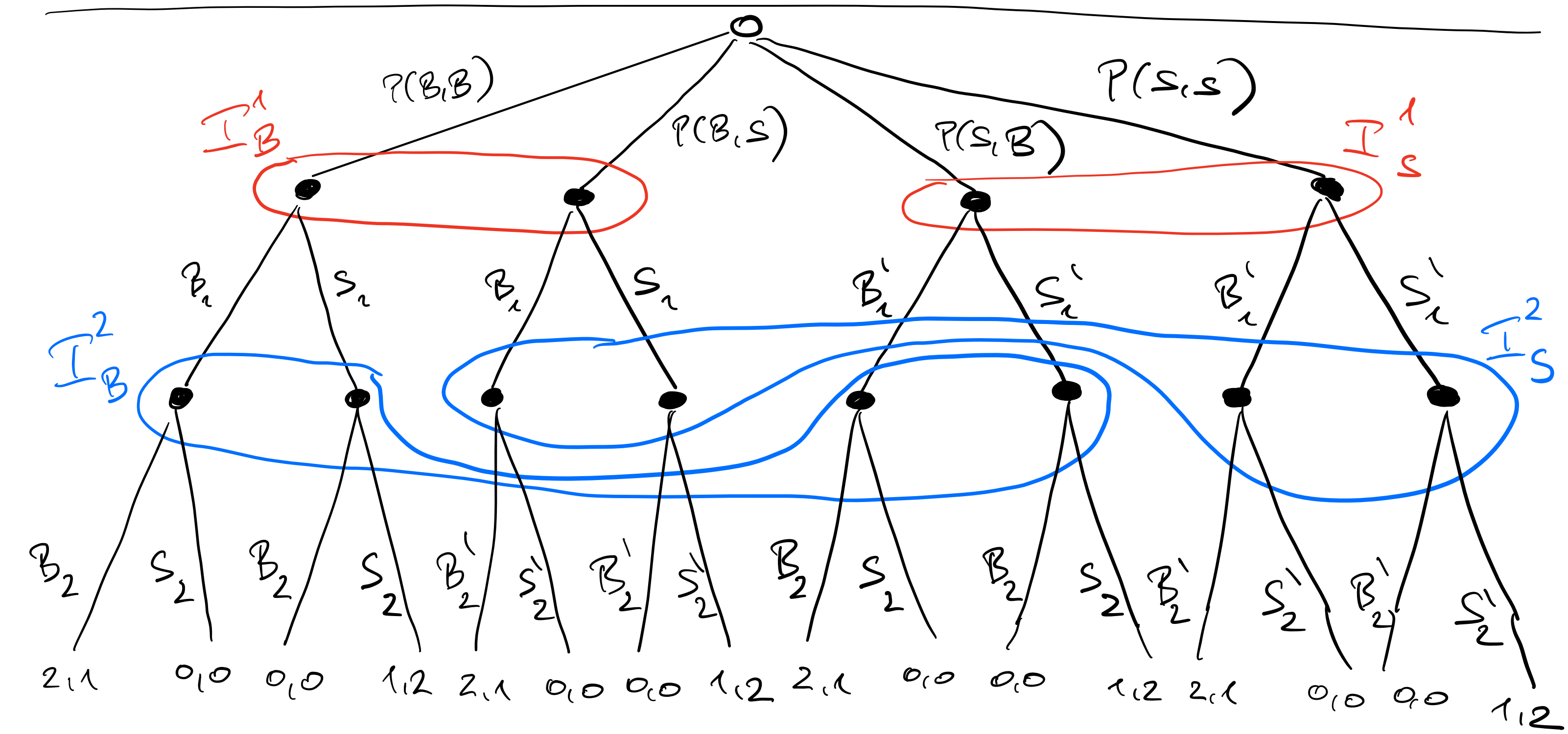
Metagame for the correlated equilibrium



- Expected utility of player 1 for following advice B is $P(B_2|B)\cdot 2 + P(S_2|B)\cdot 0$
- Expected utility of player 1 for not following advice B is $P(B_2|B)\cdot 0 + P(S_2|B)\cdot 1$

CE, equivalently (proof)

Idea: Write $P(s_{-i}|s_i) = \dfrac{P(s_{-i},s_i)}{P(s_i)}$ and note that the denominator doesn't depend on s_{-i}. Finally, note that if $P(s_i)=0$, then necessarily $P(s_{-i},s_i)=0$, so the inequality 2. from Proposition is trivially true.

