Auctions

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Introduction to Auctions

Auctions: Traditional

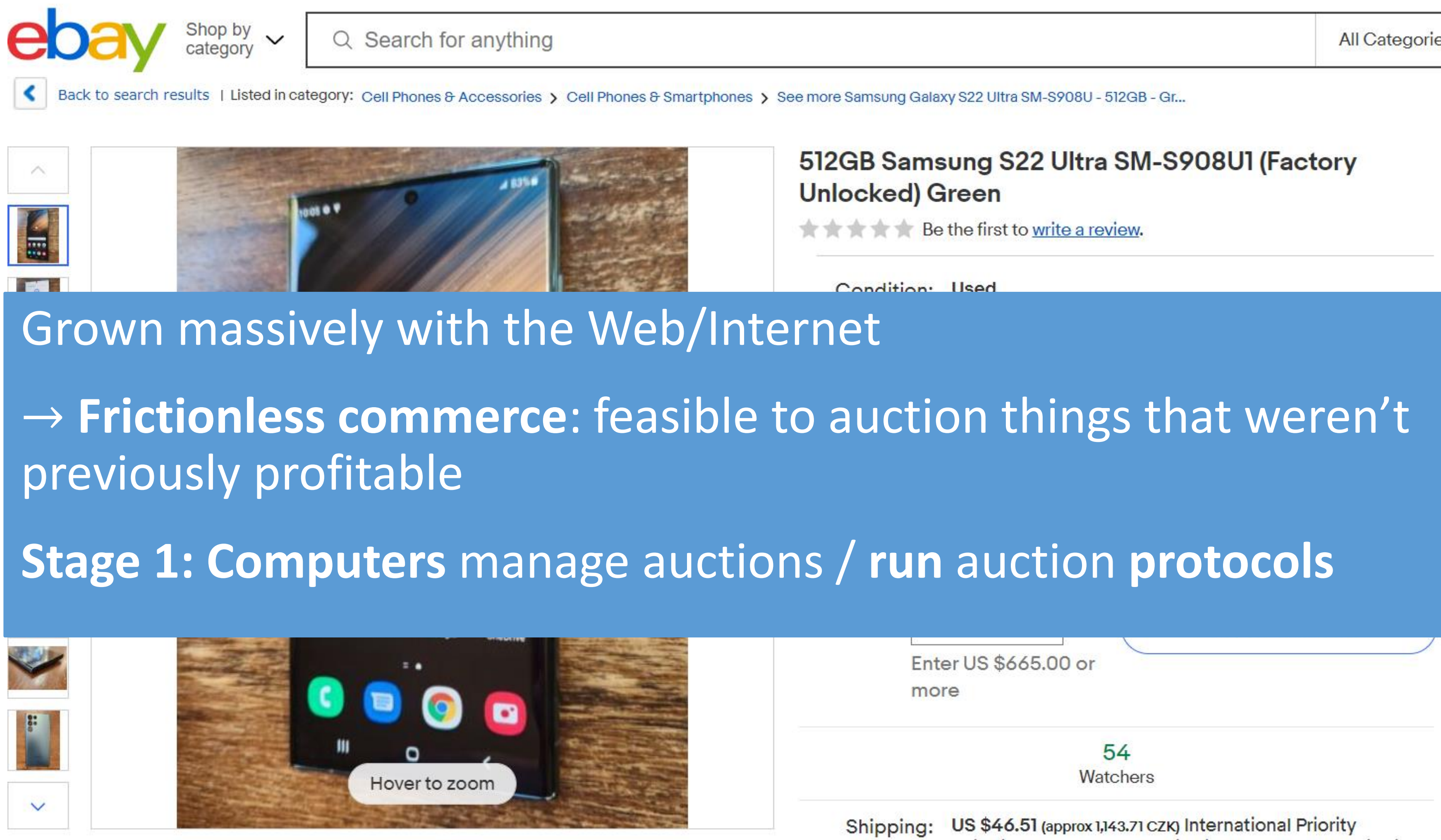
Auctions used in Babylon as early as 500 B.C.

