Digital Image

(B4M33DZO, Summer 2024)

Lecture 8:

Image Registration 1

`https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start`

Daniel Sýkora & Ondřej Drbohlav

Department of Cybernetics

Faculty of Electrical Engineering

Czech Technical University in Prague

Image registration

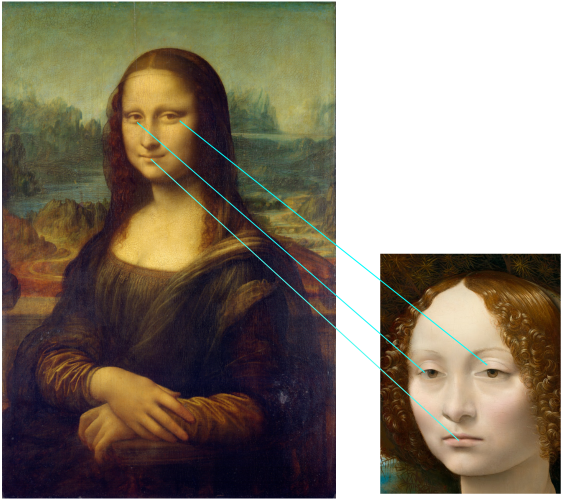
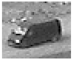


Image registration



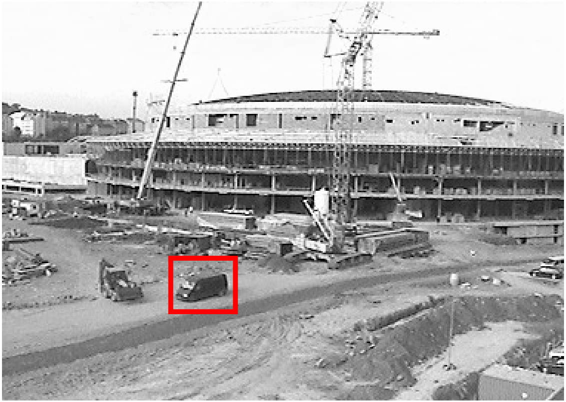


Image similarity

Sum of absolute differences (SAD): $\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$

Image similarity

Sum of absolute differences (SAD): $\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$

Sum of squared differences (SSD): $\sum_x \sum_y (\mathbf{A}[x,y] - \mathbf{B}[x,y])^2$

Image similarity

Sum of absolute differences (SAD): $$\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$$

Sum of squared differences (SSD): $$\sum_x \sum_y (\mathbf{A}[x,y] - \mathbf{B}[x,y])^2$$

$$\sum_x \sum_y \mathbf{A}[x,y]^2 - 2 \cdot \mathbf{A}[x,y] \cdot \mathbf{B}[x,y] + \mathbf{B}[x,y]^2$$

Image similarity

Sum of absolute differences (SAD): $\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$

Sum of squared differences (SSD): $\sum_x \sum_y (\mathbf{A}[x,y] - \mathbf{B}[x,y])^2$

$$\sum_x \sum_y \mathbf{A}[x,y]^2 - 2 \cdot \underbrace{\mathbf{A}[x,y] \cdot \mathbf{B}[x,y]} + \mathbf{B}[x,y]^2$$

Cross-correlation:

$$\sum_x \sum_y \mathbf{A}(x,y) \cdot \mathbf{B}(x,y)$$

Image similarity

Sum of absolute differences (SAD): $\sum_x \sum_y |\mathbf{A}[x,y] - \mathbf{B}[x,y]|$

Sum of squared differences (SSD): $\sum_x \sum_y (\mathbf{A}[x,y] - \mathbf{B}[x,y])^2$

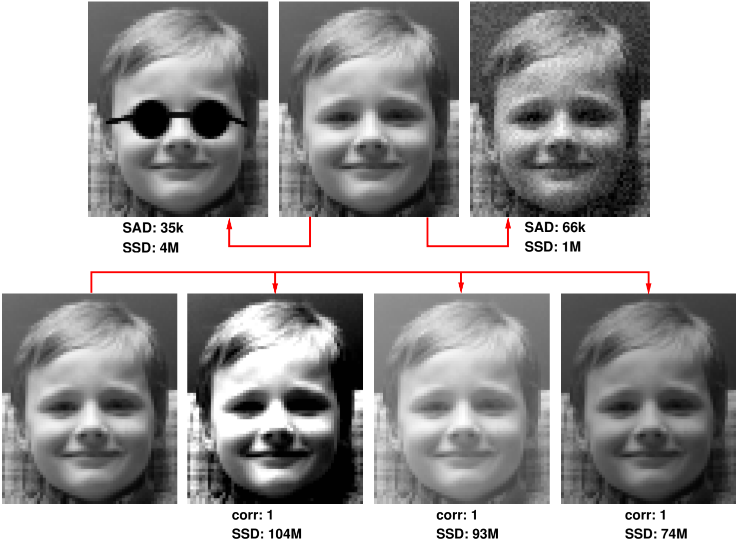
$$\sum_x \sum_y \mathbf{A}[x,y]^2 - 2 \cdot \mathbf{A}[x,y] \cdot \mathbf{B}[x,y] + \mathbf{B}[x,y]^2$$

Normalized cross-correlation:

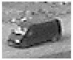
$$\frac{1}{\sigma_{\mathbf{A}} \sigma_{\mathbf{B}}} \sum_x \sum_y \left(\mathbf{A}(x,y) - \hat{\mathbf{A}}\right) \cdot \left(\mathbf{B}(x,y) - \hat{\mathbf{B}}\right)$$

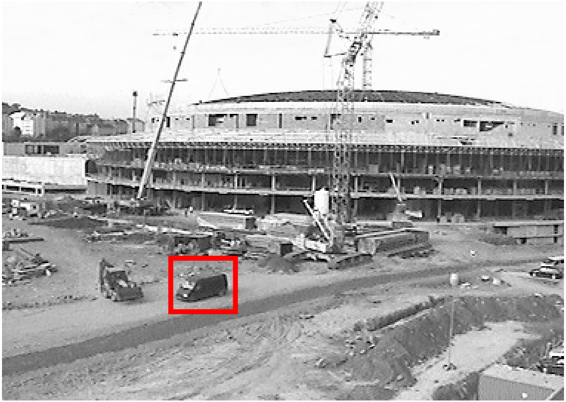
(invariant to brightness & contrast changes)

Image similarity



Translation





Translation

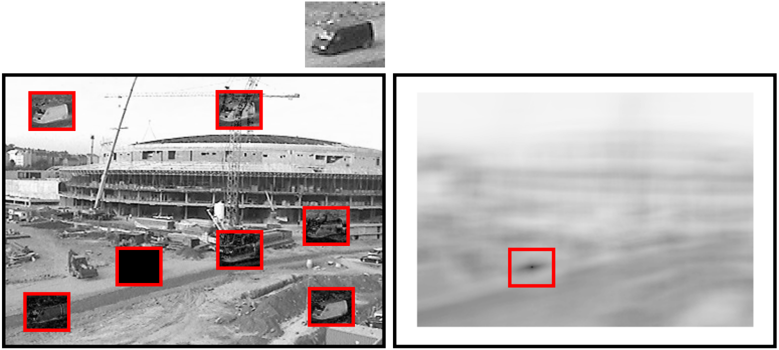
Minimize similarity metric $\circ$ over all possible shifts:

$$\arg\min_{[s,t]} \sum_x \sum_y \mathbf{A}[x+s, y+t] \circ \mathbf{B}[x,y]$$

Translation

Minimize similarity metric ∘ over all possible shifts:

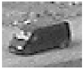
$$\arg\min_{[s,t]} \sum_x \sum_y \mathbf{A}[x+s, y+t] \circ \mathbf{B}[x,y]$$

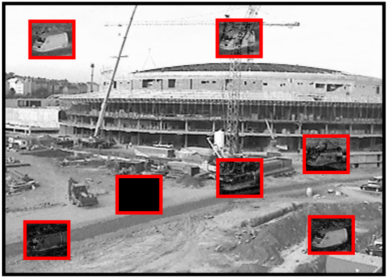


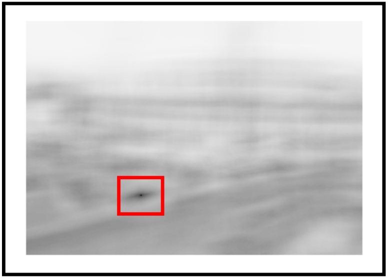
Translation

Minimize similarity metric $\circ$ over all possible shifts:

$$\arg\min_{[s,t]} \sum_x \sum_y \mathbf{A}[x+s, y+t] \circ \mathbf{B}[x,y]$$







Problem: complexity of block matching is $\mathcal{O}(|\mathbf{A}| \cdot |\mathbf{B}|)$.

Speed-up for SSD

SSD ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, global minimum):

$$\arg\min_{[s,t]} \sum_{x=0}^{w} \sum_{y=0}^{h} (\mathbf{A}[x+s, y+t] - \mathbf{B}[x,y])^2 =$$

Speed-up for SSD

SSD ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, global minimum):

$$\arg\min_{[s,t]} \sum_{x=0}^{w} \sum_{y=0}^{h} \left(\mathbf{A}[x+s, y+t] - \mathbf{B}[x,y]\right)^2 =$$

$$\arg\min_{[s,t]} \left(\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 - 2 \sum_{x=0}^{w} \sum_{y=0}^{h} \mathbf{A}[x+s, y+t] \cdot \mathbf{B}[x,y] \right)$$

Speed-up for SSD

SSD ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, global minimum):

$$\arg\min_{[s,t]} \sum_{x=0}^{w} \sum_{y=0}^{h} \left(\mathbf{A}[x+s, y+t] - \mathbf{B}[x,y]\right)^2 =$$

$$\arg\min_{[s,t]} \left(\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 - 2 \sum_{x=0}^{w} \sum_{y=0}^{h} \mathbf{A}[x+s, y+t] \cdot \mathbf{B}[x,y] \right)$$

Summed area table:

$$\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 = \Sigma[s,t] - \Sigma[s+w,t] - \Sigma[s,t+h] + \Sigma[s+w,t+h]$$

Speed-up for SSD

SSD ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, global minimum):

$$\arg\min_{[s,t]} \sum_{x=0}^{w} \sum_{y=0}^{h} \left(\mathbf{A}[x+s, y+t] - \mathbf{B}[x,y]\right)^2 =$$

$$\arg\min_{[s,t]} \left(\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 - 2 \sum_{x=0}^{w} \sum_{y=0}^{h} \mathbf{A}[x+s, y+t] \cdot \mathbf{B}[x,y] \right)$$

Summed area table:

$$\sum_{x=s}^{s+w} \sum_{y=t}^{t+h} \mathbf{A}[x,y]^2 = \Sigma[s,t] - \Sigma[s+w,t] - \Sigma[s,t+h] + \Sigma[s+w,t+h]$$

Fourier convolution theorem:

$$\sum_{x=0}^{w} \sum_{y=0}^{h} \mathbf{A}[x+s, y+t] \cdot \mathbf{B}[x,y] = \mathcal{F}^{-1}\left\{\mathcal{F}\{\mathbf{A}\} \cdot \mathcal{F}\{\mathbf{B}^{+}\}\right\}[s,t]$$

Phase Correlation

Phase correlation ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, local minimum):

$$\mathbf{A}[x,y] * \delta(s,t) = \mathbf{B}[x,y] \quad \Longleftrightarrow \quad \mathcal{A}[u,v] \cdot e^{2\pi i(us+vt)} = \mathcal{B}[u,v]$$

Phase correlation ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, local minimum):

$$\mathbf{A}[x,y] * \delta(s,t) = \mathbf{B}[x,y] \quad \Longleftrightarrow \quad \mathcal{A}[u,v] \cdot e^{2\pi i(us+vt)} = \mathcal{B}[u,v]$$

$$\frac{\mathcal{A}^*[u,v] \cdot \mathcal{B}[u,v]}{|\mathcal{A}[u,v]|^2} = e^{2\pi i(us+vt)} \quad \Longrightarrow \quad \delta(s,t)$$

