Selected Topics in AI

Practical SAT Solving

Martin Suda
(based on slides of Mikoláš Janota and João Marques-Silva)

4 May 2026

Outline

Propositional Logics as a Formal Language

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Tautologyhood

A formula φ is *logically valid* (a tautology) iff for every assignment A we have $A \models \varphi$.

Further Important Notions

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A set of formulas Ψ logically entails a formula φ, written $\Psi \models \varphi$,
iff for every assignment A: if $A \models \psi$ for every $\psi \in \Psi$ then $A \models \varphi$.

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Note: Equivalent formulas have the same set of satisfying assignments.

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- ... *among other things*, e.g. restarts

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Preliminaries – CNF Formulas

- Variables: $w, x, y, z, a, b, c, \ldots$
- Literals: $w, \bar{x}, \bar{y}, a, \ldots$, but also $\neg w, \neg y, \ldots$
- Clauses: disjunction of literals or set of literals
- Formula: conjunction of clauses or set of clauses
- Model (satisfying assignment): partial/total mapping from variables to $\{0, 1\}$ that satisfies formula
- Formula can be SAT/UNSAT

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- Example:

$$\mathcal{F} \triangleq (r) \wedge (\bar{r} \vee s) \wedge (\bar{w} \vee a) \wedge (\bar{x} \vee b) \wedge (\bar{y} \vee \bar{z} \vee c) \wedge (\bar{b} \vee \bar{c} \vee d)$$

  - Example models:
    - $\{r, s, a, b, c, d\}$
    - $\{r, s, \bar{x}, y, \bar{w}, z, \bar{a}, b, c, d\}$

Resolution

- Resolution rule: [DP60,R65]

$$\frac{(\alpha \vee x) \qquad (\beta \vee \bar{x})}{(\alpha \vee \beta)}$$

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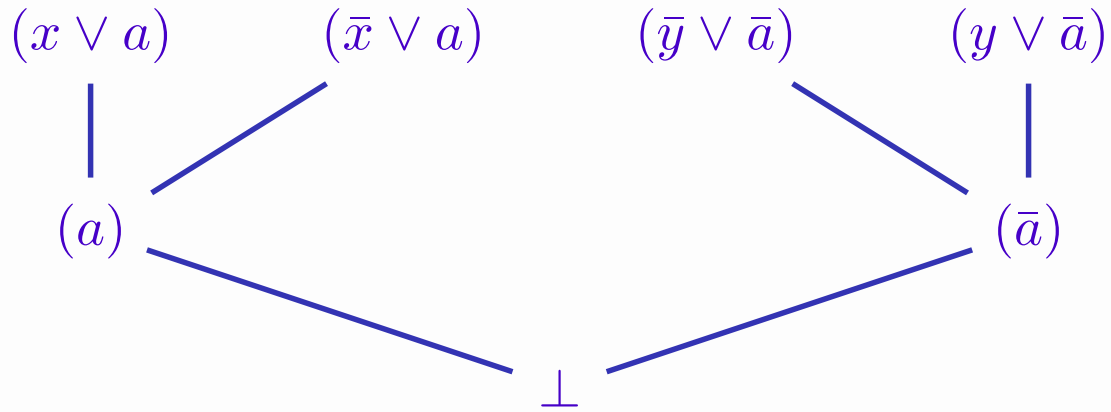

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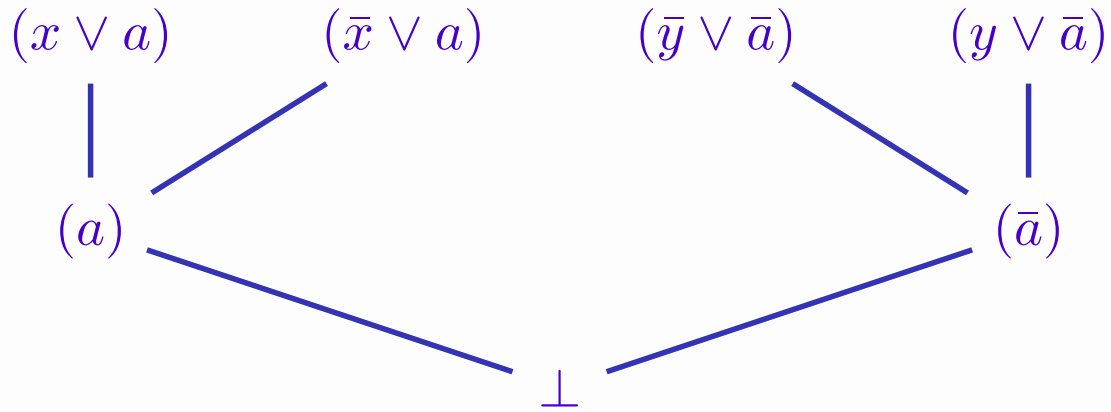

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- Self-subsuming resolution (with $\alpha' \subseteq \alpha$): [e.g. SP04,EB05]

$$\frac{(\alpha \vee x) \qquad (\alpha' \vee \bar{x})}{(\alpha)}$$

  - (α) subsumes $(\alpha \vee x)$

Resolution Proofs

- Refutation of unsatisfiable formula by iterated resolution operations produces resolution proof
- An example:
  $\mathcal{F} = (\bar{c}) \wedge (\bar{b}) \wedge (\bar{a} \vee c) \wedge (a \vee b) \wedge (a \vee \bar{d}) \wedge (\bar{a} \vee \bar{d})$

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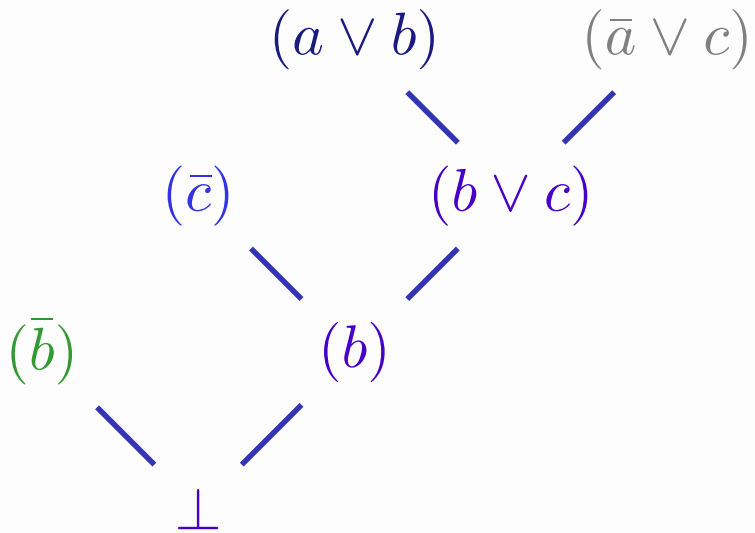

