

Lecture 9: Multi-robot mutual localization

B3M33MRS — Aerial Multi-Robot Systems

Doc, Ing. Martin Saska, Dr. rer. nat.

Labs: Ing. Tomáš Báča, Ph.D

Multi-Robot Systems group, Faculty of Electrical Engineering
Czech Technical University in Prague



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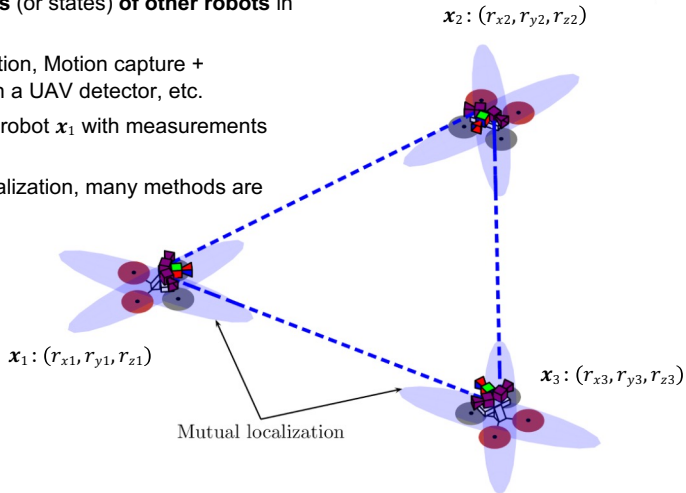


MULTI-ROBOT
SYSTEMS
GROUP

What is multi-robot mutual localization?

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- **A system that determines positions (or states) of other robots in the environment**
- Various setups: GNSS + communication, Motion capture + communication, onboard camera with a UAV detector, etc.
- For example, a system that provides robot x_1 with measurements $L_1 = \{(r_{x2}, r_{y2}, r_{z2}), (r_{x3}, r_{y3}, r_{z3})\}$
- Although different from robot self-localization, many methods are similar



Why is mutual localization important?

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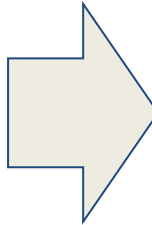
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- Collision avoidance
- Cooperation/coordination/collaboration
 - ✓ Formation control, swarming
- Mutual physical interaction
 - ✓ Aerial interception



Almost all multi-robot scenarios require mutual localization!

Source: MRS archive



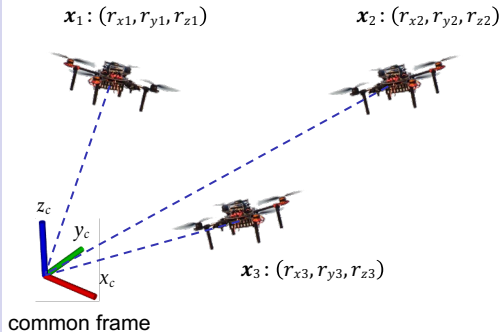
Clasification of multi-robot mutual localization

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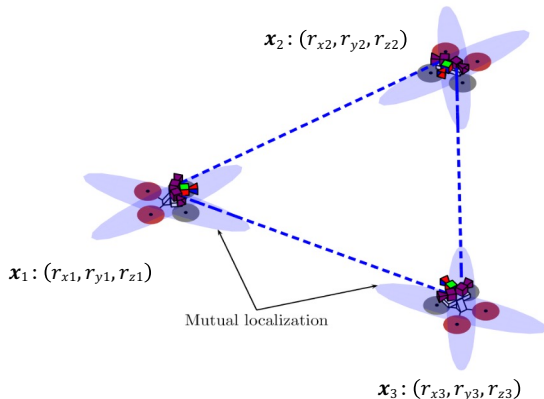
- Frame of reference
 - ✓ Absolute vs. relative localization
 - ✓ Requiring communication vs. communication-less
- Markers
 - ✓ Marker-based vs. marker-less localization
 - ✓ Active markers vs. passive markers
- Time of flight-based localization (UWB)
- Map sharing-based localization

Frame of reference: Absolute vs. relative localization

- **Absolute localization:** localization in a common coordinate frame using an external source



- **Relative localization:** localization using only onboard sensors
- Each UAV with its own coordinate frame



Absolute (global) localization

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- GNSS-based localization (GPS, Galileo, GLONASS ...)
- Motion Capture (Vicon, Optitrack, Qualisys ...)
- Map (or landmark) merging
- Pre-installed radio beacons



Source: archive of GRASP lab at UPENN

Towards a Swarm of Nano Quadrotors

Alex Kushleyev, Daniel Mellinger, and Vijay Kumar
GRASP Lab, University of Pennsylvania

Source: archive of GRASP lab at UPENN

Relative localization

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- Marker detection (visual, IR, UV, etc.)
- Radio-based multilateration (UWB)
- Marker-less detection (CNNs, RADAR, LiDAR, acoustic, etc.)
- Hybrid approaches

Decentralized Visual-Inertial-UWB Fusion for Relative State Estimation of Aerial Swarm

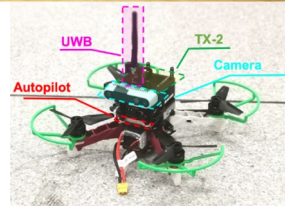
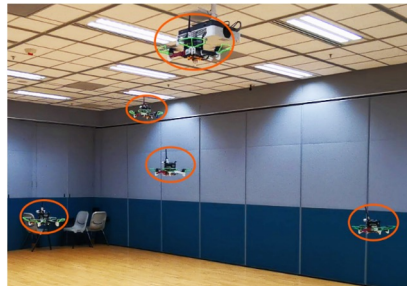
Hao Xu, Luqi Wang, Yichen Zhang, Kejie Qiu and Shaojie Shen



香港科技大學
THE HONG KONG
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AND TECHNOLOGY



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HKUST-DJI JOINT
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Xu, Hao, et al., "Decentralized visual-inertial-UWB fusion for relative state estimation of aerial swarm", *IEEE ICRA 2020*.

