Lecture 3: UAV localization

> Tomáš Báča

State estimators

Linear Kalman Filter

Kalman Filter Unscente

Kalman Filter

estimation

Odometry

Localizatio

localization Local localization

Multirotor UAV state estimation and localization B(E)3M33MRS — Aerial Multi-Robot Systems

Ing. Tomáš Báča, Ph.D.

Multi-Robot Systems group, Faculty of Electrical Engineering Czech Technical University in Prague



Lecture 3: UAV localization

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Lecture 3: UAV localization

Mathemater U.W. state excitonations and localization distributions — and or bottom for state for states (and the states) and the states of states of states) Mathematical excitonation of states of states of states of states (and the states) = and the states of states

October 7th, 2024

2024-10-07



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Obtaining UAV state for feedback control and autonomous navigation?

2024-10-07

Lecture 3: UAV local-	Closing the loop	Problems with measurements	
Tomáš Báča Introduction	 How to obtain position r, velocity r, 	 Some system states can not be measured a Some system states can not be measured c Sometimes, the measurement rate is not his 	ıt all. lirectly. igh enough for control loop.
State estimators and Filters Linear Kalman	 acceleration r, orientation R, angular velocity ω. 	Measurements tend to be noisy.Measurement precision might not be sufficient.	ient.
Filter Extended Kalman Filter			
Unscented Kalman Filter			
Attitude estimation			
Odometry Localization			
Global localization Local			
localization			
	Tomáš Báča (CTU in Prague)	Lecture 3: UAV localization	October 7th, 2024 2 / 54
L	ecture 3: UAV localization		Obtaining UAV state for feedback control and autonomous navigation?
2	Introduction		Looking une soup 1 souties and namestaneous at all Histo obstain Sone splan status cas not be manued at all position at Sone splan status cas not be manued during a colorization at Sonestime at an assessment rule is not ally small for caread loop. a solitation at Sonestime at the manues rule is not ally small for caread loop. a solitation bit and an advectory at the manues to be a notion. Manuement to be all the notion.
2024-10-0	└──Obtaining UAV state for feedba	ack control and autonomous navigation?	



• State estimator is often called state observer in the context of control systems.



• State estimator is often called state observer in the context of control systems.

Probabilistic state estimation and localization

Robot's current state

- Robot's belief of its current state.
- Probability Distribution Function (PDF), often multivariate normal distribution.

Robot's motion model

- Allows to predict robot's future state based on the current state and input.
- Transforms the current state distribution, based on input.

Robot's sensor model

- Allows to incorporate measurements into the current state probabilistically.
- Allows to create artificial measurements based on the world model and the robot's state.

[1] S. Thrun, W. Burgard, and D. Fox, Probabilistic robotics. Cambridge, Mass.: MIT Press, 2005

	Tomáš Báča (CTU in Prague)	Lecture 3: UAV localization	October 7th, 2024 3 / 54
	Lecture 3: UAV localization		Probabilistic state estimation and localization
	State estimators and Filters		Robot's current state Robot's belief of its current state. • Probability Distribution Function (PDF), often multivariane normal distribution.
2024-10-07	└──Linear Kalman Filter └──Probabilistic state estimation and		Robot's motion model Allow to prefict rebot's future state based on the current state and input. Transforms the current state distribution, based on input.
		on and localization	Robot's senser model • Also: to incorpora measurements into the curvet stated and the polabilitationly. • Also: to incorporate measurements based on the world model and the robot's state. [1] 5. Three, W. Burgeet, and D. Fao, Polabilitär: robotics: Combility, Mass. HIT Press, 2005

• All three models are considered to be stochastic.

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State estimators and Filters

Kalman Filter Extended Kalman

Unscented Kalman Filter

estimation

Localization Global

Probabilistic generative laws

Probabilistic generative laws



Báča

Robot's state is complete

• Robot's state at the time step k is all we need to predict the future:

$$\mathsf{p}(\mathbf{x}_{[k]}|\mathbf{x}_{[0:k-1]},\mathbf{u}_{[1:k]},\mathbf{z}_{[1:k-1]}) \to \mathsf{p}(\mathbf{x}_{[k]}|\mathbf{x}_{[k-1]},\mathbf{u}_{[k-1]}).$$
(1)

• The measurement of robot's state is conditionally independent on the previous states:



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• Gauss-Markov assumption states that the future and past states are decorrelated given the current state.

Probabilistic generative laws

Robot's state is complete

• Robot's state at the time step k is all we need to predict the future:

1

$$\mathsf{p}(\mathbf{x}_{[k]}|\mathbf{x}_{[0:k-1]}, \mathbf{u}_{[1:k]}, \mathbf{z}_{[1:k-1]}) \to \mathsf{p}(\mathbf{x}_{[k]}|\mathbf{x}_{[k-1]}, \mathbf{u}_{[k-1]}).$$
(1)

• The measurement of robot's state is conditionally independent on the previous states:

$$p(\mathbf{z}_{[k]}|\mathbf{x}_{[0:k-1]}, \mathbf{u}_{[1:k]}, \mathbf{z}_{[1:k-1]}) \to p(\mathbf{z}_{[k]}|\mathbf{x}_{[k]}).$$
(2)



- 2024-10-07
- Gauss-Markov assumption states that the future and past states are decorrelated given the current state.

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Filter

Linear

Kalman





- Current state is all we need to capture the past.
- Current state yields the prediction of the future state.
- The future states is measured.
- The prediction and the measurement are combined to form the estimate of the future state.



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Lecture 3: UAV localization

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State estimators and Filters

Linear Kalman Filter

- Extended Kalman Filtor
- Unscented Kalman

Attitude

Odometry

Localization

Global localization Local localization

• Developed in \approx 1960 at NASA.

- Optimal state estimator for linear models.
- Minimum Mean-Square Error estimator.

Models

- State: Multivariate Gaussian
- Sensor model: added noise $\mathcal{N}(\mathbf{0},\mathbf{R})$
- Motion model: linear model with added noise $\mathcal{N}(\mathbf{0},\mathbf{Q})$

Two-stage algorithm

- **Prediction**: propagation of robot's state and its uncertainty through the model.
- **Correction**: update of the robot's state and its uncertainty using measurements.

How to derive it?

• B3M35OFD, Estimation, filtering and detection



Estimator for the Linear Time Invariant (LTI) system

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l inear Kalman Filter

Filter

Discrete stochastic LTI System

- State at the time step k: $\mathbf{x}_{[k]} \in \mathbb{R}^n$.
- Input at the time step k: $\mathbf{u}_{[k]} \in \mathbb{R}^m$.

 $\mathbf{x}_{[k+1]} = \mathbf{A}\mathbf{x}_{[k]} + \mathbf{B}\mathbf{u}_{[k]} + \mathbf{w}_{[k]},$ (3)

where:

- System matrix: $\mathbf{A} \in \mathbb{R}^{n \times n}$,
- Input matrix: $\mathbf{B} \in \mathbb{R}^{n \times m}$.
- Process noise $\mathbf{w}_{[k]} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$,
- Process covariance matrix: $\mathbf{Q} \in \mathbb{R}^{n \times n}_{> 0}$.

Goal of the estimator

To estimate the tuple $\mathbf{x}^*_{[k]}, \mathbf{P}_{[k]}$, where

- x^{*}_[k] is the state vector estimate,
- $\mathbf{P}_{[k]} \in \mathbb{R}^{n \times n}$ is the state covariance.

