A4M36PAH - Plánování a hry

# From one to many: Multiagent Planning

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# Multiagent systems

## Overview

- systems consisting of agents
- an agent is a bounded entity
- the entities interact with each other
- generally no limitations on what an agent is (robots, humans, programs, ...)

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## For scope of this lecture

- agents $\sim$ intelligent programs
- interaction $\sim$ message passing

# Multiagent systems

## Technically

- agents $\sim$ a computational thread or process running ideally on its own processor (core)
- interaction $\sim$ inter-process sending of messages (potentially over network)

# Constraint Satisfaction Problems (CSPs)

## What is a CSP?

- finite set of variables $v_1, v_2, \ldots, v_k$
- non-empty domain of possible values for each variable $D_{v_1}, D_{v_2}, \ldots, D_{v_k}$
- finite set of constraints $C_1, C_2, \ldots C_m$
- each constraint $C_i$ specifies allowable combinations of values for subsets of variables
- a solution is an assignment of values to all variables that satisfies all constraints

# Constraint Satisfaction Problems (CSPs)

## Example

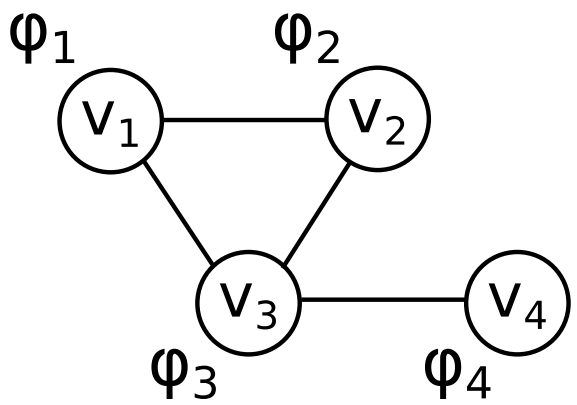
- variables:
  - $v_1 \in \{0, 1, 2\} = D_{v_1}$
  - $v_2 \in \{3, 4\} = D_{v_2}$
  - $v_3 \in \{5, 6\} = D_{v_3}$
  - $v_4 \in \{5, 6\} = D_{v_4}$
- constraints:
  - $C_1 = \{v_1 = 0 \Rightarrow v_2 = 4\}$
  - $C_2 = \{v_1 = 1 \Rightarrow v_2 = 3\}$
  - $C_3 = \{v_2 = 3 \Rightarrow v_3 = 5\}$
  - $C_4 = \{v_3 = 5 \Rightarrow v_1 \neq 0 \wedge v_1 \neq 2\}$
  - $C_5 = \{v_3 = v_4\}$
- solution:
  - ?,+?

# Distributed Constraint Satisfaction Problems (DisCSPs)

## Distribution of CSP

- each agent $\varphi_i \in \Phi$ is responsible for one variable $v_i$
- constraints are over more agents according to their variables
- an agent interaction graph of the agents is based on their variables and the constraints

Interaction graph of the previous example:



# Detour to Graph theory

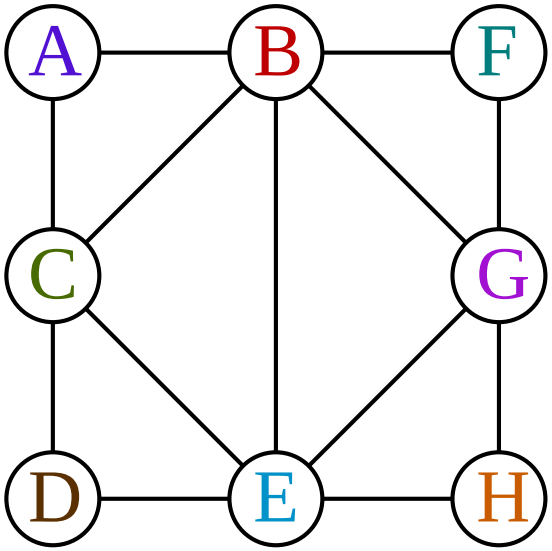
## Graph tree-decomposition, width and tree-width

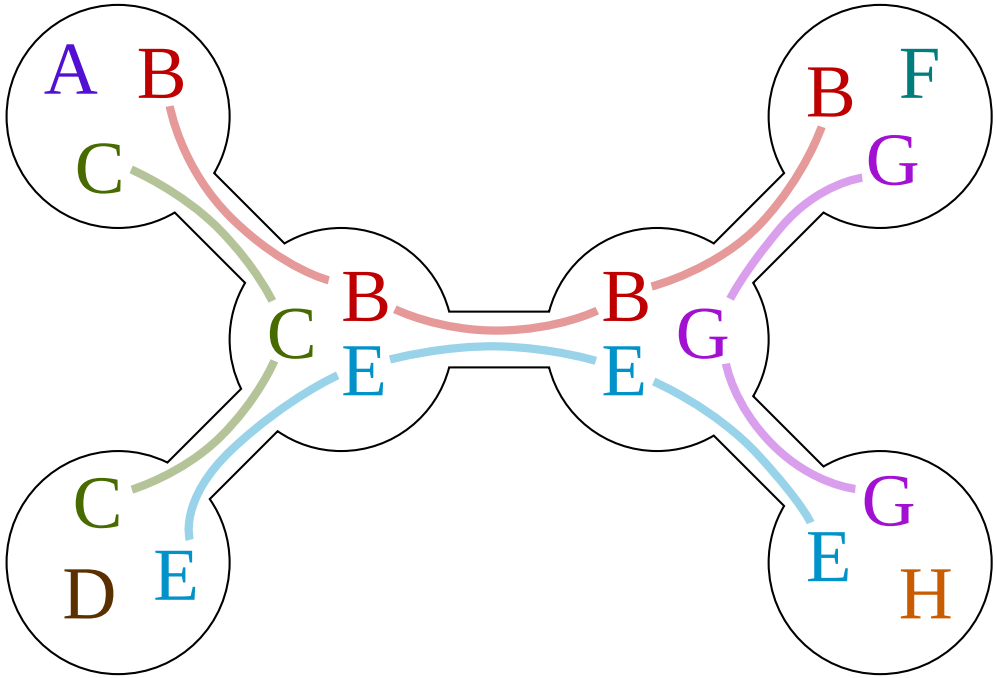
# Graph tree-decomposition, width and tree-width

## Definitions

- a tree-decomposition of a graph $G$ is $(T, W)$, where $T$ is a tree and $W = (W_t : t \in V(T))$ satisfies:
  - $\bigcup_{t \in V(T)} W_t = V(G)$ (each graph vertex is associated with at least one tree node)
  - $\forall uv \in E(G) \exists t \in V(T)$ s.t. $u, v \in W_t$ (vertices are adjacent in the graph only when the corresponding subtrees have a node in common)
  - if $t' \in T[t, t'']$, then $W_t \cap W_{t''} \subseteq W_{t'}$(the nodes associated with vertex form a connected subset of $T$.)
- the width of a graph is $\max(|W_t| - 1 : t \in V(T))$
- the tree-width of $G$ is the minimum width of a tree-decomposition of $G$