Bach or Stravinski

NE induce CE:

	B	S	
B	1	0	1
S	0	0	0
	1	0	

0	0	0
0	1	1
0	1	

2/9	4/9	6/9
1/9	2/9	3/9
3/9	6/9	

Weak SE may not exist

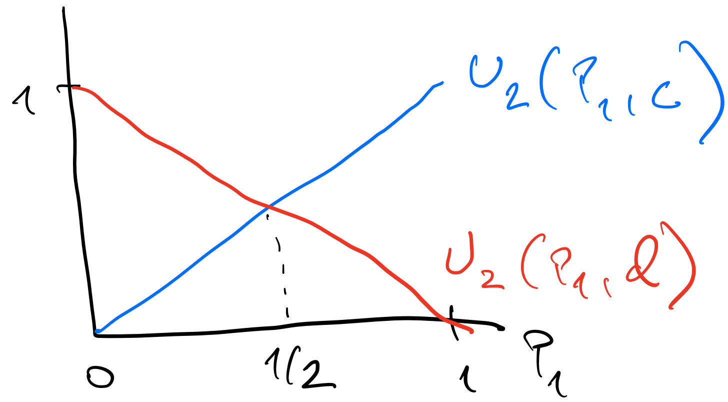
	c	d
a	2,1	4,0
b	1,0	3,1

Best response of follower to a mixed strategy of leader:

$p_1 := p_1(a)$

$U_2(p_1, c) = p_1$

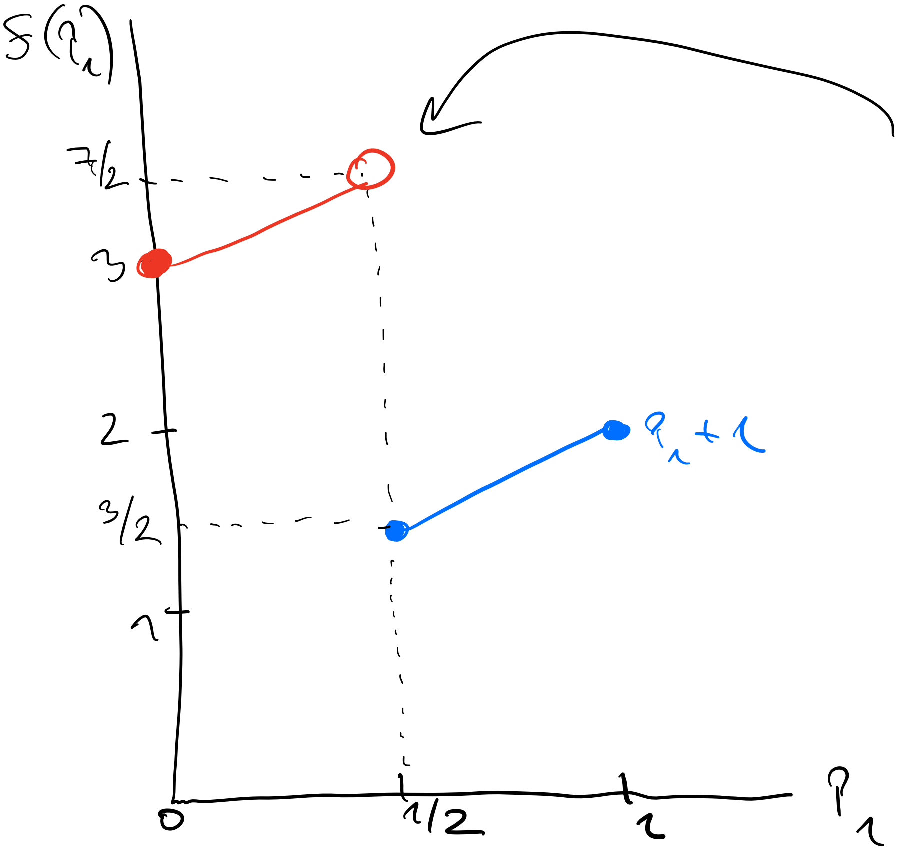
$U_2(p_1, d) = 1 - p_1$



Function $S(p_1) := \min_{s_2 \in BR_2(p_1)} U_1(p_1, s_2)$

$U_1(p_1, c) = 2p_1 + 1 - p_1 = p_1 + 1$

$U_1(p_1, d) = 4p_1 + 3(1 - p_1) = p_1 + 3$



Function S has no maximum only supremum!

SE in 2-player zero-sum games

Let $t_2 \in BR_2(p_1)$:

$$U_2(p_1, t_2) = \max_{s_2 \in S_2} U_2(p_1, s_2)$$

$$\iff$$

$$\boxed{U_1(p_1, t_2)} = \min_{s_2 \in S_2} U_1(p_1, s_2)$$

Weak SE: leader solves the problem

$$\max_{p_1 \in \Delta_1} \min_{t_2 \in BR_2(p_1)} \boxed{U_1(p_1, t_2)} =$$

$$= \max_{p_1 \in \Delta_1} \min_{t_2 \in BR(p_1)} \min_{s_2 \in S_2} U_1(p_1, s_2) =$$

$$= \max_{p_1 \in \Delta_1} \min_{s_2 \in S_2} U_1(p_1, s_2)$$

Strong SE: analogously, leader solves

$$\max_{p_1 \in \Delta_1} \max_{t_2 \in BR_2(p_1)} \boxed{U_1(p_1, t_2)} =$$

$$= \max_{p_1 \in \Delta_1} \max_{t_2 \in BR_2(p_1)} \min_{s_2 \in S_2} U_1(p_1, s_2)$$

$$= \max_{p_1 \in \Delta_1} \min_{s_2 \in S_2} U_1(p_1, s_2)$$

Bach or Stravinski, revisited

	c	d
a	2,1	0,0
b	0,0	1,2

Let the row player be the leader and $p_1 \in \Delta_1$ be the commitment.

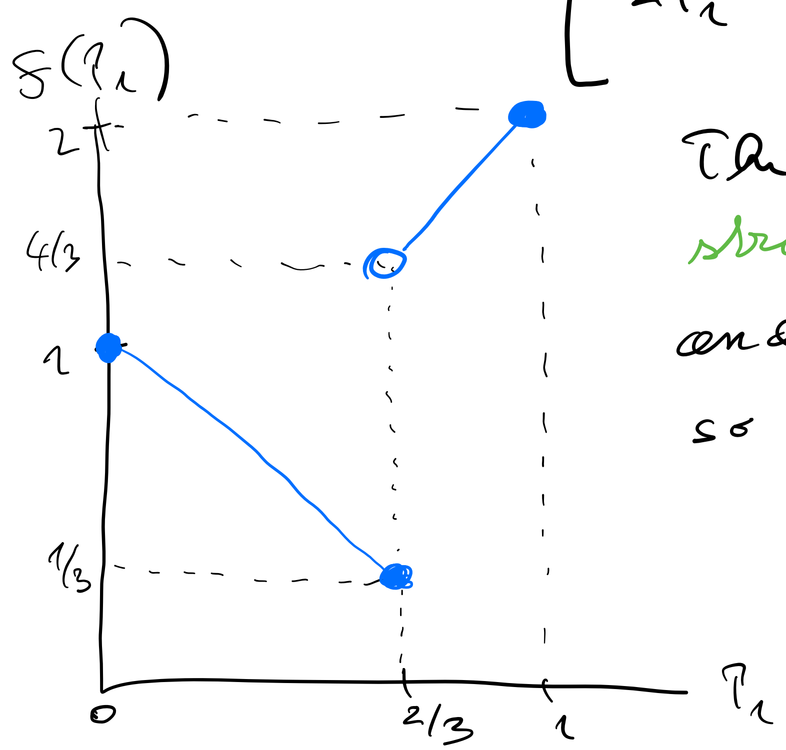
Then $U_2(p_1, c) = p_1$, where $p_1 := p_1(a)$, and $U_2(p_1, d) = 2 - 2p_1$. This means that

$$BR_2(p_1) = \begin{cases} d, & 0 \le p_1 < 2/3 \\ \{c, d\}, & p_1 = 2/3 \\ c, & 2/3 < p_1 \le 1 \end{cases}$$

We analyze function

$$f(p_1) := \min_{s_2 \in BR_2(p_1)} U_1(p_1, s_2)$$

$$= \begin{cases} 1 - p_1 & 0 \le p_1 \le 2/3 \\ 2p_1 & 2/3 < p_1 \le 1 \end{cases}$$



This implies that strong SE = weak SE, and $p_1^* = 1$, $s_2^* = c$, so this is a pure NE.

Advertising competition

Two competing firms (row / column) can either advertise or not. Advertising is costly but can steal customers if the other firm doesn't advertise.

	a	n
a	2,2	4,1
n	1,4	3,3

The game has a unique NE (a,a) since the strictly dominant strategy for each firm is to advertise.

Now, the market regulator wants to send private recommendations generated by a probability distribution P.

Let's check when P is a CE:

(1) $p(a,a) + 3(a,n) \leq 2p(a,a) + 4(a,n)$

(2) $2P(n,a) + 4(n,n) \leq P(n,a) + 3P(n,n)$

(3) $P(a,a) + 3P(n,a) \leq 2P(a,a) + 4P(n,a)$

(4) $2P(a,n) + 3P(n,n) \leq P(a,n) + 3P(n,n)$

$$\left.\begin{array}{lll} (4) & \Rightarrow & p(a,n)=0 \\ (3) & \Rightarrow & p(n,n)=0 \\ (2) & \Rightarrow & p(n,a)=0 \end{array}\right\} \Rightarrow p(a,a)=1$$

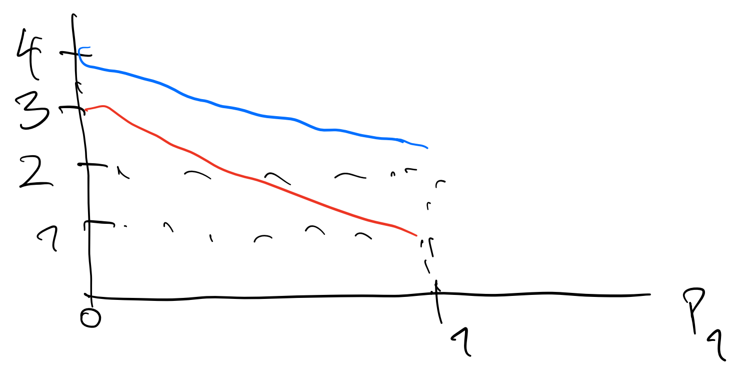
This means that the only CE is the unique NE. There is no point in coordination for this game since the firms have strictly dominant strategies.

What if the row firm chooses first?

Then $U_2(P_1, a) = 2P_1 + 4(1-P_1) = 4-2P_1$

$U_2(P_1, n) = 1P_1 + 3(1-P_1) = 3-2P_1$

where $P_1 := P_1(a)$.



This implies that $BR_2(P_1) = \{a\}$
$\forall P_1 \in [0,1]$

The leader (row firm) maximises

$$U_1(p_1, a) = 2p_1 + (1-p_1) = p_1 + 1$$

$$\Downarrow$$

$$p_1 = 1$$

The unique SE (weak/strong)

is (a, a).