- SAT solvers can generate resolution proofs from learned clauses

[ZM03]

Unit Propagation

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 & \wedge & (\bar{r} \vee s) & \mathcal{B} \\
 & \wedge & (\bar{w} \vee a) & \mathcal{C} \\
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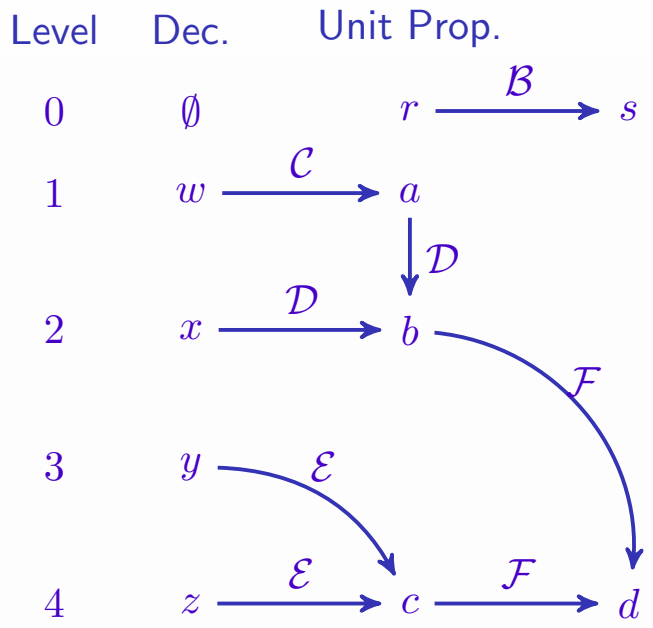

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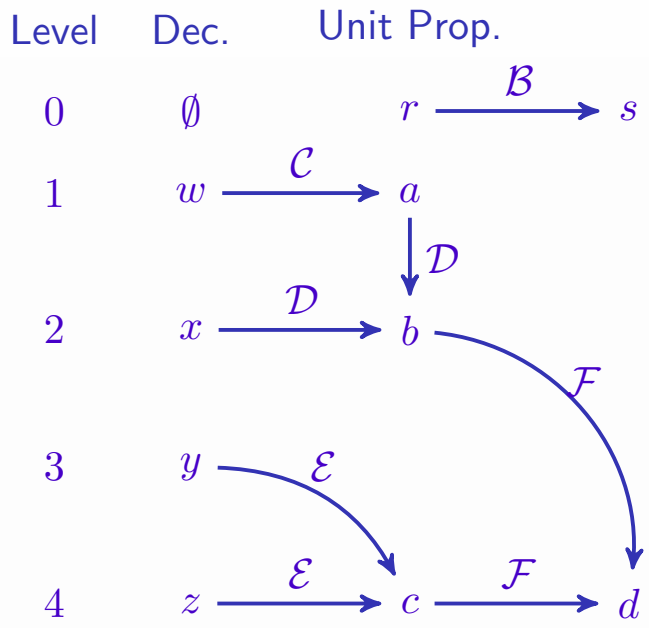

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- Additional definitions:
  - Antecedent (or reason) of an implied assignment
    - $(\bar{b} \vee \bar{c} \vee d)$ for d
  - Associate assignment with decision levels
    - $w = 1 @ 1$, $x = 1 @ 2$, $y = 1 @ 3$, $z = 1 @ 4$
    - $r = 1 @ 0$, $d = 1 @ 4$, ...

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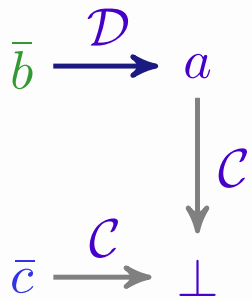
0	$\emptyset$	

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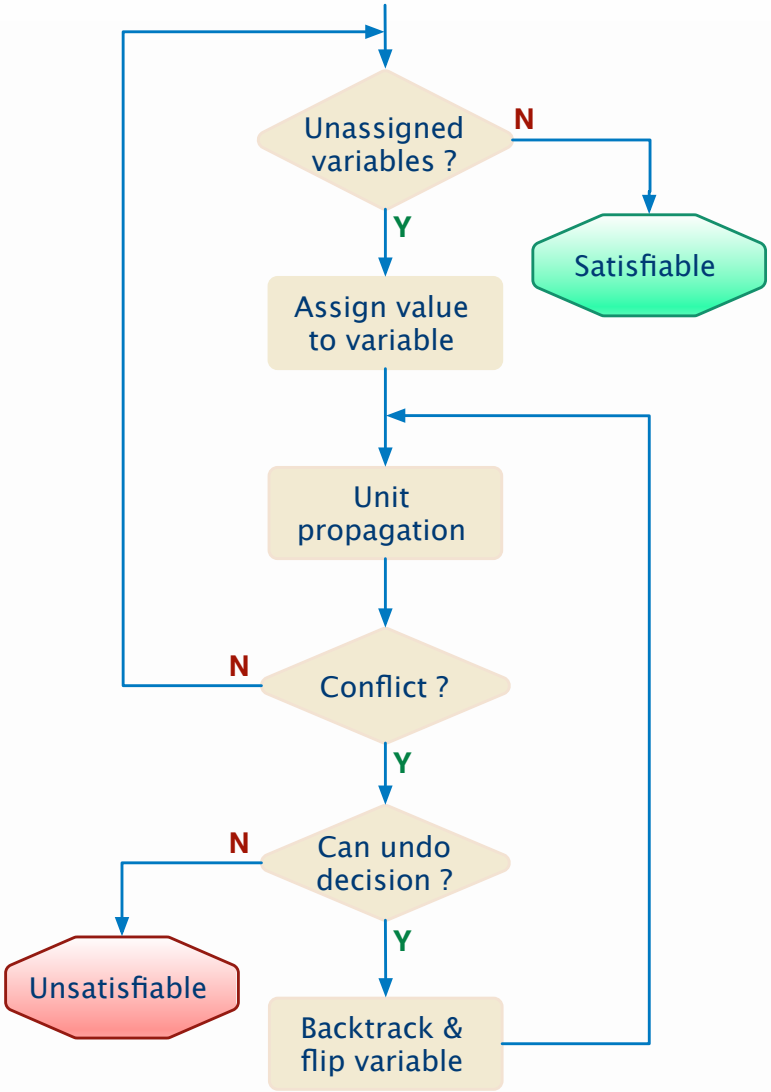
The DPLL Algorithm