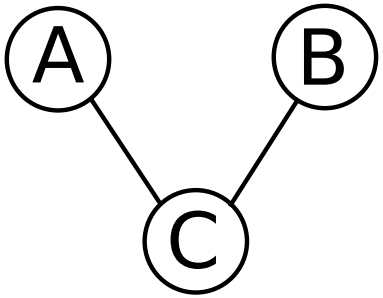
# Graph tree-decomposition, width and tree-width



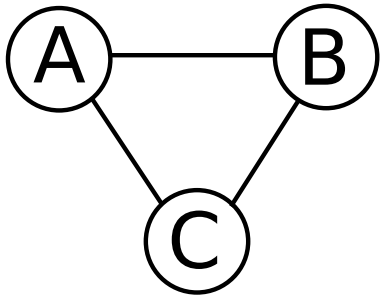



# Graph tree-decomposition, width and tree-width

tw(G)=1




tw(G)=2

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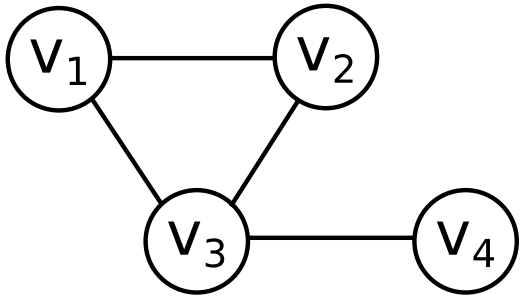
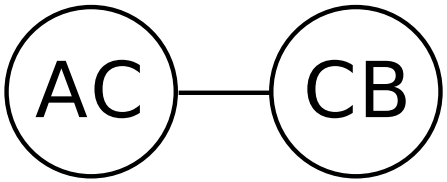
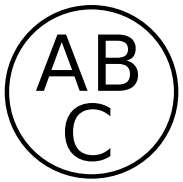


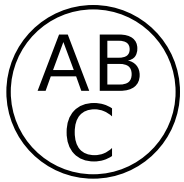

w(G)=1




homework

w(G)=2



# Graph tree-decomposition, width and tree-width

## Formally

- $tw(G) < 1 \Leftrightarrow G$ is forest (or tree or a series) graph
- $tw(G) < 2 \Leftrightarrow G$ is series-parallel graph
- $tw(G_n^{k \times k-\text{grid}}) = k = \sqrt{n}$ for a grid graph of $n$ vertices [bigger homework (optional)]
- $tw(K_n) = n - 1$ for a complete graph of $n$ vertices

## Informally

- tree-width of a graph determines its "cliquishness" (opposite of linearity or "treeness")
- in the DisCSP problems the tree-width of the interaction graph is related to coupling of the problem

End of the detour

(not end of the lecture, though)

# Distributed Constraint Satisfaction Problems (DisCSPs)

## Solving (Dis)CSP

- more families of algorithms solving CSP
- *Adaptive Tree Consistency (ATC)* – based on tree-decomposition of the underlying constraint graph, can be described as message passing among the variables (i.e., agents)

## Complexity of ATC

- proven that time complexity of ATC is

$$O(kD^{\omega+1})$$

- $k$ corresponds to number of CSP variables and therefore number of agents in DisCSP
- $D = \max_{i=1}^{k} D_i$, i.e., size of the largest domain (because of asymptotic complexity)
- $\omega$ is tree-width of the constraint graph (corresponds to the agent interaction graph)

# Planning for loosely coupled multiagent systems

[R. Brafman, C. Domshlak: From one to many: Planning for loosely coupled multiagent systems, In *Proceedings of ICAPS*'08, 2008]

# Motivation

## Logistics planning

Deliver packages using vehicles (trucks, airplanes, ships) operating in/between different countries/regions/cities

- Classical benchmark for "single-agent" planning
- Classic example of a distributed system $\rightsquigarrow$ vehicle $=$ agent

## (Informal) Question

Can we exploit the fact that the domain describes a naturally distributed system to make planning more efficient?

## (Ultimate) Answer

YES, we can solve distributed components independently

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# Basic Motivation/Intuition

## $k$-agents MA Systems (Logistics domain example)

### Fully decoupled

Vehicles are a priori responsible for different packages

Same as planning $k$ times for a single agent
$\rightsquigarrow$ linear time-complexity growth
(exp($k$) time-complexity reduction)

### Fully coupled

Vehicles have to move the same packages and maybe coordinate on loads/unloads

Same as planning for a single "$k$-times larger" agent
$\rightsquigarrow$ exp($k$) time-complexity growth
(no reduction in time-complexity)

# “Loose Coupling” is a Loose Concept

## Questions

1. How to measure the coupling level of a MA system?

2. Is there an algorithm that
   1. can handle any “coupling level”, yet
   2. is guaranteed to benefit from lower “coupling level”

# Centralized Planning for MA Systems
## Problem Statement

### Our Focus Here

**Input** Planning problem for a set of $k$ collaborative agents

**Question** To what extent is planning for such a MA system harder than solving individual planning problems of each of the agents in isolation?

**Approach** Theoretical. Try to formulate an algorithm that is tractable under reasonable conditions.

# Centralized Planning for MA Systems

## Our Focus Here

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| Input | Planning problem for a set of $k$ collaborative agents |
| Question | To what extent is planning for such a MA system harder than solving individual planning problems of each of the agents in isolation? |
| Approach | Theoretical. Try to formulate an algorithm that is tractable under reasonable conditions. |

# Main Ideas

## A New Graphical Model

Potential (positive and negative) interactions between the agents' individual abilities (= actions)

## System coupling-level

Define an interaction graph of the system

Nodes = agents

Edges = agents need (to coordination with) each other

Parameter $\omega \rightsquigarrow$ tree-width of interaction graph

# Main Ideas

## A New Graphical Model

Potential (positive and negative) interactions between the agents' individual abilities (= actions)

## System coupling-level

Parameter $\omega \rightsquigarrow$ tree-width of interaction graph

## Problem coupling-level

Some problems require more coordination than others!

Parameter $\delta \rightsquigarrow$ minmax number of times a single agent needs some other agent to do something for it

# Main Ideas

## System coupling-level

Parameter $\omega \rightsquigarrow$ tree-width of interaction graph

## Problem coupling-level

Parameter $\delta \rightsquigarrow$ minmax number of times a single agent needs some other agent to do something for it

## Algorithm

- Fix the agents' commitments to each other
  $\rightsquigarrow$ *careful selection of language matters!*
- Let each agent independently plan "in-between" commitments
- Use iterative deepening to extend the number of per-agent commitments if needed

# Agent Actions

## Logistics planning

Deliver packages using vehicles (trucks, airplanes, ships) operating in/between different countries/regions/cities

- Actions $\texttt{move}(v, from, to), \texttt{load}(p, v, at), \texttt{unload}(p, v, at)$
- Agents: vehicles
- Vehicle agent actions:
  moving it, loading into it, unloading from it

## From STRIPS to MA-STRIPS

Everything is the same, except that
actions are partitioned between the agents

# From STRIPS to MA-STRIPS

**Definition**

A STRIPS problem is given by a quadruple $\Pi = \langle P, A, I, G \rangle$, where:

- $P$ is a finite set of *atoms*, $I \subseteq P$ is the *initial situation*, and $G \subseteq P$ encodes the *goal* situations,
- Each action $a \in A$ is given by $\langle \mathsf{pre}(a), \mathsf{add}(a), \mathsf{del}(a) \rangle$.

# From STRIPS to MA-STRIPS

## Definition

An MA-STRIPS problem for a system of agents $\Phi = \{\varphi_i\}_{i=1}^{k}$ is given by a quadruple $\Pi = \langle P, \{A_i\}_{i=1}^{k}, I, G\rangle$, where:

- $P$ is a finite set of *atoms*, $I \subseteq P$ is the *initial situation*, and $G \subseteq P$ encodes the *goal* situations,
- For $1 \leq 1 \leq k$, $A_i$ is the set of actions that the agent $\varphi_i$ is capable of performing. Each action $a \in A = \bigcup A_i$ is given by $\langle \mathsf{pre}(a), \mathsf{add}(a), \mathsf{del}(a)\rangle$.

# Solving MA-STRIPS Problems

## Standard Approaches

1. Compile into a single-agent STRIPS problem
   - ☹ Lose all structure and obtain k-times larger "agent"
   - ☹ Worst-case complexity exponential in description size or shortest plan (depending on search strategy)

2. Try to solve as much as possible locally and compose the resulting individual agent plans
   - ☹ What can we say about it?

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   - ☹ What can we say about it?

# A Closer Look at Agent Actions

## Private vs. Non-Private

**Private** affect and depend only on that agent

**Non-Private** all the rest

## Logistic planning

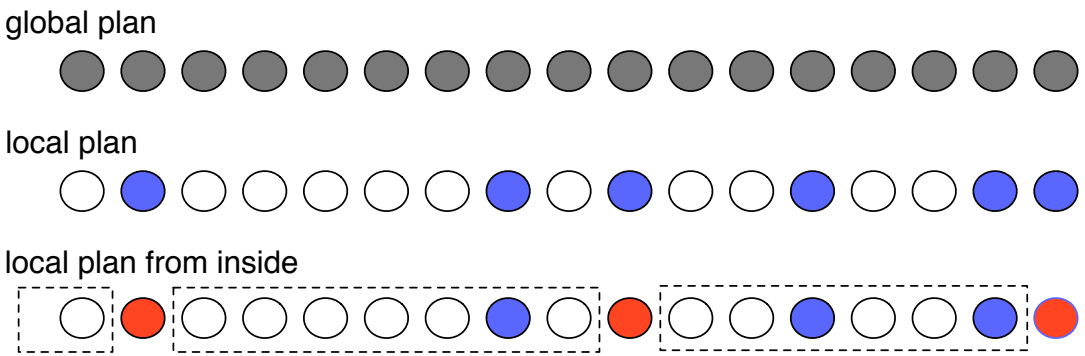
- Move actions are private
  (influence and influenced only by vehicle location)
- Loading into/unloading from a vehicle is non-private
  $\leadsto$ except if the package location is private to the vehicle!

# A Closer Look at Agent Subplans

## Private vs. Non-Private

**Private** affect and depend only on that agent

**Non-Private** all the rest



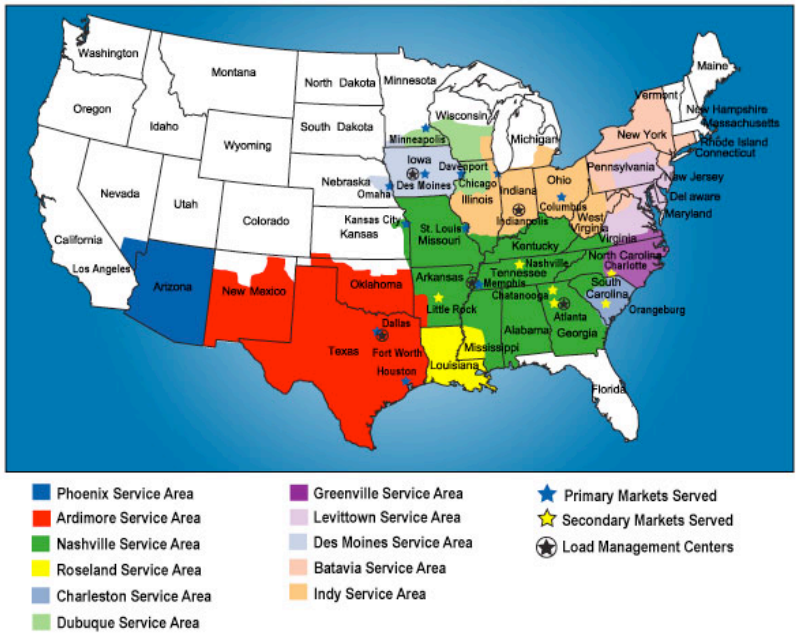

- non-private actions in the plan $\rightsquigarrow$ coordination points
- arbitrarily long sequences of private actions between adjacent non-private actions

# Example: Logistics

## Logistics

- imagine vehicles moving on a large map
- each vehicle has a service region

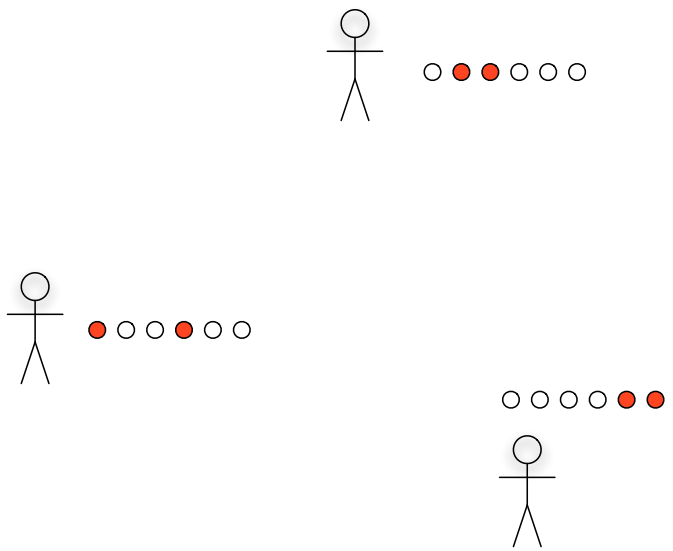
$\rightsquigarrow$ between each load/unload action, there are multiple `move` actions by the vehicle



# Main Idea

## "Algorithm"

1. Find a good choice of coordination points
2. Solve $k$ local planning problems over the private actions of the agents only



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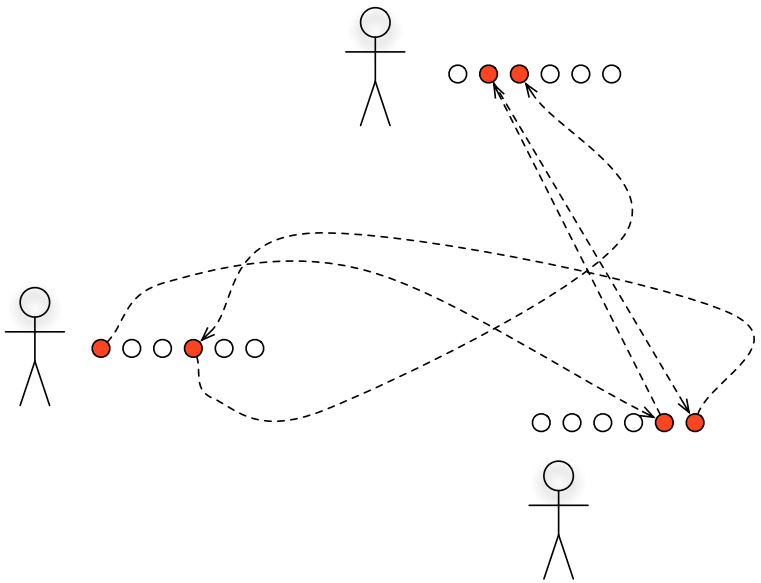

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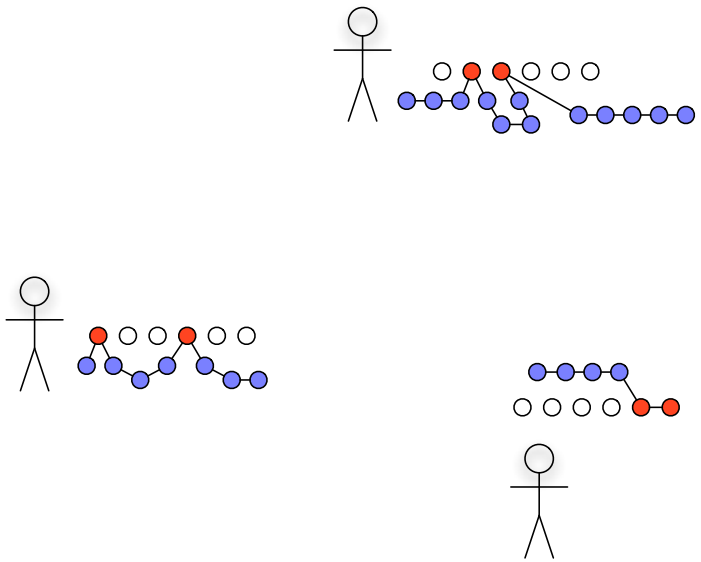
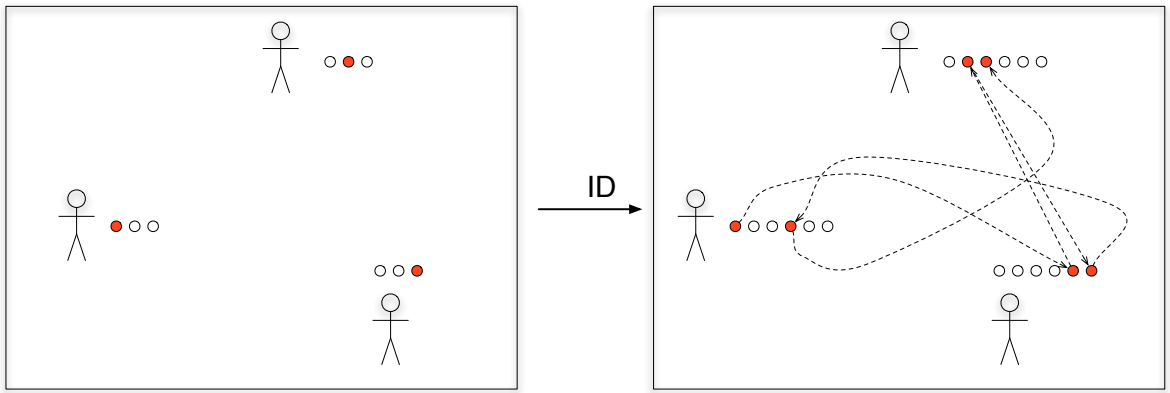


# Main Idea

## "Algorithm"

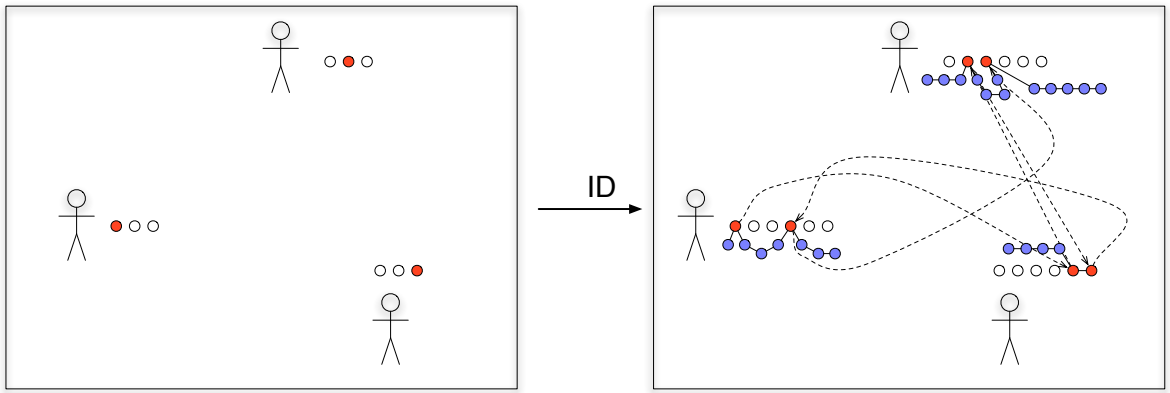
1. Find a good choice of coordination points
   - Iterative deepening on $\delta$ — # of coord-points per agent
   - For each choice of $\delta$
     - Define a CSP whose solutions are consistent assignments to the coordination points
2. Solve $k$ local planning problems over the private actions



# Main Idea

## "Algorithm"

1. Find a good choice of coordination points
2. Solve $k$ local planning problems over the private actions
   - purely independent phase $\rightsquigarrow$ unary constraints
   - can be reduced to regular STRIPS planning



# Complexity

## The complexity is derived from

- number of agents $k$ in set $\Phi$ with public actions $A_i^{pub}$
- maximal complexity of the local planning $\mathcal{I}$ with a cost function for switching from regular planning $f(\cdot)$
- number of "coordination" CSPs we have to solve (corresponds to $\delta$)
- solving each "coordination" CSP $O(kD^{\omega+1})$
- length of the "coordination" plan $k\delta$

The idea of $k\delta$:

$$\begin{array}{l} \alpha: \\ \beta: \end{array} \begin{pmatrix} a_1^\alpha & * & a_2^\alpha & * & a_3^\alpha & * \\ * & a_1^\beta & * & a_2^\beta & * & a_3^\beta \end{pmatrix}.$$

# Complexity

Size of agent's domain is:

$$|D_i| = \sum_{d=1}^{\delta} \binom{k\delta}{d} \cdot |A_i^{pub}|^d = O((k\delta|A_i^{pub}|)^{\delta+1}).$$

## Terms

- $\binom{k\delta}{d}$ represents all possible combinations of $d$ virtual time points for the public actions (e.g., for $d = 2, k\delta = 6$ there are 15 of them $\{(1,2),(1,3),\ldots,(1,6),(2,3),(2,4),\ldots,(5,6)\}$)
- $|A_i^{pub}|^d$ represents all possible public action sequences of length $d$ (e.g, for $d = 2$ and $|A_i^{pub}| = 2$ they are $\{a_1a_1, a_1a_2, a_2a_1, a_2a_2\}$)
- the summed up result represent the number of all possible coordination sequences for one agent.