Measurement model

• Vector of measurements: $\mathbf{z} \in \mathbb{R}^p$.

$$\mathbf{z}_{[k]} = \mathbf{H}\mathbf{x}_{[k]} + \mathbf{v}_{[k]}, \tag{4}$$

where:

- Measurement-state mapping: $\mathbf{H} \in \mathbb{R}^{p imes n}$,
- Measurement noise: $\mathbf{v}_{[k]} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$,
- Measurement noise covariance: $\mathbf{R} \in \mathbb{R}_{>0}^{p \times p}$.

	Tomáš Báča (CTU in Prague)	Lecture 3: UAV localization	Octobe	r 7th, 2024	7 / 54
	Lecture 3: UAV localization		Estimator for the Linear Time Invariant (LT	1) system	
-10-07	State estimators and Filters Linear Kalman Filter Estimator for the LTI syst	em	$\label{eq:constraints} \begin{split} \hline Discrete stochastic: LTI System \\ & Stars at the time spic k: u_{N_{1}}\in\mathbb{R}^{m_{1}},\\ & toper at the time spic k: u_{N_{1}}\in\mathbb{R}^{m_{1}},\\ & u_{N_{1}}:=Ax_{N_{1}}:=Ax_{N_{1}}:=Ax_{N_{1}}:=Ax_{N_{1}}:=Ax_{N_{1}}:\\ & ubare:\\ & $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$	Measurement model • Vector of measurement: a c: a _{jk1} = Hx _{jk1} + where: • Measurement moles: vaping: • Measurement moles: vaping >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>	\mathbb{R}^{p} . $\mathbf{v}_{(0)}$, (4) $\mathbf{H} \in \mathbb{R}^{p \times n}$, $(0, \mathbf{R})$, $\mathbf{c} \in \mathbb{R} \in \mathbb{R}_{>0}^{p \times p}$.
2024			Goal of the estimator To estimate the tuple $\mathbf{x}_{[p_i]}^{i}, \mathbf{P}_{[p_i]}$, where • $\mathbf{x}_{[p_i]}^{i}$ is the state vector estimate, • $\mathbf{P}_{[p_i]} \in \mathbb{R}^{n \times n}$ is the state covariance.		

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State estimator and Filter

Linear Kalman Filter

Extended Kalman Filter Unscented

Attitude

Odometr

Localizatio

localization Local localization

Prediction

• Given
$$\mathbf{x}_{[k]}^{*}, \mathbf{P}_{[k]}, \mathbf{u}_{[k]}$$
:
 $\mathbf{x}_{[k+1]}^{*} = \mathbf{A}\mathbf{x}_{[k]}^{*} + \mathbf{B}\mathbf{u}_{[k]}$ (5)

$$\mathbf{P}_{[k+1]} = \mathbf{A}\mathbf{P}_{[k]}\mathbf{A}^{\intercal} + \mathbf{Q}$$

Correction

• Given $\mathbf{P}_{[k]}$, calculate the Kalman gain:

$$\mathbf{K}_{[k]} = \mathbf{P}_{[k]} \mathbf{H}^{\mathsf{T}} \left(\mathbf{H} \mathbf{P}_{[k]} \mathbf{H}^{\mathsf{T}} + \mathbf{R} \right)^{-1}.$$
(7)

• Given $\mathbf{x}^*_{[k]}, \mathbf{P}_{[k]}, \mathbf{z}_{[k]}, \mathbf{K}_{[k]}$, update the state and its covariance:

$$\mathbf{x}_{[k]}^* := \mathbf{x}_{[k]}^* + \mathbf{K}_{[k]} \left(\mathbf{z}_{[k]} - \mathbf{H} \mathbf{x}_{[k]}^* \right), \qquad (8)$$

$$\mathbf{P}_{[k]} := \left(\mathbf{I} - \mathbf{K}_{[k]}\mathbf{H}\right)\mathbf{P}_{[k]},\tag{9}$$

Lecture 3: UAV localization State estimators and Filters Linear Kalman Filter LKF	$g_{2,k}^{(2)}$ \mathbf{R}) ⁻¹ . (7) is the state and $\mathbf{h}_{(k)}^{(1)}$). (8) (9)

(6)



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 $-\mathbf{P}_{12}\mathbf{H}^{2}$

The cycle eval • At the men

 $= \mathbf{A} \mathbf{s}_{[k]}^*$

 $\mathbf{P}[\mathbf{x}] = (\mathbf{I} - \mathbf{K}[\mathbf{x}]\mathbf{H})\mathbf{F}$

Lecture 3: UAV localization State estimators and Filters Linear Kalman Filter 2024-10-07 Linear Kalman Filter (LKF)



- More often, the prediction and correction happen asynchronously.
- Corrections can be even caused by variety of sources at different rate.
- The prediction step is often being evaluated at fixed rate. The state obtained at the prediction step is used for control.



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Tomáš Báča (CTU in Prague)	Lecture 3: UAV localization	October 7th, 2024 11 / 54
Lecture 3: UAV localization		Linear Kalman Filter (LKF) — Example
State estimators and Filters Linear Kalman Filter	F) — Example	Example 11 years, $\Delta t = 8.05$ $= \begin{bmatrix} 1 \\ - \end{bmatrix} = \begin{bmatrix} 1 \\ $





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Lecture 3: UAV localization		Linear Kalman Filter (LKF) — Example
State estimators and Filters		Example 11 system: $\Delta t = 0$ fits $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1$
Dinear Kalman Filter (LKF) — Example	$\begin{array}{c} \hline \\ \hline \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $



z - Hx. where H r 2""" ent mapping matrix H = [1 0]

2024-10-07





UAV local-

Linear Kalman

Filter



2024-10-07

UAV localization

Linear Kalman Filter (LKF) — Example

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Linear Kalman





UAV local-

Linear Kalman



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Estimating hidden states

- Hidden states are often used to model sensor biases.
- LKF can estimate them if they are observable.



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	Lecture 3: UAV localization		Linear Kalman Filter (LKF) — Example 2	
	State estimators and Filters		Estimating hidden states - Hidden states an other used to model sense biases. - LKF can estimate them 2 they are observable.	
-07	Linear Kalman Filter		LTI System diagram	ample LTI system, $\Delta t = 0.01 \text{ s}$
4-10	└──Linear Kalman Filter (LKF	F) — Example 2		$\mathbf{x} = \begin{bmatrix} \hat{x} \\ \hat{x}_1 \\ \hat{x}_1 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} \hat{x}_d \end{bmatrix}, (15)$ $\begin{bmatrix} 1.0 & 0.01 & 0.0 & 0.0 \\ 0 & 0.01 & 0 & 0.0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
02			/	$\mathbf{A} = \begin{bmatrix} 0.0 & 0.0 & 1.0 & 1.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.99 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.01 \end{bmatrix}. (16)$

- Sensor bias estimation is heavily used to estimate nonzero offsets of sensors such as gyroscopes and accelerometers.
- Wind speed can be estimated as a bias in UAV acceleration.

Extended Kalman Filter (EKF) (EKF)

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Extended Kalman Filter

Filter

What if we have a non-linear model?

- UAV Rotational dynamics.
- Ackermann vehicle.
- Differential car-like model.
- ... almost anything engineering-related in the real world.

Linearization?

- Needs an operation point.
- A single operation point is hard-to-find with most models.

Let's linearize more

- Extended Kalman Filter (EKF).
- De-facto standard in aviation and inertial navigation.