Frame of reference: Absolute vs. relative localization

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- **Absolute localization**

- + Usually little or no drift
- + Easy to use from the user's perspective
- + Can be extremely accurate and low-delay
- + Simplifies multi-robot planning
- **Often relies on existing infrastructure**
(satellites, MoCap cameras, the beacons)
- **Requires communication between robots**
to share the measurements/the map

- **Relative localization**

- + **no reliance on infrastructure**
- + **communication not necessary**
- + may work almost in any environment
- accuracy/drift varies
- can suffer from false positives → **lower reliability**

Markers: Marker-based vs. marker-less localization

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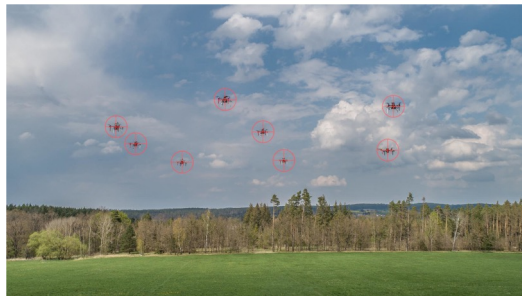
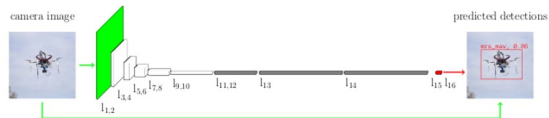
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- **Marker-based localization:** group members equipped with distinct markers
- Visual markers for cameras, IR markers for MoCap, UWB beacons, active/passive markers



Source: MRS archive

- **Marker-less localization:** group members localized without markers
- detection based on visual appearance, 3D shape, noise, radio reflectivity, ...



Types of multi-robot mutual localization: Markers

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- **Marker-based localization**

- + Simpler to implement
- + Usually **more reliable**
- + Usually **more accurate**
- + Easier to estimate relative orientation
- **Not always applicable** (e.g. non-cooperating targets)
- Markers take up space & weight (active markers also energy)
- Often difficult to localize from an arbitrary direction

- **Marker-less localization:**

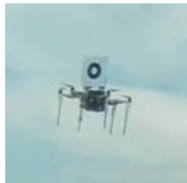
- + **Can be used for non-cooperating targets**
- + Bio-inspired approach
- A harder recognition problem to solve
- **More complex** approaches required
- Usually less robust and less accurate

Passive markers

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- WhyCON
- WhyCode
- ArUco
- QR
- Color-based markers
- Other fiducials

- + Simple to implement
- + Cheap and easy to install
- Dependent on light conditions
- Large size for significant range
- Heavy, non-aerodynamic, complicated maintenance
- Directional dependent



Source: MRS archive



Source: Ulrich's archive



Evangeliou, N., Chaikalis, D., Tsoukalas, A., & Tzes, A., "Visual Collaboration Leader-Follower UAV-Formation for Indoor Exploration", *Frontiers in Robotics and AI*, 8, 2021.

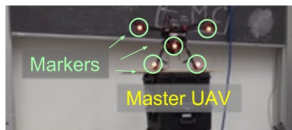


Tron, R., Thomas, J., Loianno, G., Polin, J., Kumar, V. & Daniilidis, K., "Vision-based formation control of aerial vehicles", *RSS*, 2014.

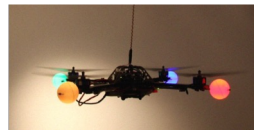
Active markers

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- Monochrome visible light LEDs
 - RGB LEDs
 - Infrared LEDs
 - Ultraviolet LEDs
-
- + Lighting-independent
 - + Small image - typically points → less computationally demanding
 - + Higher reliability
 - + Smaller footprint
 - + Usually less precise pose estimate
 - Complex implementation (HW and SW)
 - Power consumption



L. Teixeira, F. Maffra, M. Moos and M. Chli, "VI-RPE: Visual-Inertial Relative Pose Estimation for Aerial Vehicles", *IEEE RAL*, vol. 3, no. 4, pp. 2770-2777, 2018.



Achtelik, M., Zhang, T., Kuhnlenz, K., & Buss, M., "Visual tracking and control of a quadcopter using a stereo camera system and inertial sensors", *IEEE ICMA*, 2019.



Source: MRS archive



Faessler, M., Mueggler, E., Schwabe, K., & Scaramuzza, D., "A monocular pose estimation system based on infrared leds", *IEEE ICRA*, 2014.