Stage 0: No automation

Tuna Fish Auction

Property Auction

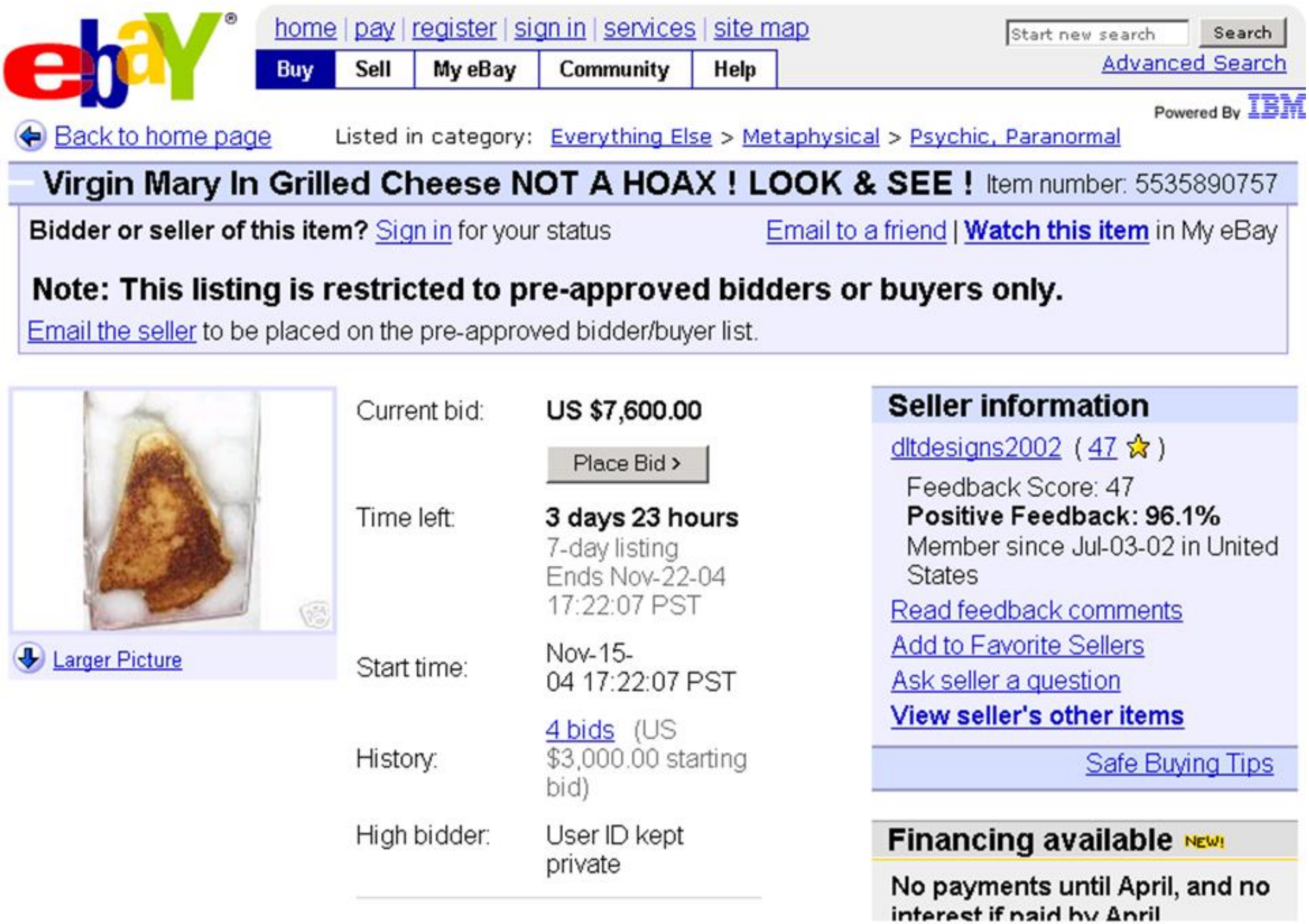
Auctions: Partial Automation



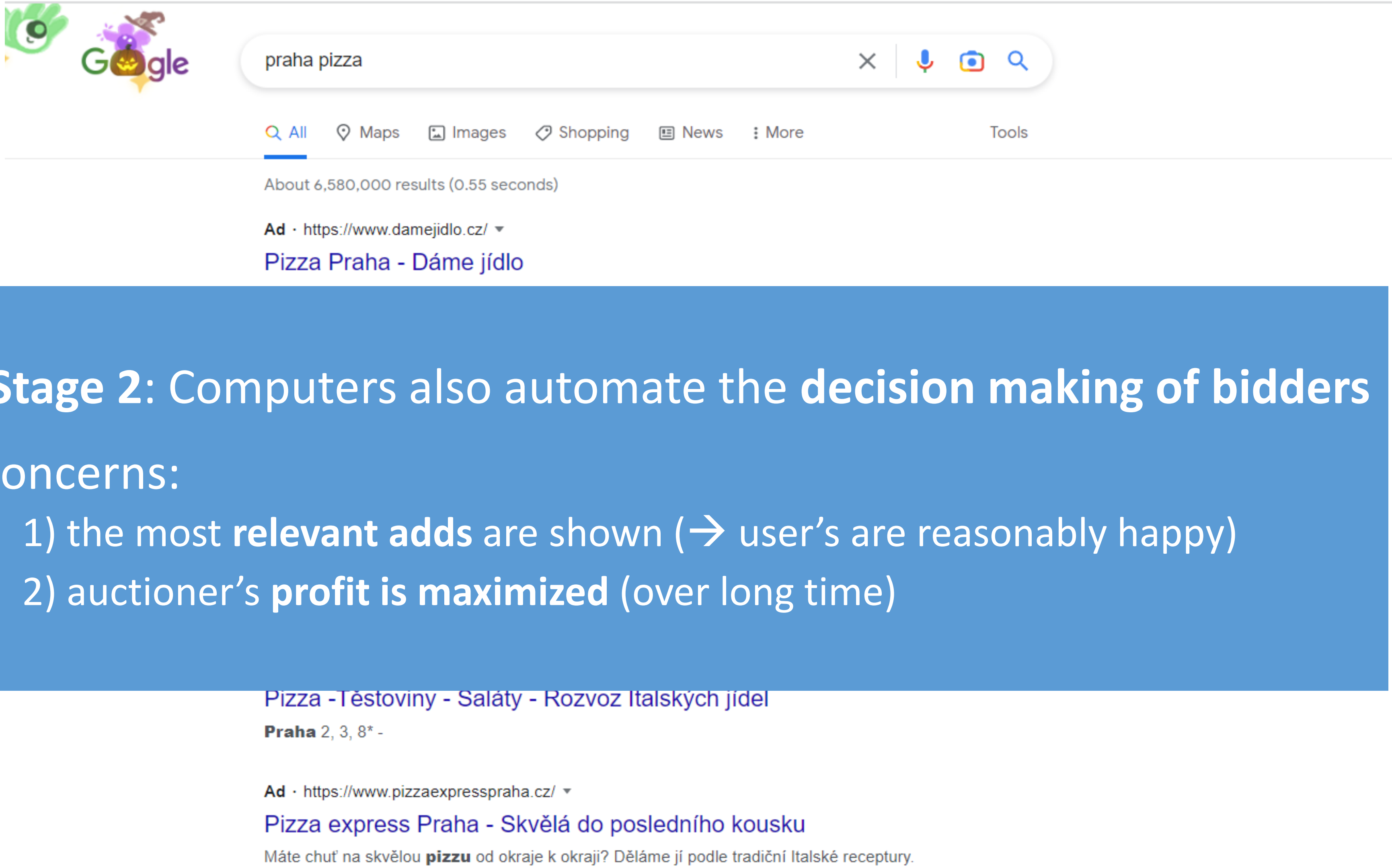
Grown massively with the Web/Internet

→ **Frictionless commerce**: feasible to auction things that weren't previously profitable

