1. Assume two parallel lines p and q in the space. Line p passes through points $\vec{X}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{X}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Line q passes through points $\vec{Y}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{Y}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Find the vanishing point M of the lines p a q in the image by the camera with the projection matrix

$$\mathrm{P} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\lambda_1 \vec{m}_1 = P\vec{X}_1 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\lambda_2 \vec{m}_2 = P\vec{X}_2 = \dots\dots = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\lambda_3 \vec{n}_1 = P\vec{Y}_1 = \dots\dots = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$$\lambda_4 \vec{n}_2 = P\vec{Y}_2 = \dots\dots = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$\vec{p}', \vec{q}' \sim p, q$ IN IMAGE

- $\vec{m}_1, \vec{m}_2, \vec{n}_1, \vec{n}_2$ DEFINED UP TO SCALE

$\rightarrow$ WE CAN ELIMINATE λ_i

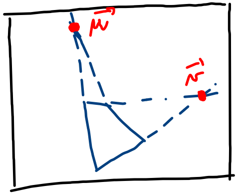
$$\vec{p}' = \vec{m}_1 \times \vec{m}_2 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{q}' = \vec{n}_1 \times \vec{n}_2 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$M = \vec{p}' \times \vec{q}' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

2. Let us have two vanishing points in $\vec{u} = [0,0]^\top$, $\vec{v} = [2,0]^\top$, which come from the image of an observed rectangle. Find all values of parameter a in the matrix

$$\mathtt{K} = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad K^{-1} = \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad K^{-T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$$



of a camera which captured the image.

$$\vec{v}_{1\beta}^\top \mathtt{K}^{-\top}\mathtt{K}^{-1}\vec{v}_{2\beta} = 0 \quad 11.40$$

$$\vec{v}_{1\beta}^\top \omega\, \vec{v}_{2\beta} = 0 \quad 11.41$$

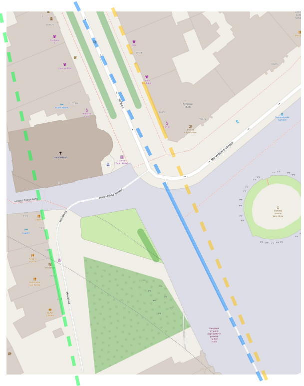
$$\omega = K^{-T} K^{-1} = \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & 0 \\ -a & 0 & a^2+1 \end{bmatrix}$$

$$\rightarrow \vec{u}^\top \omega \vec{v} = 0$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & 0 \\ -a & 0 & a^2+1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = a^2 - 2a + 1 \rightarrow \underline{\underline{a = 1}}$$

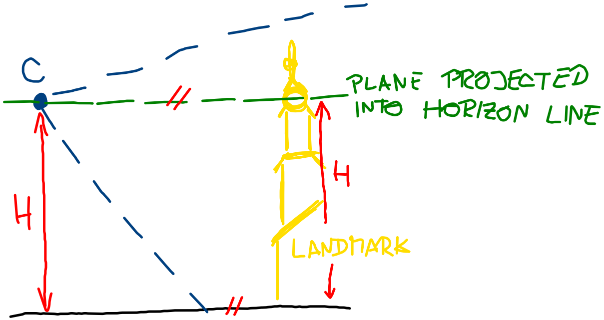
A) Are various linear features visible in the image (e.g. green, blue and yellow lines) parallel in the scene?

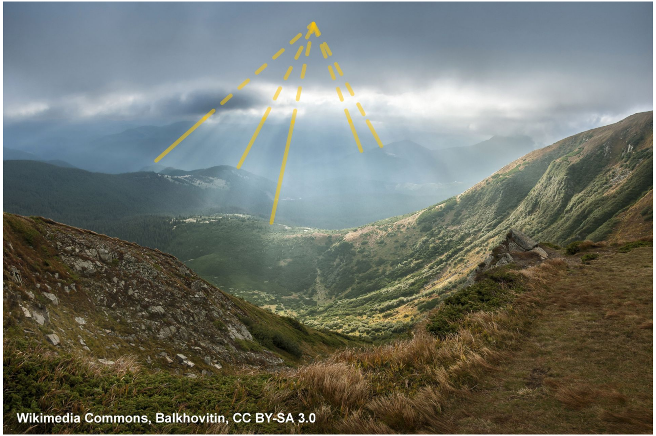
- All parallel lines in the scene have a common vanishing point in the image. Here green, blue and yellow lines have each it's own vanishing point, so they are not parallel in the scene (you can check in the map).



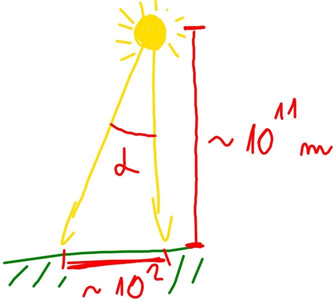
B) What was the height of the camera center above the ground relative to the landmarks in the scene?

- We can construct a horizon in the image - join of vanishing points of lines, which lie in planes parallel with the ground plane (here all four color cases). We can imagine the horizon as a projection of plane. This plane is parallel with the ground plane and has the same offset from the ground plane as camera center, so it marks in the image features, which are in the same height above the ground plane as the camera center.





The light rays from the Sun are almost parallel in the scene, so there will be a single vanishing point in the image, which can be used as an estimation of the position of the Sun in the image.



$$\alpha = \mathrm{TAN}\,\frac{10^2}{10^{11}} \doteq 0$$