Marker-less relative localization

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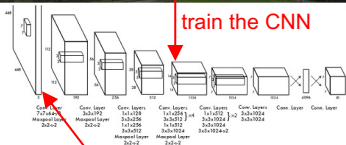
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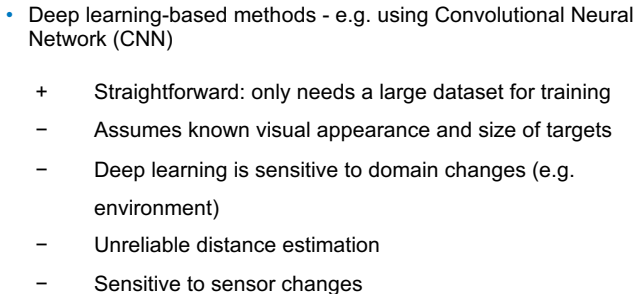
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- Typically used if the target
 - ✓ is too small to carry markers
 - ✓ is non-cooperative (i.e. collision avoidance with non-team members)
 - ✓ is hostile (i.e. intruder/attacker drone)
- In comparison with marker-based methods, it is
 - ✓ harder to estimate full pose (e.g., orientation estimation)
 - ✓ harder to provide unique IDs of targets (can be mitigated by multi-target tracking)
 - ✓ less restrictive usage

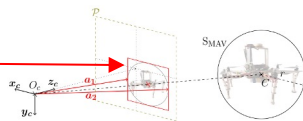
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feed images to the CNN



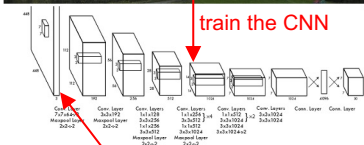
the CNN estimates bounding boxes



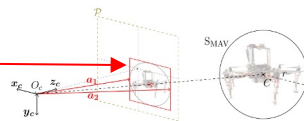
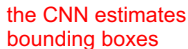
calculate relative positions of the targets
using a calibrated camera projection model

Source of images: Redmon's and MRS archives

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feed images to the CNN



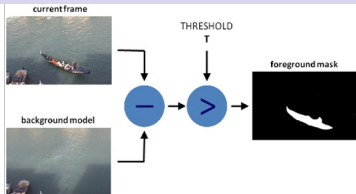
calculate relative positions of the targets
using a calibrated camera projection model

- Deep learning-based methods (a typical approach)
 - ✓ A large labeled dataset for training and validation
 - ✓ A neural network (usually a CNN such as YOLO)
- The main steps of the detection algorithm:
 - ✓ Train the CNN to detect the desired targets in images
 - ✓ Feed the CNN with camera images
 - ✓ Use the detections to estimate targets' relative positions

Source of images: Redmon's and MRS archives

Marker-less relative localization - visual detection

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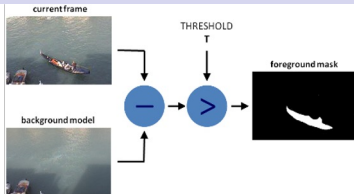
https://docs.opencv.org/3.4/d1/dc5/tutorial_background_subtraction.html



J. Li, D. H. Ye, ... C. Bouman, "Multi-target detection and tracking from a single camera in Unmanned Aerial Vehicles (UAVs)", *IROS*, 2016.

- Background subtraction steps:
 - ✓ Detect salient features in subsequent images
 - ✓ Calculate the transformation between corresponding features
 - ✓ Obtain the background model by transforming the 2nd image
 - ✓ Subtract the background model from the 1st image
 - ✓ Filter the resulting differential image
 - ✓ Detect and classify moving objects in the differential image
- Optical flow can also be used to remove the background
- A classifier is often used to further filter the detected objects
- + Can detect distant and small targets
- + Agnostic to the target's visual appearance
- Unreliable distance estimation

Marker-less relative localization - visual detection



https://docs.opencv.org/3.4/d1/dc5/tutorial_background_subtraction.html



J. Li, D. H. Ye, ... C. Bouman, "Multi-target detection and tracking from a single camera in Unmanned Aerial Vehicles (UAVs)", *IROS*, 2016.

- Visual tracking steps:
 - ✓ Initialize with a bounding box of the target in an image
 - ✓ In the subsequent image, find a visually similar shape nearby
 - ✓ Estimate the target's motion to predict its position in the next frame
 - ✓ Repeat from the second step
- Related to multi-target tracking (later in this lecture)
 - + Can associate the targets and fill out frames when detection fails
 - Can latch onto a visually similar background shape

Marker-less relative localization - spatial detection

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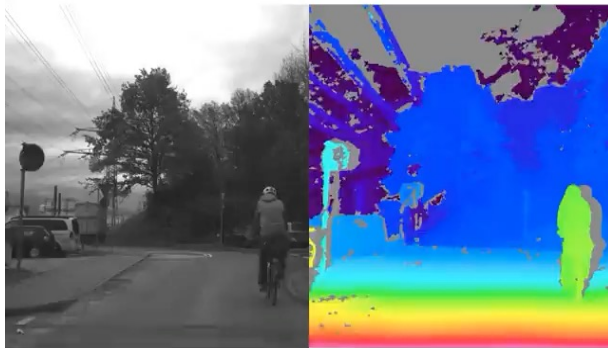
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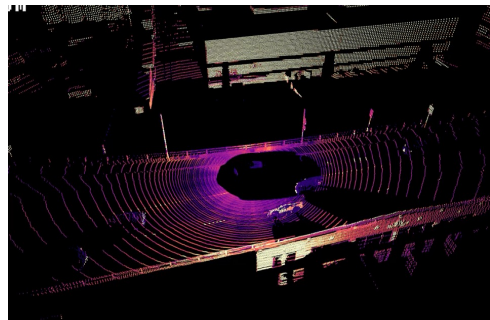
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- Spatial information about the environment provided by stereo cameras and 3D LiDARs used for flying object detection
- Flying objects “stand out” in spatial sensor modalities as compared to visual



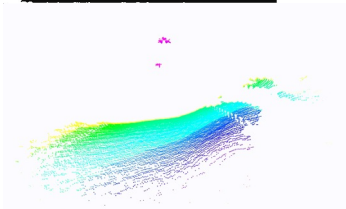
<https://nerian.com>



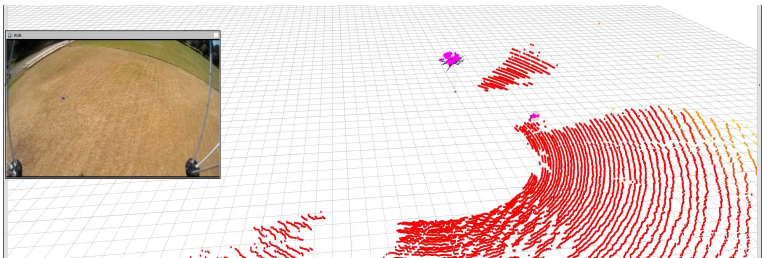
<https://ouster.com/products/scanning-lidar/os1-sensor/>

Marker-less relative localization - spatial detection

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- + Appearance-agnostic
- + Implicit estimation of the target's relative 3D position
- Ad-hoc methods that are not always reliable (ongoing research)



M Vrba, D Heřt and M Saska, "Onboard Marker-Less Detection and Localization of Non-Cooperating Drones for Their Safe Interception by an Autonomous Aerial System.", *IEEE RAL*, 2019.

Ultra-Wideband (UWB) localization

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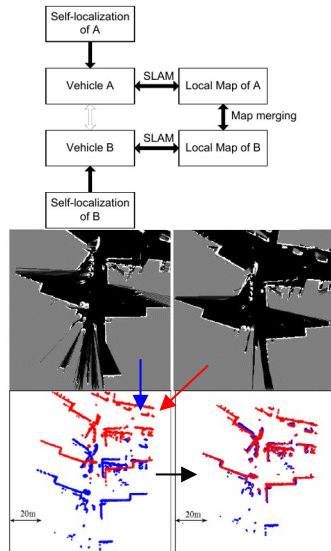
- UWB beacons placed on the UAVs and optionally in the environment
- ToF-based range measurements between the beacons provided by the UWB system
- Relative positions of beacons multilaterated – similar approach as in task 02
- UWB provides:
 - ✓ Good accuracy and robustness to interference and reflections provided by UWB - compared to e.g. Bluetooth
 - ✓ Precise and reliable localization especially when combined with other methods

Map merging

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- Neighboring agents sharing their own maps
- Only salient features/landmarks can be transmitted to reduce bandwidth
- Each agent tries to localize itself in the map of the other robots (similarly to normal SLAM algorithms)
- Requires communication
- Susceptible to local minima when merging
- A global map provided for free

H. Li and F. Nashashibi, "Multi-vehicle cooperative localization using indirect vehicle-to-vehicle relative pose estimation", *CVES*, 2012.



Multi-target tracking and filtration for localization - motivation

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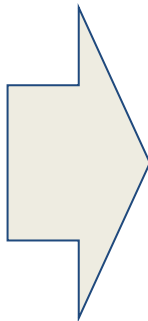
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- Relative localization methods can produce noisy measurements and false positives
- Not all methods provide unique identification of the detected targets
- Required estimating more states than the localization methods provide
- Fusing multiple different localization methods together to improve accuracy, robustness, and reduce delay



Multi-target tracking and filtration algorithms with a motion model of the robot can help!

Multi-target tracking and filtration for localization

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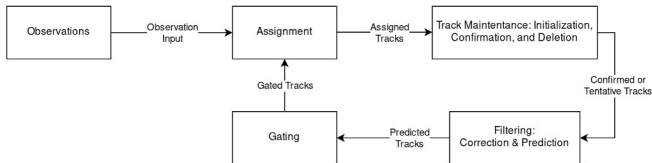
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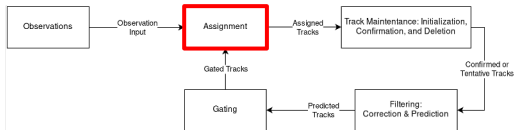
- Issues to be solved in Multi-Target Tracking (MTT):
 - ✓ An unknown number of relevant objects (UAVs in our case) in the observed area
 - ✓ Objects appearing and disappearing (e.g. when entering/leaving the area)
 - ✓ Anonymous inaccurate position estimates of the objects
 - ✓ Tracking the objects – identification of which detection belongs to which object
 - ✓ Filtering the position estimates to obtain a better state estimation of each object
- Input of an MTT algorithm in one iteration: a set of detections and their corresponding position estimates = a single observation
- Desired output of an MTT algorithm: filtered state estimates of the identified objects in the observed area = confirmed tracks



<https://www.mathworks.com/help/fusion/ug/introduction-to-multiple-target-tracking.html>

Multi-target tracking and filtration for localization

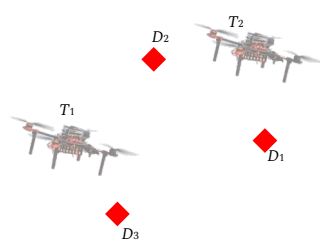
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<https://www.mathworks.com/help/fusion/ug/introduction-to-multiple-target-tracking.html>

- How to assign the detections to the tracks? – **Global Nearest Neighbor (GNN)**:

- ✓ An example of a **greedy method** using track-detection distance:
- 1. Start with any valid assignment of detections to tracks
- 2. Let $i := 1$ be the index of the first assigned pair
- 3. For an assigned pair $\{T_i, D_i\}$, find $j \geq i$ so that after swapping D_i with D_j the sum of distances between all assigned pairs is minimized. Swap D_i with D_j
- 4. Increment i and repeat step 3 for all pairs $\{1, \dots, n\}$



