

Digital Image

(B4M33DZO, Summer 2024)

Lecture 4:

Linear Filtering

<https://cw.fel.cvut.cz/wiki/courses/b4m33dzo/start>

Daniel Sýkora & Ondřej Drbohlav

Department of Cybernetics

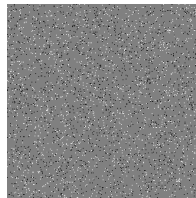
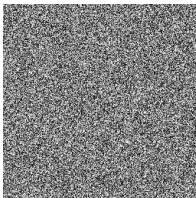
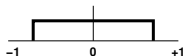
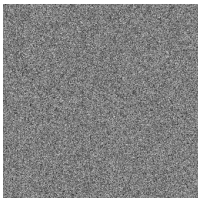
Faculty of Electrical Engineering

Czech Technical University in Prague

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Model of additive noise: $H = F + N$



gaussian



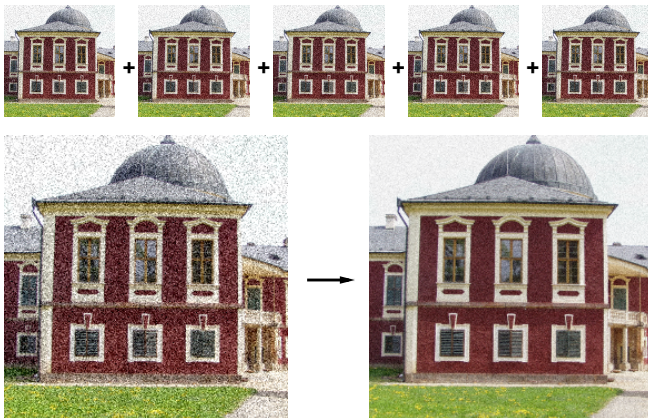
uniform



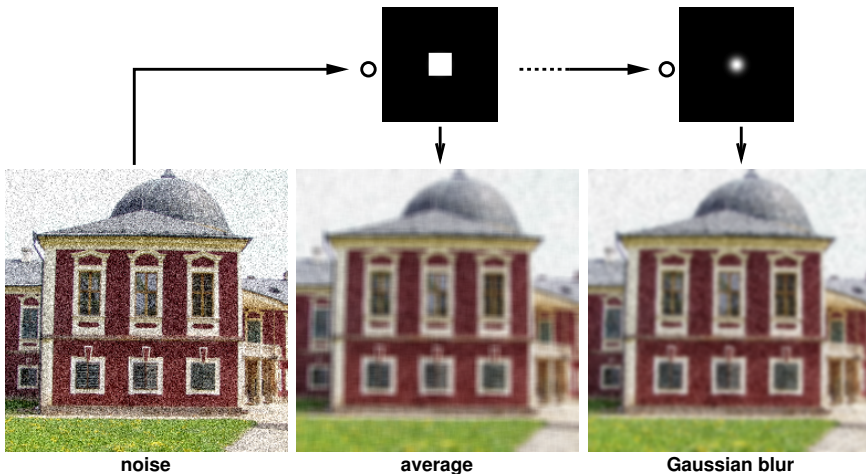
impulsive

Average of k images (degraded by Gaussian noise $\mathbf{N} \sim \mathcal{N}(0, \sigma^2)$):

$$\text{pdf} \left\{ \frac{1}{k} \sum_{i=1}^k n_i \right\} = \mathcal{N} \left(0, \frac{\sigma^2}{k} \right)$$

