1 Description Logics

1.1 Formal Ontologies

Formalizing Ontologies

- We heard about
  - RDF,
  - ontologies as “some shared knowledge structures often visualized through UML-like diagrams” ...
- But how to check they are designed correctly? How to reason about the knowledge inside?
- No single language – many graphical/textual languages ranging from informal to formal ones can be used, e.g. *relational algebra*, *Prolog*, *RDFS*, *OWL*, *topic maps*, *thesauri*, *conceptual graphs*

Logics for Ontologies

- propositional logic

  Example

  “John is clever.″ $\Rightarrow$ ¬“John fails at exam.″

- first order predicate logic

  Example

  $(\forall x)(Clever(x) \Rightarrow \neg((\exists y)(Exam(y) \wedge Fails(x, y))))$.

- modal logic

  Example

  $\Box((\forall x)(Clever(x) \Rightarrow \Diamond\neg((\exists y)(Exam(y) \wedge Fails(x, y)))))$.

- ... what is the meaning of these formulas ?

Logics for Ontologies (2)

Logics are defined by their

- Syntax – to *represent* concepts (*defining symbols*)
- Semantics – to capture meaning of the syntactic constructs (*defining concepts*)
- Proof Theory – to enforce the semantics

Logics trade-off
A logical calculus is always a trade-off between *expressiveness* and *tractability of reasoning.*

Propositional Logic

Example
How to check satisfiability of the formula $A \vee (\neg(B \wedge A) \vee B \wedge C)$?

syntax – atomic formulas and $\neg$, $\wedge$, $\vee$, $\Rightarrow$

semantics ($\models$) – an interpretation assigns true/false to each formula.

proof theory ($\vdash$) – resolution, tableau

complexity – NP-Complete (Cook theorem)

First Order Predicate Logic

Example
What is the meaning of this sentence ?

$$(\forall x_1)((Student(x_1) \wedge (\exists x_2)(GraduateCourse(x_2) \wedge isEnrolledTo(x_1, x_2))) \\ \Rightarrow (\forall x_3)(isEnrolledTo(x_1, x_3) \Rightarrow GraduateCourse(x_3)))$$

$$Student \sqcap \exists isEnrolledTo.GraduateCourse \sqsubseteq \forall isEnrolledTo.GraduateCourse$$

First Order Predicate Logic – quick informal review

syntax – constructs involve

term (variable x, constant symbol $JOHN$, function symbol applied to terms $fatherOf(JOHN)$)

axiom/formula (predicate symbols applied to terms $hasFather(x, JOHN)$, possibly glued together with $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\forall$,$\exists$)

universally closed formula formula without free variable ($(\forall x)(\exists y)hasFather(x, y) \wedge Person(y)$)

semantics – an interpretation (with valuation) assigns:

- **domain element** to each term
- **true/false** to each closed formula

proof theory – resolution; *Deduction Theorem*, *Soundness Theorem*, *Completeness Theorem*

complexity – undecidable (Goedel)

Open World Assumption

OWA
FOPL accepts Open World Assumption, i.e. whatever is not known is not necessarily false.

As a result, FOPL is *monotonic*, i.e.

monotonicity
No conclusion can be invalidated by adding extra knowledge.

This is in contrary to relational databases, or Prolog that accept Closed World Assumption.

1.2 Towards Description Logics

Languages sketched so far aren't enough ?