Multi-target tracking and filtration for localization

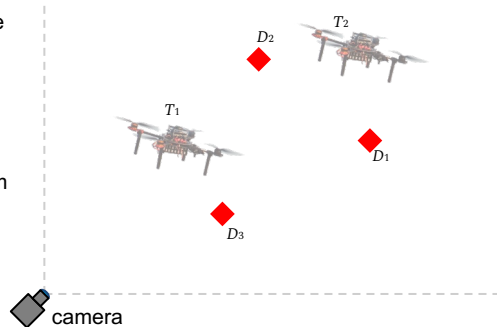
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<https://www.mathworks.com/help/fusion/ug/introduction-to-multiple-target-tracking.html>

- How to assign the detections to the tracks? – **Global Nearest Neighbor (GNN):**

- ✓ The greedy method provides a reasonable feasible solution, although not necessarily the optimal solution
- ✓ But in most of the cases good enough
- ✓ Asymptotic complexity: $O(N^2)$
- ✓ An optimal solution - using the Hungarian algorithm with $O(N^3)$ asymptotic complexity



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<https://www.mathworks.com/help/fusion/ug/introduction-to-multiple-target-tracking.html>

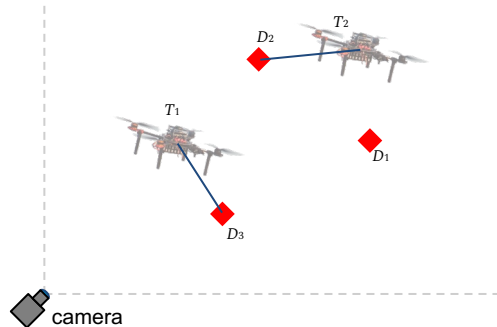
- How to assign the detections to the tracks? – **Global Nearest Neighbor (GNN):**

- ✓ Using Euclidean distance between the track and detection:

$$D1 \leftrightarrow \emptyset$$

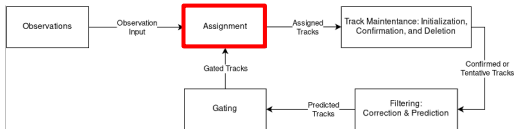
$$D2 \leftrightarrow T2$$

$$D3 \leftrightarrow T1$$



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- How to assign the detections to the tracks? – **Global Nearest Neighbor (GNN):**

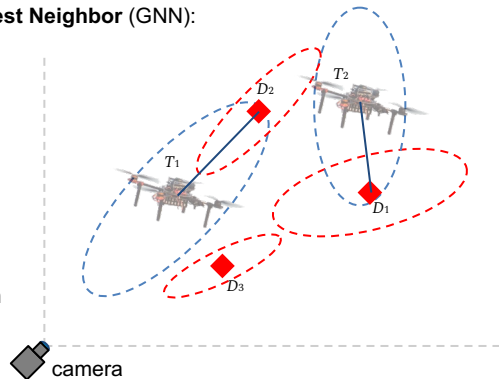
- ✓ Using track-detection distance scaled by track and detection uncertainties:

$$D1 \leftrightarrow T2$$

$$D2 \leftrightarrow T1$$

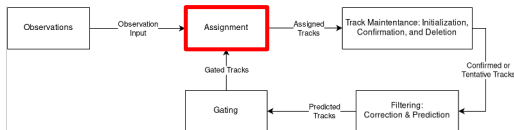
$$D3 \leftrightarrow \emptyset$$

- ✓ Taking known uncertainties into account - statistical improving the algorithm
- ✓ Applying only track uncertainties or even detection uncertainties



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- How to assign the detections to the tracks? – **Joint Probabilistic Data Association (JPDA)**:

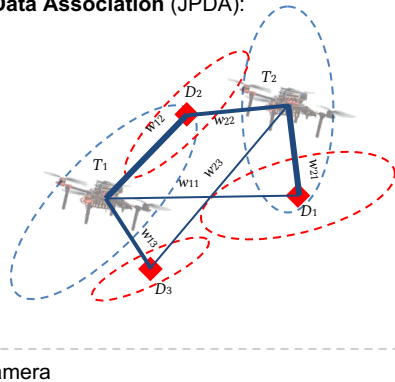
- ✓ Multiple detections used to update a single track (no hard associations):

$$w_{11}D_1 + w_{12}D_2 + w_{13}D_3 \leftrightarrow T_1$$

$$w_{21}D_1 + w_{22}D_2 + w_{23}D_3 \leftrightarrow T_2$$

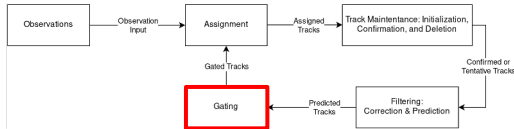
with $w_{ij} = d(T_i, D_j)$, where the distance function $d(\cdot)$ is Euclidean, Mahalanobis, measurement likelihood, etc.

