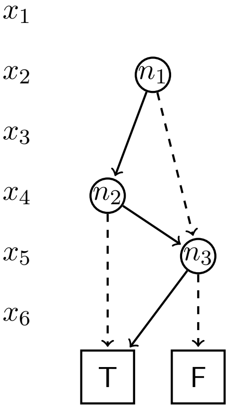

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- $Count(n_3) = 2$
- $Count(n_2) =$

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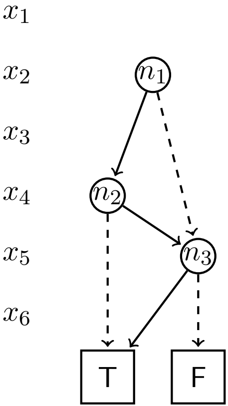

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- $Count(n_2) = Count(n_3) + 2^2 = 6$
- $Count(n_1) =$

How to Count

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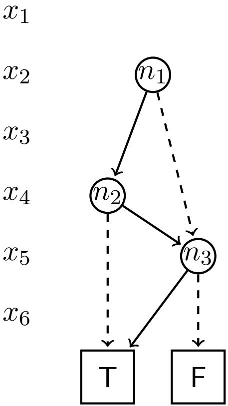

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- $Count(n_2) = Count(n_3) + 2^2 = 6$
- $Count(n_1) = 2 \cdot Count(n_2) + 2^2 Count(n_3) = 20$
- Total:

How to Count

Question!

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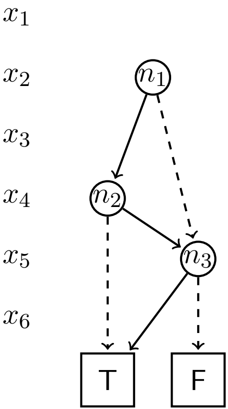

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- $Count(n_3) = 2$
- $Count(n_2) = Count(n_3) + 2^2 = 6$
- $Count(n_1) = 2 \cdot Count(n_2) + 2^2 Count(n_3) = 20$
- Total: 40

$\rightarrow$Each node needs to be processed only once

BDD Operations: And

- Recursive function over two BDDs f and g
- Base case(s): f (or g) is a constant:
  - $(f = \top) \implies f \wedge g = g$
  - $(f = \bot) \implies f \wedge g = \bot$
- Recursive case $(v = var(f) = var(g))$

$$f = (v \wedge f_1) \vee (\neg v \wedge f_2), \qquad g = (v \wedge g_1) \vee (\neg v \wedge g_2)$$

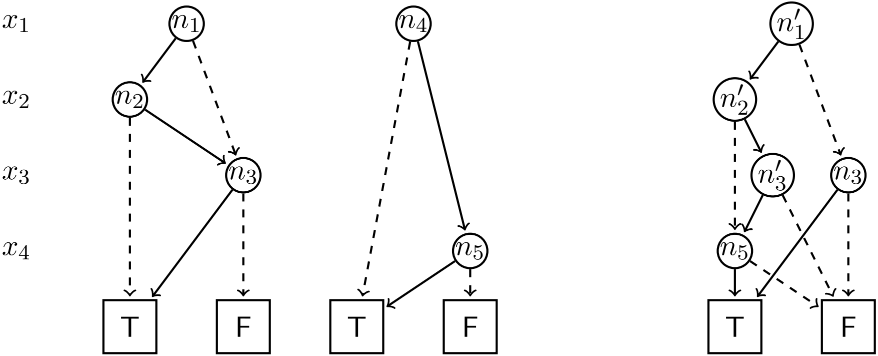
$$f \wedge g = (f_1 \wedge g_1, f_2 \wedge g_2)$$