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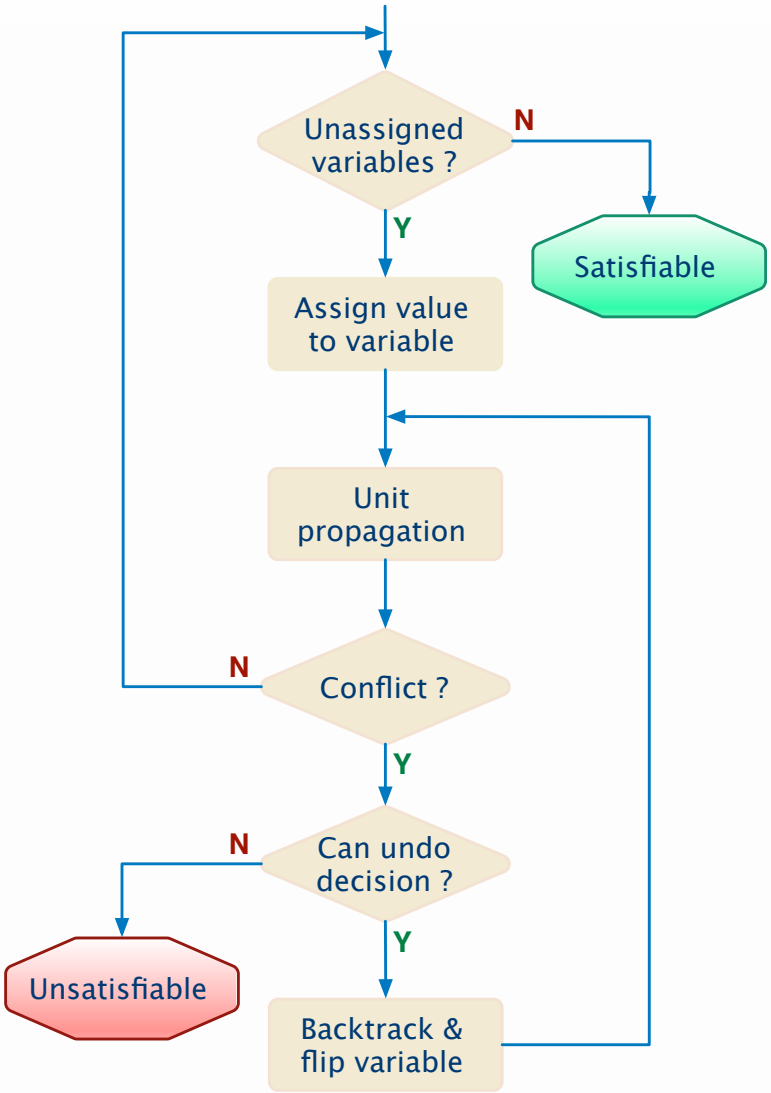

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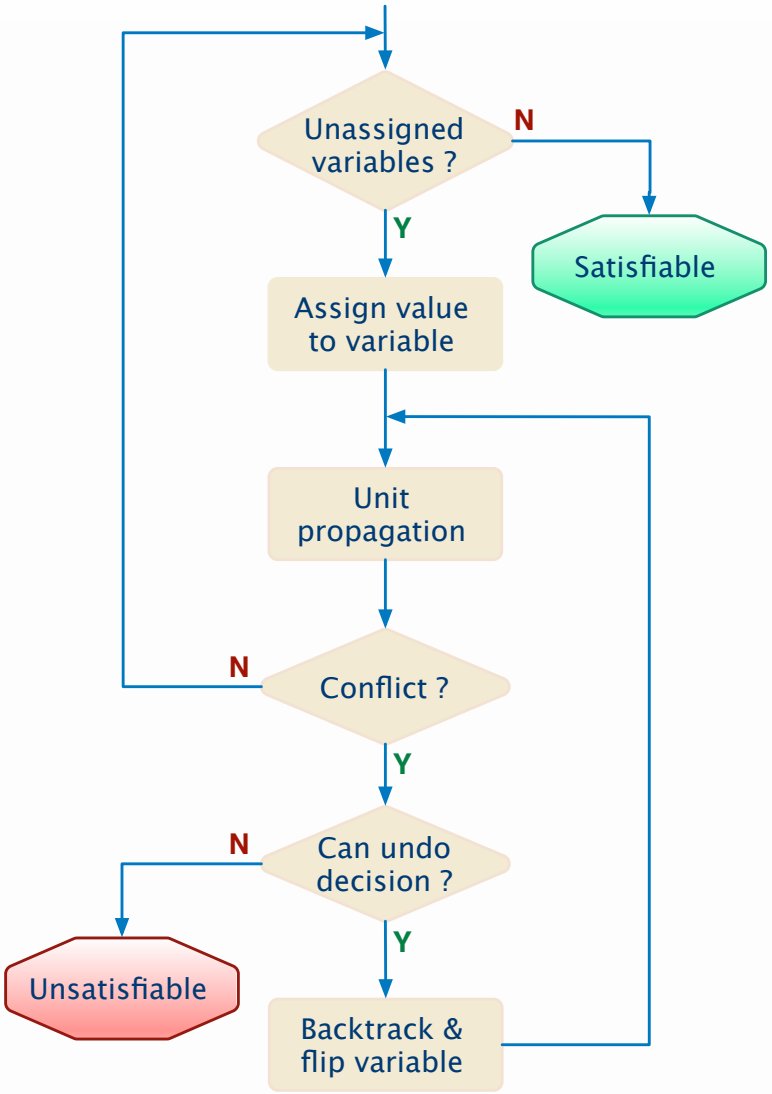

