Elements of Geometry for Computer Vision and Computer Graphics

Lecture 7: The Projective Plane

Translation of Euclid’s Elements by Adalardus Bathensis (1080–1152)
Comparing Geometrical and Algebraic Models

<table>
<thead>
<tr>
<th>Point position</th>
<th>Projection</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Geometrical model in aff. space</td>
</tr>
<tr>
<td>$X \notin \sigma$</td>
<td>one point of $\pi$</td>
</tr>
<tr>
<td>$C \neq X \in \sigma$</td>
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<tr>
<td>$X = C$</td>
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## Comparing Geometrical and Algebraic Models

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<td>$\eta \neq 0, \vec{x}<em>\beta = \begin{bmatrix} u \ v \ 1 \end{bmatrix}$, $(\vec{x}</em>\beta \neq \vec{0})$</td>
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<td>$X = C$</td>
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<td>$\eta \neq 0, \vec{x}_\beta = \vec{0}$</td>
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1. We can always assume $\eta \neq 0$.

2. The "projection" of $C$ is represented by the zero vector while the projections of all other points are represented by non-zero vectors.

3. The algebraic projection model can represent projections of all points in the affine space.

4. The geometrical projection model is less capable than the algebraic projection model since it can't model the projection of points in $\sigma$ different from $C$. 

Torsten Sattler
### Comparing Geometrical and Algebraic Models

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1. We can always assume $\eta \neq 0$.
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Torsten Sattler
Q: Which points project to point(s) in the image plane using the geometric model of perspective projections in affine space?

1. A point $X$ in the plane $\sigma$ parallel to the image plane $\pi$.
2. The camera center $C$.
3. A point $X$ in the plane $\pi$. 
Q: Which points project to point(s) in the image plane using the geometric model of perspective projections in affine space?

1. A point $X$ in the plane $\sigma$ parallel to the image plane $\pi$.
2. The camera center $C$. ✔
3. A point $X$ in the plane $\pi$. ✔
The Real Projective Plane - Geometrical Model

\[ \mathbb{A}^3 \]

\[ \mathbb{A}^2 \]

\[ \mathbb{A}^3 \]

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The Real Projective Plane - Algebraic Model
Lines of the Real Projective Plane
Lines of the Real Projective Plane

ideal line
Quiz

cw.fel.cvut.cz/b212/courses/gvg/start → BRUTE → GVG  Geometry of Computer Vision and Graphics

Q: Mark ideal points (points at infinity).

1. $[0 \ 0 \ 0]^T$
2. $[1 \ 0 \ 0]^T$
3. $[0 \ 1 \ 0]^T$
4. $[0 \ 0 \ 1]^T$
Q: Mark ideal points (points at infinity).

1. $[0 \ 0 \ 0]^T$
2. $[1 \ 0 \ 0]^T$
3. $[0 \ 1 \ 0]^T$
4. $[0 \ 0 \ 1]^T$
Q: Mark homogenous coordinates representing the point \([0 1]^T\) in the affine plane.

1. \([1 0 1]^T\)
2. \([0 101 1]^T\)
3. \([0 101 101]^T\)
4. \([0 1 0.5]^T\)
Q: Mark homogenous coordinates representing the point \( [0 \ 1]^T \) in the affine plane.

1. \([1 \ 0 \ 1]^T\)  ✔
2. \([0 \ 101 \ 1]^T\)
3. \([0 \ 101 \ 101]^T\)  ✔
4. \([0 \ 1 \ 0.5]^T\)