- Recursive case $(var(f) < var(g))$

$$f \wedge g = (f_1 \wedge g, f_2 \wedge g)$$

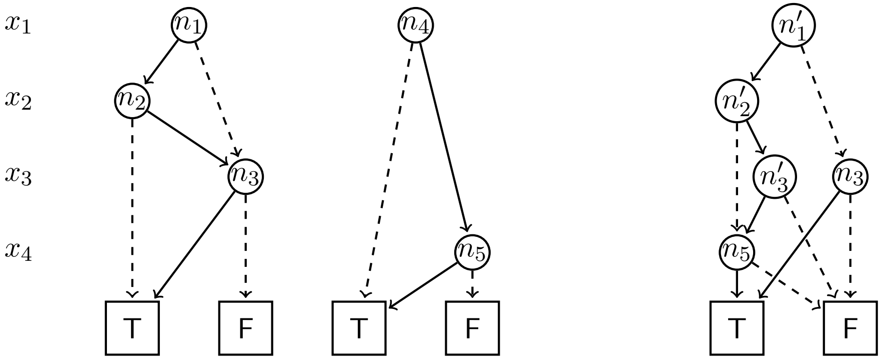
$\rightarrow$Ensure that nodes are not duplicated

BDD Operations: And



$\rightarrow$Complexity?

BDD Operations: And



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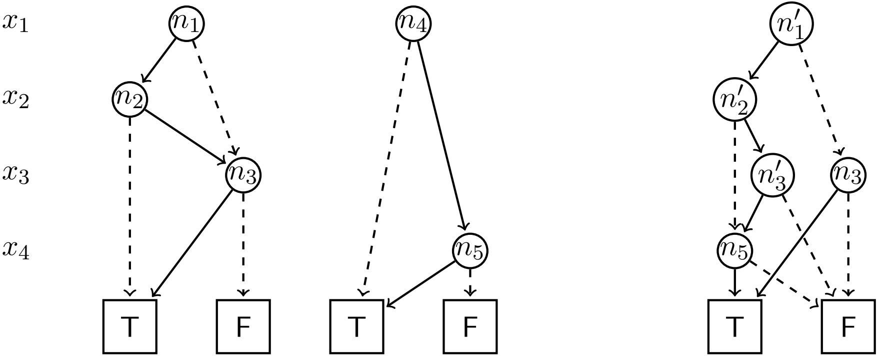
BDD Operations: And

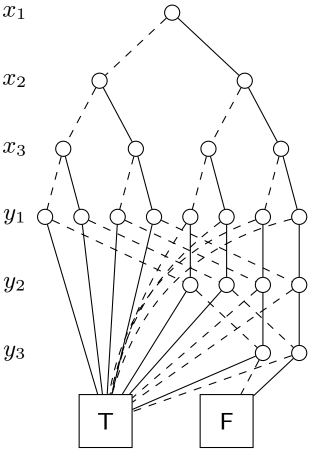


→Complexity? Quadratic, $O(|f||g|)$
→Important to keep a cache $(n_i, n_j) \rightarrow n_k$ to avoid duplicate work

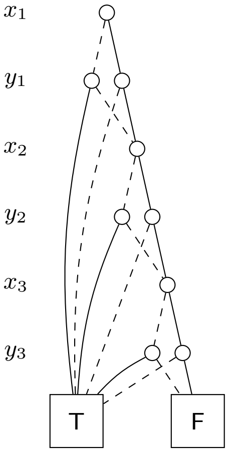
BDD Variable Ordering

$$(x_1 \neq y_1) \vee (x_2 \neq y_2) \vee (x_3 \neq y_3)$$

Exponential ($> 2^{n+1}$)



Polynomial ($3n + 2$)



Practical Strategies for a Good Variable Ordering

Static Variable Ordering: Put causally-related variables close [KE11]

$$\text{Choose ordering } o \text{ that minimizes } \sum_{v_i, v_j \in CG} d_o(v_i, v_j)^2$$

→No strong theoretical guarantees [KH13] but compares well against other alternatives [BRKM91, CHP93, Mai09, MWBSV88, MIY90]

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Dynamic Variable Ordering: variable re-ordering to minimize the size of the BDDs generated so far

- Finding the optimal BDD ordering is NP-hard [Bry86]
- But practical approximations (based on local-search) exist [Rud93].
- Applied in planning with good results by dynamic-Gamer [KH14]

Complexity Results

BDD Representation of Interesting Sets of States [EK11]

Goal States/Reachable states	Polynomial	Exponential
Polynomial	Gripper	Blocksworld N-puzzle
Exponential	Connect-4 Tic-tac-toe Gomoku	

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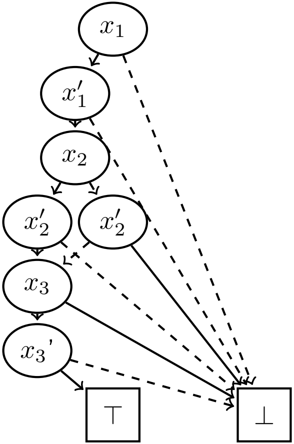
Can variable orderings schemas based on the causal graph give us theoretical guarantees for the size of BDDs in the search? [KH14]
$\rightarrow$Mostly not.