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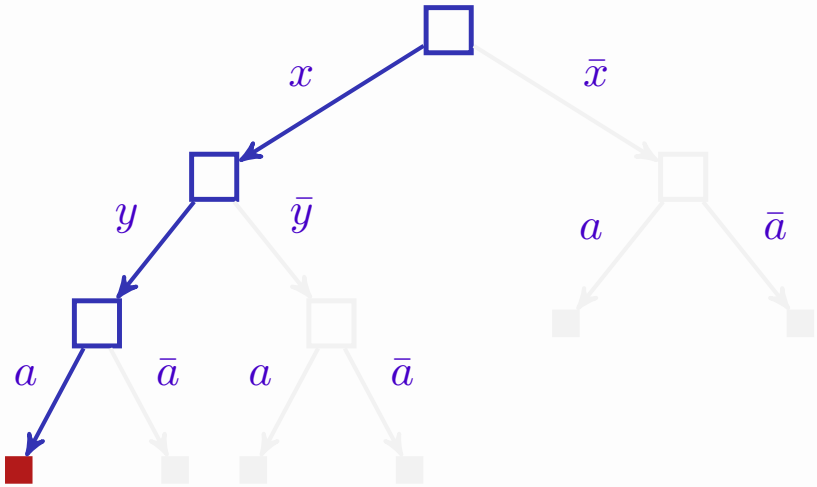

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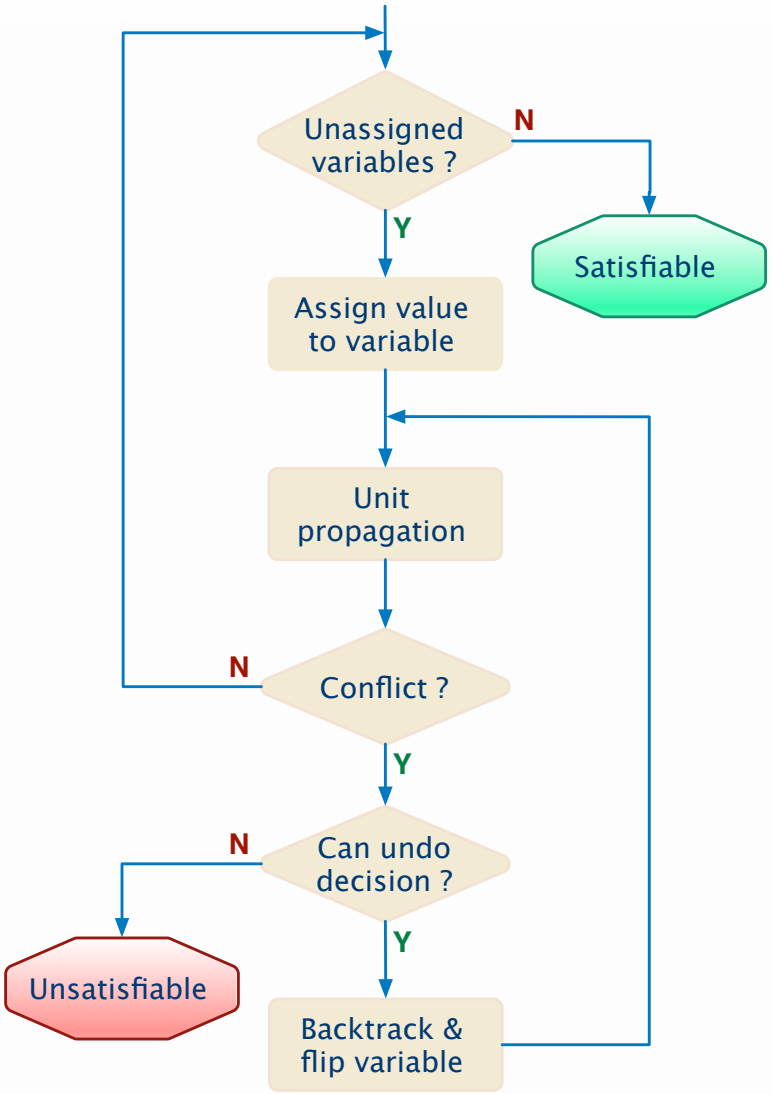

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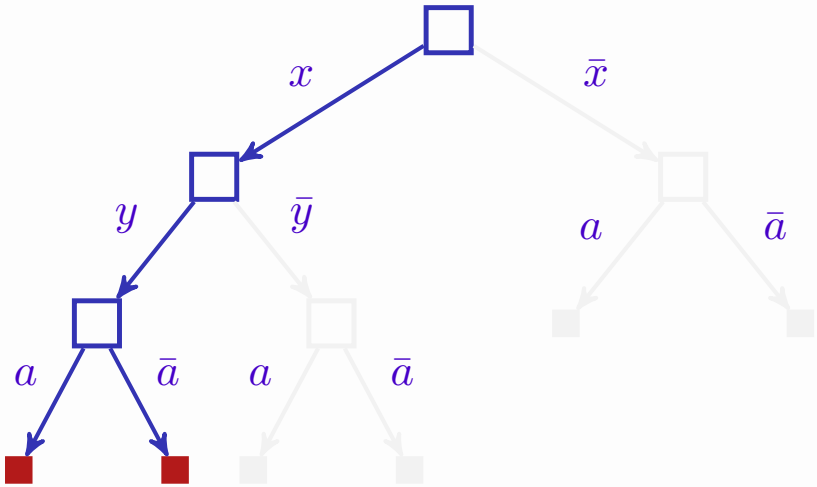