Stage 1: Computers manage auctions / **run** auction **protocols**


ebay
home | pay | register | sign in | services | site map
Start new search
Search
Advanced Search
Buy
Sell
My eBay
Community
Help
Powered By IBM
Back to home page
Listed in category: Everything Else > Metaphysical > Psychic, Paranormal
Virgin Mary In Grilled Cheese NOT A HOAX ! LOOK & SEE ! Item number: 5535890757
Bidder or seller of this item? Sign in for your status
Email to a friend | Watch this item in My eBay
Note: This listing is restricted to pre-approved bidders or buyers only.
Email the seller to be placed on the pre-approved bidder/buyer list.
Larger Picture
Current bid: US $7,600.00
Place Bid >
Time left: 3 days 23 hours
7-day listing
Ends Nov-22-04 17:22:07 PST
Start time: Nov-15-04 17:22:07 PST
History: 4 bids (US $3,000.00 starting bid)
High bidder: User ID kept private
Seller information
dltdesigns2002 (47)
Feedback Score: 47
Positive Feedback: 96.1%
Member since Jul-03-02 in United States
Read feedback comments
Add to Favorite Sellers
Ask seller a question
View seller's other items
Safe Buying Tips
Financing available NEW!
No payments until April, and no interest if paid by April

Auctions: (Almost) Full automation



Stage 2: Computers also automate the **decision making of bidders**

Concerns:

1) the most **relevant adds** are shown (→ user's are reasonably happy)
2) auctioner's **profit is maximized** (over long time)

What is an Auction?

*An **auction** is a protocol that allows **agents** (=bidders) to indicate their **interests** in one or more **resources** (=items or goods) and that uses these indications of interest to determine both an **allocation** of the resources and a set of **payments** by the agents.* [Shoham & Leyton-Brown 2009]

Auctions use employ **cardinal preferences** to express interest .

Auctions are mechanisms **with money**.

Auctions can be viewed as **games** of a specific structure.

Why Auctions?

Market-based price setting: for objects of unknown value, the value is dynamically assessed by the market!

Flexible: any object type can be allocated

Can be **automated**

- use of simple rules reduces complexity of negotiations
- well-suited for computer implementation

Revenue-maximising and **efficient allocations** are achievable

Auctions Rules

Auctions are **structured negotiations** governed by **auction rules (→** *rules of the game)*

Bidding rules

How **offers (bids)** are made:

- by whom
- when
- what their content is

Clearing rules

Who gets which goods (**allocation**) and what money changes hands (**payment**).

Information rules

What information about the state of the negotiation is *revealed* **to whom** and **when**.

Lots of Applications

Industrial procurement

Transport and logistics

Energy markets

Cloud and grid computing

Internet auctions

(Electromagnetic spectrum allocation)

... and counting!

Types of Auctions

multi-unit
multiple indistinguishable items (A, A, A)

single-good

multi-item
multiple different items (A,B,C)

Combinatorial auctions

multi-attribute
items characterized by multiple attributes (A=a_1, a_2, a_3)

Single-Item Auctions

Basic Auction Mechanisms

English

Japanese

Dutch

First-Price

Second-Price

(All pay auction)

English Auction

1. Auctioneer starts the bidding (at some **reservation price**)
2. Bidders then shout out **ascending** prices (with minimum increments)
3. Once bidders stop shouting, the *high bidder* gets the good at that price

Japanese Auctions

Same as an English auction except that the auctioneer calls out the prices

1. All bidders start out **standing**
2. When the price reaches a level that a bidder is not willing to pay, that bidder **sits down;** once a bidder sits down, they **can't get back up.**
3. The **last** person **standing** gets the good

Dutch Auction

1. The auctioneer starts a clock at some high value; it descends
2. At some point, a bidder shouts “mine!" and gets the good at the price shown on the clock

Good when items need to be sold **quickly** (similar to Japanese)