Efficient Image Computation: Variable Ordering

Variable Ordering: Interleave variables x and x'



$\rightarrow$The TR of an action has linear size on the number of variables

Efficient Image Computation

Transition Relation Partitioning [BCL91, JVB08]

Given a set of K actions with the same cost, replace $T_i(x, x')$ and $T_j(x, x')$ by $T_i(x, x') \vee T_j(x, x')$

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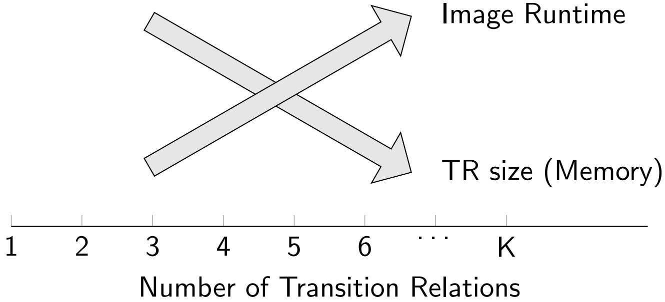

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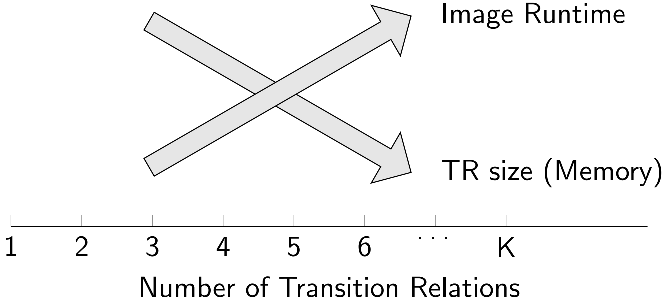

→Merge as many TRs as possible given the available memory [TEK13, TAKE17]

Uses of Decision Diagrams

In classical planning:

- Symbolic search (this lecture)
- Representation of state-dependent action costs [GKM15]
- Subsumption of partial states [AFB14]
- Dominance pruning [TH15]

Also very important in other areas:

- Model checking
- Hardware design and verification

BDD Packages

Library	Language	Reference
CUDD	C/C++	[Som]
CacBDD	C++	[LSX13]
BuDDy	C	[CWWG]
CAL	C	
Sylvan	C	[vDvdP15]
JDD	Java	[Vah]
BeeDeeDee	Java	[LMS14]

→Not clear best performer. CUDD, BuDDy, CacBDD have good results in symbolic model-checking [vDHJ+15].
→There are interfaces that adapt these libraries for other languages like Java, Python, Haskell, . . .

Agenda

How to do Search

We have all the necessary ingredients to do search:

- Representation of the initial state and goal
- Operation to generate successors: image operation
- Operation to check if goal has been reached: $S \wedge G \neq \emptyset$

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- Representation of the initial state and goal
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$\rightarrow$We can devise versions of the standard search algorithms that take advantage of the symbolic representation to compute the successors from a set of states at the same time

Symbolic Breadth-First Search

Input: Planning Task $\Pi = (V, A, I, G)$
$S_0 \leftarrow I$;
$C \leftarrow \emptyset$;
$i \leftarrow 0$;
while $S_i \neq \emptyset$ **do**
    if $S_i \wedge G$ **then**
        return Plan ;
    end
    $C \leftarrow C \vee S_i$;
    $S_{i+1} \leftarrow image(S_i, TR) \wedge \neg C$;
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return Unsolvable ;