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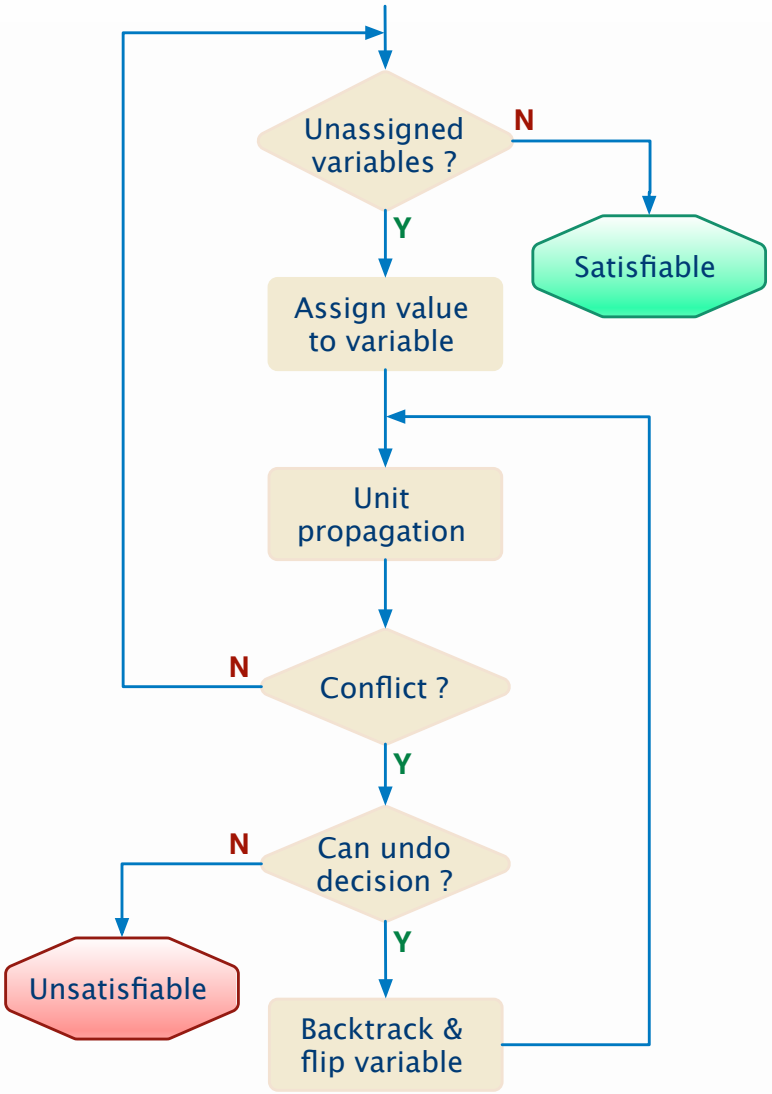

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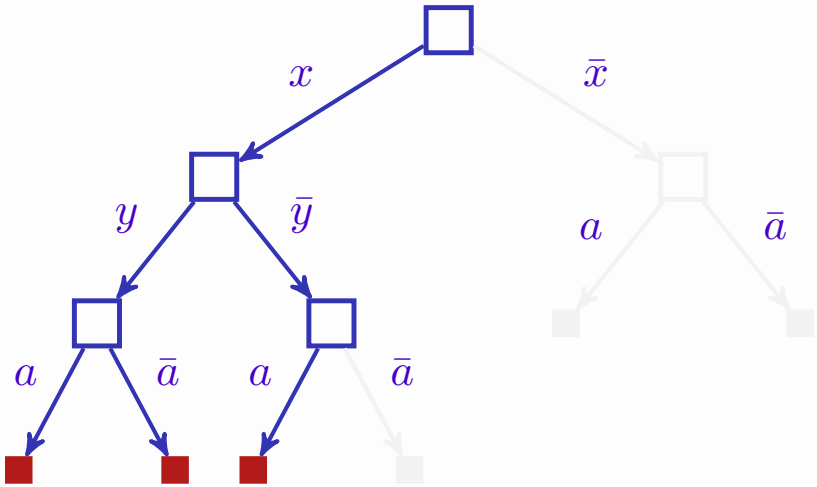

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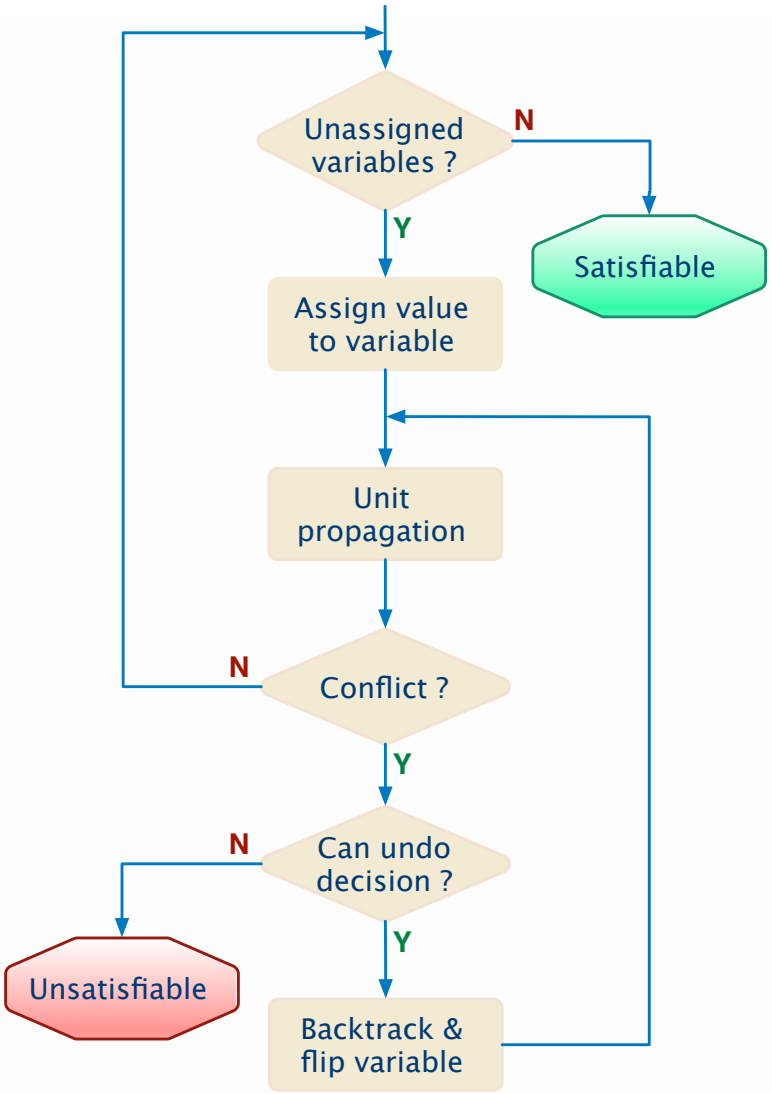

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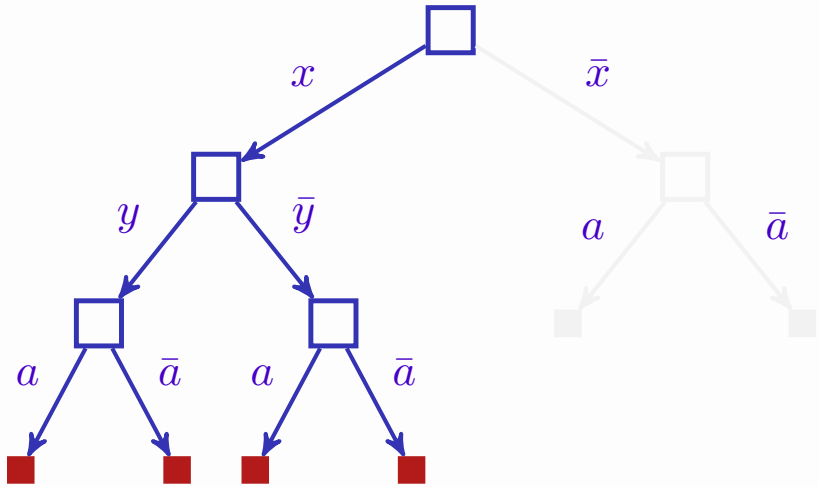
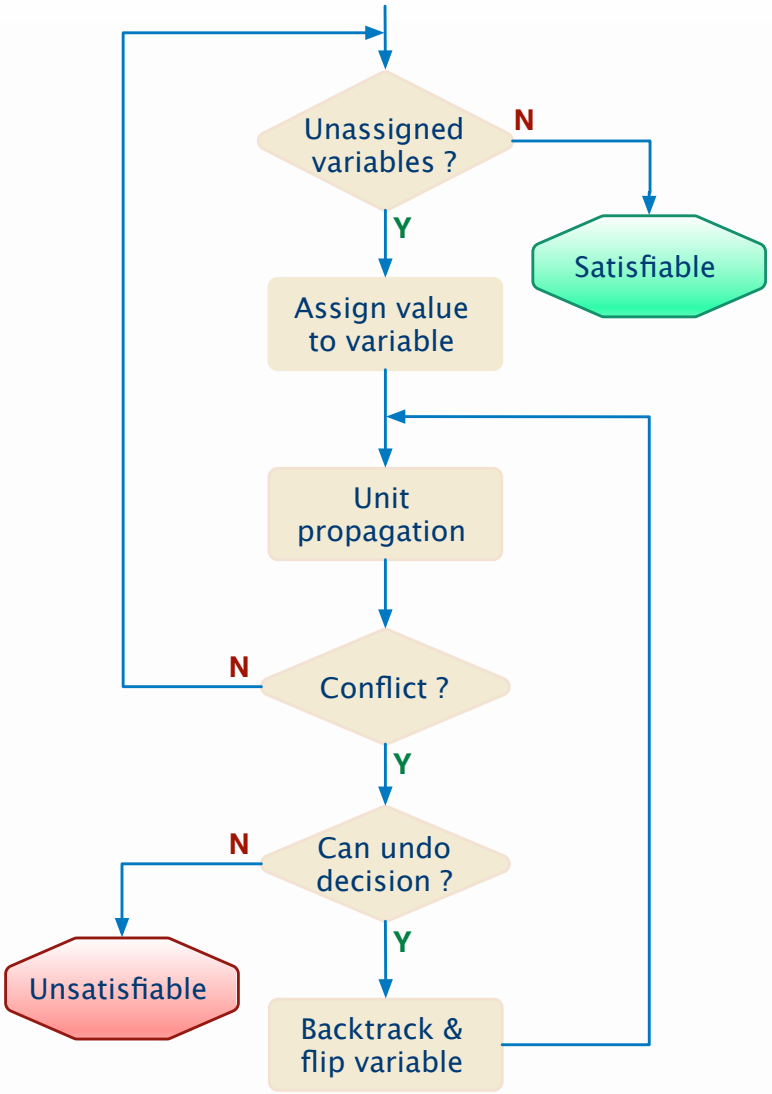


The DPLL Algorithm



$$\mathcal{F} = (x \vee y) \wedge (a \vee b) \wedge (\bar{a} \vee b) \wedge (a \vee \bar{b}) \wedge (\bar{a} \vee \bar{b})$$

Level	Dec.	Unit Prop.
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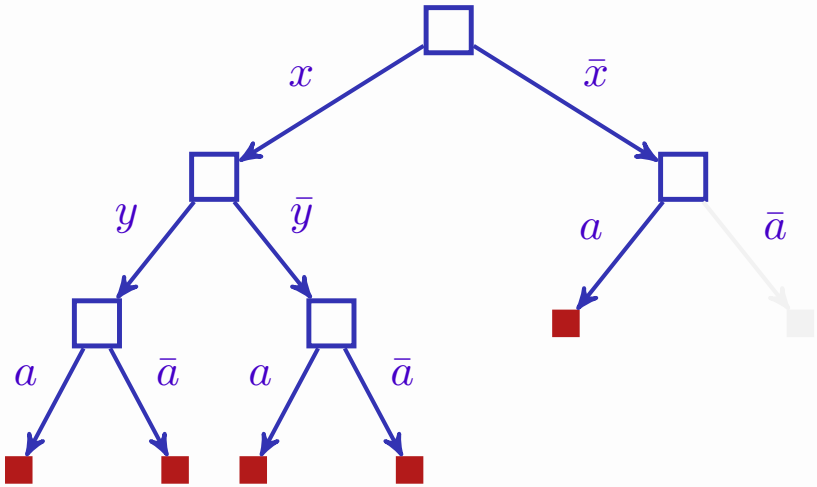

