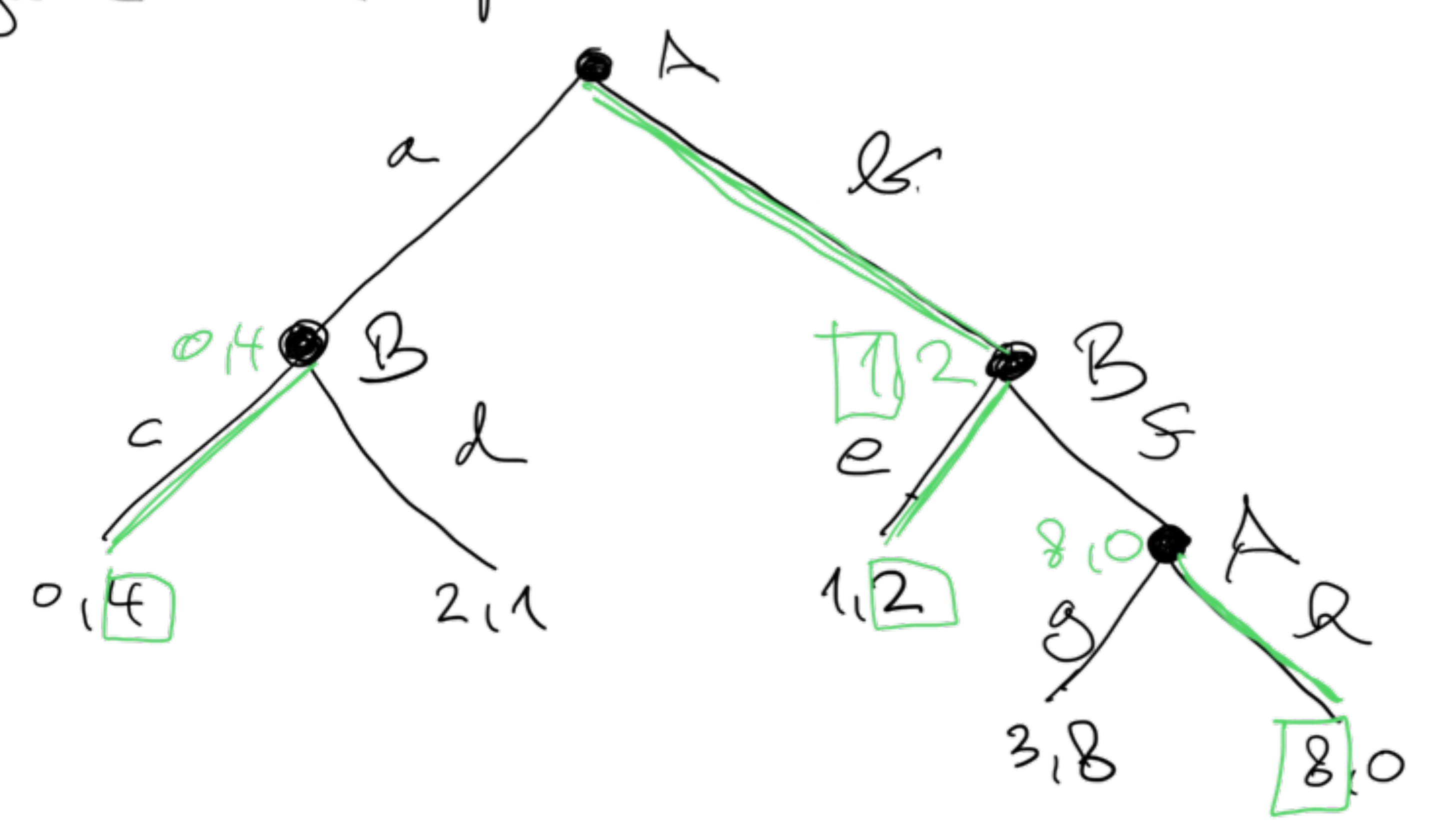
① A game with perfect information



	ce	cf	de	df
ag	0,4	0,4	2,1	2,1
ah	0,4	0,4	2,1	2,1
bg	1,2	3,8	1,2	3,8
bh	$\boxed{1,2}$	8,0	1,2	8,0

NE

No strictly dominated strategy.

② Iterative elimination and weak dominance

	x	y	z
a	10,0	5,1	4,−2
b	10,1	5,0	1,−1

Order 1: Eliminate b, then x and z ⇒ (a,y)

Order 2: Eliminate z. There are no other dominated strategies.

③ Truthful bidding in 2nd price auction

Claim: $u_i(v_i, b_{-i}) \geq u_i(b_i, b_{-i})$

$\forall i \in N, \forall b_{-i}, b_i$

Proof.

First, assume that $b_i < v_i$, that is, player i is underbidding.

We distinguish 3 cases:

(i) $\bar{b}_i < b_i$

$$\Downarrow$$

$$u_i(v_i, b_{-i}) = u_i(b_i, b_{-i}) = v_i - \bar{b}_i$$

(ii) $\bar{b}_i = b_i$

$$u_i(v_i, b_{-i}) = v_i - b_i$$

$$u_i(b_i, b_{-i}) = \frac{v_i - b_i}{k}, \quad k \geq 2$$

(iii) $\bar{b}_i > b_i$

$$u_i(v_i, b_{-i}) \geq 0$$

$$u_i(b_i, b_{-i}) = 0$$

We proceed analogously when $b_i > v_i$.

It is instructive to draw the dependence of the function $u_i(v_i, b_{-i})$ on b_{-i}. Note that the function depends only on $\bar{b}_i$, so we may write

$$u_i(v_i, \bar{b}_i) := u_i(v_i, b_{-i}).$$

We get: