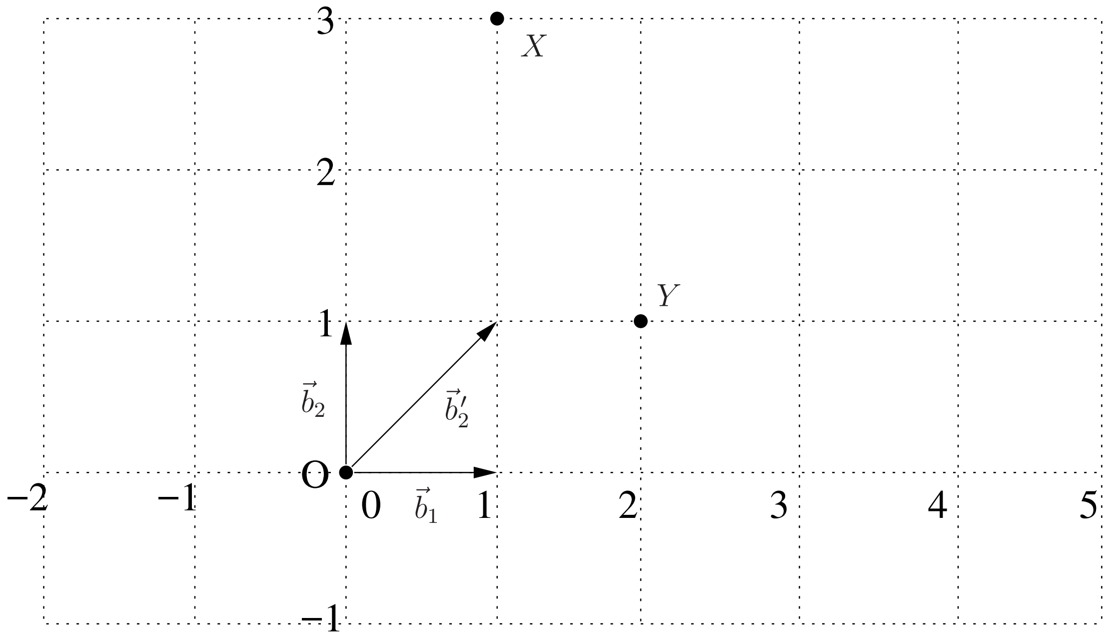
PRO'2020 EX—03 EN

Name:	**Points:**

1. The following figure shows coordinate system $\sigma = (O, \beta)$, with basis $\beta = (\vec{b}_1, \vec{b}_2)$.



   (a) Find the coordinate system $\sigma' = (O', \beta')$, $\beta' = (\vec{b}'_1, \vec{b}'_2)$ with basis vector $\vec{b}'_2$, which is shown (as a representative of a free vector) in the figure, so that the vectors $\vec{x}'_{\beta'}$, $\vec{y}'_{\beta'}$ representing points X, Y have the following coordinates

$$\vec{x}'_{\beta'} = \begin{bmatrix} -1/2 \\ 2 \end{bmatrix}, \vec{y}'_{\beta'} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix},$$

   and plot them into the figure.

   (b) Write the coordinates of the vectors of β in basis β'.

   (c) Write the coordinates of the vectors of β' in basis β.

2. Consider bases $\beta = (\vec{b}_1, \vec{b}_2)$ and $\beta' = (\vec{b}'_1, \vec{b}'_2)$ of a two dimensional linear space

$$\vec{x}_\beta = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \vec{x}_{\beta'} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \vec{b}'_{1\beta} = \begin{bmatrix} . \\ 2 \end{bmatrix} \quad \vec{b}'_{2\beta} = \begin{bmatrix} 1 \\ . \end{bmatrix}$$

   What are the coordinates of vectors β in basis β'.

3. Find all 2×2 rotation matrices $\mathtt{R}$ such that there holds

$$\mathtt{R}\,\mathtt{R} = \mathtt{R}$$

   Explain why there are no other such matrices.

4. Find the eigenvalues and eigenvectors of matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Find all unit quaternions representing the inverse rotation of the rotation represented by the unit quaternion

$$\vec{q} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & \frac{2}{5} & \frac{2}{5} \end{bmatrix}$$

6. Let us apply the motion with $\mathtt{R} = \begin{bmatrix} 3/5 & 0 & a \\ 0 & 1 & b \\ 4/5 & 0 & c \end{bmatrix}$ and $\vec{o}_\beta = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ twice on the space. Find a, b and the axis of motion of the composite motion.

7. Construct a Groebner basis for the following system of polynomials

$$\begin{aligned} x^2\,y^2 + y - 1 &= 0 \\ x^2\,y + x &= 0 \end{aligned}$$

Find the solutions to the system.

8. Let us have a manipulator with four axes of motion as in the picture below. On the picture
   (a) draw coordinate systems of bodies according to the Denavit-Hartenberg notation
   (b) draw all parameters that are needed to describe the manipulator using Denavit-Hartenberg notation along with their orientations.
   (c) Write down the D-H table of the robot parameters.