# Complexity

Time complexity of the unary internal-planning constraints:

$$O(f(\mathcal{I}) \cdot k \cdot \max_{i \in \Phi} |D_i|) = O(f(\mathcal{I}) \cdot k(k\delta|A^{pub}|)^{\delta+1}) = O_{ipc},$$

## Terms

- $f(\mathcal{I}) \cdot k$ the internal planning has to be run by each agent
- asymptotically (in worst case) $\max_{i \in \Phi} |D_i|$ domains has to be planned by all agents
- asymptotically (in worst case) $|A_i^{pub}|$ for all agents are all public actions $|A^{pub}|$

# Complexity

Time complexity of the coordination constraints:

$$O(k \cdot \max_{i \in \Phi} |D_i|^{\omega+1}) = O(k(k\delta|A^{pub}|)^{\delta\omega+\epsilon}) = O_{cc},$$

## Terms

- based on ATC algorithm time complexity $O(kD^{\omega+1})$
- $D = \max_{i \in \Phi} |D_i|$
- $\epsilon = \delta + \omega + 1$is dominated by $\delta\omega$
- asymptotically (in worst case) $|A_i^{pub}|$ for all agents are all public actions $|A^{pub}|$

# Final Complexity

Time final time complexity bound of the multiagent planning:

$$O_{ipc} + O_{cc} = O(f(\mathcal{I}) \cdot k(k\delta|A^{pub}|)^{\delta+1} + k(k\delta|A^{pub}|)^{\delta\omega+\epsilon}).$$

The exponential bounds can be therefore expressed as:

$$f(\mathcal{I}) \cdot \exp(\delta) + \exp(\delta\omega)$$

## Not exponentially dependent on

Algorithm complexity has no direct exponential dependence on the number of agents $k$, has no direct exponential dependence of the length of the individual plans of the agents and has no direct exponential dependence of the size of the original planning problem.

## Exponentially dependent on

Algorithm complexity is exponentially dependent on number of coordination points, i.e., length of the coordination plan and on tree-width of the agent interaction graph.

# Multiagent $A^*$

[R. Nissim, R. Brafman: Multi-Agent A* for Parallel and Distributed Systems, In *Proceedings of HDIP Workshop (ICAPS)*, 2012]

# Multiagent Distributed/Parallel A*

## Overview

- based on partition of actions from the previous slides
- private/public actions (respecting privacy)
- A* expansion only of agent's own actions
- distributed optimal search
- distributed termination detection
- currently most efficient distributed planning approach

# Our approach – optimal forward search

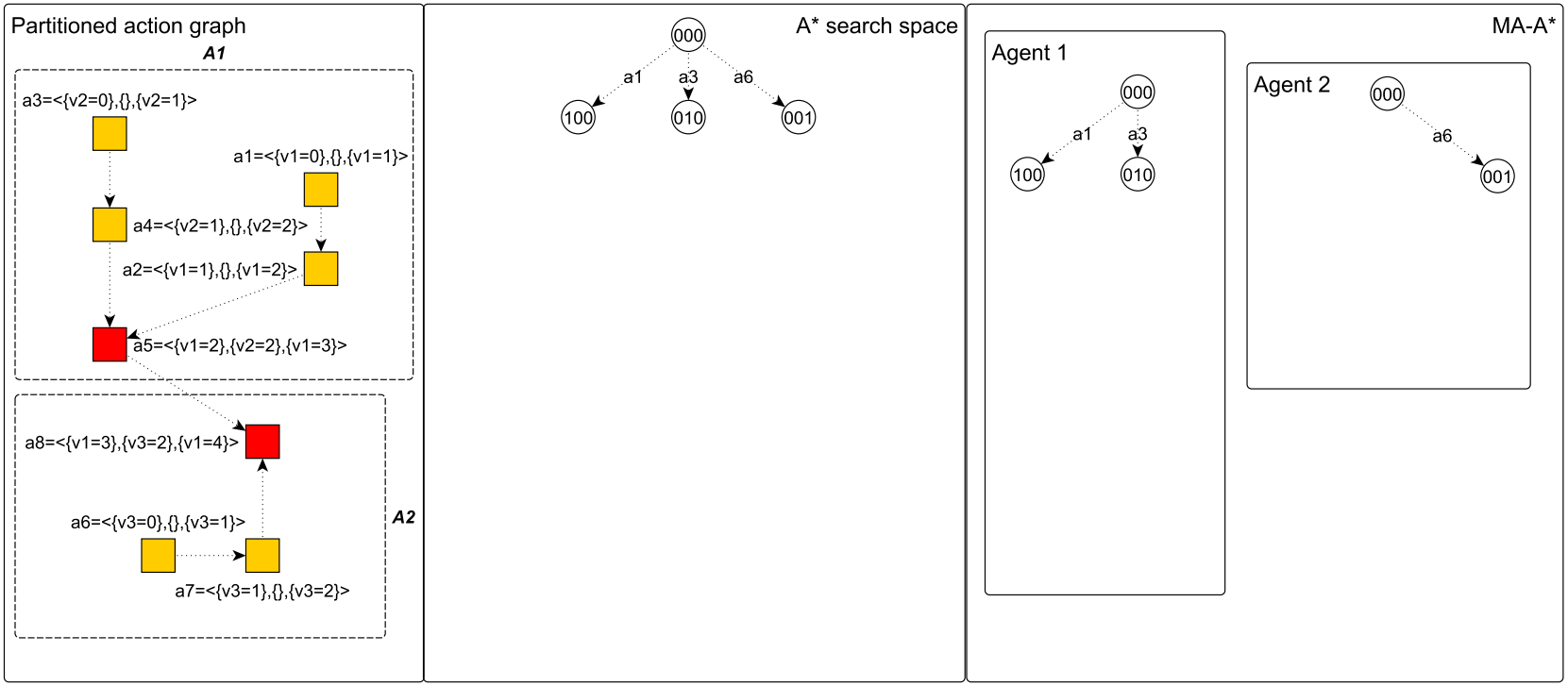
Each agent runs an A*-like search separately, using its own open/closed list. In each iteration, the agent performs:

## MA-A*

- Receive messages and insert states into open list.
- Retrieve first node $n$ from open list.
- If $n$ is a solution, perform distributed optimality check.
- Expand $n$ using the agent's own actions only.
- Compute h-value and add to open list all children $n'$.
- If $n'$ was obtained by applying a public action
  - then send $n'$ to all agents to which $n'$ is relevant.

Messages contain the full state $n'$, its $g$ and $h$-values, and its creating action.