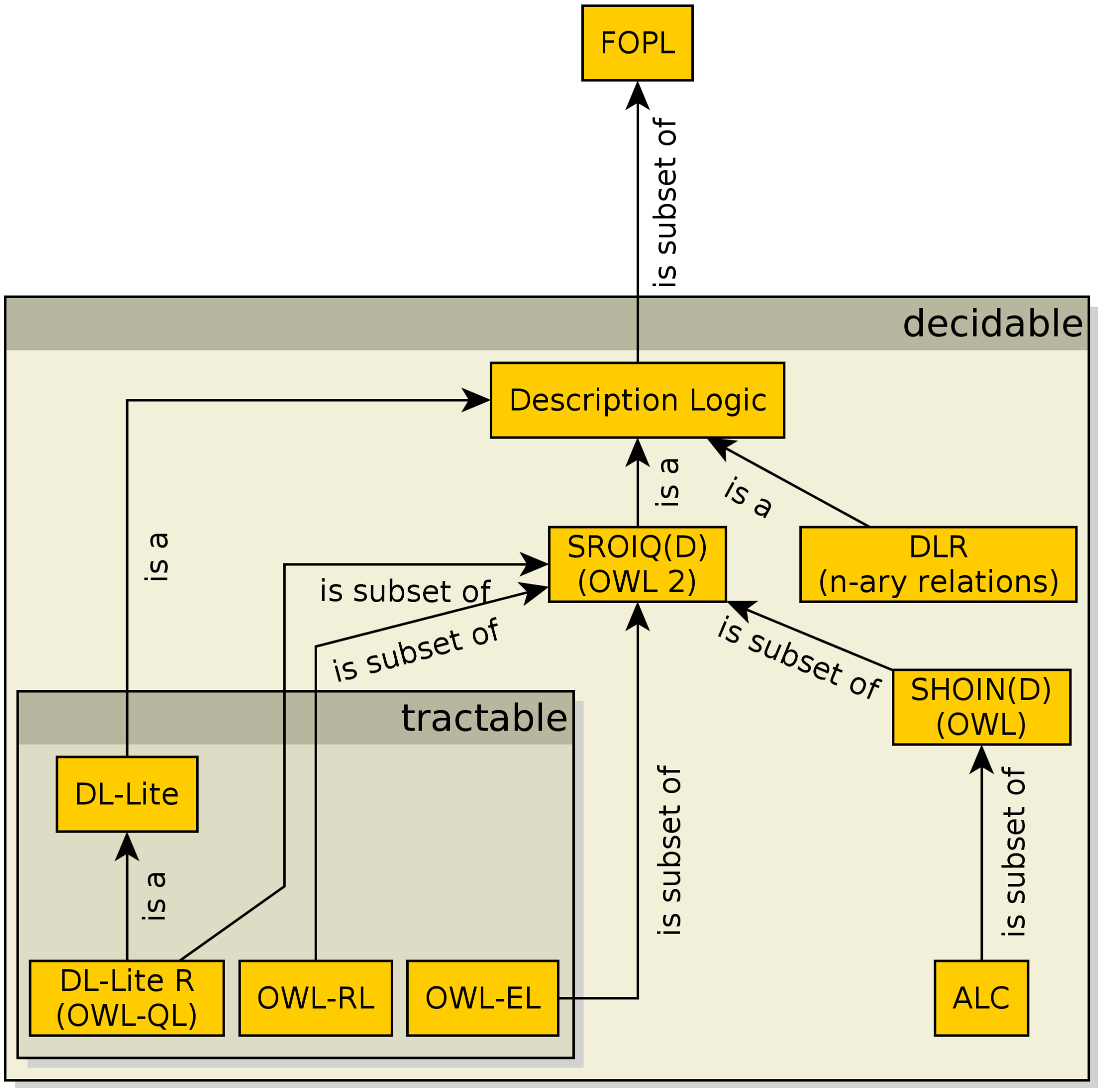
- Why not First Order Predicate Logic ?
  - ☹ FOPL is undecidable – many logical consequences cannot be verified in finite time.
  - – We often do not need full expressiveness of FOL.
- Well, we have Prolog – wide-spread and optimized implementation of FOPL, right ?
  - ☹ Prolog is not an implementation of FOPL – OWA vs. CWA, negation as failure, problems in expressing disjunctive knowledge, etc.

What are Description Logics ?

Description logics (DLs) are (almost exclusively) decidable subsets of FOPL aimed at modeling *terminological incomplete knowledge.*

- first languages emerged as an experiment of giving formal semantics to semantic networks and frames. First implementations in 80's – KL-ONE, KAON, Classic.

- 90's $\mathcal{ALC}$
- 2004 $\mathcal{SHOIN}(\mathcal{D})$ – OWL
- 2009 $\mathcal{SROIQ}(\mathcal{D})$ – OWL 2



1.3 $\mathcal{ALC}$ Language

Concepts and Roles

- Basic building blocks of DLs are :

(atomic) concepts - representing (named) *unary predicates* / classes, e.g. $Parent$, or $Person \sqcap \exists hasChild \cdot Person$.