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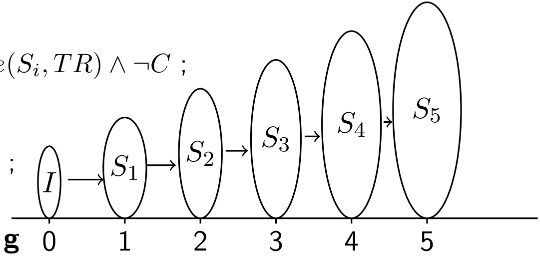

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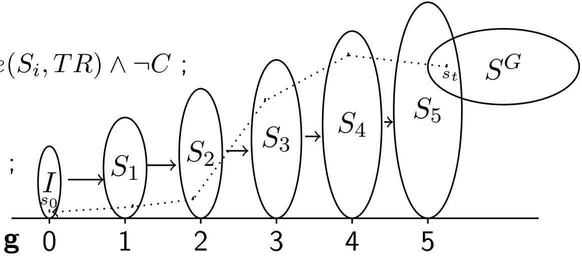

Symbolic Uniform-Cost Search

Expand set of states S_i with minimum g-value i

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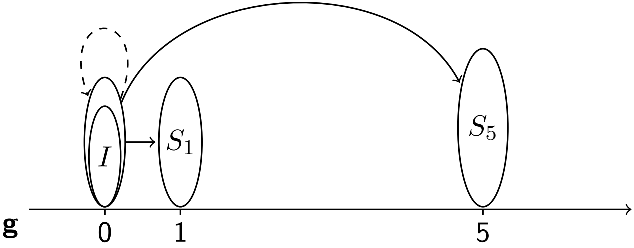

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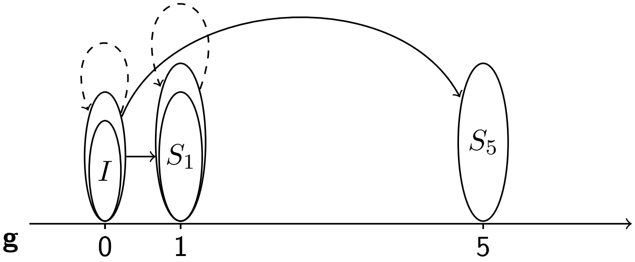

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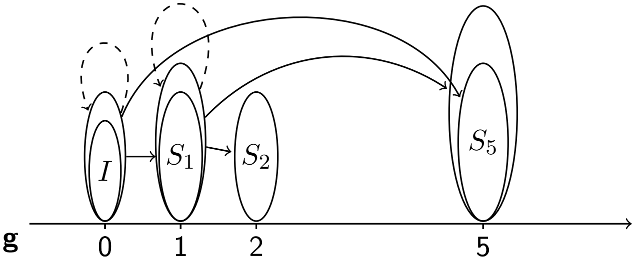

Symbolic Backward Uniform-Cost Search

We can perform the search in backward direction:

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- Use pre-image instead of image operation

Challenges:

1. Multiple goal states
2. Subsumption of partial states
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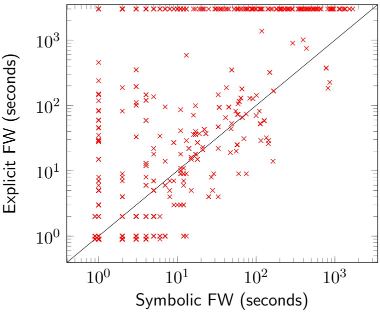
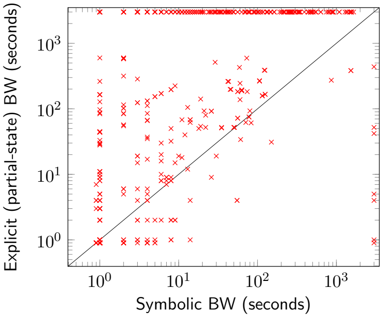
2. Subsumption of partial states $\rightarrow$Not a problem in symbolic search!
3. Spurious states $\rightarrow$Solution: state-invariant pruning [TAKE17]
   - Compute state invariants, e.g., h^2 mutexes
   - Encode the set of spurious states as a BDD
   - Remove spurious states from the goal and the TRs

Symbolic Uniform-Cost Search: Results



Forward



Backward

Symbolic Bidirectional Uniform-Cost Search

- Do forward and backward search at the same time
- Decide forward or backward direction at each step

$g_f = 0$ $g_b = 0$

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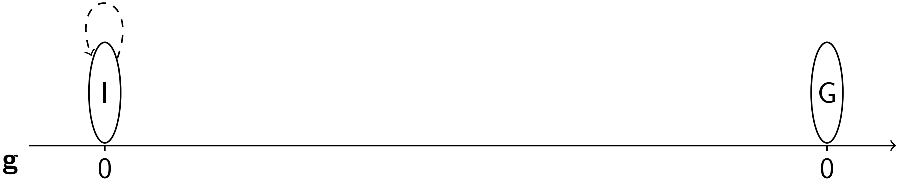
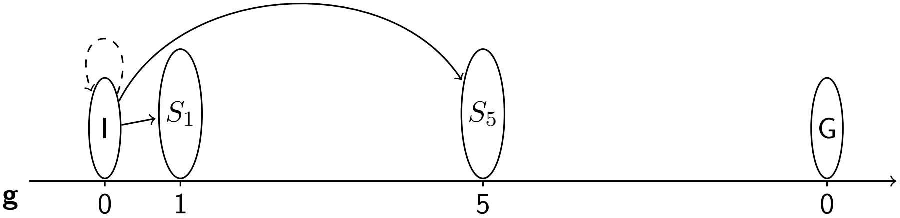


Symbolic Bidirectional Uniform-Cost Search

- Do forward and backward search at the same time
- Decide forward or backward direction at each step
- Stop when $g_f + g_b + min_{a \in A} c(a) \geq Sol$

$g_f = 1$ $g_b = 0$



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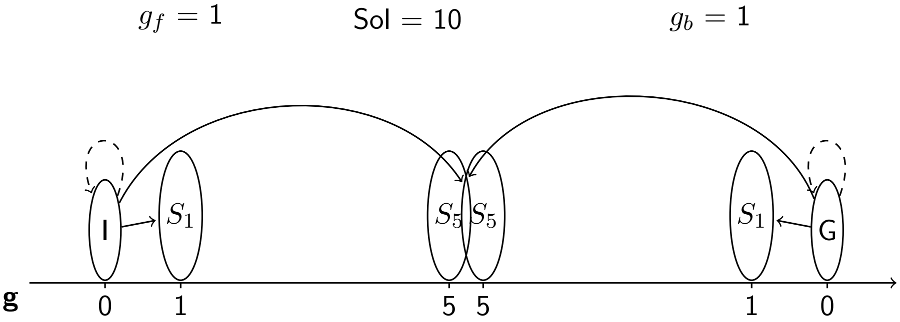

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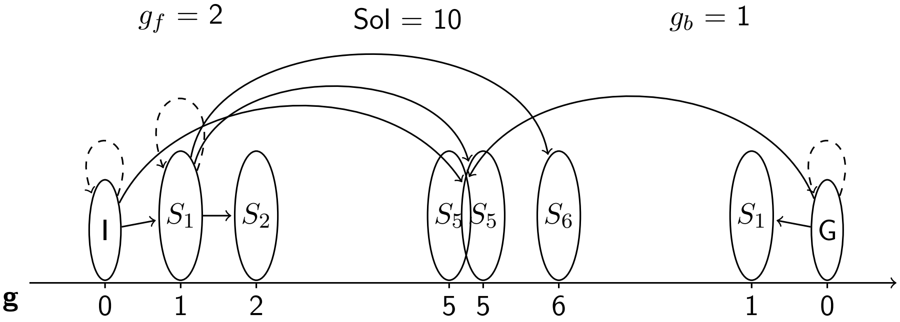