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Extended Kalman Filter (EKF)

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Extended

Kalman Filter

Discrete stochastic system

- State at the time step k: $\mathbf{x}_{[k]} \in \mathbb{R}^n$.
- Input at the time step k: $\mathbf{u}_{[k]} \in \mathbb{R}^m$.

$$\mathbf{x}_{[k+1]} = f(\mathbf{x}_{[k]}, \mathbf{u}_{[k]}) + \mathbf{w}_{[k]}, \tag{17}$$

where:

- f() is differentiable,
- Process noise $\mathbf{w}_{[k]} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$,
- Process covariance matrix: $\mathbf{Q} \in \mathbb{R}_{>0}^{n \times n}$.

Measurement model

• Vector of measurements: $\mathbf{z} \in \mathbb{R}^p$.

$$\mathbf{z}_{[k]} = h(\mathbf{x}_{[k]}) + \mathbf{v}_{[k]},\tag{18}$$

where:

- Measurement-state mapping: $h(): \mathbb{R}^p \to \mathbb{R}^n$ is differentiable,
- Measurement noise: $\mathbf{v}_{[k]} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{R}
 ight)$,
- Measurement noise covariance: $\mathbf{R} \in \mathbb{R}_{>0}^{p \times p}$.

	Tomáš Báča (CTU in Prague)	Lecture 3: UAV localization	October 7th, 2024	17 / 54
2024-10-07	Lecture 3: UAV localization State estimators and Filters Extended Kalman Filter		Extended Kalman Fiber (EKF) Direct stratubule: years - for at the data is $u_{01} \ge u_{11} \ge u_{12}$ - grant data into out $u_{12} \ge u_{12} \ge u_{12} = f(u_{01}, u_{01}) + u_{01}$ - alar - for an early $u_{12} \ge u_{12} \ge u_{12} \ge u_{12}$ - Proven stratube, with $u_{12} \ge u_{12}$.	(17)
	└── Extended Kalman Filter (E	EKF)	$\label{eq:second} \begin{array}{l} \hline Measurement model \\ & \mbox{ victor of measurement: } x \in \mathbb{R}^n, \\ & \mbox{ algorithms} \\ & \mbox{ where extra straphing } (h) : \mathbb{R}^n \to \mathbb{R}^n \mbox{ is differentiable}, \\ & \mbox{ Measurement subs} : \mbox{ vigorithms} \in \mathbb{R} \cap \mathbb{R}^n, \\ & \mbox{ Measurement subs} : \mbox{ constraint } \mathbb{R} \cap \mathbb{R}^n, \\ & \mbox{ Measurement subs} : \mbox{ constraint } \mathbb{R} \cap \mathbb{R}^n, \end{array}$	(18)

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Extended Kalman Filter

Prediction

• Given $\mathbf{x}_{[k]}^*, \mathbf{P}_{[k]}, \mathbf{u}_{[k]}$:

$$\mathbf{x}_{[k+1]}^* = f(\mathbf{x}_{[k]}^*, \mathbf{u}_{[k]})$$
(19)
$$\mathbf{P}_{[k+1]} = \mathbf{F}_{[k]} \mathbf{P}_{[k]} \mathbf{F}_{[k]}^\mathsf{T} + \mathbf{Q},$$
(20)

$$\mathbf{P}_{[k+1]} = \mathbf{F}_{[k]} \mathbf{P}_{[k]} \mathbf{F}_{[k]}^{\mathsf{T}} + \mathbf{Q}, \qquad (20)$$

where

$$\mathbf{F}_{[k]} = \left. \frac{\partial f}{\partial \mathbf{x}} \right|_{\mathbf{x}_{[k]}^*, \mathbf{u}_{[k]}}$$
(21)

is the Jacobian of f() evaluated at the $\mathbf{x}^*_{[k]}, \mathbf{u}_{[k]}$.

Correction

• Given $\mathbf{P}_{[k]}$, calculate the Kalman gain:

$$\mathbf{K}_{[k]} = \mathbf{P}_{[k]} \mathbf{H}_{[k]}^{\mathsf{T}} \left(\mathbf{H}_{[k]} \mathbf{P}_{[k]} \mathbf{H}_{[k]}^{\mathsf{T}} + \mathbf{R} \right)^{-1}.$$
 (22)

+ Given $\mathbf{x}^*_{[k]}, \mathbf{P}_{[k]}, \mathbf{z}_{[k]}, \mathbf{K}_{[k]}$, update the state and its covariance:

$$\mathbf{x}_{[k]}^* := \mathbf{x}_{[k]}^* + \mathbf{K}_{[k]} \left(\mathbf{z}_{[k]} - h(\mathbf{x}_{[k]}^*) \right), \qquad (23)$$

$$\mathbf{P}_{[k]} := \left(\mathbf{I} - \mathbf{K}_{[k]}\mathbf{H}_{[k]}\right)\mathbf{P}_{[k]},\tag{24}$$

where

$$\mathbf{H}_{[k]} = \left. \frac{\partial h}{\partial \mathbf{x}} \right|_{\mathbf{x}^*_{[k]}} \tag{25}$$

is the Jacobian of h() evaluated at the $\mathbf{x}_{[k]}^*$.

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Lecture 3: UAV localization		Estended Kalman Filter (EKF)
Extended Kalman Filter	KF)	$\label{eq:constraints} \begin{array}{c} \text{constraints}\\ \hline & \text{constraints}\\ \hline & & \text{constraints} \\ & & & \text{constraints} \\ & & & & & & & \text{constraints} \\ & & & & & & & \text{constraints} \\ & & & & & & & \text{constraints} \\ & & & & & & & \text{constraints} \\ & & & & & & & \text{constraints} \\ & & & & & & & & \text{constraints} \\ & & & & & & & & & \text{constraints} \\ & & & & & & & & & & & \\ & & & & & & $

Extended Kalman Filter (EKF) - properties



ization

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Extended Kalman Filter

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EKF Properties

• Optimality? no

Lecture 3: UAV localization

October 7th, 2024 Extended Kalman Filter (EKF) - properties

EKF Properties

State estimators and Filters

Extended Kalman Filter

Extended Kalman Filter (EKF) - properties

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Extended Kalman Filter (EKF) — properties

Extended Kalman Filter (EKF) — properties

Lecture 3:	
UAV local- ization	EKF Properties
Tomáš	

- Optimality? no
- Stability? not guaranteed

and Filters			
Linear Kalman Filter			
Extended Kalman Filter			
Unscented Kalman Filter			
Attitude estimation			
Odometry			
Localization Global localization Local localization			
	Tomáš Báča (CTU in Prague)	Lecture 3: UAV localization	October 7th, 2024 1
1	ecture 3: UAV localization		Extended Kalman Filter (EKF) — properties
	State estimators and Filters		EKF Properties • Optimality? no • Stability? Not guaranteed
	- Extended Kalman Filter		

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Extended Kalman Filter (EKF) — properties

Lecture 3:		
UAV local- ization	EKF Properties	
Tomáš Báča	• Optimality? no	

- Optimality? no
- Stability? not guaranteed
- Ease of use? far from it

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Kalı	man
E des	

Extended Kalman Filter

Filter

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October 7th, 2024

Extended Kalman Filter (EKF) - properties EKF Properties Optimality? no Stability? not guaranteed Ease of use? far from it

State estimators and Filters

Extended Kalman Filter

-Extended Kalman Filter (EKF) — properties

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Extended Kalman Filter (EKF) - properties

EKF Properties

- Optimality? no
 - Stability? not guaranteed
 - Ease of use? far from it

EKF Problems

- f(), and h() needs to be differentiable.
- f(), and h() are linearized *blindly* in each state.
- EKF is sensitive to model inaccuracies.
- EKF is sensitive to poor initialization.