(atomic) roles - represent (named) *binary predicates* / relations, e.g. $hasChild$

individuals - represent ground terms / individuals, e.g. $JOHN$

- Theory $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ (in OWL refered as Ontology) consists of a

  TBOX $\mathcal{T}$ - representing axioms generally valid in the domain, e.g. $\mathcal{T} = \{Man \sqsubseteq Person\}$

  ABOX $\mathcal{A}$ - representing a particular relational structure (data), e.g. $\mathcal{A} = \{Man(JOHN), loves(JOHN, M$

- DLs differ in their expressive power (concept/role constructors, axiom types).

Semantics, Interpretation

- as $\mathcal{ALC}$ is a subset of FOPL, let's define semantics analogously (and restrict interpretation function where applicable):
- **Interpretation** is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is an interpretation domain and $\cdot^{\mathcal{I}}$ is an interpretation function.
- Having *atomic* concept A, *atomic* role R and individual a, then

$$\begin{aligned} A^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \\ R^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \\ a^{\mathcal{I}} &\in \Delta^{\mathcal{I}} \end{aligned}$$

$\mathcal{ALC}$ (= attributive language with complements)

Having concepts C, D, atomic concept A and atomic role R, then for interpretation $\mathcal{I}$:

concept	*concept*$^{\mathcal{I}}$	*description*
$\top$	$\Delta^{\mathcal{I}}$	(universal concept)
$\bot$	$\emptyset$	(unsatisfiable concept)
$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	(negation)
$C_1 \sqcap C_2$	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$	(intersection)
$C_1 \sqcup C_2$	$C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$	(union)
$\forall R \cdot C$	$\{a \mid \forall b((a,b) \in R^{\mathcal{I}} \implies b \in C^{\mathcal{I}})\}$	(universal restriction)
$\exists R \cdot C$	$\{a \mid \exists b((a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}})\}$	(existential restriction)

TBOX

axiom	$\mathcal{I} \models$ axiom iff	*description*
$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$	(inclusion)
$C_1 \equiv C_2$	$C_1^{\mathcal{I}} = C_2^{\mathcal{I}}$	(equivalence)

ABOX (UNA = unique name assumption[1])

axiom	$\mathcal{I} \models$ axiom iff	*description*
$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$	(concept assertion)
$R(a_1, a_2)$	$(a_1^{\mathcal{I}}, a_2^{\mathcal{I}}) \in R^{\mathcal{I}}$	(role assertion)

[1] two different individuals denote two different domain elements

$\mathcal{ALC}$ – Example

Example
Consider an information system for genealogical data integrating multiple geneological databases. Let's have atomic concepts $Person, Man, GrandParent$ and atomic role $hasChild$.

- Set of persons that have just men as their descendants (if any)
  - $Person \sqcap \forall hasChild \cdot Man$
- How to define concept $GrandParent$? (specify an *axiom*)
  - $GrandParent \equiv Person \sqcap \exists hasChild \cdot \exists hasChild \cdot \top$
- How does the previous axiom look like in FOPL ?

$$\forall x \, (GrandParent(x) \equiv (Person(x) \wedge \exists y \, (hasChild(x, y) \\ \wedge \exists z \, (hasChild(y, z)))))$$

$\mathcal{ALC}$ Example – $\mathcal{T}$

Example

$$\begin{aligned}
Woman &\equiv Person \sqcap Female \\
Man &\equiv Person \sqcap \neg Woman \\
Mother &\equiv Woman \sqcap \exists hasChild \cdot Person \\
Father &\equiv Man \sqcap \exists hasChild \cdot Person \\
Parent &\equiv Father \sqcup Mother \\
Grandmother &\equiv Mother \sqcap \exists hasChild \cdot Parent \\
MotherWithoutDaughter &\equiv Mother \sqcap \forall hasChild \cdot \neg Woman \\
Wife &\equiv Woman \sqcap \exists hasHusband \cdot Man
\end{aligned}$$