No information is revealed during auction

First-, Second-Price Sealed Bid Auctions

First-price sealed bid auction

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount of **his bid**

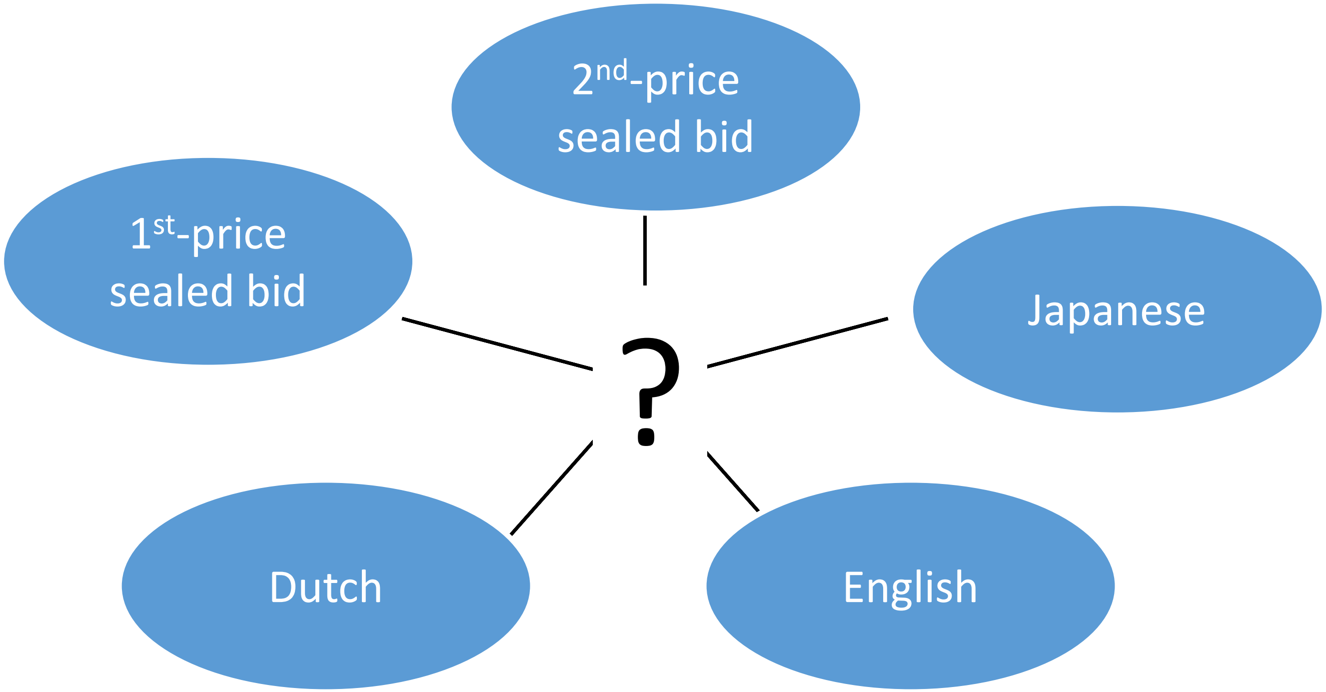
Second-price sealed bid auction (**Vickerey** auction)

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount bid by **the second-highest** bidder

Intuitive Comparison

	English	Dutch	Japanese	1^{st}-Price	2^{nd}-Price
Duration	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
Info Revealed	2^{nd}-highest val; bounds on others	winner's bid	all val's but winner's	none	none
Jump bids	yes	n/a	no	n/a	n/a
Price Discovery	yes	no	yes	no	no

Analysing Auctions



Are there fundamental similarities / differences between mechanisms described?

Two Problems

Analysis of auction mechanisms

- determine the properties of a given auction mechanism
- methodology: treat auctions as (extended-form) *Bayesian games* and analyse players' (i.e. bidders') strategies

Design of auction mechanisms

- design the auction mechanism (i.e. the game for the bidders) with the desirable properties
- methodology: apply mechanism design techniques

Payoff

Agent's payoff
from participating in an auction

If winner

payoff* =
valuation of the item −
price paid for the item

* Also termed **surplus**

If *not* winner

payoff=**zero**[1]

Individual rationality: the agent never bids higher than its valuation

[1] not in the all pay auction

Risk Attitudes

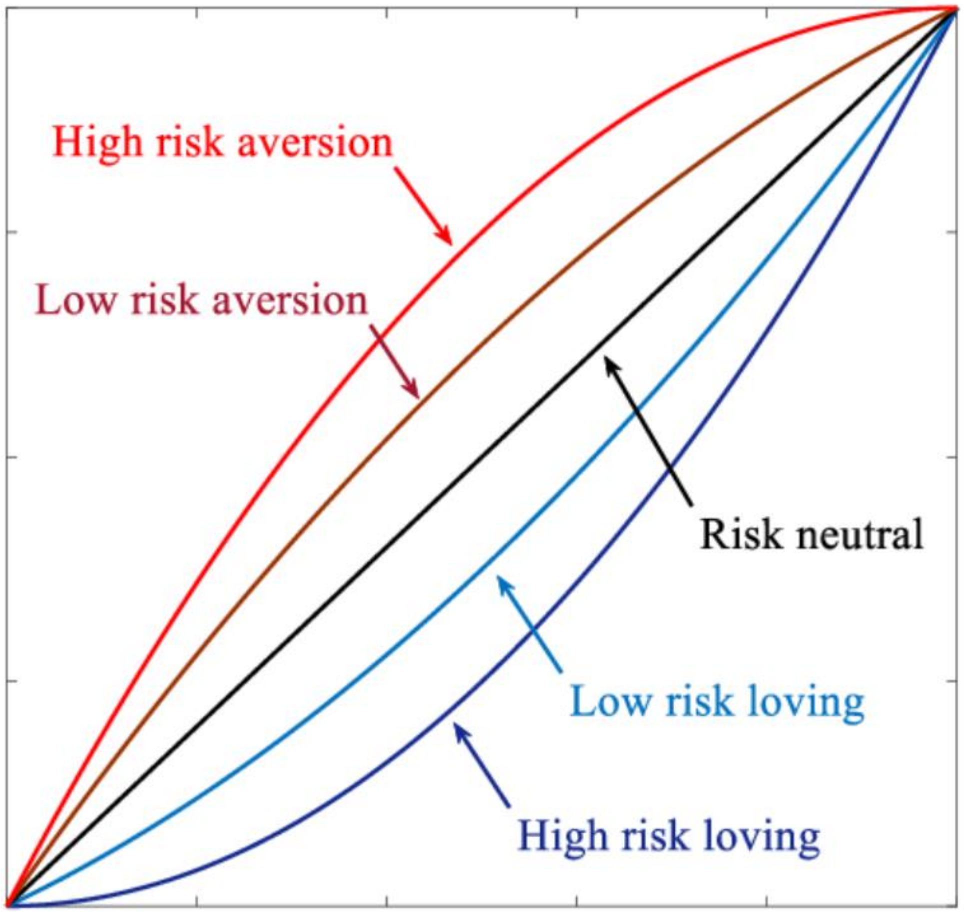
Risk neutrality: the payoff is a *linear function* of the difference between the item's valuation and the price paid