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State estimators and Filters

Linear Kalman Filter

Extended Kalman Filter

Kalman Filter

Attitude

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Localizatio

Global localization Local

Extended Kalman Filter (EKF) - properties

EKF Properties

- Optimality? no
- Stability? not guaranteed
- Ease of use? far from it

EKF Problems

- f(), and h() needs to be differentiable.
- f(), and h() are linearized *blindly* in each state.
- EKF is sensitive to model inaccuracies.
- EKF is sensitive to poor initialization.

EKF mind-set problem

- How EKF deals with non-linearity? EKF works with the original state Probability Distribution Function (PDF) and a degraded model description.
- What about we swap it around? Let's transform a degraded state PDF through the original model.

Tomáš Báča (CTU in Prague) Lecture 3: UAV localization October 7th, 2024 19 / 54 Lecture 3: UAV localization Ented Klain Filer (BV) - properties Ented Klain an Filer (BV) - properties Image: Comparison of the comp

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Kalman Filter

Extended Kalman Filter

Kalman Filter

Attitude

estimatio

Odometry

Unscented Kalman Filter (UKF)

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State estimator

- Linear Kalman Filter
- Extended Kalman Filter

Unscented Kalman Filter

- Attitude stimation
- Odometry
- Localization Global localization Local localization

Unscented Kalman Filter	ι	Inscented	Kalman Filter
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- Published in early 2000s by Uhlmann et al.
- Uses the full nonlinear model f(), h().
- Does not linearize, therefore, f(), h() can be arbitrary.
- More *elegant solution* than EKF.
- More *robust* than EKF.
- [2] E. A. Wan and R. Van Der Merwe, "The unscented Kalman filter for nonlinear estimation," in Adaptive Systems for Signal Processing, Communications, and Control Symposium, IEEE, IEEE, 2000, pp. 153– 158

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Lecture 3: UAV localization		Unscented Kalman Filter (UKF)
State estimators and Filters		Unscented Kalman Filter Publicked is say 2000 by Uhinase et al. I lise the filt ordering model (70 kf)
Unscented Kalman Filter		 Does not liouxin, therefore, f(i, bi) can be arbitrary. More adapter assists than RKF. More voluent than RKF.
11UKF		[2] E. A. Wan and R. Van Der Manne, "The ancoretal Kalman fiber for earliesar estimation," in Adaptin Systems for Signal Proceeding, Communications, and Control Symposium, IEEE, IEEE, 2020, pp. 103– 108


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-Unscented Transform



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State estimators and Filters

Linear Kalman Filter

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Unscented Kalman Filter

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Lecture 3: UAV localization		Unscanted Transform
State estimators and Filters		Unscented transform — transforming sigma points through $h()$
Unscented Kalman Filter		

Lecture 3: UAV localization

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State estimators and Filters

Linear Kalman Filter

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Unscented Kalman Filter

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	Lecture 3: UAV localization		Unscented Transform
	State estimators and Filters		Uncented transform — transforming agona points through h()
4-10-07	Unscented Kalman Filter		and the second s
	Unscented Transform		
202			G





Unscented Kalman Filter (UKF) — Algorithm

Lecture 3 UAV localization

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Unscented Kalman

Filter

[2]

Prediction step

- 1. Calculate the sigma points for $\mathbf{x}_{[k]}^{*}, \mathbf{P}_{[k]}.$
- 2. Propagate the sigma points through f().
- 3. Reconstruct $\mathbf{x}^*_{[k+1]}, \mathbf{P}_{[k+1]}$

Correction step

- 1. Calculate the sigma points for $\mathbf{x}_{[k]}^*, \mathbf{P}_{[k]}$.
- 2. Propagate the sigma points through h() to obtain the expected measurement z^* .
- 3. Reconstruct the mean and covariance of the expected measurement.
- 4. Calculate cross-covariance between the measurement z and the expected measurement z^* .
- 5. Calculate the Kalman gain using the cross-covariance.
- 6. Update the mean and covariance \mathbf{x}^*, \mathbf{P} .

E. A. Wan and R. Van Der Merwe, "The unscented Kalman filter for nonlinear estimation," in Adaptive Systems for Signal Processing, Communications, and Control Symposium, IEEE, IEEE, 2000, pp. 153-158

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Lecture 3: UAV localization	
State estimators and Filters Unscented Kalman Filter Unscented Kalman Filter (UKF) — Algorithm	ap P(a); agk A() to obtain the ance of the supected on the resonances to and the organ constance. (cv, P. are estimation," in Adaptiv are estimation, in a Adaptiv

Unscented Kalman Filter (UKF) — properties

Lecture 3:
UAV local-
ization

Tomáš Báča

State estimators

Linear Kalman Filter

Extended Kalman Filter Unscented

Kalman Filter

Attitude estimation

Odometry

Localizatio

Local localization

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UKF Properties

• Optimality? still no

Lecture 3: UAV localization

October 7th, 2024 23/

Unscented Kalman Filter (UKF) — properties

UKF Properties • Optimality? etil no

—State estimators and Filters

Unscented Kalman Filter

Unscented Kalman Filter (UKF) — properties

2024-10-07

Unscented Kalman Filter (UKF) - properties

• Stability? still not guaranteed, but much better than EKF

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Filter

Filter

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UKF Properties

• Optimality? still no

Lecture 3: UAV localization

October 7th, 2024

State estimators and Filters

Unscented Kalman Filter

-Unscented Kalman Filter (UKF) — properties

2024-10-07

Unscented Kalman

Unscented Kalman Filter (UKF) - properties

UKF Properties • Optimality? still no • Stability? still not guaranteed, but such better than EK

Unscented Kalman Filter (UKF) — properties

• Stability? still not guaranteed, but much better than EKF

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State estimator and Filte

Linear Kalman Filter Extended

Kalman Filter Unscented

Kalman Filter

estimation

Odometry

Localizatio

Global localization Local

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UKF Properties

• Optimality? still no

• Ease of use? yes

Lecture 3: UAV localization

October 7th, 2024 23 /

Unscented Kalman Filter (UKF) — properties

UKF Properties Optimality? ctill no Stability? ctill not guarant Ease of use? yes

State estimators and Filters

Unscented Kalman Filter

Unscented Kalman Filter (UKF) — properties

2024-10-07

Unscented Kalman Filter (UKF) — properties

UKF Properties

- Optimality? still no
- Stability? still not guaranteed, but much better than EKF
- Ease of use? yes

UKF Benefits

- No need to derive Jacobians.
- h() and f() can be arbitrary.
- Only the implementation of h() and f() needs to be supplied.

UKF Problems

- Does not have many.
- Mathematical soundness of operations needs to be checked (square rooting of P).

	Tomáš Báča (CTU in Prague)	Lecture 3: UAV localization	October 7th, 2024	23/54
	Lecture 3: UAV localization		Unscented Kalman Filter (UKF) — properties	
10-07	State estimators and Filters Unscented Kalman Filter Unscented Kalman Filter (UKF) — properties		UKF Proparties • Optimality? will no • Stability? will not guaranteed, but such better than EKF • East of use' yes	
		(UKF) — properties	UKF Benefits • No and to derive Jacobiane. • A(1) and (1) can be arbitrary. • Only the implementation of A(1) and f(1) ends to be supplied.	
2024-			UKF Problems • Does not have many • Mathematical coundress of operations needs to be checked (opener rooting of P).	

Lecture 3: UAV localization

Tomáš Báča

ntroducti

estimators and Filter Linear

Filter Extended Kalman

Filter Unscented Kalman

Filter

estimation

Lecture 3: UAV localization

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State estimators and Filter

Kalman Filter Extended

Filter Unscented Kalman

Filter

estimation

Global localization



The state vector is

 $\mathbf{x} = \begin{bmatrix} \mathbf{r}^{\mathsf{T}}, \dot{\mathbf{r}}^{\mathsf{T}}, \eta, \dot{\eta} \end{bmatrix}^{\mathsf{T}},$

where

- $\mathbf{r}^\mathcal{W}:$ the 2D position in the world frame,
- $\dot{\mathbf{r}}^{\mathcal{B}}$: the 2D velocity in the body frame,
- η : the heading,
- $\dot{\eta}$: the heading rate.



 $\mathbf{z} = egin{bmatrix} \mathbf{r}^\intercal, \dot{\mathbf{r}}^\intercal, \dot{\eta} \end{bmatrix}^\intercal$



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Lecture 3: UAV local	ization		Un	iscented Kalman Filter (UKF) — example	
State estimators	and Filters		S T	tate vector	System illustration
⊳ Unscented k	alman Filter		-	$\mathbf{x} = [\mathbf{r}^{T}, \hat{\mathbf{r}}^{T}, \eta, \dot{\eta}]^{T},$ (26) data • \mathbf{r}^{W} : the 2D position in the world frame,	- Long
입 Unscent	Unscented Kalman Filter (UKF) — example		 i^{at}: the 2D velocity in the body forms, i; the heading, i; the heading rate. 	$\mathbf{r}, \mathbf{R}^{\dagger}$ $\mathbf{\hat{r}} = \mathbf{e} \hat{\mathbf{b}}_{1}$	
024				feasurement vector $\mathbf{z} = [\mathbf{r}^{T}, \mathbf{r}^{T}, \mathbf{t}]^{T}$ (27)	
CN					



Tomáš Báča

State estimators

Linear Kalman Filter

Kalman Filter Unscented

Kalman Filter

estimation

Odometry

Localization Global localization Local localization



	Tomáš Báča (CTU in Prague)	Lecture 3: UAV localization	October 7th,	2024 24 / 54
	Lecture 3: UAV localization		Unscented Kalman Filter (UKF) — example	
	State estimators and Filters		Motion model, Δt $\begin{bmatrix} s \\ s \end{bmatrix} = \begin{bmatrix} s_{[1]} + \Delta H_{[2]}s^{2}_{[2]} \end{bmatrix}$ System	illustration
20	Unscented Kalman Filter		$\mathbf{x}_{[k+1]} = \begin{bmatrix} q \\ q \\ q \end{bmatrix}_{[k+1]} = \begin{bmatrix} q_{[k]} & \mathbf{x}_{[k]} \\ q_{[k]} & \mathbf{x}_{[k]} \\ q_{[k]} \end{bmatrix}$, (24)	
-10-	Unscented Kalman Filter	(UKF) — example	$\mathbf{R}_{[N]} = \begin{bmatrix} \cos \eta_{[N]} & -\sin \eta_{[N]} \\ \sin \eta_{[N]} & \cos \eta_{[N]} \end{bmatrix}, (27) \end{bmatrix}$	r,R1 r=vb1
2024			$\begin{array}{c} \hline \\ \hline \\ \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	*



2024-10-07

Lecture 3: UAV localization

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State estimators and Filters

Linear Kalman Filter

Kalman Filter Unscented

Kalman Filter

estimation

Localization

localization Local State estimation of a car

- Nonlinear car-like model (similar to the previous example).
- Unscented Kalman Filter.
- Car observed by a single camera.



Video: https://youtu.be/BSNUOd61teY

[3] T. Baca, P. Stepan, B. Spurny, D. Hert, R. Penicka, M. Saska, et al., "Autonomous Landing on a Moving Vehicle with an Unmanned Aerial Vehicle," *Journal of Field Robotics*, vol. 36, pp. 874–891, 5 2019

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10-07	Lecture 3: UAV localization State estimators and Filters Unscented Kalman Filter Unscented Kalman Filter	(UKF) — example 2	Uncertain Kalana Fibre (UKP) — example 2 Marcana Kalana Fibre (UKP) — example 2 Marc
202-			Video: https://ywww.he/k88004ittef [3] T. Baca, P. Sespan, B. Spuny, D. Hert, R. Penicka, M. Saska, et al., "Autoenneuet Landing on a Meving Welicide with an Useramed Aerial Welick," <i>Journal of Field Robotics</i> , vol. 36, pp. 874–891, 5 2019

UAV Attitude estimation

Lecture 3: UAV local-



UAV Onboard Sensors

- Gyroscope:
 - 3-axis MEMS.
 - Measured intrinsic angular rate.
 - Sufficient for attitude rate control.
 - 3-axis MEMS.
 - Measures proper acceleration.
 - Gravity model might be needed for precise
- "Magnetometer":
 - Measures external magnetic field.
 - Magnetic field model needed for precise navigation.
- All above are often part of the Inertial Measurement Unit (IMU).
- All above need calibration.
- All above benefit from temperature stabilization.

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Lecture 3: UAV localization		UAV Attitude estimation	
Attitude estimation		Retation+translation Dynamics model μα r = 3 ÷ + ∞ × 3 ∞ (24) R = 8Ω (24) W = 1 × 0 ∞ (24) W = 1 × 0 ∞ (24)	Gyroscope Gyroscope
UAV Attitude estimation	n	wine $-\frac{1}{m} HT_r + g^*$ (11) What do is meet? Applier which $r_r = 0$. Otherstoin: R	3-sole MEMS. Measures proper acceleration. Gravity model might be seeded for precise reassignion. "Magnetaments": 3-sole Measures enternal magnetic field.
2024		Static extinator •) Cast and the second s	Magnetic fail model needed for precise nangezion. All above are often part of the inertial Measurement Unit (IMU). All above read calibration. All above benefit from temperature stabilization.

Odometry

Etymology

ization Tomáš Báča

Filter

Odometry

Lecture 3:

UAV local-

Odo-metry = Measuring of steps \rightarrow measuring of where we are based on the steps we took.

Ground robot analogy

- Ground robots have encoders in theirs wheels.
- Encoders' outputs represent intrinsic velocity.
- Integrating encoders from the last known position is called dead reckoning.

Can we do odometry using the IMU

- Not in general: double-integration of acceleration will drift with increasing velocity.
- Can be done with very precise instruments and models: in aerospace.
- Definitely not with the consumer-level sensors in most UAVs: would not lead to a stable flight.

How can we do odometry then?

• We need to go derivative higher from acceleration: to **velocity**.

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Lecture 3: UAV localization		Odomitry
		Etymology
		Odo-metry = Measuring of steps -> measuring of where we are based on the steps we took.
Odometry		Ground robot analogy
		Ground robots have encoders in theirs wheels.
		Encodem' autputs represent intrinsic velocity.
		Integrating encodent from the last known position in carled dead reckaring
o Odometry		Can we do edemetry using the IMU
7		 Not in general: double-integration of acceleration will drift with increasing velocity.
4		 Cas be done with very precise instruments and models: in aerospace.
N		 Definibily not with the consumer-level sensors in most UAWs: would not lead to a stable flight.
		How can we do odometry then?
		 We need to go derivative higher from acceleration: to velocite.

• To be complete: double integration of acceleration will lead to quadratic drift in position, if the accelerometer exhibits non-zero bias.

