GVG'2021 Exam-02 EN

1. Consider the cameras with the (scaled) camera projection matrices

$$\mathtt{P} = \begin{bmatrix} b & 0 & 1 & a \\ 0 & a & b & 1 \\ 0 & 0 & a & b \end{bmatrix}$$

   Find all values of parameters $a, b \in \mathbb{R}$ such that $\mathtt{P}$ projects the point

$$\vec{X}_\delta = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

   from the space onto the ideal point of the line $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^\top$.

2. Consider the camera with the camera projection matrix

$$\mathtt{P} = \begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

   Find the calibration matrix $\mathtt{K}$, rotation $\mathtt{R}$ and the camera center $\vec{C}_\delta$ of the given camera.

3. Assume two parallel lines p and q in the space. Line p passes through points $\vec{X}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{X}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Line q passes through points $\vec{Y}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{Y}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Find the vanishing point of the lines p a q in the image by the camera with the projection matrix

$$\mathtt{P} = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

4. Let us have cameras with projection matrices

$$\mathtt{P}_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad \mathtt{P}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

   Find all points which are projected to the point $[1, -1, 0]^\top$ in the first image and to the point $[-1, 1, 0]^\top$ in the second image.

5. Let us have two images bound by a fundamental matrix

$$\mathtt{F} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

   Point X projects in the first image into point $[1, 1]^\top$ and in the second image on a line $[1, 1, 1]^\top$. Write the coordinates of a point, into which X projects in the second image.

6. Find all homographies between two images of a calibrated camera ($\mathtt{K} = \mathtt{I}$) that is rotating around its projection center (i.e. its projection center doesn't change) and which maps points $[0, 0]^\top$ on $[0, 0]^\top$ and $[1, 0]^\top$ on $[0, 1]^\top$.