Risk seeking (also **risk loving)**: the payoff is a *convex* function of the difference (aggressively seeking high gains is prioritized)

Risk aversion: the payoff is a *concave* function of the difference (conservatively ensuring at least some gains is prioritized)

Risk Attitudes

payoff
(as a function
of surplus)



surplus

Valuation Models

Independent private value (IPV)	Correlated value
An agent A's valuation of the good is **independent from other agent's** valuation of the good (e.g. a taxi ride to the airport).	Valuations of the good are **related between agents** (typically the more other agents are prepared to pay, the more the agent A prepared to pay – e.g. purchase of items for later resale).

Bayesian Game

Definition (Bayesian Game)

A Bayesian game is a tuple $\langle N, A, \Theta, p, \boldsymbol{u} \rangle$ where

- N is the set of **players**
- $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, Θ_i is the **type space** of player i
- $A = A_1 \times A_2 \times \cdots \times A_n$ where A_i is the **set of actions** for player i
- $p: \Theta \mapsto [0,1]$ is a **common prior over types**
- $\boldsymbol{u} = (u_1, \ldots, u_n)$, where $u_i: \mathrm{A} \times \Theta \mapsto \mathbb{R}$ is the **utility function** of player i

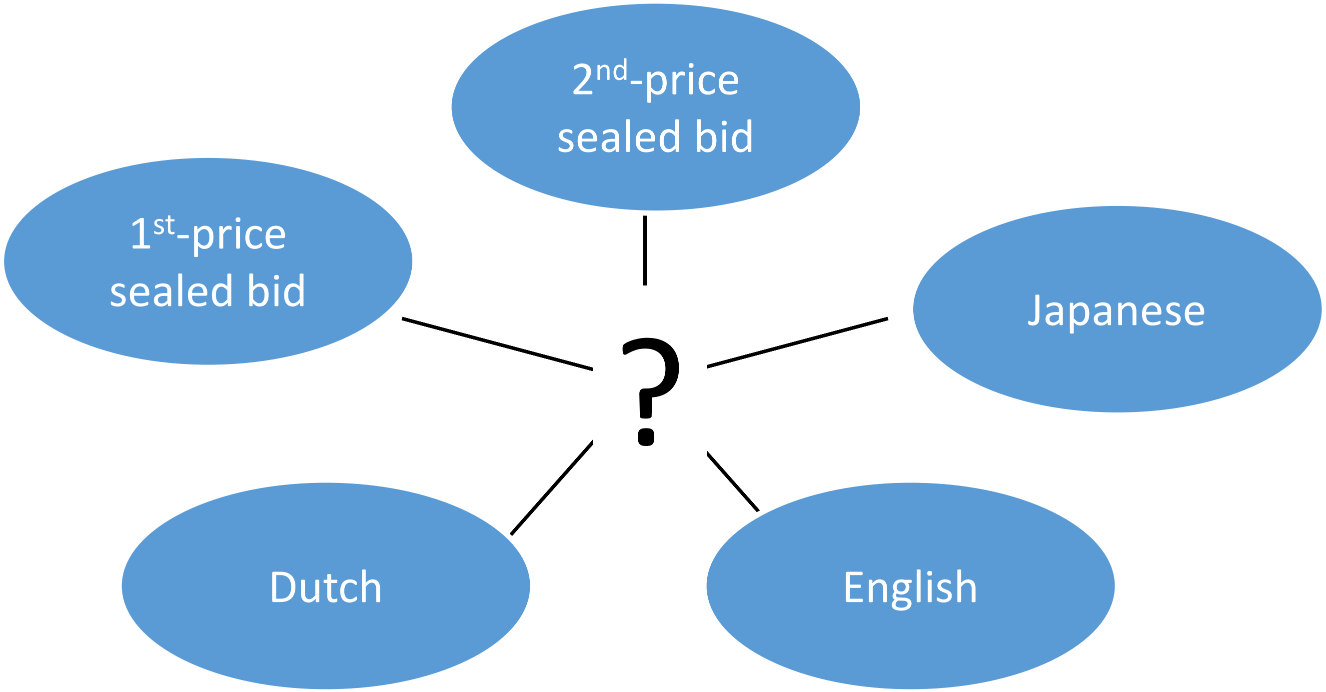
We assume that all of the above is **common knowledge** among the players, but the type of an agent **is private** (i.e. only known by that agent).

Relation to (sealed bid) Auctions

Sealed bid auction under IPV is a Bayesian game in which

- player i's **actions** correspond to its **bids** $\widehat{v_i}$
- player types Θ_i correspond to players' **private valuations** v_i over the auctioned item(s)
- the **payoff** of player i corresponds to: ***i*'s valuation of the item $\boldsymbol{v_i}$ – price paid** (if winner); zero otherwise.

Similar analogies for more complicated auction mechanisms.



Are there fundamental similarities / differences between mechanisms described?

Bidding in Second-Price Sealed Bid Auction

Bidding in Second Price Sealed Bid Auction

How should agents bid in the second-price sealed bid auctions?

Bidding in Second Price Sealed Bid Auction

Theorem

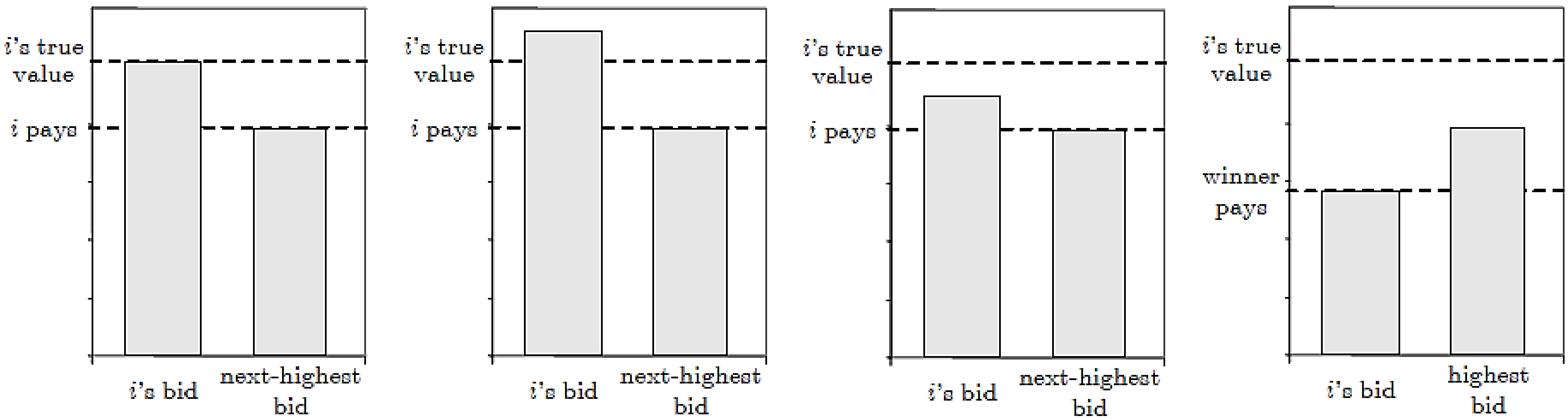
Truth-telling is a **dominant strategy** in a second-price sealed bid auction (assuming independent private values – IPV).

Proof: Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

- Bidding honestly, i would win the auction
- Bidding honestly, i would lose the auction

Second-Price Sealed Bid Proof

Bidding honestly, i is the winner



If i bids higher, he will still win and still pay the same amount

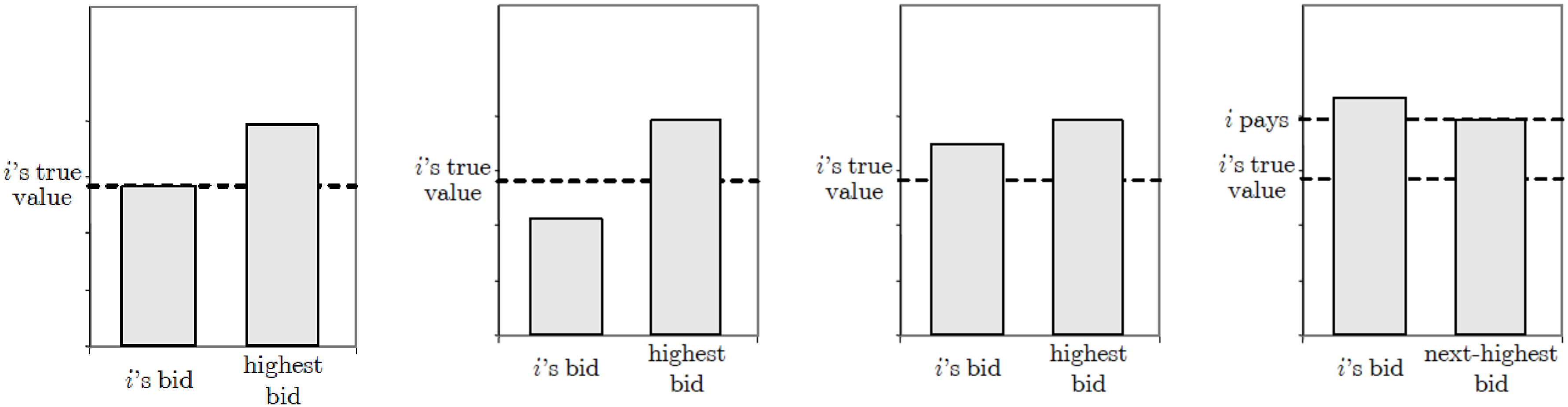
If i bids lower, he will either still win and still pay the same amount. . .

... or lose and get the payoff of zero.

➔ *There is a disadvantage bidding lower and no advantage bidding higher*

Second-Price Sealed Bid Proof

Bidding honestly, i is not the winner



If i bids lower, he will still lose and still pay nothing

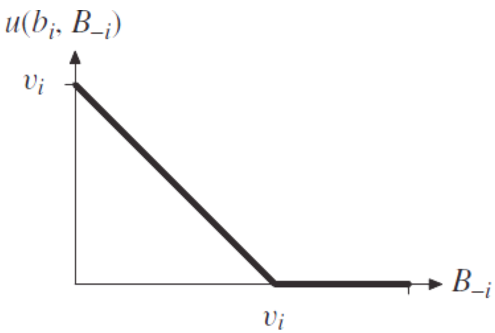
If i bids higher, he will either still lose and still pay nothing...

... or win and pay more than his valuation ($\Rightarrow$ negative payoff).

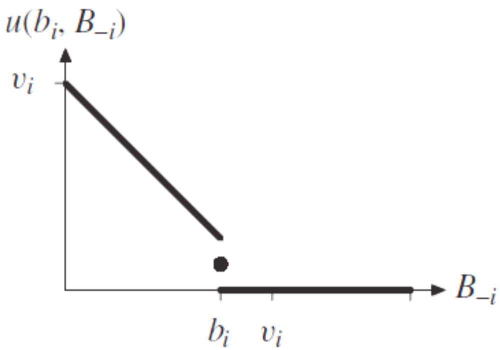
➔ *There is a disadvantage bidding higher and no advantage bidding lower*

Second-Price Sealed Bid Proof (alternative)

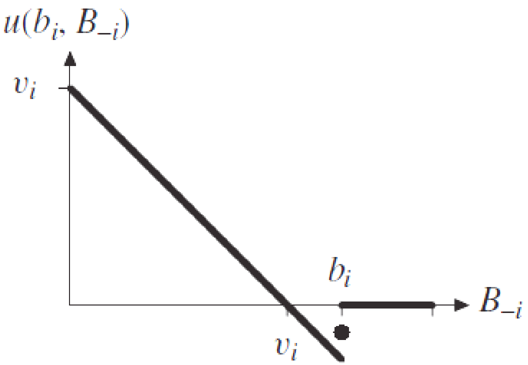
$u(b_i, B_{-i})$ … i's bidder payoff when bidding b_i and when the highest of the other bidders (except i) is B_{-i}.



$b_i = v_i$
(i.e. i bids its valuation)



$b_i < v_i$
(i.e. i bids lower than its valuation)



$b_i > v_i$
(i.e. i bids higher than its valuation)

(from Maschler, page 93 and 94)

Second-Price Sealed Bid

Advantages:

- **Truthful** bidding is dominant strategy
- No incentive for counter-speculation
- Computational efficiency

Disadvantages:

- Lying auctioneer
- Bidder collusion self-enforcing
- Reveals true valuations
- Not revenue maximizing

Bidding in First-Price Sealed Bid Auctions

Dutch and First-price Sealed Bid

Strategically equivalent: an agent bids without knowing about the other agents' bids (i.e. difference are technical implementation)

- a bidder must decide on the amount he's willing to pay, conditional on having placed the highest bid

Differences

- First-price auctions can be held **asynchronously**
- Dutch auctions are **fast**, and require **minimal communication**
  - only **one bit** needs to be transferred from the bidders to the auctioner

Bidding in Dutch / First Price Sealed Bid

How should bidders bid in these auctions?

There's a **trade-off** between **probability of winning** vs. **amount paid** upon winning (and thus the winner's surplus)

➔ Bidders don't have a **dominant strategy** any more:

Individually optimal strategy depends on the **assumptions** about **others' valuations.**

We have a Bayesian game → **Bayes-Nash equilibrium**: a strategy profile that maximizes the expected payoff for each player given their **beliefs** and given the strategies played by the other players.

Equilibrium Strategy

Assume a **first-price auction** with **two risk-neutral bidders** whose valuations are drawn independently and **uniformly** at random from the interval [0, 1] - what is the equilibrium strategy?

→ $\left(\frac{1}{2}v_1, \frac{1}{2}v_2\right)$ is the Bayes-Nash equilibrium strategy profile

Interim expected utility

Given a Bayesian game (N, A, Θ, p, u) with *finite* sets of players, actions, and types, player i's ***interim* expected utility** with respect to *type* θ_i and a *mixed strategy profile* s is

$$EU_i(s|\theta_i) = \sum_{\theta_{-i} \in \Theta_{-i}} p(\theta_{-i}|\theta_i) \sum_{a \in A} \left(\prod_{j \in N} s_j(a_j|\theta_j) \right) u_i(a, \theta_i, \theta_{-i})$$

Proof

Assume that bidder 2 bids $\frac{1}{2}v_2$, and that bidder 1 bids s_1.

Bidder 1 wins when $\frac{1}{2}v_2 < s_1$, gaining payoff $v_1 - s_1$, but loses when $v_2 > 2s_1$ and then gets utility 0 (we can ignore the case where the agents have the same valuation, because this occurs with probability zero).

$$EU_1(s|v_1) = \int_0^{2s_1}(v_1 - s_1)dv_2 + \int_{2s_1}^{1}(0)dv_2 =$$

$$= (v_1 - s_1)v_2|_0^{2s_1} = 2v_1s_1 - 2s_1^2$$

Proof Continued

We can determine bidder 1's best response to bidder 2' strategy by taking the derivative and setting it to zero:

$$\frac{\partial}{\partial s_1}(2v_1 s_1 - 2s_1^2) = 0$$

$$2v_1 - 4s_1 = 0$$

$$s_1 = \frac{1}{2}v_1$$

Thus, when player 2 is bidding half her valuation, player 1's best reply is to **bid half his valuation** (and analogously for player 2, given the symmetry of the game)

Equilibrium in Dutch / First Price Sealed Bid Auctions

Theorem

In a first-price sealed bid auction with n **risk-neutral** agents whose valuations $v_1, v_2, \dots, v_n$ are **independently** drawn from a **uniform distribution** on the **same** bounded **interval** of the real numbers, the **unique** symmetric **equilibrium** is given by the **strategy profile** $\left(\frac{n-1}{n}v_1, \dots, \frac{n-1}{n}v_n\right)$.

The more players, **the harder to win** and the lower the expected surplus.

$\Rightarrow$ Dutch / FPSB auctions **not incentive compatible,** i.e., there are incentives to **counter-speculate**.