Odometry on UAVs - Optical Flow

• Means of calculating velocity from RGB camera

• Requires distance measurement to fix the

• Very common on most commercial platforms.

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State estimators and Filters

Optical flow

footage.

• Downwards-facing camera.

• Parrot AR Drone (2010)

absolute velocity.

• Relatively robust.

Not very accurate.

- Kalman Filter Extended
- Filter Unscented Kalman
- Filter
- estimation
- Odometry
- Localization Global localization Local localization

PX4 Flow

- Ultrasound rangefinder
- GreyScale camera
- Embedded µcontroller

Flowdeck v2

- IR ToF rangefinder
- GreyScale camera
- Embedded µcontroller





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Odometry on UAVs — Optical Flow



Odometry on UAVs — Feature-based visual odometry

Visual Inertial Odometry

- Combination of feature matching and IMU predictions.
- Does not require a rangefinder.
- Requires proper camera calibration.
- Requires high-resolution and high-rate cameras.
- Global shutter is necessary.
- Robustness is still to be desired (for UAVs).

Feature matching [5]

Features detectors:

- Invariance in transformations and lighting.
- Edges, Corners, Blobs.
- SURF, FAST, SIFT, MSER (Matas et al. [4]).

Feature descriptors:

• SURF, SIFT, BRIEF.

Odometry



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Lecture 3: UAV localization Odometry

2024-10-07

Odometry on UAVs - Feature-based visual odometry

 SURF, FAST, SIFT, MSER (M SURF, SIFT, BRIEF

v on UAVs -

Lecture 3: UAV local-

ization

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Odometry on UAVs - Feature-based visual odometry



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State estimato and Filte

Linear Kalman Filter

Kalman Filter Unscente

Filter Attitude

Odometry

ocalization Global localization Local localization



Video: https://youtu.be/EVreW6VDT6U

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2024-10-07

Odometry on UAVs — Feature-based visual odometry



October

Odometry on UAVs - Feature-based visual odometry



Video: https://youtu.be/f00V9fnvnEw

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Odometry

2024-10-07

-Odometry on UAVs — Feature-based visual odometry



Odometry on UAVs — LiDAR odometry

• Scans the environment in stacked rings.

• Organized/unorganized list of 3D points. • Can contain meta information (reflectivity,

UAV local-Lidar ization

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Lecture 3:

Odometry

- PointCloud features
 - 3D corners, 3D edges.

• 2D or 3D.

color).

Active sensor: Infra-red.

• Has mechanical parts. • Requires obstacles to be close.

PointCloud data structure

Facets of polyhedra.



Ouster LiDAR Field of View

Figure 1: source: http://ouster.com



Odometry on UAVs — LiDAR odometry

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- Odometry

Iterative Closest Point (ICP)

- Pointcloud registration method.
- Assumption: Not that many points have changed between two consecutively-measured point clouds.
- Minimizing sum of squares of the closest points on two pointclouds.
- Many variants and implementations exist.
- Outlier rejection is important.
- Algorithm:
 - 1. compute point-to-point correspondences,
 - 2. optimize for the rotation and translation.
 - 3. move the pointcloud,
 - 4. repeat.

Showcase of LiDAR odometry



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etry on UAVs — LiDAR o

Lecture 3: UAV localization Odometry

2024-10-07

Odometry on UAVs - LiDAR odometry





Localization

Lecture 3: UAV localization

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State estimators

Linear Kalman Filter Extended

Kalman Filter Unscenter

Attitude

Odometry

Localization

localization Local localization

Localization

The means of obtaining the 3D position of the robot in the world coordinate fame.

Why?

- Global localization is needed for global navigation:
 - for building accurate 3D maps of the environment,
 - for using the maps for navigation.
- Localization is needed for any meaningful interaction of a robot with its world.

Where is the state of the art?

• Depends heavily on the use case.

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l	ecture 3: UAV localization		Localization
	Localization		Localization The mass of obvious the 1D position of the other in the world coordinate frame
			Why?
10-0	Localization		 Ginal localization is needed for global randgetons: for building accessors 3D mays of the servicement, for using the maps for majorities. Localization is needed for any managingful interaction of a rabot with its world.
4			Where is the state of the art?
ġ			Depends heavily on the use case.
50			

Global outdoor UAV Localization — GNSS



ization

Tomáš Báča

State estimator

- Linear Kalman Filter
- Extended Kalman Filter
- Kalman Filter

Attitude estimation

estimation

Localization

localization Local localization



- Earth's satellite constellations.
- GPS, GLONAS, Galileo, BeiDou.

Properties

- Most-often 10 Hz 3D position output.
- Needs clear sky view.
- Beware of Solar activity (lonosphere).
- Beware of reflections (buildings).
- Requires magnetometer.

Influence of ionosphere on GNSS

GPS Satellite

Upgrade: Realtime Kinematics (RTK)

- Works directly with the GPS carrier wave signal.
- Fixed base-station on a tripod for relaying carrier wave phase.
- The UAV is equipped with RTK-compatible antenna and radio receiver.



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Lecture 3: UAV localization Localization Global localization Global outdoor UAV Local Global outdoor UAV Local	ization — GNSS	<section-header><section-header><section-header><list-item><section-header><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></list-item></section-header></list-item></section-header></section-header></section-header>

• Sensitive to El-Mag interference (USB 3.0).

Global indoor UAV Localization - Motion capture

Lecture 3: UAV localization

Tomáš Báča

State estimator and Filte

Linear Kalman Filter Extended Kalman

Filter Unscented Kalman

Attitude

estimation

Localization

localization Local localization

Marker-based localization

- Pre-set IR camera system.
- IR lighting.
- Retro-reflective markers.
- Popular for control theory research.

Qualisys motion capture cameras

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Lecture 3: UAV localization

- -Localization
 - - -Global indoor UAV Localization Motion capture

Lecture 3: UAV localization

Motion capture output

- Rigid body's position and velocity, 200 Hz.
- Almost no noise, can be used directly for feedback.

UAV equipped with retro-reflective markers





2024-10-07

Global indoor UAV Localization — Motion capture

Vijay Kumar's TED talk

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Báča

State estimators

Linear Kalman Filter Extender

Kalman Filter Unscente Kalman

Attitude

Odometr

Localizati Global

localization Local localization



Video: https://youtu.be/4ErEBkj_3PY

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 Localization
 Global localization
 Market W Lo

Global UAV Localization — Indoor GPS

Lecture 3: UAV local-

Radio beacons Ultrasound beacons ization Pre-set environment with radio beacons. Tomáš Pre-set environment with ultrasound beacons. Báča Known beacon locations. Known beacon locations. Terabee RTPS radio beacons Marvelmind ultrasound beacons Clobal localization Figure 2: Source: Marvelmind Figure 3: Source: Terabee Tomáš Báča (CTU in Prague) Lecture 3: UAV localization Lecture 3: UAV localization Global UAV L Localization Global localization 2024-10-07 Global UAV Localization — Indoor GPS

- These systems are very unreliable and are more suitable for ground vehicles.
- Ground vehicles do not need constant precise localization for stabilization, therefore, they cope much better with measurement outages than UAVs.

Global UAV Localization — Indoor GPS





Local UAV Localization — SLAM

Lecture 3: UAV localization

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SLAM — Simultaneous Localization and Mapping

- Creating a map of a priori unknown environment while being localized in the same map.
- Chicken-and-egg problem.
- The holy grail problem in mobile robotics.
- Two options:
 - Online SLAM computes the current robot pose.
 - Full SLAM recovers the whole history of the robot poses.

