Surname and name: _

Task	1	2	3	4	5	6	7	Total
Maximum	2	2	2	2	2	5	10	25
Points								

- 1. (2 pts) Player 1 moves first (a or b). Player 2 then moves without observing Player 1's choice; her two decision nodes (after a and after b) form one information set, and at that infoset she chooses c or d. How many pure strategies does Player 2 have?
 - (a) 2
 - (b) 3
 - (c) 4
 - (d) 5
 - (e) It depends on Player 1's action

Solution:

- (a) Player 2 has one infoset with two actions, hence exactly 2 pure strategies.
- 2. (2 pts) Two-player game (payoffs are Row, Column):

$$\begin{array}{c|cccc} & L & C & R \\ U & 3,1 & 1,3 & 2,0 \\ M & 2,2 & 2,3 & 3,1 \\ D & 1,0 & 0,2 & 1,0 \\ \end{array}$$

After iterated elimination of strictly dominated strategies for either player, which profile is the unique pure Nash equilibrium of the remaining game?

- (a) (U, L)
- (b) (U, C)
- (c) (M, L)
- (d) (M,C)
- (e) No pure NE remains

(d)

3. (2 pts) Row's payoff matrix is

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -2 \end{pmatrix}.$$

Let Column mix with $y = (y_1, y_2, y_3)$ (over columns) and let w be the game value for Row. Which linear program is correct for Column?

- (a) Minimize w s.t. $y_1 y_2 + 2y_3 \le w$; $2y_2 2y_3 \le w$; $y_1 + y_2 + y_3 = 1$; $y_1, y_2, y_3 \ge 0$.
- (b) Maximize w s.t. $y_1 y_2 + 2y_3 \ge w$; $2y_2 2y_3 \ge w$; $y_1 + y_2 + y_3 = 1$; $y_1, y_2, y_3 \ge 0$.
- (c) Minimize w s.t. $y_1 y_2 + 2y_3 \ge w$; $2y_2 2y_3 \ge w$; $y_1 + y_2 + y_3 \le 1$; $y_1, y_2, y_3 \in \mathbb{R}$.
- (d) Minimize w s.t. for each column $j: \sum_i a_{ij}y_i \leq w; y_1 + y_2 = 1; y_1, y_2, y_3 \geq 0$.
- (e) Maximize w s.t. $A^{\top}y \geq w\mathbf{1}$; $y_1 + y_2 + y_3 = 1$; $y_1, y_2, y_3 \geq 0$.

Solution:

(a)

4. (2 pts) The game:

$$\begin{array}{c|cc} & c & d \\ a & 4,2 & 0,3 \\ b & 1,1 & 3,0 \end{array}$$

In the unique mixed-strategy NE, let p be the probability Row plays a, and q the probability Column plays c. What are (p,q)?

- (a) (0.5, 0.5)
- (b) (1/3, 1/3)
- (c) (0.25, 0.75)
- (d) (0.75, 0.25)
- (e) No mixed-strategy NE exists

Solution:

(a)

5. (2 pts) Consider the game

$$\begin{array}{c|cc} & c & d \\ a & 2,2 & 0,0 \\ b & 0,0 & 1,1 \end{array}$$

Which joint distribution p over action profiles is a correlated equilibrium that is not induced by any Nash equilibrium?

(a)
$$p(a,c) = p(b,d) = 0.5$$

- (b) p(a,c) = 1
- (c) Uniform probability distribution p over all four strategy profiles
- (d) p(a,d) = p(b,c) = 0.5
- (e) p(a,b) = 1/9, p(a,c) = p(b,c) = 2/9, p(b,d) = 4/9

Solution:

(a)

- 6. Consider the game $a \begin{tabular}{c|c} c & d \\ \hline $2,2$ & $0,3$ \\ \hline b & $3,0$ & $1,1$ \\ \hline \end{tabular}$
 - (a) (3 pts) Find all Nash equilibria of this game.
 - (b) (2 pts) Formulate the problem of finding the correlated equilibrium maximizing the expected payoff of Row as an optimization problem.

Solution:

(a) Pure NE are (a, d) and (b, c). In the unique mixed NE, Row player a with probability 2/3 and Column playes c with probability 1/2. (b) Let x = p(a, c), y = p(a, d), z = p(b, c), and w = p(b, d). Maximize x + 3z + w subject to

$$2x \ge 3x + y$$
, $3z + w \ge 2z$, $2x \ge 3x + z$, $3y + w \ge 2y$

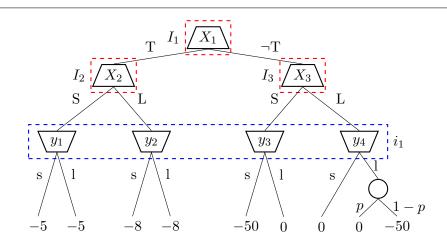
and x + y + z + w = 1, where x, y, z, w > 0.

7. Consider the following game between a traveller and a ticket inspector.

There are two routes the participants can choose from: a short route and a long route. The tickets cost \$5 and \$8, respectively. The traveller can either pay for the ticket or ride without one. If caught by the inspector without a ticket, the traveller must pay a fine of \$50. Unbeknownst to the inspector, when the traveller chooses the long route, there is a probability p that they never meet.

- (a) (5 pts) Formulate this scenario as an extensive-form two-player zero-sum game.
- (b) (5 pts) Write down the sequence-form linear program for the minimizing 2nd player (inspector), which has a Nash equilibrium as a solution.

Solution:



Player 2 (Minimizer) LP:

$$\min_{r_2,v} \quad v(I_1)$$

s.t.

$$r_2(s) + r_2(l) = 1$$

$$v(I_1) \ge v(I_2)$$
 (P1's action 'T' at I_1)

$$v(I_1) \ge v(I_3)$$
 (P1's action '¬T' at I_1)

$$v(I_2) \ge -5 \cdot r_2(s) - 5 \cdot r_2(l)$$
 (P1's action 'S' at I_2)

$$v(I_2) \ge -8 \cdot r_2(s) - 8 \cdot r_2(l)$$
 (P1's action 'L' at I_2)

$$v(I_3) \ge -50 \cdot r_2(s)$$
 (P1's action 'S' at I_3)

$$v(I_3) \ge -50(1-p) \cdot r_2(l)$$
 (P1's action 'L' at I_3)