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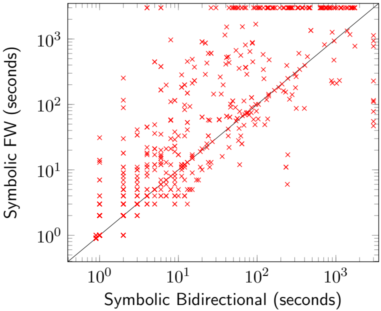
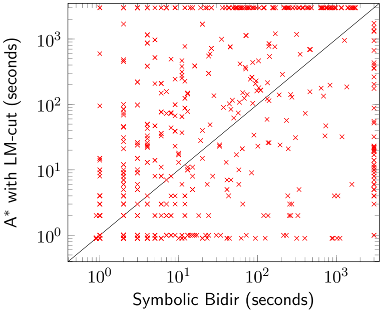


vs Symbolic Forward



vs A* with LM-cut

Agenda

Planning Heuristics

Definition *A heuristic h is a function $h : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$. Its value $h(s)$ for a state s is referred to as the state's heuristic value, or h value.*

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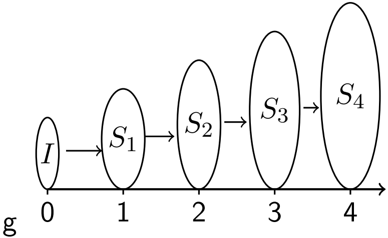
Definition *For a state $s \in S$, the perfect heuristic value h^* of s is the cost of an optimal plan for s, or ∞ if there exists no plan for s.*

$\rightarrow$ Heuristic functions h estimate the remaining cost h^*.

How to Exploit Heuristics in Symbolic Search?



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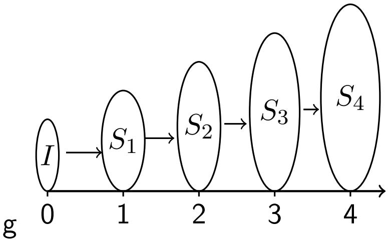

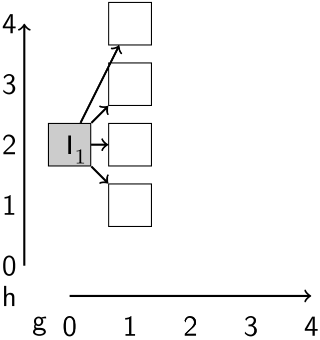

Split a BDD into subsets of states according to their h-value!

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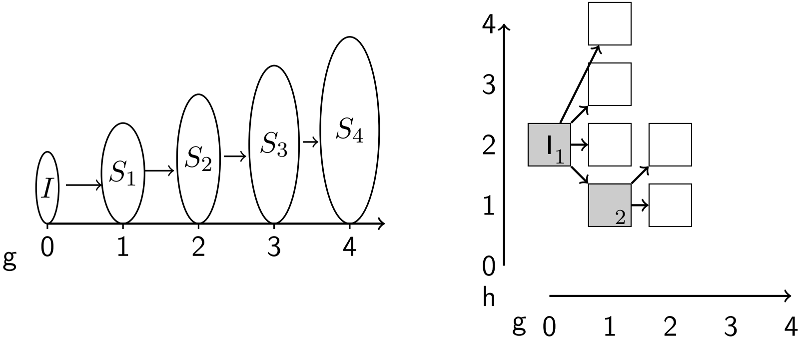

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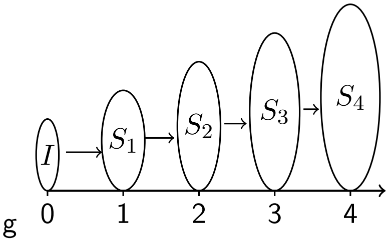

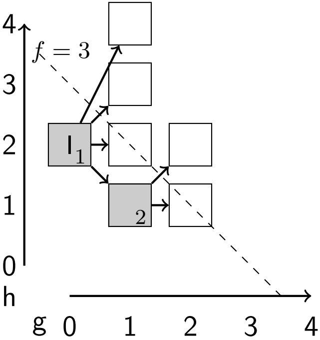

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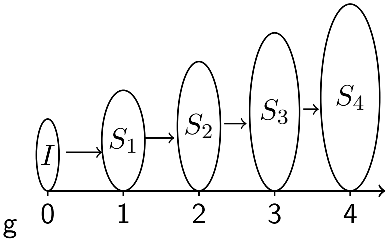