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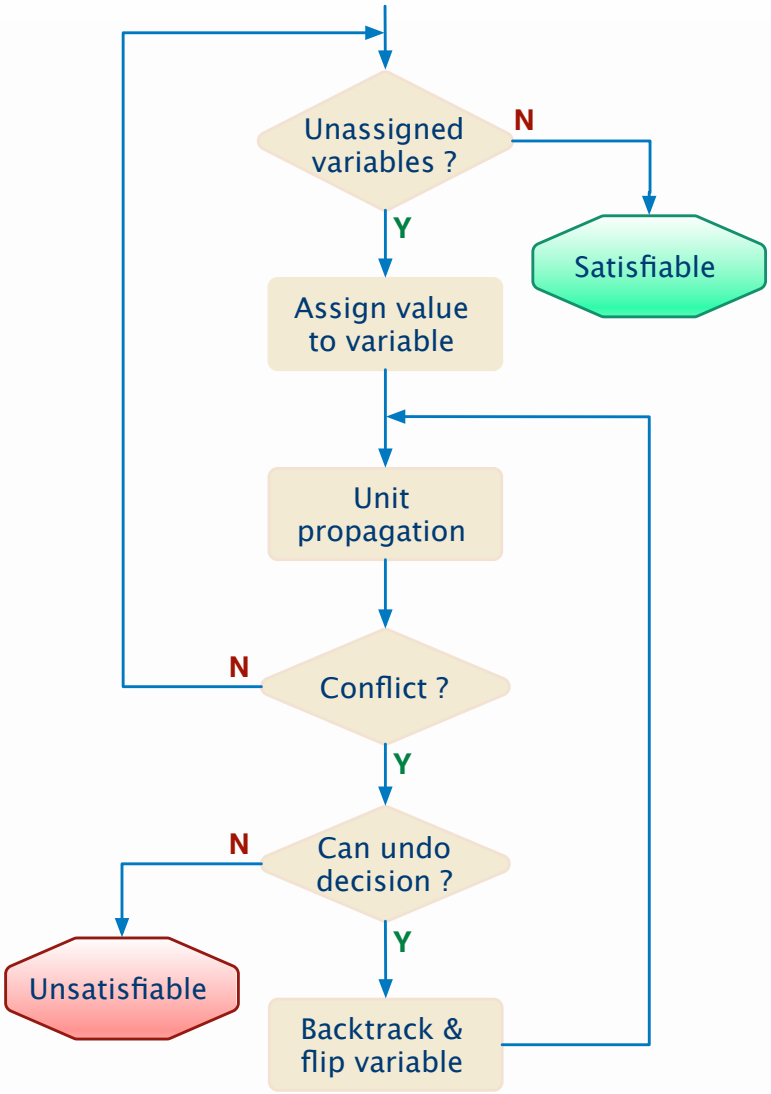

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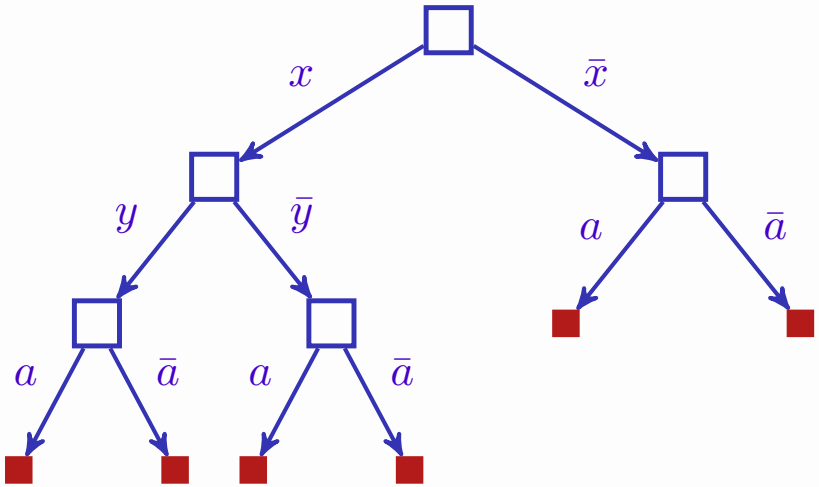
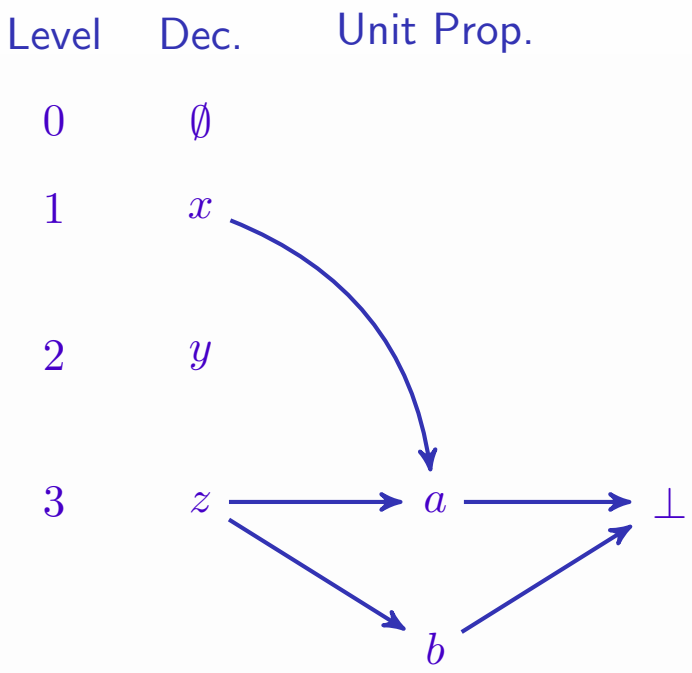


Outline

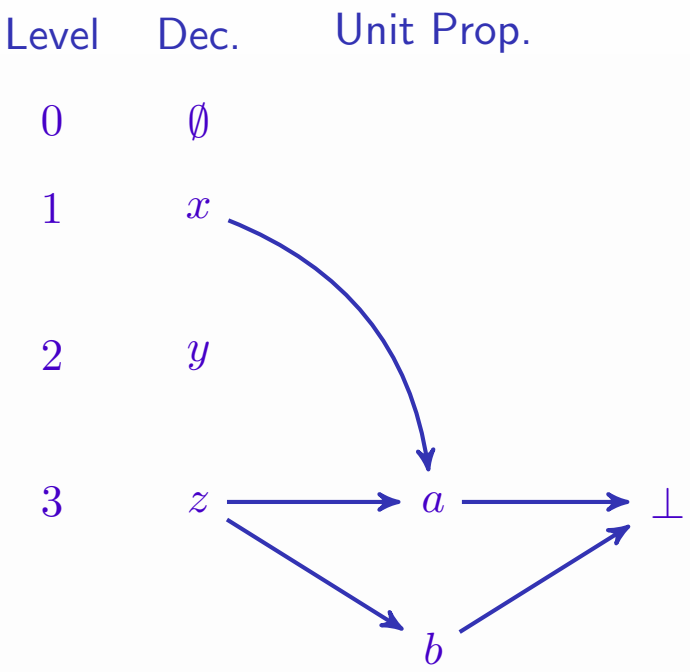
What is a CDCL SAT Solver?

- Extend DPLL SAT solver with: [DP60,DLL62]
  - Clause learning & non-chronological backtracking [MSS96,BS97,Z97]
    - Exploit UIPs [MSS96,SSS12]
    - Minimize learned clauses [SB09,VG09]
    - Opportunistically delete clauses [MSS96,MSS99,GN02]
  - Search restarts [GSK98,BMS00,H07,B08]
  - Lazy data structures
    - Watched literals [MMZZM01]
  - Conflict-guided branching
    - Lightweight branching heuristics [MMZZM01]
    - Phase saving [PD07]
  - ...