Equilibrium in Dutch / First Price Sealed Bid Auctions

For non-uniform valuation distributions: Each bidder should bid **the expectation of the second-highest valuation**, conditioned on the assumption that his own valuation is the highest.

Equilibrium in more general cases?

Note we only **verified** the equilibrium.

What about more general assumptions?

→ We need to guess the equilibrium and it gets more complicated as we relax the assumptions about the distributions of valuations (non-uniformity, no symmetry etc.).

Even determining a Nash equilibrium exists gets difficult.

This because auctions are **non-continuous games**: even a small variation in the bid amount can lead to not-winning and thus large changes in the payoff.

Bidding in English and Japanese Auctions

English and Japanese Auctions Analysis

A much more complicated **strategy space**

- extensive-form game
- bidders are able to condition their bids on information revealed by others
- in the case of English auctions, the ability to place jump bids

Intuitively, though, the **revealed information** does not make any **difference** in the **independent-private value** (IPV) setting.

English and Japanese Auctions Analysis

Theorem

Under the IPV model, it is a **dominant strategy** for bidders to bid **up to** (and not beyond) their valuations in both Japanese and English auctions.

In correlated-value auctions, it can be worthwhile to counter-speculate.

Seller’s Revenue

Which auction should the seller choose?

Expected Seller's Revenue (First Price Sealed Bid Auction)

$$\mathrm{E}\left(\max\left\{\frac{V_1}{2},\frac{V_2}{2}\right\}\right) = \frac{1}{2}\mathrm{E}(\max\{V_1,V_2\} =?$$

Let $Z = \max\{V_1, V_2\}$

$$F_Z(z) = \mathrm{P}(Z \leq z) = \mathrm{P}(\max\{V_1,V_2\} \leq \mathrm{z}) = \mathrm{P}(V_1 \leq z) \cdot \ \mathrm{P}(V_2 \leq z) = \mathrm{z}^2$$

$$\Rightarrow f_Z(z) = \begin{cases} 2z & \text{if } 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{1}{2}E(Z) = \frac{1}{2}\int_0^1 z f_z(z)dz = \int_0^1 z^2 dz = \frac{1}{3}z^3\Big|_0^1 = \mathbf{\frac{1}{3}}$$

Expected Seller's Revenue

$\mathrm{E}(\min\{V_1, V_2\}) = ?$

Note:

$$\min\{V_1, V_2\} + \max\{V_1, V_2\} = V_1 + V_2$$

Hence:

$$\mathrm{E}(\min\{V_1, V_2\}) + \mathrm{E}\,(\max\{V_1, V_2\}) = \mathrm{E}(V_1) + \mathrm{E}(V_2) = \frac{1}{2} + \frac{1}{2} = 1$$

We already calculated (previous slide):

$$\mathrm{E}\,(\max\{V_1, V_2\}) = \mathrm{E}(Z) = \frac{2}{3}$$

Hence:

$$\mathrm{E}\,(\min\{V_1, V_2\}) = \mathbf{\frac{1}{3}}$$

Seller's Revenue

Corrolary (Expected Seller's Revenue)

In the symmetric case with two risk-neutral bidders and IPV, the **expected seller's revenue** from the first-price and second-price sealed bid auction is **the same**.

Somewhat surprising and far from self-evident.

Revenue Equivalence

In fact, holds in more general.

Theorem (Revenue Equivalence)

Assume that each of n **risk-neutral** agents has an **independent private valuation** for a single good at auction, drawn from a **common cumulative distribution** $F(v)$ that is **strictly increasing** and **atomless** on $[\underline{v}, \overline{v}]$. Then any auction mechanism in which

1. the good will be allocated to the agent with the highest valuation; and
2. any agent with valuation $\underline{v}$ has an expected utility of zero

yields the **same expected revenue**, and hence results in any bidder with valuation v making the **same expected payment**.

Revenue Equivalence

Informally: As long as two mechanism **allocate in the same way** and they **do not charge anything** to the agent with the lowest valuation, the **rest of payment functions** is the same.

You cannot get **extra money** from bidder without changing the allocation function or the payment to the lowest-valued bidder.

In fact, the revenue equivalence holds beyond IPV and single good.

Assuming bidders are risk neutral and have independent private valuations, **all the auctions** we have spoken about so far—English, Japanese, Dutch, and all sealed bid auction protocols—are **revenue equivalent**.

Applying Revenue Equivallence

Expected value of the k^{th}-largest of n IID draws* from $[0, v_{max}]$:

$$\frac{n+1-k}{n+1} v_{max}$$

Expected seller's revenue in the second-price auction (with IID valuations):

$$\frac{n-1}{n+1} v_{max}$$

* termed *k-th order statistics*

Applying Revenue Equivallence

Both second-price and first-price auction **satisfies** the conditions of the revenue equivalence theorem.

Thus, a bidder in the **first-price auction** must **bid his expected payment** conditional on being the **winner of a second price auction**.

If v_i is the high value, there are $n-1$ other values drawn from the uniform distribution on $[0, v_i]$. The expected value of the second-highest bid is therefore the first-order statistics of $n-1$ draws from $[0, v_i]$, which is

$$\frac{n-1}{n} v_{max}$$

We still need to verify the above is an equilibrium
(the revenue equivalence theorem does not state not every revenue-equivalent strategy profile is an equilibrium)

Auctions Summary

Auctions are mechanisms for **allocating scarce resource** among **self-interested agent**

Mechanism-design and game-theoretic perspective

Many auction mechanisms: English, Dutch, Japanese, First-price sealed bid, Second-price sealed bid

Desirable properties: truthfulness, efficiency, optimality, ...

Rapidly expanding list of **applications** worth billions of dollars

Reading:

- [Shoham] – Chapter 11