Popular approaches

- EKF SLAM,
- Fast SLAM (Particle filter),
- PoseGraph SLAM (Bundle Adjustment),
- Factor Graph SLAM.

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Lecture 3: UAV localization

Local UAV Localization - SLAM Lecture 3: UAV localization SLAM — Simultaneous Localization and Mapping Crusting a map of a priori unknown environment while I Chicken-and-ogg problem. The holy goal problem in mobile robotics. Two patient Localization Local localization Online SLAM — computes the current robot pose.
 Full SLAM — recovert the whole history of the robot pos Local UAV Localization — SLAM Popular approaches EKF SLAM, Fast SLAM (Particle filter),
 PoseGraph SLAM (Bundle Ac
 Factor Graph SLAM.

October 7th,

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2024-10-07

Local localization

Local UAV Localization — EKF SLAM

EKF SLAM

- Online SLAM.
- The first SLAM solution, now mostly history.
- The LKF state vector contains:
 - The robot's state (r_x, r_y, r_η) ,
 - The map of landmarks $(l_{x,n}, l_{y,n})$.
- Assumption: landmark association is solved.
- Capable of loop closure (revisiting places should help).
- Computationally intractable for large maps.

2D EKF SLAM Illustration

[1] S. Thrun, W. Burgard, and D. Fox, Probabilistic robotics. Cambridge, Mass.: MIT Press, 2005

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Lecture 3: UAV localization		Local UAV Localization — EKF SLAM	
Localization Local localization	- EKF SLAM	EXPE SEAM The first EAM The first EAM statistics, new neutry history The first EAM statistics, new neutry history The first EAM statistics, new neutry The neutry of industrials (L ₁ , n ₁ , n ₂) The neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₂ , n ₃) the neutry of industrials (L ₁ , n ₁ , n ₂) the neutry of industrials (L ₁ , n ₂) the neutry of industrials (L ₁ , n ₂) the neutry of industrials (L ₁ , n ₂) the neutry of industrials (L ₁ , n ₂) the neutry of industrials (L ₁ , n ₂) the neutry of industrials (L ₁ , n ₂) the neutry of industrials (L ₁ , n ₂) the neutry of industrials (L ₁ , n ₂) the neutry of industrials (L_1) the neutry	THE EXP SLAMI Indextoon

Lecture 3: UAV localization

Báča

State estimato

Linear Kalman Filter

Filter Unscente

Filter

estimatio

Odometry

Local

Localization

localization



Kalman	3. Calculation of the expected measurement: which
Filter	landmarks should be observed and where.
Unscented Kalman	4 Massurements landmark acception
Filter	4. Measurement: landmark association.

Local

Local UAV Localization — EKF SLAM

Lecture 3: UAV localization

Tomáš Báča

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Lecture 3: UAV localization		Local UAV Localization — EKF SLAM	2D EKF SLAM Illustration
Localization Local localization Local UAV Localization — Local UAV Localization —	EKF SLAM	Algorithm . To see the search of a map as a lotted and . The search of	Vdec: kerges.//puets.ka/vC89848714



bel(x)

× Figure & Source: Probabilistic robustion, Theor et al. [1]

Local UAV Localization — Fast SLAM

Lecture 3: UAV localization

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Local

localization

Local UAV Localization — Fast SLAM

Particle filter — Illustration

- Weight is put to the particles which could generate such measurements.
- Particles move to next generation: weighted particles have higher chance to survive and to multiply.





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Lecture 3:

UAV local-

State

Linear Kalman

Extended Kalman Filter

Kalman Filter

Attitude estimation

Odometry

Localization

Global localization

localization

Local UAV Localization — Fast SLAM

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Filter

Local

Particle filter — Illustration

- The robot moves in the physical world.
- We apply the control input to each particle and move it as well.



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Lecture 3: UAV localization		Local UAV Localization — Fast SLAM
Localization Local localization Local UAV Localization Local UAV Localization —	Fast SLAM	Particle frame. Headson • The stage frame and the physical and mean it is used. • The stage frame. Headson • Beadson Headson • Figure 1. Summary Readmann strates, from et al. [1]

Local UAV Localization — Fast SLAM




Local UAV Localization — Fast SLAM

• The robot moves in the physical world.

Local UAV Localization — Fast SLAM

• We apply the control input to each particle and move it as well.

Particle filter — Illustration

Local localization



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Local

localization



• We apply the defection marks to mark polynomia and marks in

Finant & Source: Probabilistic robustics, Thrun et al. [1].

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Local UAV Localization — Fast SLAM



Local UAV Localization — Pose Graph SLAM

Lecture 3: UAV localization

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- ntroductio
- State estimator and Filter
- Linear Kalman Filter
- Extended Kalman Filter
- Kalman Filter
- Attitude estimatio
- Odometr
- Localizatio Global localization
- Local localization

• Special case of a Bayes network.

- Constructed as bi-parted graph.
- Two types of nodes:
 - poses,
 - landmarks.
- Edges:

Pose graphs

- motions: constraints between poses,
- **observations** constraints between poses and landmarks.
- Inference from the graph forms a nonlinear least-squares optimization.

Local UAV Localization — Pose Graph SLAM

- Mostly used by visual SLAMs.
- E.g., ORB-SLAM [6], LSD-SLAM [7].

Pose graph illustration



Inference from the graph forms a Inarr-oquares optimization. Mostly used by visual SLAMs. E.g., ORII-SLAM [6], LSD-SLAM

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2024-10-07

Local UAV Localization — Factor Graph SLAM

• Special case of a Bayes network.

• Constructed as bi-parted graph.

least-squares optimization.

VIRAL-SLAM.

Often used with LiDAR Simultaneous

• E.g., LIO-SAM, MILIOM, LVI-SAM,

Localization and Mappings (SLAMs).

 Constraints originating from IMU, Loop closure constraints,

 Global navigation constraints (Global Navigation Satellite System (GNSS)). Inference from the graph forms a nonlinear

• Two types of nodes:

 variables. • factors.