# Running example



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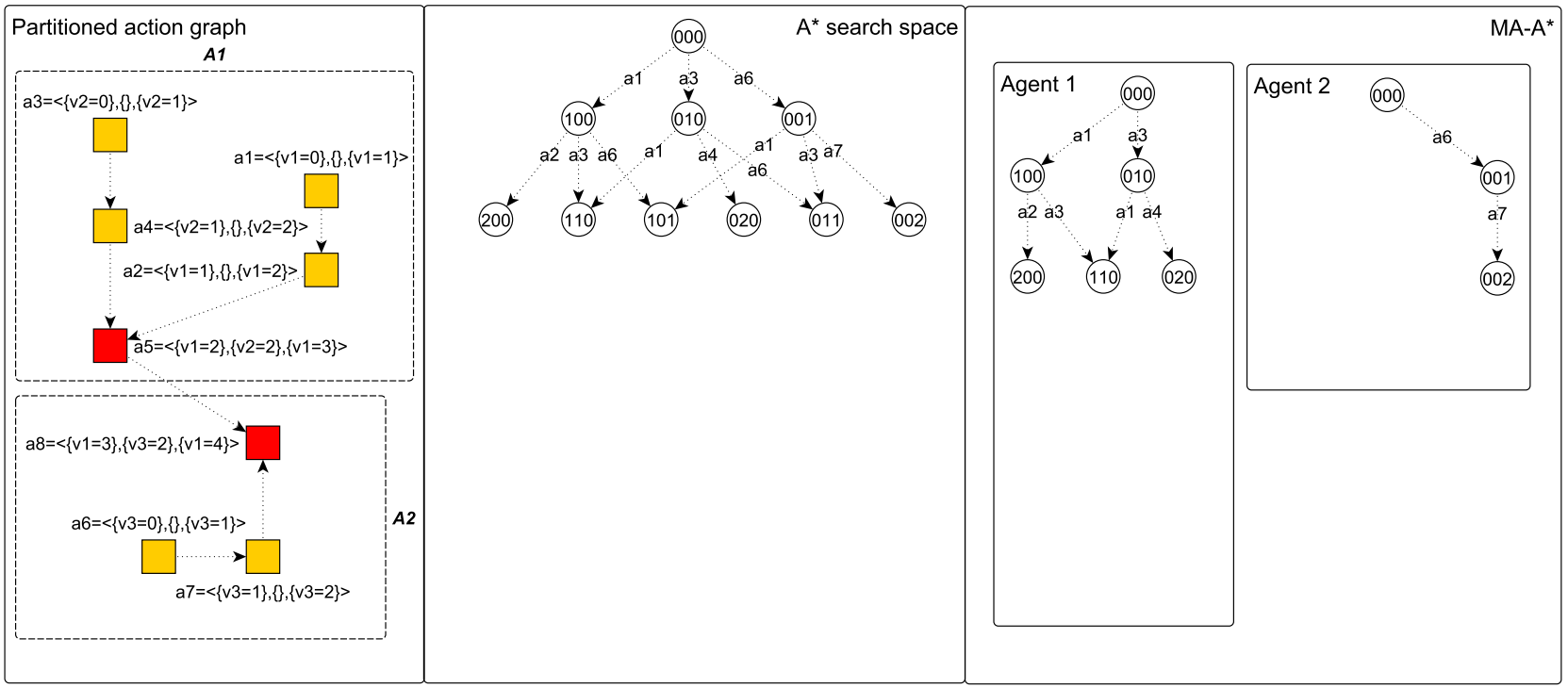

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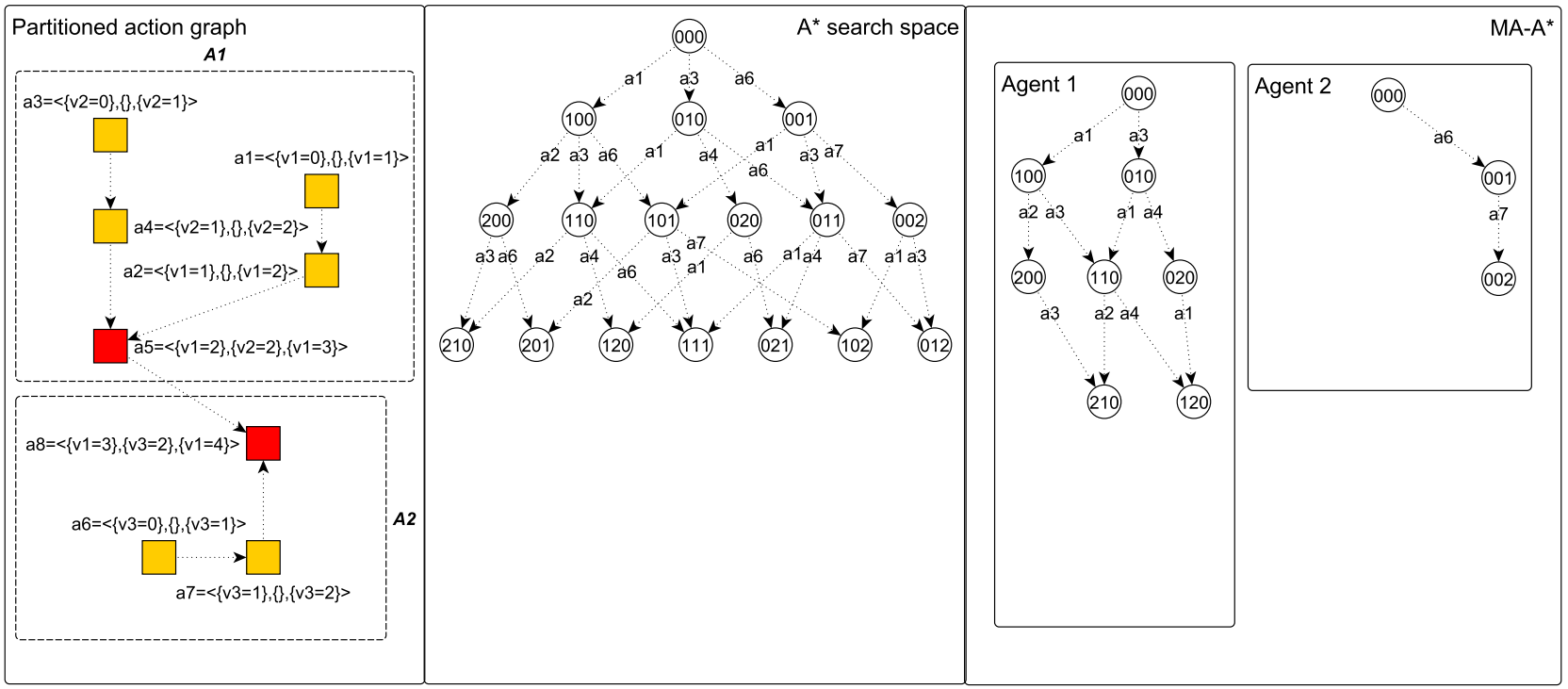

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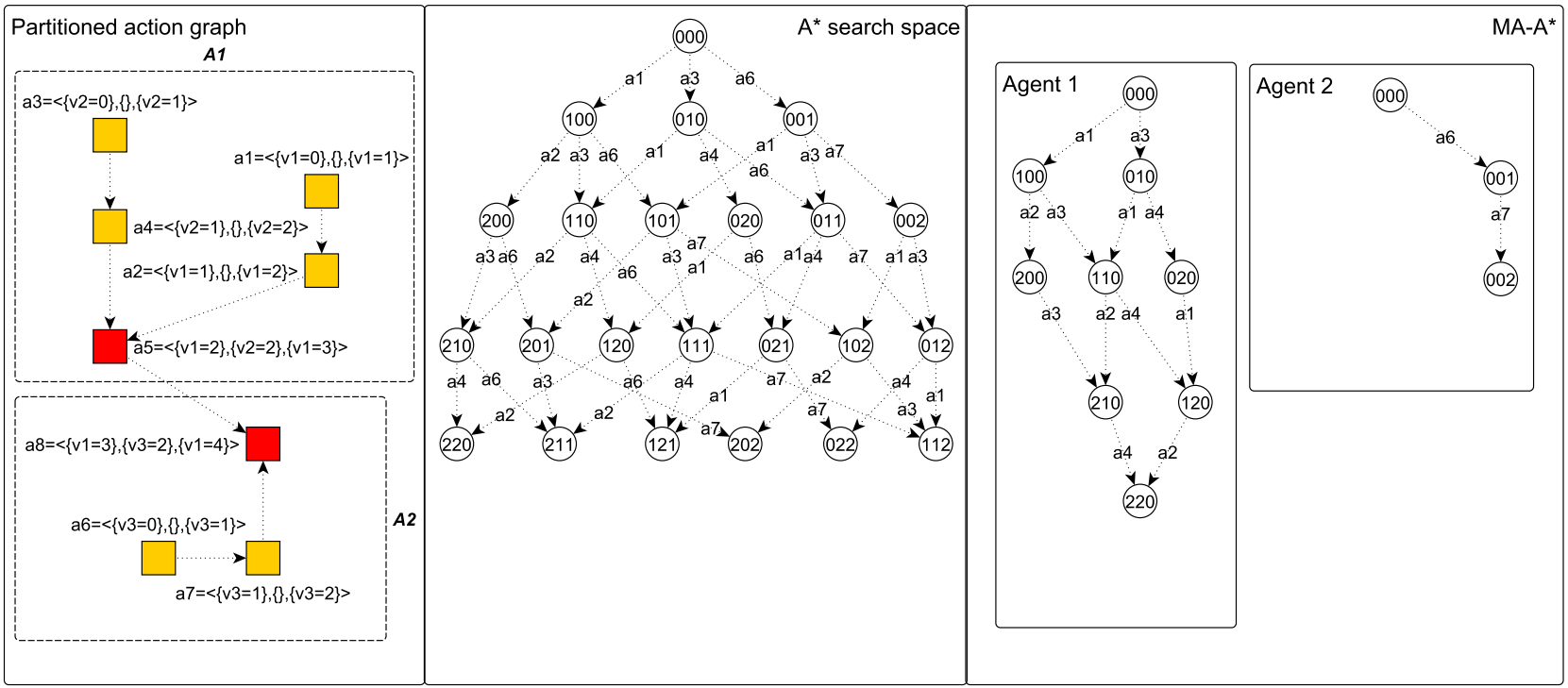

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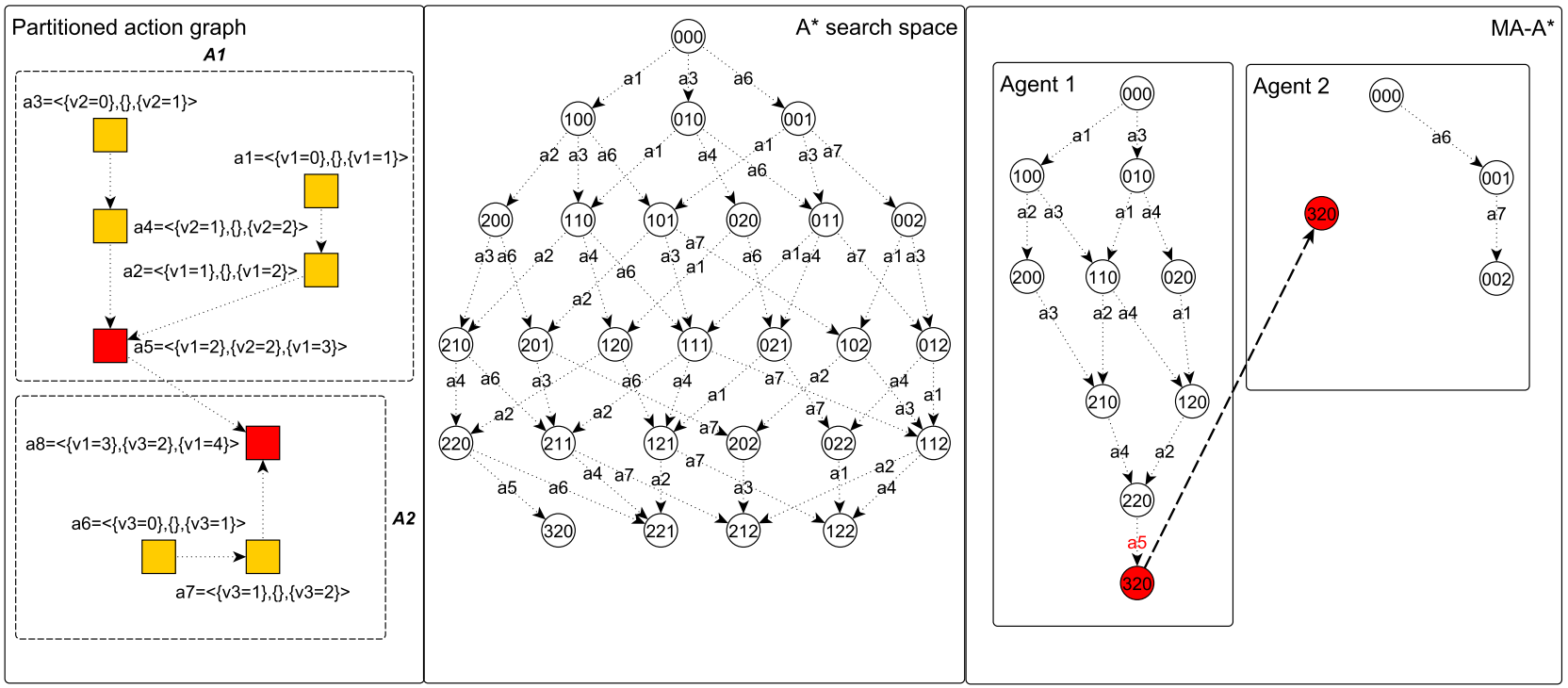