Interpretation – Example

Example

- Consider a theory $\mathcal{K}_1 = (\{GrandParent \equiv Person \sqcap \exists hasChild \cdot \exists hasChild \cdot \top\}, \{GrandParent$
  Find some model.
- a model of $\mathcal{K}_1$ can be interpretation $\mathcal{I}_1$:
  - $\Delta^{\mathcal{I}_1} = Man^{\mathcal{I}_1} = Person^{\mathcal{I}_1} = \{\text{John}, \text{Phillipe}, \text{Martin}\}$
  - $hasChild^{\mathcal{I}_1} = \{(\text{John}, \text{Phillipe}), (\text{Phillipe}, \text{Martin})\}$

- $GrandParent^{\mathcal{I}_1} = \{\text{John}\}$
- $JOHN^{\mathcal{I}_1} = \{\text{John}\}$

- this model is finite and has the form of a tree with the root in the node John :

$$\boxed{\text{Person, Man, GrandParent: John}} \xrightarrow[\text{hasChild}]{} \boxed{\text{Person, Man: Phillipe}} \xrightarrow[\text{hasChild}]{} \boxed{\text{Person, Man : Martin}}$$

Shape of DL Models

The last example revealed several important properties of DL models:

Tree model property (TMP)

Every consistent $\mathcal{K} = (\{\}, \{C(I)\})$ has a model in the shape of a *rooted tree*.

Finite model property (FMP)

Every consistent $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ has a *finite model*.

Both properties represent important characteristics of $\mathcal{ALC}$ that significantly speed-up reasoning.

In particular (generalized) TMP is a characteristics that is shared by most DLs and significantly reduces their computational complexity.

Example – CWA $\times$ OWA

Example

ABOX
$hasChild(JOCASTA, OEDIPUS)$ $\quad$ $hasChild(JOCASTA, POLYNEIKES)$
$hasChild(OEDIPUS, POLYNEIKES)$ $\quad$ $hasChild(POLYNEIKES, THERSANDROS)$
$Patricide(OEDIPUS)$ $\quad$ $\neg Patricide(THERSANDROS)$

Edges represent role assertions of $hasChild$; red/green colors distinguish concepts instances – $Patricide$ a $\neg Patricide$

$$\begin{tikzcd} JOCASTA \arrow[rr] \arrow[dr] & & POLYNEIKES \arrow[r] & THERSANDROS \\ & OEDIPUS \arrow[ur] & & \end{tikzcd}$$

Q1 $(\exists hasChild \cdot (Patricide \sqcap \exists hasChild \cdot \neg Patricide))(JOCASTA)$,

$$JOCASTA \longrightarrow \bullet \longrightarrow \bullet$$

Q2 Find individuals x such that $\mathcal{K} \models C(x)$, where C is

$$\neg Patricide \sqcap \exists hasChild^- \cdot (Patricide \sqcap \exists hasChild^- \cdot \{JOCASTA\})$$

What is the difference, when considering CWA ?

$$JOCASTA \longrightarrow \bullet \longrightarrow x$$

Logical Consequence

For an arbitrary set S of axioms (resp. theory $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, where $S = \mathcal{T} \cup \mathcal{A}$) :

Model

$\mathcal{I} \models S$ if $\mathcal{I} \models \alpha$ for all $\alpha \in S$ ($\mathcal{I}$ is a model of S, resp. $\mathcal{K}$)

Logical Consequence

$S \models \beta$ if $\mathcal{I} \models \beta$ whenever $\mathcal{I} \models S$ (β is a logical consequence of S, resp. $\mathcal{K}$)

- S is consistent, if S has at least one model

1.4 From $\mathcal{ALC}$ to OWL(2)-DL

Extending $\ldots\mathcal{ALC}$ **...**

- We have introduced $\mathcal{ALC}$. Its expressiveness is higher than the expressiveness of the propositional calculus, still it lacks many constructs needed for practical applications.
- Let's take a look, how to extend $\mathcal{ALC}$ while preserving decidability.

Extending $\ldots\mathcal{ALC}$ **... (2)**

$\mathcal{N}$ (Number restructions) are used for restricting the number of successors in the given role for the given concept.

syntax (concept)	semantics
$(\geq\ n\ R)$	$\left\{ a \,\middle\|\, \left\| \{b \mid (a,b) \in R^{\mathcal{I}}\} \right\| \geq n \right\}$
$(\leq\ n\ R)$	$\left\{ a \,\middle\|\, \left\| \{b \mid (a,b) \in R^{\mathcal{I}}\} \right\| \leq n \right\}$
$(=\ n\ R)$	$\left\{ a \,\middle\|\, \left\| \{b \mid (a,b) \in R^{\mathcal{I}}\} \right\| = n \right\}$

Example

- Concept $Woman \sqcap (\leq\ 3\ hasChild)$ denotes women who have at most 3 children.
- What denotes the axiom $Car \sqsubseteq (\geq\ 4\ hasWheel)$?
- ... and $Bicycle \equiv (=\ 2\ hasWheel)$?

Extending . . . $\mathcal{ALC}$ ... (3)

$\mathcal{Q}$ (Qualified number restrictions) are used for restricting the number of successors *of the given type* in the given role for the given concept.

syntax (concept)	semantics
$(\geq\ n\, R\, C)$	$\left\{ a \middle\vert\ \left\vert \{b \mid (a,b) \in R^{\mathcal{I}} \wedge b^{\mathcal{I}} \in C^{\mathcal{I}}\} \right\vert \geq n \right\}$
$(\leq\ n\, R\, C)$	$\left\{ a \middle\vert\ \left\vert \{b \mid (a,b) \in R^{\mathcal{I}} \wedge b^{\mathcal{I}} \in C^{\mathcal{I}}\} \right\vert \leq n \right\}$
$(=\ n\, R\, C)$	$\left\{ a \middle\vert\ \left\vert \{b \mid (a,b) \in R^{\mathcal{I}} \wedge b^{\mathcal{I}} \in C^{\mathcal{I}}\} \right\vert = n \right\}$

Example

- Concept $Woman \sqcap (\geq\ 3\, hasChild\, Man)$ denotes women who have at least 3 sons.
- What denotes the axiom $Car \sqsubseteq (\geq\ 4\, hasPart\, Wheel)$?
- Which qualified number restrictions can be expressed in $\mathcal{ALC}$?

Extending . . . $\mathcal{ALC}$ ... (4)

$\mathcal{O}$ (Nominals) can be used for naming a concept elements explicitely.

syntax (concept)	semantics
$\{a_1, \ldots, a_n\}$	$\{a_1^{\mathcal{I}}, \ldots, a_n^{\mathcal{I}}\}$

Example

- Concept $\{MALE, FEMALE\}$ denotes a gender concept that must be interpreted with at most two elements. Why at most ?
- $Continent \equiv \{EUROPE, ASIA, AMERICA, AUSTRALIA, AFRICA, ANTARCTICA\}$?

Extending . . . $\mathcal{ALC}$ ... (5)

$\mathcal{I}$ (Inverse roles) are used for defining role inversion.

syntax (role)	semantics
R^-	$(R^{\mathcal{I}})^{-1}$

Example

- Role $hasChild^-$ denotes the relationship $hasParent$.
- What denotes axiom $Person \sqsubseteq (=\ 2\, hasChild^-)$?
- What denotes axiom $Person \sqsubseteq \exists hasChild^- \cdot \exists hasChild \cdot \top$?

Extending ... $\mathcal{ALC}$ ... (6)

$\cdot^{trans}$ (Role transitivity axiom) denotes that a role is transitive. Attention – it is not a transitive closure operator.

syntax (axiom)	semantics
$trans(R)$	$R^{\mathcal{I}}$ is transitive

Example

- Role $isPartOf$ can be defined as transitive, while role $hasParent$ is not. What about roles $hasPart$, $hasPart^{-}$, $hasGrandFather^{-}$?
- What is a transitive closure of a relationship ? What is the difference between a transitive closure of $hasDirectBoss^{\mathcal{I}}$ and $hasBoss^{\mathcal{I}}$.

Extending ... $\mathcal{ALC}$ ...(7)

$\mathcal{H}$ (Role hierarchy) serves for expressing role hierarchies (taxonomies) – similarly to concept hierarchies.

syntax (axiom)	semantics
$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$

Example

- Role $hasMother$ can be defined as a special case of the role $hasParent$.
- What is the difference between a concept hierarchy $Mother \sqsubseteq Parent$ and role hierarchy $hasMother \sqsubseteq hasParent$.