Lecture 3: UAV localization

Factor graphs

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Local localization



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Lecture 3: UAV local-

ization Tomáš

2024-10-07

List of SO	TA Visual	SLAM	algorithms	[9]
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Daca	Summary of topological mapping and localization solutions based on global image descriptors.					Table 5 Summary of templorized manning and localization solutions based on local factors						
	References	Camera	Map	Tasks	Environment	Descriptor	Bafarans or	ipping and localiz	Man Man	rd on local leatures.	Environment	Fastura
Introduction	Winters [16]	Omnidir	Торо	Map + Loc	Indoors	PCA	References	camera	map	143763	Liwitonment	reature
	Gaspar [17]	Omnidir	Торо	Map + Loc	Indoors	PCA	Kosecka [97–99]	Mono	Topo	Map + Loc	Indoors	SIFT
State	Ulrich [18]	Omnidir	Торо	Map + Loc	In + Out	Colour hist.	Zhang [100]	Mono	Topo	Map + Loc	Indoors	SIFI
otate	Werner [46]	Omnidir	Торо	SLAM	Indoors	Colour hist.	Zhang [101]	Omeidie	Topo	Man i Lan	Indoors	SIF I
estimators	Kosecka [19]	Mono	Topo	Map + Loc	Indoors	Gradient orien. hist.	Kybski [102]	Mono	Topo	Map + Loc	Outdoorr	SIET
and Filters	Bradley [20]	Mono	Topo	Map + Loc	Outdoors	WGOH	Sabatta [104]	Omnidir	Topo	Map + Loc	Indoors	SIFT
and meets	weiss [21]	Mono	Topo	Map + Loc	Outdoors	WGII	Johns [105]	Mono	Topo	Map + Loc	Indoors	SIFT
Linear	Wang[22]	Mono	Topo	Map + Loc	In + Out	OACH	Kawewong [106,107]	Omnidir	Topo	SLAM	In + Out	PIRF (SIFT)
12 - Long and	Pionobis [23]	Mono	Topo	LOC	Indoors	Receptive neid nist.	Tongo rasit [108]	Omnidir	Topo	SLAM	In + Out	PIRF (SURF)
Rannan	Singh [96]	Omnidia	Topo	Map + Loc	Un L Out	Cist Omni nist	Morioka [109]	Omnidir	Hybrid	SLAM	Indoors	3D-PIRF(SURF)
Filter	Pituarto [40]	Omnidir	Topo	Mapping	Indoorr	Omni mit	Andreasson [90]	Omnidir	Topo	Map + Loc	Indoors	KLT/M-SIFT
Estended	Sunderhauf [26]	Mono	Topo	SI AM	Outdoors	BRIFF-gist	Valgren [110]	Omnidir	Торо	Mapping	Indoors	KLT/M-SIFT
Extended	Arrow [53]	Omnidir	Topo	Man + Inc	Outdoors	IDB	Valgren [111]	Omnidir	Торо	Mapping	In + Out	SIFT
Kalman	Arrown [55]	Stereo	Topo	Man + Inc	Outdoors	D-IDB	Valgren [112]	Omnidir	Торо	Loc	Outdoors	SIFT/SURF
Filter	Liu [50]	Мопо	Торо	SLAM	Outdoors	Gist	Ascani [113]	Omnidir	Торо	Loc	In + Out	SIFT/SURF
	Chapoulie [51]	Sphere	Topo	Map + Loc	In + Out	Gist	Anati [114]	Omnidir	Торо	Map + Loc	In + Out	SIFT
Unscented	Chapoulie [27]	Sphere	Торо	Map + Loc	In + Out	Spherical harmonics	Zivkovic [115]	Omnidir	Hybrid	Map + Loc	Indoors	SIFT
Kalman	Lamon [28]	Omnidir	Topo	Loc	Indoors	Fingerprints	Booi [116]	Omnidir	Hybrid	Map + Loc	Indoors In i Out	SIFI
Filter	Tapus [56,57]	Omnidir	Торо	Map + Loc	Indoors	Fingerprints	Danub [118]	Omnidir	Hybrid	Map + Loc	Indoors	SUPE
	Liu [29]	Omnidir	Торо	Map pin g	Indoors	FACT	Blanco [119 120]	Stereo	Hybrid	SIAM	Indoors	SIFT
Asstanda	Liu [30]	Omnidir	Торо	Map pin g	Indoors	DP-FACT	Tally [121]	Omnidir	Hybrid	Man + Loc	Indoors	SIFT
	Menegatti [31,32]	Omnidir	Topo	Map + Loc	Indoors	Fourier signatures	Tully [122]	Omnidir	Hybrid	SLAM	Indoors	SIFT
estimation	Paya [58]	Omnidir	Topo	Map + Loc	Indoors	Fourier signatures	Segvic [123]	Mono	Hybrid	Map + Loc	Outdoors	SIFT/Harris/MSER
	Kanganathan [59]	Omnidir	Topo	Mapping	Indoors	Founer signatures	Ramisa [124]	Omnidir	Topo	Map + Loc	Indoors	MSER/SIFT/GLOH
	Millord [60]	Omnidia	Hybrid	SLAM	Outdoors	Colour segmentation	Badino [125]	Mono	Hybrid	Map + Loc	Outdoors	SURF/U-SURF
	Milford 1241	Mone	Hubrid	SLAM	Outdoors	Coroni nist.	Dayoub [126]	Omnidir	Торо	Map + Loc	Indoors	SURF
	Character 1	Mono	Hoheid	SLOW	Outdoors	Scan intensity proc	Bacca [127,128]	Omnidir	Торо	Map + Loc	Indoors	SIFT/SURF
Localization	Giover [62]	Omnidir	Hybrid	SLAM	In + Out	2D Haarawaalat dag	Bacca [129]	Omnidir	Торо	SLAM	Indoors	Edges
Localization	Badino [38]	Mono	Hybrid	Man + Inc	Outdoors	M/LSURF	Romero [130,131]	Omnidir	Topo	SLAM	Outdoors	MSER
Global	Xu [64]	Mono	Hybrid	Map + Loc	Outdoors	WI-SURF	Majdik [132]	Mono	Торо	Loc	Outdoors	ASIFT
la sa lina ti sa	Laterahn [39]	Mono	Hybrid	SLAM	Outdoors	DIRD	Saedan [133]	Omnidir	Hybrid	SLAM	Indoors	wavelets
localization	Nourani [40]	Mono	Topo	Map + Loc	In + Out	OFM/OFSC	Mashai [134]	Omnidir	Topo	Man + Loc	Indoors	ACIET
Local	Milford [35,65,66]	Mono	Topo	SLAM	Outdoors	Normalized patches	Carcia Fidaleo [126]	Mono	Topo	si a M	In + Out	SLIPE
la antination	Pepperell [67]	Mono	Topo	SLAM	Outdoors	Normalized patches	Carcia-Fidalgo [130]	Mono	Topo	SIAM	In + Out	SIFT
localization	Wu [68]	Mono	Topo	Map + Loc	Outdoors	Binarized patches	our cru Huargo [157]	mond	1010	Jurund	m ; ou	

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Lecture 3: UAV localization

Lecture 3: UAV localization

Localization Local localization

October 7th, 2024 48 Local UAV Localization --- Visual SLAMs List of SOTA Visual SLAM algorit

• The research field of visual SLAMs is huge and also very popular.

Local UAV Localization — Visual SLAMs

- Almost everyone can contribute, because you only need a camera to start working.
- Rarely anything works in the real world and onboard a UAV.

Local UAV Localization — Visual SLAMs



- The video showcases RGBD SLAM.
- RTAB-Map can also utilize other sources of data to build a map, e.g., LiDAR pointclouds.

Local UAV Localization — LiDAR SLAMs



Tomáš Báča

State estimators and Filters

Linear Kalman Filter Extended Kalman

Filter Unscente Kalman

Attitude

Odometr

Localization Global localization Local localization





- The field of LiDAR SLAMs is also very active and rich.
- Similarly, true SLAMs are rarely used on UAVs, mostly due to the SLAMs' computational demands.

Coupled Odometry + Localization



- Open-VINS odometry for fast state estimation.
- Particle filter for re-localization in a known height map.

Localization — Summary

- UAVs most commonly operate outdoors, therefore, GNSS localization the most common.
- Commercial platforms are capable of onboard odometry (most often visual), however, that is used for stabilization and to aid human pilots with control in GNSS-denied environments.
- SLAMs are mostly the **subject of research** and are not reliable enough to use the UAVs to their full potential.
- Multi-modal SLAMs and geometries are probably the future. Fusion of different sensor modalities (Visual, LiDAR, Radar, InfraRed) will increase the reliability and robustness.

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2024-10-07	Localization Local localization Localization — Summary		EVBN mat unserval years action, fination, CHDS backstate the same canase. Constraint and there are quark of advant actioners (near data wais), there is a defined action of the same of the s	, itaal,

UAV localization Tomáš Báča

Lecture 3:

State estimators and Filter

Kaiman Filter Extended Kalman Filter

Unscented Kalman Filter

Attitude estimation

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