Conflict-Driven Clause Learning

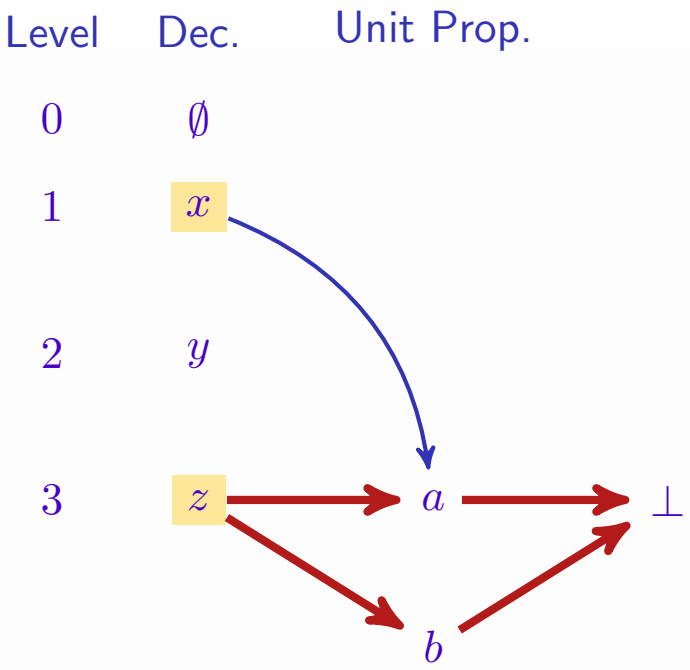


Conflict-Driven Clause Learning



- Analyze conflict

Conflict-Driven Clause Learning



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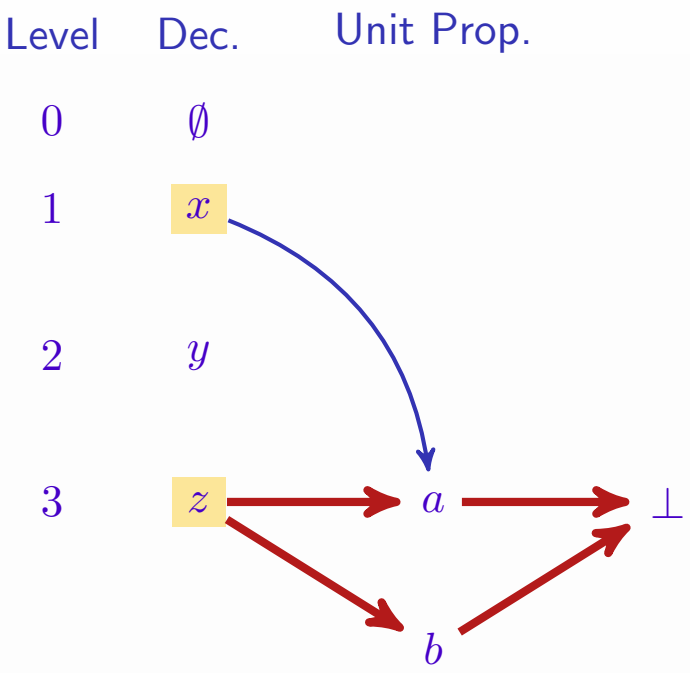
  - Reasons: x and z
    - Decision variable & literals assigned at lower decision levels

Conflict-Driven Clause Learning



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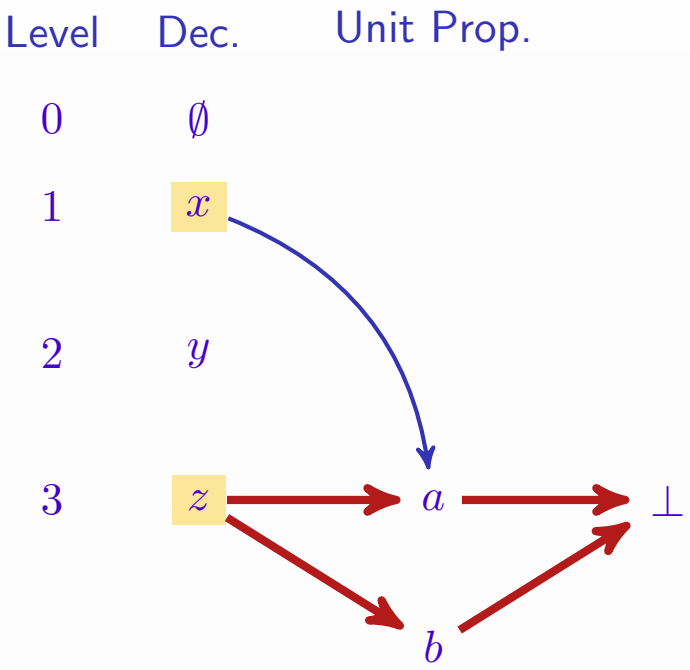

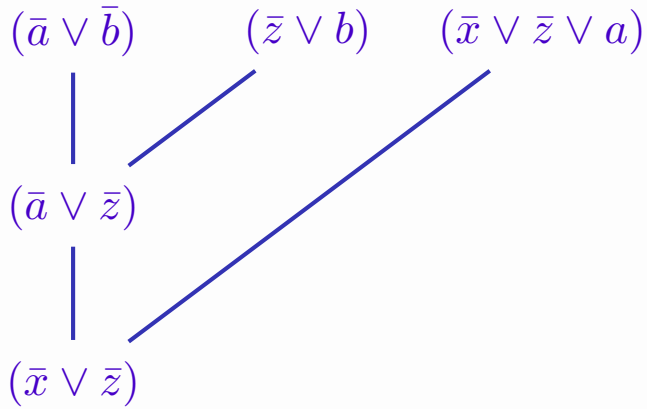

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  - Reasons: x and z
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  - Create **new** clause: $(\bar{x} \vee \bar{z})$
- Can relate clause learning with resolution
  - Learned clauses result from (**selected**) resolution operations

Practicals: Exercise

Encode into CNF the k-graph coloring problem

- k is a parameter of the encoding
- use provided colab as a starting point

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Advanced: encode the K-clique problem

- i.e., does the input graph has a clique of size K?
- hint: look at `pysat`'s `CardEnc.atmost(lits=[...], bound=k, encoding=...)`