Phase Correlation

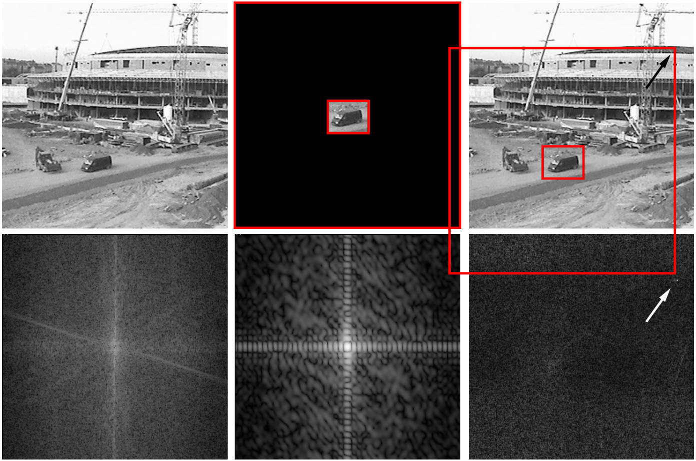
Phase correlation ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, local minimum):

$$\mathbf{A}[x,y] * \delta(s,t) = \mathbf{B}[x,y] \quad \Longleftrightarrow \quad \mathcal{A}[u,v] \cdot e^{2\pi i(us+vt)} = \mathcal{B}[u,v]$$

$$\frac{\mathcal{A}^*[u,v] \cdot \mathcal{B}[u,v]}{|\mathcal{A}[u,v] \cdot \mathcal{B}[u,v]|} = e^{2\pi i(us+vt)} \quad \Longrightarrow \quad \delta(s,t)$$

Phase Correlation

Phase correlation ($\mathcal{O}(|\mathbf{A}| \cdot \log |\mathbf{A}|)$, **local minimum):**



Rotation

Polar transformation:

rotation $\Longrightarrow$ translation on y-axis

Rotation

Polar transformation:

rotation $\Longrightarrow$ translation on y-axis

$$r = \sqrt{(x - x_c)^2 + (y - y_c)^2}$$

$$\alpha = \arctan \frac{y - y_c}{x - x_c}$$

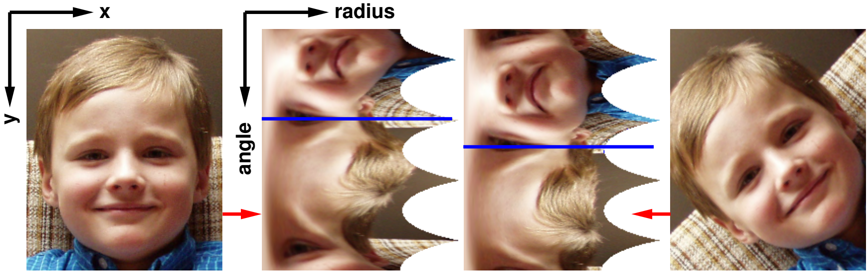
Rotation

Polar transformation:

rotation $\Longrightarrow$ translation on y-axis

$$r = \sqrt{(x - x_c)^2 + (y - y_c)^2}$$

$$\alpha = \arctan \frac{y - y_c}{x - x_c}$$



Rotation & Scale

Log-polar transformation:

scale	$\Longrightarrow$	**translation on x-axis**
rotation	$\Longrightarrow$	**translation on y-axis**

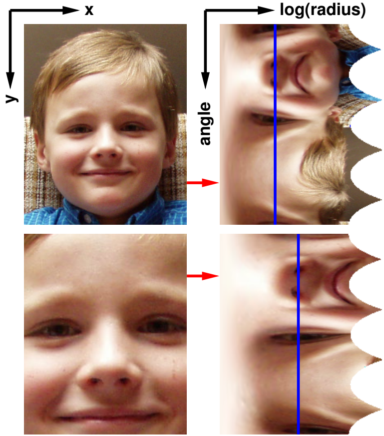
Rotation & Scale

Log-polar transformation:

scale $\Longrightarrow$ translation on x-axis
rotation $\Longrightarrow$ translation on y-axis

$$r = \log\left(\sqrt{(x - x_c)^2 + (y - y_c)^2}\right)$$

$$\alpha = \arctan\frac{y - y_c}{x - x_c}$$



Rotation & Scale & Translation

Fourier-Mellin transformation:

1. **Estimate rotation and scale using frequency domain.**
2. **Resample image to have the same rotation and scale.**
3. **Use standard phase correlation to estimate translation.**

Rotation & Scale & Translation

Fourier-Mellin transformation:

1. **Estimate rotation and scale using frequency domain.**
2. **Resample image to have the same rotation and scale.**
3. **Use standard phase correlation to estimate translation.**