Extending ... $\mathcal{ALC}$ ... (8)

$\mathcal{R}$ (role extensions) serve for defining expressive role constructs, like role chains, role disjunctions, etc.

syntax	semantics
$R \circ S \sqsubseteq P$	$R^{\mathcal{I}} \circ S^{\mathcal{I}} \sqsubseteq P^{\mathcal{I}}$
$Dis(R,S)$	$R^{\mathcal{I}} \cap S^{\mathcal{I}} = \emptyset$
$\exists R \cdot Self$	$\{a \vert (a,a) \in R^{\mathcal{I}}\}$

Example

- How would you define the role $hasUncle$ by means of $hasSibling$ and $hasParent$?
- how to express that R is transitive, using a role chain ?
- Whom does the following concept denote $Person \sqcap \exists likes \cdot Self$?

Global restrictions

- *Simple roles* have no (direct or indirect) subroles that are either *transitive* or are defined by means of property chains

$$\begin{aligned} hasFather \circ hasBrother &\sqsubseteq hasUncle \\ hasUncle &\sqsubseteq hasRelative \\ hasBiologicalFather &\sqsubseteq hasFather \end{aligned}$$

  hasRelative and *hasUncle* are not simple.

- Each concept construct and each axiom from this list contains only *simple roles*:
  - number restrictions – $(\geq n\ R)$, $(= n\ R)$, $(\leq n\ R)$ + their qualified versions
  - $\exists R \cdot Self$
  - functionality/inverse functionality (leads to number restrictions)
  - irreflexivity, asymmetry, and disjoint object properties.

Extending ... $\mathcal{ALC}$ ... – OWL-DL a OWL2-DL

- From the previously introduced extensions, two prominent decidable supersets of $\mathcal{ALC}$ can be constructed:
  - $\mathcal{SHOIN}$ is a description logics that backs OWL-DL.
  - $\mathcal{SROIQ}$ is a description logics that backs OWL2-DL.
  - Both OWL-DL and OWL2-DL are semantic web languages – they extend the corresponding description logics by:

    syntactic sugar – axioms NegativeObjectPropertyAssertion, AllDisjoint, etc.

    extralogical constructs – imports, annotations

    data types – XSD datatypes are used

Rules and Description Logics

- How to express e.g. that "A cousin is someone whose parent is a sibling of your parent." ?
- ... we need rules, like

$$\begin{aligned} hasCousin(?c_1, ?c_2) \leftarrow \quad & hasParent(?c_1, ?p_1), hasParent(?c_2, ?p_2), \\ & Man(?c_2), hasSibling(?p_1, ?p_2) \end{aligned}$$

- in general, each variable can bind **domain elements** (i.e. elements of the interpretation domain, not only named individual); however, such version is *undecidable.*

  DL-safe rules

  DL-safe rules are decidable conjunctive rules where each variable **only binds individuals** (not domain elements themselves).

Other extensions

Modal Logic introduces *modal operators* – possibility/necessity, used in multiagent systems.

Example

- ($\Box$ represents e.g. the "believe" operator of an agent)

$$\Box(Man \sqsubseteq Person \sqcap \forall hasFather \cdot Man) \tag{1.1}$$

- As $\mathcal{ALC}$ is a syntactic variant to a multi-modal propositional logic, where each role represents the accessibility relation between worlds in Kripke structure, the previous example can be transformed to the modal logic as:
-

$$\Box(Man \implies Person \wedge \Box_{hasFather} Man) \tag{1.2}$$

Vague Knowledge - fuzzy, probabilistic and possibilistic extensions

Data Types ($\mathcal{D}$) allow integrating a data domain (numbers, strings), e.g. $Person \sqcap \exists hasAge \cdot 23$ represents the concept describing "23-years old persons".

References

Bibliography

[1] * Vladimír Mařík, Olga Štěpánková, and Jiří Lažanský. *Umělá inteligence 6 [in czech], Chapters 2-4.* Academia, 2013.

[2] * Franz Baader, Diego Calvanese, Deborah L. McGuinness, Daniele Nardi, and Peter Patel-Schneider, editors. *The Description Logic Handbook, Theory, Implementation and Applications, Chapters 2-4.* Cambridge, 2003.

[3] * Enrico Franconi. *Course on Description Logics.* http://www.inf.unibz.it/ franconi/dl/course/, cit. 22.9.2013.