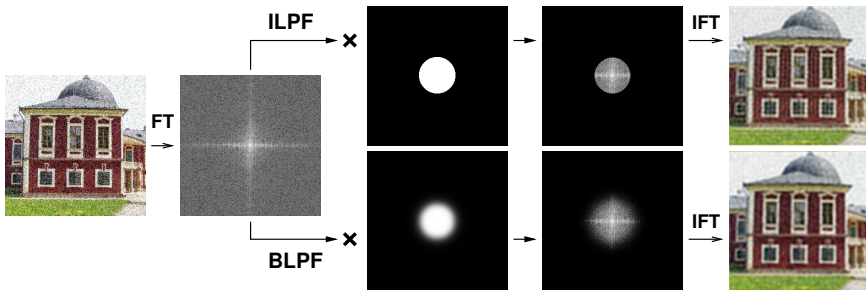


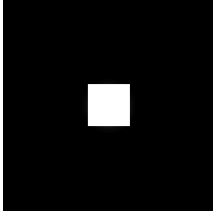
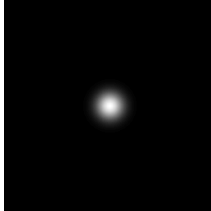
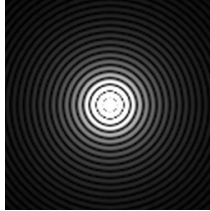
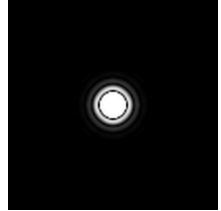
Weighted average of neighbor pixels (Box & Gaussian kernel):



Low-pass filtering (ideal low-pass filter & Butterworth filter):

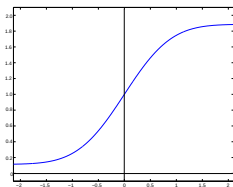
$$\text{ILPF}(u, v) = \begin{cases} 1, & \sqrt{u^2 + v^2} < \sigma; \\ 0, & \text{elsewhere.} \end{cases} \quad \text{BLPF}(u, v) = \frac{1}{1 + (\sqrt{u^2 + v^2}/\sigma)^{2\gamma}}$$



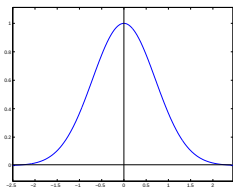
Box**Gauss****ILPF****BLPF**

Model of edge (in 1D):

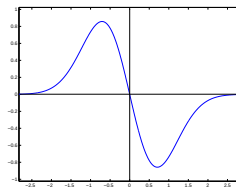
$$f(x)$$



$$f'(x)$$



$$f''(x)$$



edge position:

$$\arg \max_x f'(x)$$

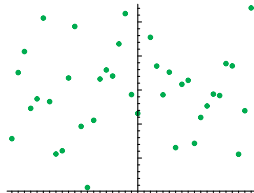
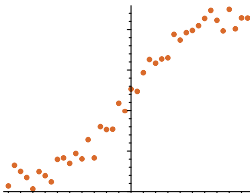
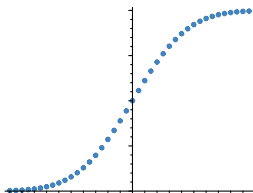
$$f''(x) = 0$$

Discrete case with noise:

$$f[x]$$

$$h[x] = f[x] + n[x]$$

$$h'[x] = h[x] - h[x - 1]$$



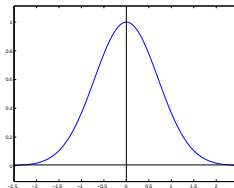
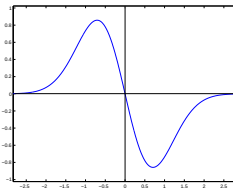
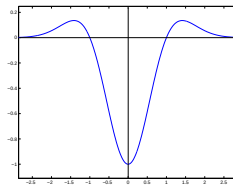
Problem: how to estimate f' when f is corrupted by noise n ?

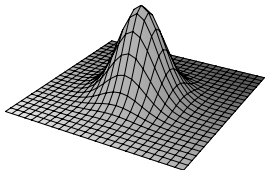
Solution: suppress noise by convolution with g and differentiate.