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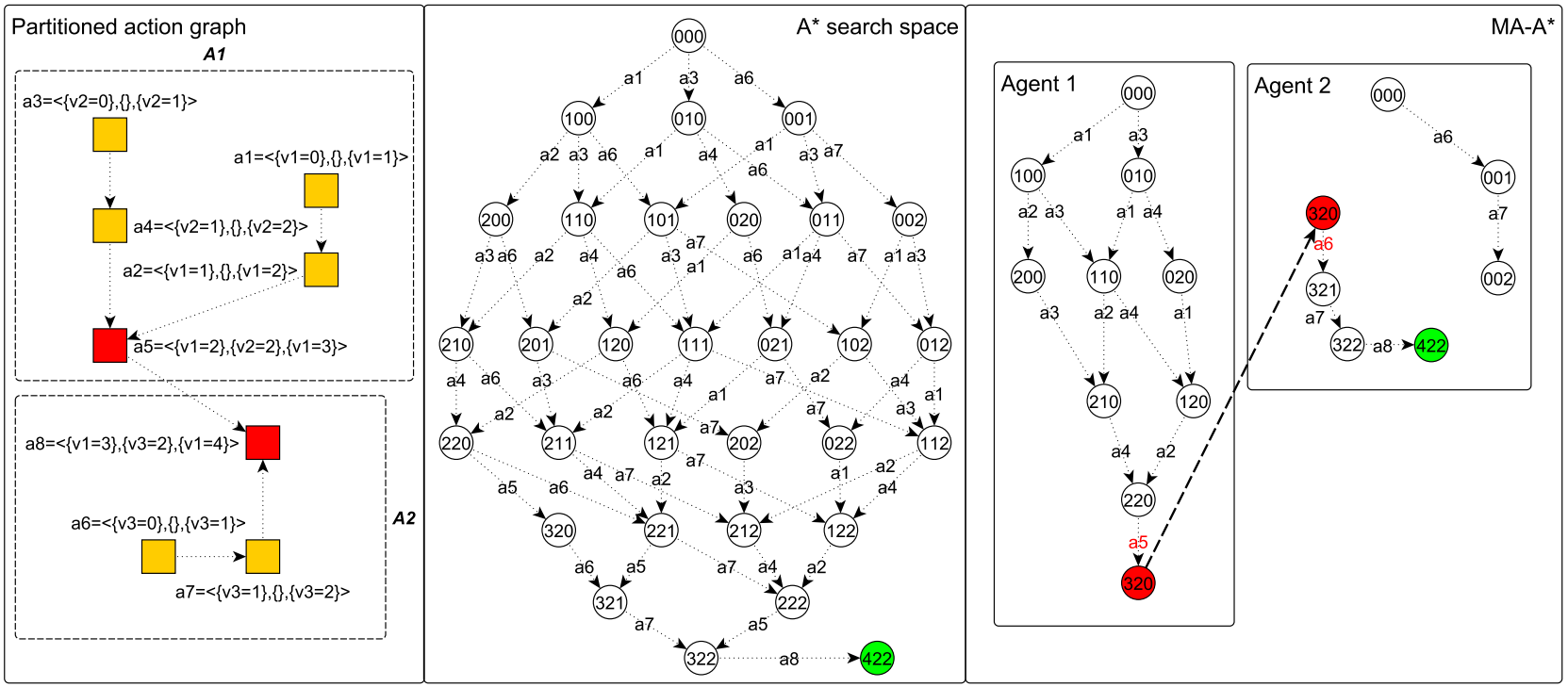

# Relevancy of messages

- A state $s$ is relevant to an agent if it has a public action for which all public preconditions hold in $s$.
- When some agent performs a private action, other agents' view of the system relevant to them has not changed!
- Sending only states for which the creating action is public, maintains optimality.
- This effectively prunes many equivalent parts of the search space $\Longrightarrow$ may result in fewer expansions than centralized A$^*$.

# Experimental results

| Problem (# of processors) | # agents | LM-cut heuristic A* time | LM-cut heuristic A* expands | LM-cut heuristic MAP-A* time | LM-cut heuristic MAP-A* expands | LM-cut heuristic MAP-A* eff. | Merge&Shrink heuristic A* time | Merge&Shrink heuristic A* init-h | Merge&Shrink heuristic MAD-A* time | Merge&Shrink heuristic MAD-A* init-h | Merge&Shrink heuristic MAD-A* eff. | Planning First time | Planning First $\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| logistics7-1(4) | 4 | 55.3 | 155289 | 27 | 504102 | 0.51 | 0 | 44 | 57 | 36 | 0 | – | N/A |
| logistics8-1(4) | 4 | 24.1 | 43665 | 13 | 168545 | 0.46 | 0.1 | 44 | 49.5 | 37 | 0 | – | N/A |
| logistics10-0(4) | 5 | 203 | 193846 | 66.6 | 627314 | 0.76 | 81.7 | 43 | – | 36 | 0 | – | N/A |
| Rovers3(2) | 2 | 0 | 50 | 0 | 90 | 1.00 | 0 | 9 | 0 | 6.5 | 1.00 | 0.3 | 2 |
| Rovers4(2) | 2 | 0 | 9 | 0.04 | 68 | 0 | 0 | 8 | 0 | 6 | 1.00 | 0.2 | 1 |
| Rovers5(2) | 2 | 8.8 | 37397 | 1.8 | 18975 | 2.44 | 11.7 | 20 | 4 | 10.5 | 1.46 | 9.3 | 3 |
| Rovers6(2) | 2 | – | – | 236 | 2255393 | $\infty$ | – | 27 | 325 | 17.5 | $\infty$ | – | N/A |
| Rovers7(3) | 3 | 6.7 | 18315 | 1 | 12929 | 2.23 | 55.9 | 14 | 7.2 | 9 | 2.59 | 38.5 | 3 |
| Rovers8(4) | 4 | – | – | 154 | 1271971 | $\infty$ | – | 15 | – | 10 | N/A | – | N/A |
| Rovers12(4) | 4 | 12.1 | 15222 | 0.9 | 10704 | 3.36 | 119 | 16 | 22.2 | 8.25 | 1.34 | – | N/A |
| Rovers14(4) | 4 | – | – | 598 | 5311741 | $\infty$ | – | 17 | – | 11 | N/A | – | N/A |
| satellites5(3) | 3 | 1.3 | 1174 | 0.1 | 793 | 4.33 | 7 | 13 | 3.8 | 8 | 0.61 | 52.3 | 2 |
| satellites6(3) | 3 | 3.5 | 2976 | 0.2 | 1650 | 5.83 | 23.5 | 18 | 9.2 | 9.3 | 0.85 | 457 | 3 |
| satellites7(4) | 4 | 94.5 | 36652 | 12.4 | 53465 | 1.91 | – | 14 | 343 | 9.5 | $\infty$ | – | N/A |
| satellites8(4) | 4 | – | – | 94.8 | 345667 | $\infty$ | – | 15 | – | 10 | N/A | – | N/A |
| satellites9(4) | 5 | – | – | 105 | 2132756 | $\infty$ | – | 18 | – | 11 | N/A | – | N/A |
| satellites10(4) | 5 | – | – | 61.8 | 95192 | $\infty$ | – | 17 | – | 10 | N/A | – | N/A |
| zenotravel9(3) | 3 | 72.1 | 15408 | 20 | 29321 | 1.20 | 56.7 | 19 | 370 | 14 | 0.05 | – | N/A |
| zenotravel10(3) | 3 | 16.1 | 1587 | 4.3 | 3453 | 1.25 | 26.7 | 22 | – | 17 | 0 | – | N/A |
| zenotravel12(3) | 3 | 458 | 41311 | 57 | 41819 | 2.68 | – | – | – | – | N/A | – | N/A |
| zenotravel13(3) | 3 | – | – | 382 | 185827 | $\infty$ | – | – | – | – | N/A | – | N/A |

**Experimental results**: Comparison of centralized A*, MA-A* in its parallel (MAP-A*) and distributed (MAD-A*) settings, and Planning First. Runtime (in sec.), number of expanded nodes and efficiency values (speedup/# of processors) are shown.