- ✓ Used to simplify the Assignment problem
- ✓ Can deal with multiple detections corresponding to the same object, track overlapping, etc.
- ✓ Number of track-detection combinations growing with the number of tracks and detections



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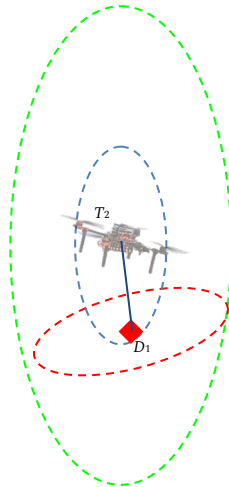
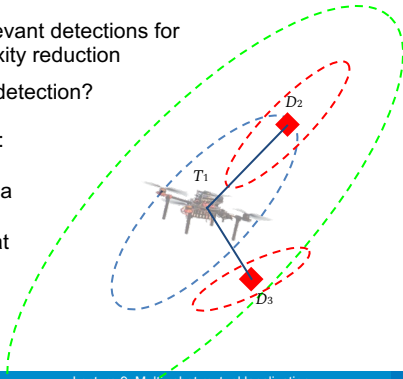
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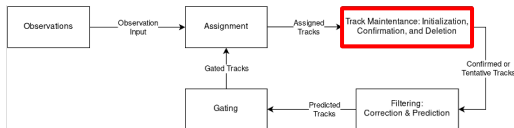
- Complexity of the Assignment problem increasing with many tracks and detections!

- ✓ Considering only relevant detections for each track – complexity reduction
- ✓ What is a “relevant” detection?
- ✓ A possible approach: Using the track’s uncertainty to select a gating volume and ignore detections that lie outside of it



Multi-target tracking and filtration for localization

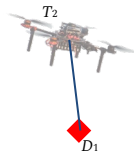
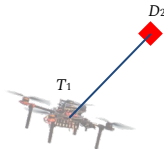
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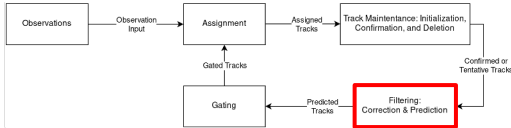
- How to deal with new objects entering the observed area and old objects leaving the area?

- ✓ Some detections not assigned to any track and vice-versa
- ✓ Unassigned detections used to initialize new tracks (but only tentative – to avoid false positives)
- ✓ Tracks not updated for a long time are deleted
- ✓ Tracks with enough assigned detections are confirmed
- ✓ Considering uncertainty etc.



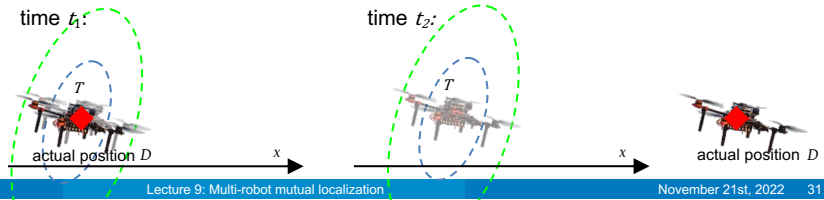
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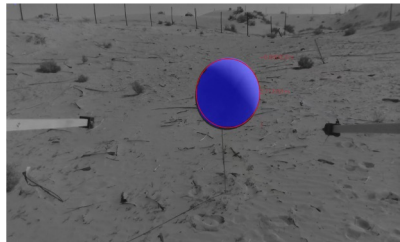
- What to do with noisy measurements?
- Estimating and filtering states of the targets: e.g. a Kalman Filter:
 - ✓ To improve accuracy, to provide estimation of the track uncertainties (covariance matrices)
 - ✓ To enable estimation of non-measured states of the targets (e.g. velocity, acceleration) and prediction of future movement, e.g. using a state-space motion model of a mass point in 3D



Monocular distance estimate

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- Relative 3D position task divided into relative bearing and range (distance) estimation sub-tasks
 - ✓ Bearing calculated from pixel position of an image of the object using a calibrated projection model
 - ✓ More difficult task is **estimating the range** if assuming a monocular system
- **Using perspective** – object image size depends on its distance from the camera
- The distance can be retrieved from
 - 1) a segment area (pixel count) vs. a known object size
 - 2) a dimension of a bounding box vs. a known object size
- Both approaches require a priori knowledge of object size and shape
- Uncertainty caused by
 - ✓ inaccurate object segmentation
 - ✓ inaccurate assumptions on the object's properties
 - ✓ inaccurate camera calibration



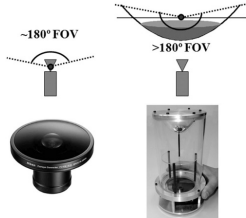
Source: MRS archive

Monocular distance estimate

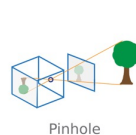
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- Common camera models:

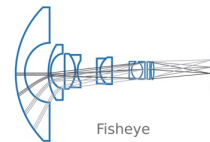
- ✓ Pinhole
- ✓ Fisheye
- ✓ Omnidirectional



Source: Immersive Media



Pinhole

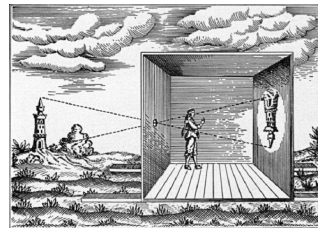


Fisheye

Source: Mathworks

- The pinhole camera model:

- ✓ The classical principle of Camera Obscura
- ✓ Computationally easy
- ✓ Assumes no lens distortion (modelled separately)



Source: blackcreek.ca

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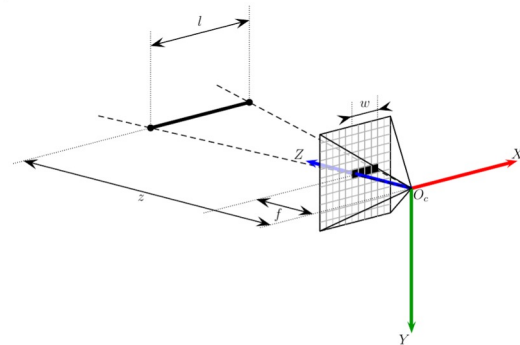
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- The pinhole camera model assumes given:

- 1) Focal distance f (in pixels): $f = f_x = f_y$
- 2) Dimension of the object's image $w := x_{IMG2} - x_{IMG1}$ [pixels]
- 3) Object is perpendicular to the optical axis Z with a horizontal dimension $l := x_2 - x_1$ [m]

- Estimated distance to the object is obtained as:

$$\frac{l}{z} = \frac{w}{f} \rightarrow z = \frac{lf}{w}$$



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- For an approximate evaluation of the depth error, we can linearize the formula
- Error caused by an incorrect object segmentation or bounding box estimation:

$$\frac{dz}{dw} = -\frac{lf}{w^2} \rightarrow e_w = -\frac{lf}{w^2}\Delta w$$

- ✓ Blur, blending with background, badly trained CNN, etc.
- ✓ A pixel averages light intensity over its surface, cameras have a limited resolution

- Error caused by an incorrect assumption about the object's size:

$$\frac{dz}{dl} = \frac{f}{w} \rightarrow e_l = \frac{f}{w}\Delta l$$

- ✓ Inaccurate size measurement, imprecise assumptions about the object's shape, etc.
- ✓ It is not always possible to measure physical dimensions of the object accurately

- Error caused by an incorrect focal length:

$$\frac{dz}{df} = \frac{l}{w} \rightarrow e_f = \frac{l}{w}\Delta f$$

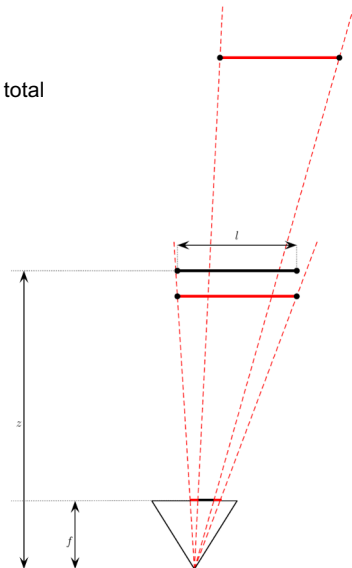
- ✓ Inaccurate calibration of the projection model, wrong projection model, lens misalignment, mechanical damage of the lens assembly, etc.

Error from object image size

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- Example 1 – **incorrect size estimation of the object's image:**

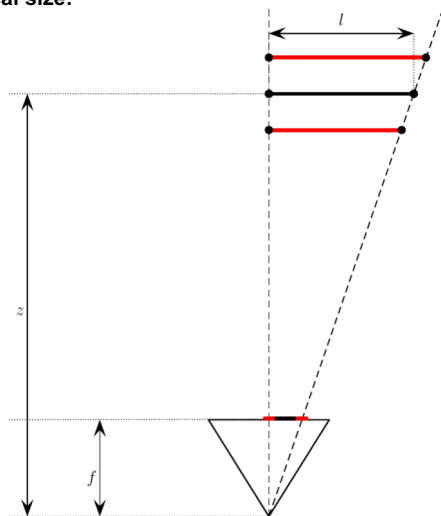
- ✓ Error is $\pm 0.5px$ on each side of the bounding box $\rightarrow \pm 1px$ in total
- ✓ Assuming an object at $z = 10m$ and camera parameters:
- ✓ $f = 200px$ (for image resolution $W = 640 \rightarrow 116^\circ$ HFoV)
- ✓ $l = 1m$ (a relatively large micro UAV)
- ✓ $w = 20px$
- ✓ $\Delta w = \pm 1px$
- ✓ $z = \frac{lf}{w} = 10m$
- ✓ $w' = w \pm 2 \cdot 0.5px = w \pm 1px$
- ✓ $e_w = \mp \frac{fl}{w^2} \Delta w = \mp \frac{200 \cdot 1}{20^2} \Delta w = \mp 0.5 \Delta w$
- ✓ $\pm 1px \rightarrow e_{w1} = \pm 0.5m$
- ✓ The error is not symmetrical (we just linearized it)!



Error from physical object geometry

- Example 2 – **incorrect assumption about the object's physical size:**

- ✓ Error is 5% of the object's true dimension
- ✓ Same assumptions as in Example 1:
- ✓ $f = 200px$
- ✓ $l = 1m$
- ✓ $w = 20px$
- ✓ $\Delta l = 0.05l$
- ✓ $z = \frac{lf}{w} = 10m$
- ✓ $l' = 1.05l; \Delta l = 0.05l = 0.05m$
- ✓ $e_l = \frac{f}{w} \Delta l = \frac{200}{20} \Delta l = 10\Delta l$
- ✓ $\pm 0.05m \rightarrow e_{l5} = \pm 0.5m$
- ✓ Note: It is a coincidence that the errors are the same, **not a rule!**



Error from camera calibration

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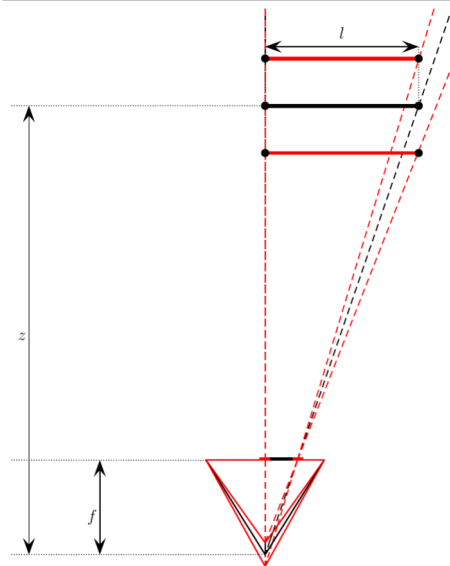
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- Example 3 – **incorrect focal length calibration:**