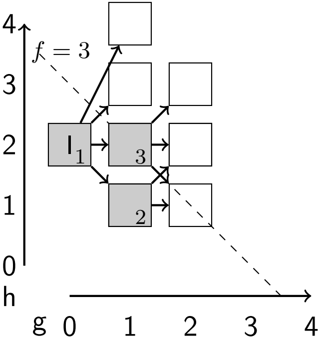

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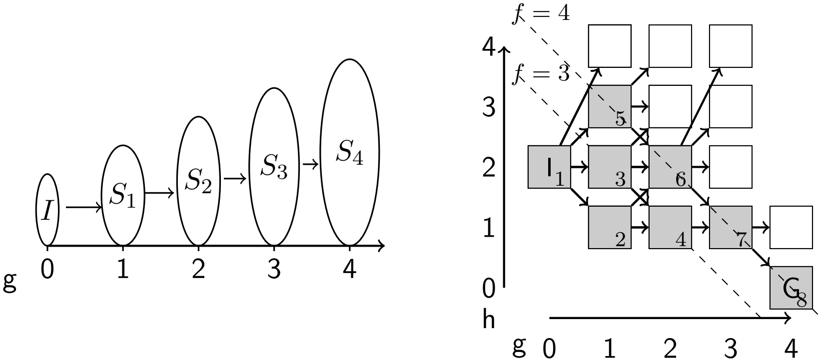

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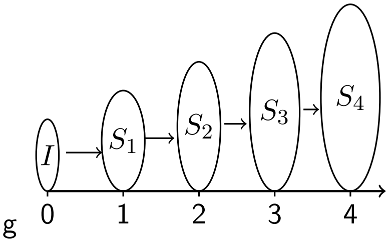

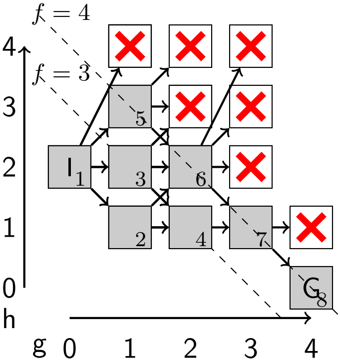

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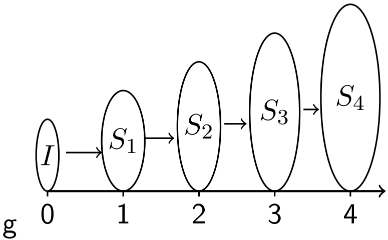

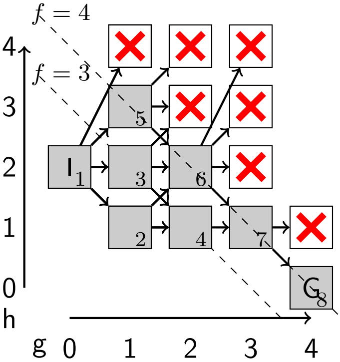

Split a BDD into subsets of states according to their h-value!

- Heuristic computation: how to evaluate a set of states?
- Does the heuristic improve the search performance?

Heuristic Computation: How to Evaluate a Set of States?

1. Iterate over all states in the BDD, computing $h(s)$

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$\boxed{H_0}$ $\boxed{H_1}$ $\boxed{H_2}$ $\boxed{H_3}$ $\boxed{H_4}$

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$$\boxed{S}$$

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$\rightarrow$Can we efficiently precompute a heuristic into BDDs?

- Yes, for some types of abstraction heuristics
- Not in the general case. Finding tractable cases is an open research question!

Agenda

Abstractions

Abstraction: *function* $\alpha : S \mapsto S^{\alpha}$. Induces an abstract state space s.t.:

1. $I^{\alpha} = \alpha(I)$.
2. $S^{\alpha G} = \{\alpha(s) \mid s \in S^{G}\}$. /* preserve goal states */
3. $T^{\alpha} = \{(\alpha(s), l, \alpha(t)) \mid (s, l, t) \in T\}$./* preserve transitions */