Differentiation can be computed analytically (in advance):

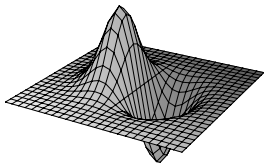
$$(f * g)' = f * g' \Rightarrow \sum_x f[x] * g'[t - x]$$

$$(f * g)'' = f * g'' \Rightarrow \sum_x f[x] * g''[t - x]$$

 $g(x)$  $g'(x)$  $g''(x)$

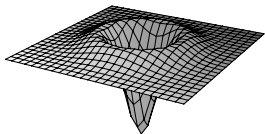


$$G(x, y) \propto \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$



$$G'_x(x, y) \propto \frac{x}{\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

$$\sqrt{(F * G'_x)^2 + (F * G'_y)^2} \arctan\left(\frac{F * G'_y}{F * G'_x}\right)$$

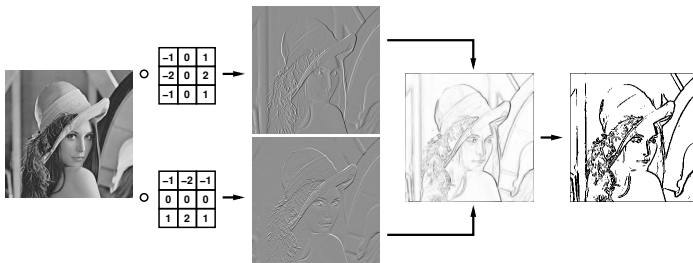


$$\nabla^2 G(x, y) \propto \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

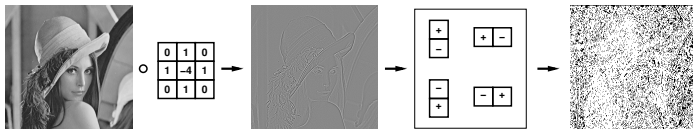
$$\nabla^2 G(x, y) = g''(x) \cdot g(y) + g(x) \cdot g''(y)$$

Simple approximations:

Sobel edge detector:



Laplacian edge detector:



Varying σ (scale-space):

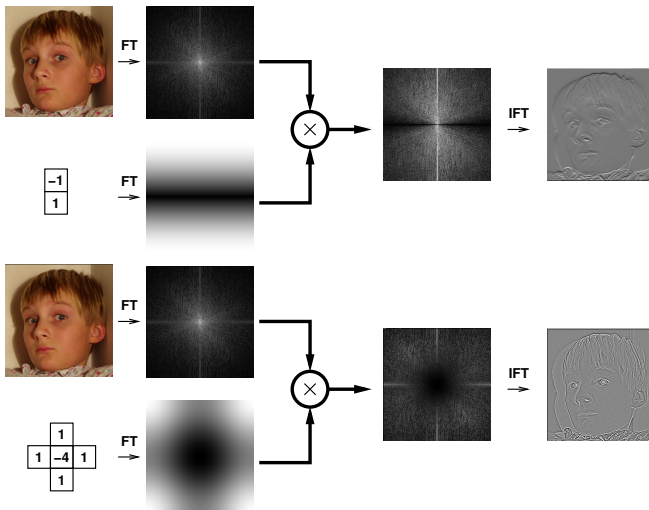
Canny edge detector (non-maxima suppression of $F * G'_\alpha$):



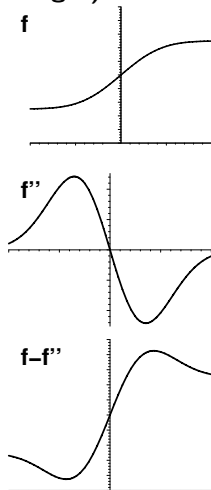
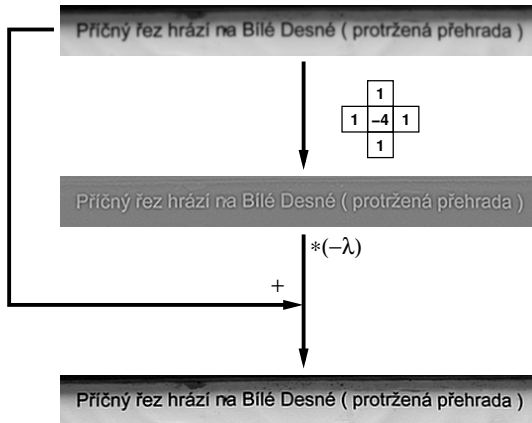
Laplacian of Gaussian zero-crossings ($F * \nabla^2 G = 0$):