- ✓ Error is 5% of the true f
- ✓ Same assumptions as in Example 1 and 2:
- ✓ $f = 200px$
- ✓ $l = 1m$
- ✓ $w = 20px$
- ✓ $\Delta f = 0.05f$
- ✓ $z = \frac{lf}{w} = 10m$
- ✓ $f' = 1.05f; \Delta f = 0.05f = 10px$
- ✓ $e_f = \frac{l}{w}\Delta f = \frac{1}{20}\Delta f = 0.05\Delta f$
- ✓ $\pm 10px \rightarrow e_{f5} = \pm 0.5m$
- ✓ Note: It is a coincidence that the errors are the same, **not a rule!**



Error from object orientation or shape

• Example 4 – **incorrect assumption about object's perpendicularity to Z:**

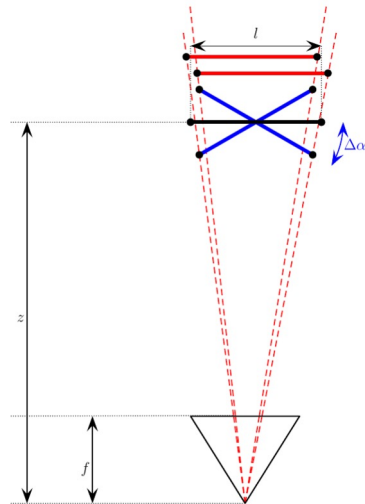
- ✓ Objects are not perpendicular in real situations
- ✓ Let assume (similarly as in the previous examples):
- ✓ $f = 200px$
- ✓ $l = 1m$
- ✓ $w = 20px$
- ✓ Object's center lies on the optical axis Z and $z \gg l$
- ✓ $\Delta\alpha = 10^\circ, 20^\circ, 30^\circ$

$$l'' \approx \cos(\Delta\alpha) l; \quad z_l'' \approx \frac{l'' f}{w} = \cos(\Delta\alpha) \frac{l f}{w}$$

$$e_\alpha \approx (1 - \cos(\Delta\alpha)) \frac{l f}{w} = (1 - \cos(\Delta\alpha)) 10m$$

- ✓ $\cos(\Delta\alpha) \leq 1 \rightarrow$ the object is **never closer than estimated**

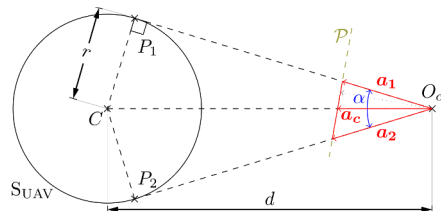
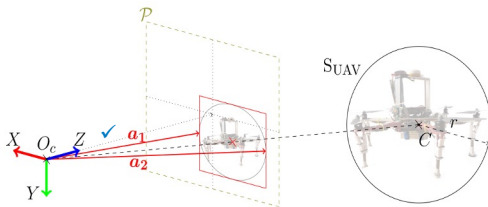
$\Delta\alpha$	e_α
$\pm 10^\circ$	-0.15m
$\pm 20^\circ$	-0.60m
$\pm 30^\circ$	-1.34m



Distance estimation for a spherical object

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- Assuming spherical objects - to tackle the perpendicularity issue
 - ✓ Complicated distance estimation formula
 - Assumptions:
 - ✓ Spherical object (not a flat line) with a known radius r
 - ✓ Known bearing vectors a_1 and a_2 corresponding to the edges of the object's bounding box
 - ✓ Removing assumptions about the projection model – to simplify the math
 - ✓ Estimating the distance d instead of the depth z - again to simplify the math
 - ✓ Using the distance to estimate the 3D position given the bearing vector a_c
- Applying the triangle $\triangle O_c C P_1$: $\alpha = \arccos\left(\frac{a_1 \cdot a_2}{\|a_1\| \|a_2\|}\right)$; $d = \frac{r}{\sin(\frac{\alpha}{2})}$



Other camera models

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- Real cameras typically don't conform to the pinhole model due to distortion
 - ✓ Barrel
 - ✓ Pincushion
- Present in omnidirectional and fisheye cameras - useful for MRS with UAVs: scene overview without an extra movement



Source: Mathworks

- Motivation
 - ✓ Mutual/relative localization is a key technique enabling safe close coordination and interaction among the UAVs sharing the same workspace
- Marker-based visual relative localization
 - ✓ Reliable approach (mainly active markers – e.g. LEDs)
 - ✓ Enables difficult-to-jam communication (secured systems of large MRS – e.g. urban mobility)
- Marker-less visual relative localization
 - ✓ General usage – also none-cooperating UAVs
 - ✓ Sense and avoid mechanisms
- Multi-target tracking and filtration
 - ✓ Important technique for real-world systems with sensory uncertainty
 - ✓ Enables estimating states not observable by the localization method or even future behavior of the observed UAVs
 - ✓ Important for large teams of anonymous UAVs – distinguishing between particles/UAVs

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[2] L. Teixeira, F. Maffra, M. Moos and M. Chli, "VI-RPE: Visual-Inertial Relative Pose Estimation for Aerial Vehicles", *IEEE RAL*, vol. 3, no. 4, pp. 2770-2777, 2018.

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