Solving Extensive-form games

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Finding Nash equilibrium in imperfect information EFG

- Existence of pure strategy Nash not guaranteed
- Convert to normal-form game and find NE there
- Backward induction does not work
- Approximative regret minimizing methods
- LP based on sequence form
- Iterative building with sequence-form LP

Sequence-form representation

- Based on sequences
- Realization plans
- Even more compressed than behavioral strategies
- Extended utility function

Sequence-form LP

$$\max_{r,v} \; v(root) \quad (1)$$

$$s.t. \quad r(\emptyset) = 1 \quad (2)$$

$$0 \leq r(\sigma_1) \leq 1 \qquad \forall \sigma_1 \in \Sigma_1 \quad (3)$$

$$\sum_{a \in \mathcal{A}(I_1)} r(\sigma_1 a) = r(\sigma_1) \qquad \forall \sigma_1 \in \Sigma_1, \forall I_1 \in \inf_1(\sigma_1) \quad (4)$$

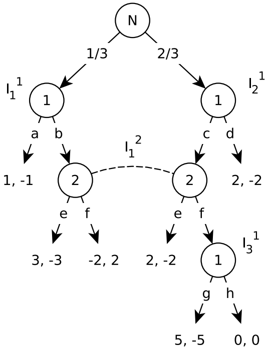
$$\sum_{I' \in \inf_2(\sigma_2 a)} v(I') + \sum_{\sigma_1 \in \Sigma_1} g(\sigma_1, \sigma_2 a) r_1(\sigma_1) \geq v(I) \qquad \forall I \in \mathcal{I}_2, \forall a \in \mathcal{A}(I), \sigma_2 = \mathrm{seq}_2(I) \quad (5)$$

- $\inf_1(\sigma_1)$ – information sets of player 1 reachable by σ_1
- $\mathrm{seq}_2(I)$ – sequence of player 2 leading to I

Example

- Small poker
  - Ante 1$
  - Deck $\{J, J, Q, Q\}$
  - Player 1 either folds or bets 2 $
  - Player 2 either calls or folds

Example 1



$g_1((a),[]) = \frac{1}{3}, g_1((b),(e)) = 1, g_1((b),(f)) = -\frac{2}{3},$
$g_1((c),(e)) = \frac{4}{3}, g_1((c,g),(f)) = \frac{10}{3}, g_1((d),[]) = \frac{4}{3},$
$g_1((c,h),(f)) = 0$

Example 1 – LP for player 1

$$
\begin{aligned}
& \max v(I_0^2) && (6)\\
& r_1([]) = 1 && (7)\\
& 0 \le r_1([]) \le 1, 0 \le r_1((a)) \le 1, 0 \le r_1((b)) \le 1, \ldots && (8)\\
I_1^1: \quad & r_1((a)) + r_1((b)) = r_1([]) && (9)\\
I_2^1: \quad & r_1((c)) + r_1((d)) = r_1([]) && (10)\\
I_3^1: \quad & r_1((c,g)) + r_1((c,h)) = r_1((c)) && (11)\\
I_0^2, \emptyset: \quad & v(I_1^2) + g_1((a),[]) \cdot r_1((a)) + g_1((d),[]) \cdot r_1((d)) \ge v(I_0^2) && (12)\\
I_1^2, e: \quad & g_1((b),(e)) \cdot r_1((b)) + g_1((c),(e)) \cdot r_1((c)) \ge v(I_1^2) && (13)\\
I_1^2, f: \quad & g_1((b),(f)) \cdot r_1((b)) + g_1((c,g),(f)) \cdot r_1((c,g)) + g_1((c,h),(f)) \cdot r_1((c,h)) \ge v(I_1^2) && (14)
\end{aligned}
$$

Example 1 – LP for player 2

$$\max v(I_0^1) \tag{15}$$

$$r_2([]) = 1 \tag{16}$$

$$0 \le r_2([]) \le 1, 0 \le r_2((e)) \le 1, 0 \le r_2((f)) \le 1 \tag{17}$$

$$I_1^2: \quad r_2((e)) + r_2((f)) = r_2([]) \tag{18}$$

$$I_0^1, \emptyset: \quad v(I_1^1) + v(I_2^1) \ge v(I_0^1) \tag{19}$$

$$I_1^1, a: \quad g_2((a), []) \cdot r_2([]) \ge v(I_1^1) \tag{20}$$

$$I_1^1, b: \quad g_2((b), (e)) \cdot r_2((e)) + g_2((b), (f)) \cdot r_2((f)) \ge v(I_1^1) \tag{21}$$

$$I_2^1, c: \quad v(I_3^1) + g_2((c), (e)) \cdot r_2((e)) \ge v(I_2^1) \tag{22}$$

$$I_2^1, d: \quad g_2((d), []) \cdot r_2([]) \ge v(I_2^1) \tag{23}$$

$$I_3^1, g: \quad g_2((c, g), (f)) \cdot r_2((f)) \ge v(I_3^1) \tag{24}$$

$$I_3^1, h: \quad g_2((c, h), (f)) \cdot r_2((f)) \ge v(I_3^1) \tag{25}$$