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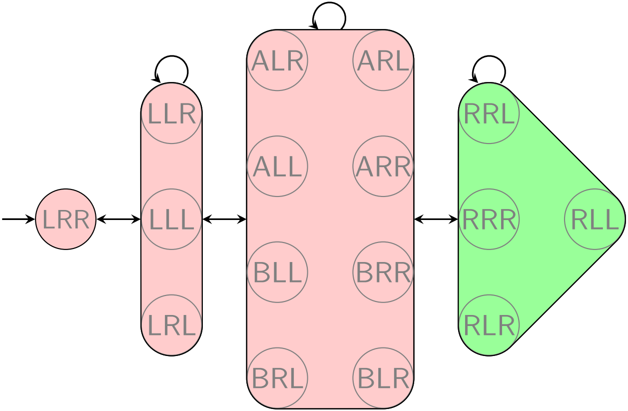
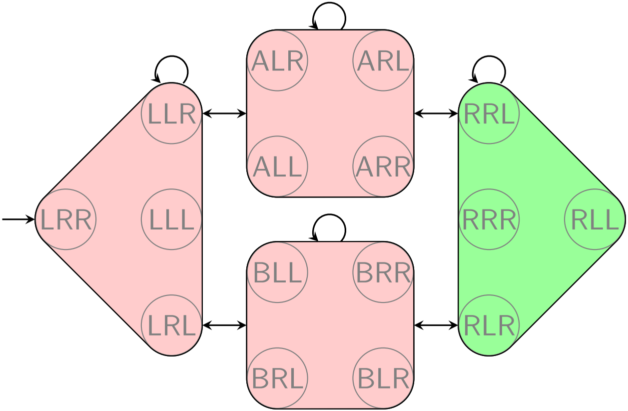


Pattern Databases [CS98, Ede01, HBH+07]

Pattern Databases: Select a subset of variables $V^\alpha \subseteq V$ (pattern). The mapping α is defined as the projection onto V^α.



Abstraction Heuristics

Use the optimal goal-distance in the abstract state space as an (admissible) estimate for the distance in the concrete state space:

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   - $\rightarrow$ Searching the entire abstract state space? That's what symbolic search is good for!
2. Store them in a look-up table

Abstraction Heuristics

Use the optimal goal-distance in the abstract state space as an (admissible) estimate for the distance in the concrete state space:

$$h(s) = h^*(\alpha(s))$$

1. Precompute h^* for all $\alpha(s) \in S^\alpha$ by performing a backward uniform-cost search in the abstract state space
   - $\rightarrow$ Searching the entire abstract state space? That's what symbolic search is good for!
2. Store them in a look-up table
   - $\rightarrow$ In the form of BDDs, so we can use them in BDDA*

Symbolic Pattern Databases

Do symbolic backward uniform-cost search with only a subset of variables

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- Do not have a limit on the number of variables to consider
- Truncate the search if it takes too much time or memory [AHS07]
- Using all variables: Symbolic Perimeter

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Do symbolic backward uniform-cost search with only a subset of variables

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- Using all variables: Symbolic Perimeter

→Really strong for heuristic search planners too [FTLB17]!

Agenda

Summary

- Symbolic search represents sets of states as Binary Decision Diagrams (BDDs) to perform search efficiently:
  - → Implicitly exploits structure of the problem
  - → Very useful for regression
- Symbolic bidirectional uniform-cost search has very good performance without using any heuristic
  - → Backward search can be understood as a (perimeter) heuristic
- Heuristics can also be used in symbolic search (though is slightly harder and not as useful as in explicit-state search)

Planners using Symbolic Search

- **MIPS**: Stefan Edelkamp and Malte Helmert
  `http://www.tzi.de/~edelkamp/mips/mips-bdd.html`
- **MIPS-XXL**: Stefan Edelkamp, Shahid Jabbar, and Mohammed Nazih. Extension to net-benefit with external planning
  `http://sjabbar.com/mips-xxl-planner`
- **BDDPlan**: Hans-Peter Strr `http://www.stoerr.net/bddplan.html`
- **Gamer** (IPC08-IPC11): Peter Kissmann and Stefan Edelkamp
  `https://fai.cs.uni-saarland.de/kissmann/planning/downloads/`
  Extensions (IPC14):
  1. cGamer: improved image computation and state invariant pruning
  2. dynamic Gamer: dynamic variable reordering
- **SymBA*** (IPC14): Based on Fast Downward
  `http://fai.cs.uni-saarland.de/torralba/software.html`

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