Computational Game Theory **Midterm Test A** Nov 7th 2023

Surname and name: ______________________________

Task	1	2	3	4	5	6	7	8	*Total*
Maximum	2	2	2	2	2	5	5	10	30
Points									

1. (2 pts) For a finite two-player zero-sum game the following is true:
   (a) The game has a unique value and a unique Nash equilibrium.
   (b) Every player has a unique best response to any mixed strategy of the opponent.
   (c) The game has a pure equilibrium.
   (d) The expected utility of player 1 is equal to the negative of the expected utility of player 2.
   (e) Maxmin strategy of player 1 may not exist.

Solution:
(d)

2. (2 pts) A correlated equilibrium in a normal-form game:
   (a) Is a probability distribution over the joint strategy profiles.
   (b) May not exist.
   (c) Is unique.
   (d) Is a Nash equilibrium in the associated extensive-form game.
   (e) Cannot be always computed by linear programming.

Solution:
(a)

3. (2 pts) For every finite normal-form game the following is true:
   (a) Nash equilibrium may not exist.
   (b) There might be infinitely-many Nash equilibria.
   (c) Nash equilibrium can be computed by linear programming.
   (d) Every Nash equilibrium yields the same utility for a given player.
   (e) There is a Nash equilibrium whose components are not best responses to other players' strategies.

Solution:

(b)

4. (2 pts) The two-player zero-sum game

5	8
5	1

   (a) Has no pure equilibrium.
   (b) Has value 5.
   (c) Has no mixed strategy equilibrium.
   (d) Has value 8.
   (e) Has no value.

Solution:

(b)

5. (2 pts) In games with perfect recall:
   (a) There is always only a single Nash equilibrium.
   (b) If player i plays the same action a in each decision node h in the same infoset I, then his next immidiate decisions h' in the future also belong into the same infoset.
   (c) There is at least one pure Nash equilibrium.
   (d) Reaching each decision node h in the same infoset I for player i is preceeded by the same number of actions played by player i.
   (e) Playing an action does not reveal any information to the other players.

Solution:

(d)

6. Consider the game

	c	d
a	$8, 8$	$4, 9$
b	$9, 4$	$1, 1$

   (a) (2 pts) Find two Nash equilibria.
   (b) (3 pts) Let p and q be the correlated equilibria associated with the Nash equilibria from item (a). Is $\frac{p+q}{2}$ a correlated equilibrium? Why?

Solution:

(a) The game has two pure equilibria, (b, c) and (a, d). (b) The corresponding correlated equilibria are $p(b, c) = 1$ and $q(a, d)$. Then the probability distribution $r = \frac{p+q}{2}$ given by $r(b, c) = r(a, d) = 0.5$ is a correlated equilibrium since it is a convex combination of two correlated equilibria.

7. (5 pts) Compute the value of the zero-sum game

	c	d
a	-1	-4
b	-3	3

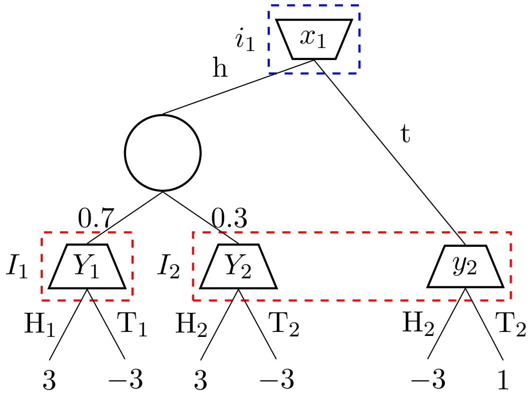
Solution:

Let $p_1 \in [0, 1]$ be the probability that the row player plays a. Then we compute the value of the game as the maximin value of the row player:

$$\max_{p_1} \min_{s_2 \in \{c,d\}} U(p_1, s_2) = \max_{p_1} \min\{2p_1 - 3, -7p_1 + 3\} = -\frac{5}{3}.$$

8. In a two-player zero-sum game *Matching Pennies*, both player's show at once one side of the coin (either heads or tails). If both show the same symbol, first player receives reward 1, otherwise the second one receives reward 1. Imagine a modified version of this game, where players choose the side of the coin sequentially starting with player 2. If the player chose heads, there is a 70% chance that the choice is revealed to the first player. Furthermore, if at least one player chooses head the reward is tripled.
   (a) (5 pts) Formulate this as an extensive-form two-player zero-sum game
   (b) (5 pts) Write down the Sequence-form Linear program for both players, which has a Nash equilibrium as a solution

Solution:



$$\max_{r_1,v_2} v_1(i_1)$$

$$r_1(H_1)+r_1(T_1)=1$$

$$r_1(H_2)+r_1(T_2)=1$$

$$v_1(i_1) \leq 0.7\cdot 3r_1(H_1) + \cdot 0.7\cdot -3r_1(T_1) + 0.3\cdot 3r_1(H_2) - 0.3\cdot 3r_1(T_2)$$

$$v_1(i_1) \leq -3r_1(h_2) + 1r_1(t_2)$$

$$\min_{r_2,v_1} v_1(I_1)+v_1(I_2)$$

$$r_2(h)+r_2(t)=1$$

$$v_1(I_1) \leq 0.7\cdot 3r_2(h)$$

$$v_1(I_1) \leq 0.7\cdot -3r_2(h)$$

$$v_1(I_2) \leq 0.3\cdot 3r_2(h) - 3r_2(t)$$

$$v_1(I_2) \leq 0.3\cdot -3r_2(h) + 1r_2(t)$$