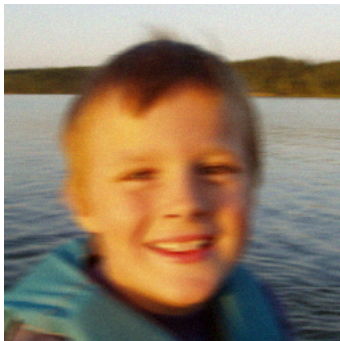


Hi-pass filtering (approximation of the 1st and 2nd derivatives):

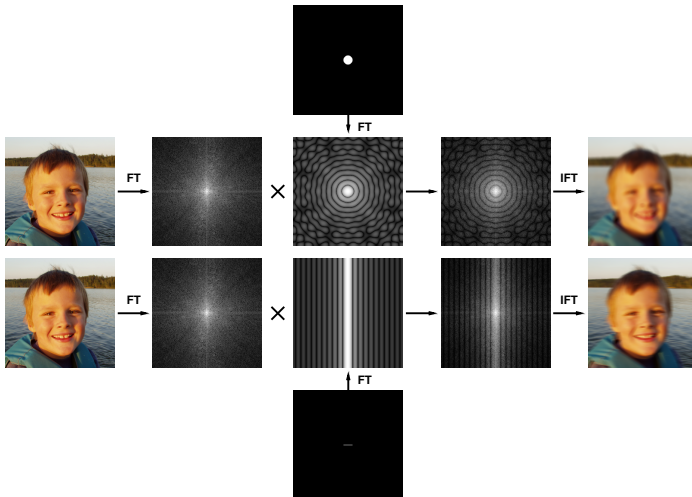


Sharpening (locally increase contrast of edges):

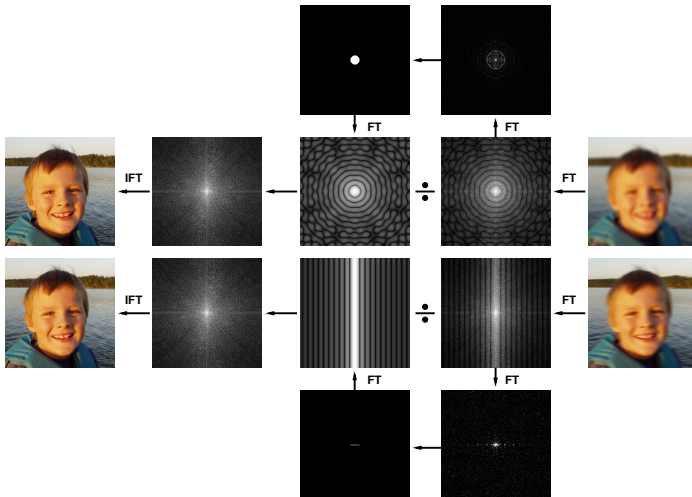




Modelling **out-of-focus** & **motion blur** in Fourier domain:



Recover image from degraded observation (**deconvolution**):



Problem: image f is also degraded by unknown additive noise n .

$$g = h * f + n \iff G = H \cdot F + N \Rightarrow F = (G - N)/H$$

Solution: find an image $\hat{f}$ such that $\|f - \hat{f}\|^2$ is minimal.

$$\hat{F}(u, v) = \frac{H^*(u, v) \cdot G(u, v)}{\|H(u, v)\|^2 + \lambda} \quad \lambda = \text{SNR}^{-1} = \frac{\|N(u, v)\|^2}{\|F(u, v)\|^2}$$

