AdaBoost

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Presentation

Motivation

AdaBoost with trees is the best off-the-shelf classifier in the world. (Breiman 1998)

Outline:

- AdaBoost algorithm
    - How it works?
    - Why it works?
- Online AdaBoost and other variants

What is AdaBoost?

AdaBoost is an algorithm for constructing a "strong" classifier as linear combination

$$f(x) = \sum_{t=1}^{T} \alpha_t h_t(x)$$

of "simple" "weak" classifiers $h_t(x) \colon \mathcal{X} \to \{-1, +1\}$.

Terminology

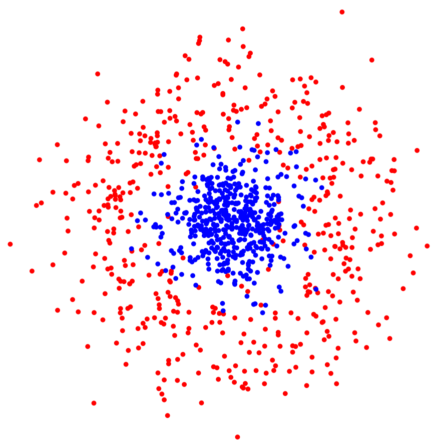
- $h_t(x)$ ... "weak" or basis classifier, hypothesis, "feature"
- $H(x) = sign(f(x))$ ... ''strong" or final classifier/hypothesis

Interesting properties

- AB is capable reducing both bias (e.g. stumps) and variance (e.g. trees) of the weak classifiers
- AB has good generalisation properties (maximises margin)
- AB output converges to the logarithm of likelihood ratio
- AB can be seen as a feature selector with a principled strategy (minimisation of upper bound on empirical error)
- AB is close to sequential decision making (it produces a sequence of gradually more complex classifiers)

The AdaBoost Algorithm

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$



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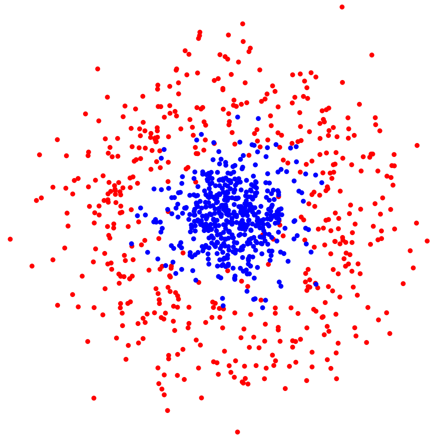

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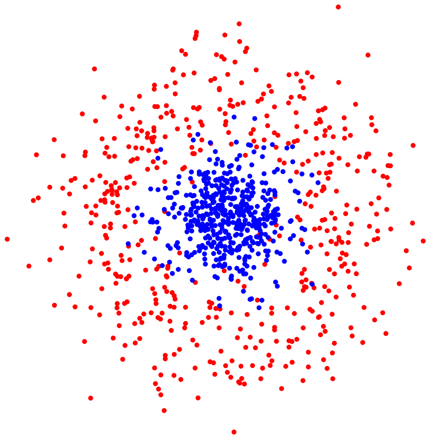

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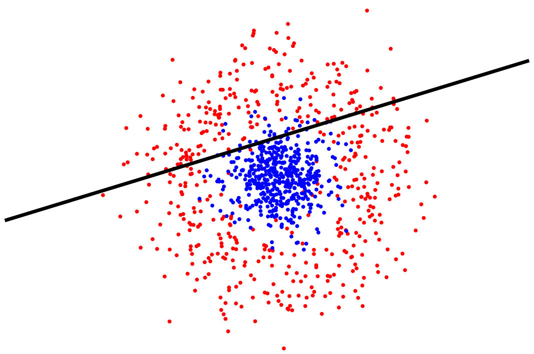
$t = 1$



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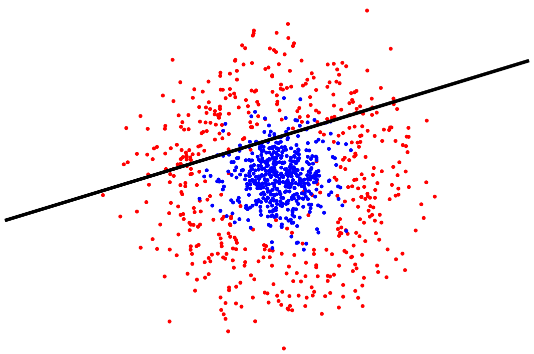

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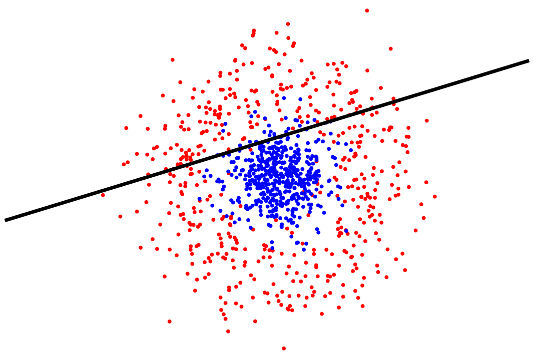

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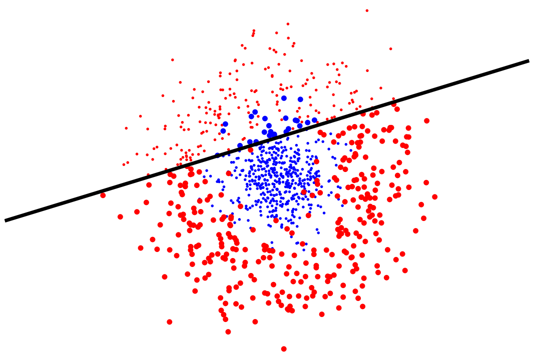
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

where Z_t is normalisation factor

$t = 1$



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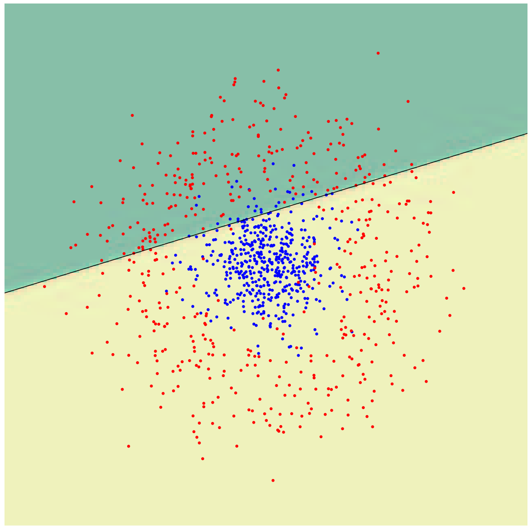
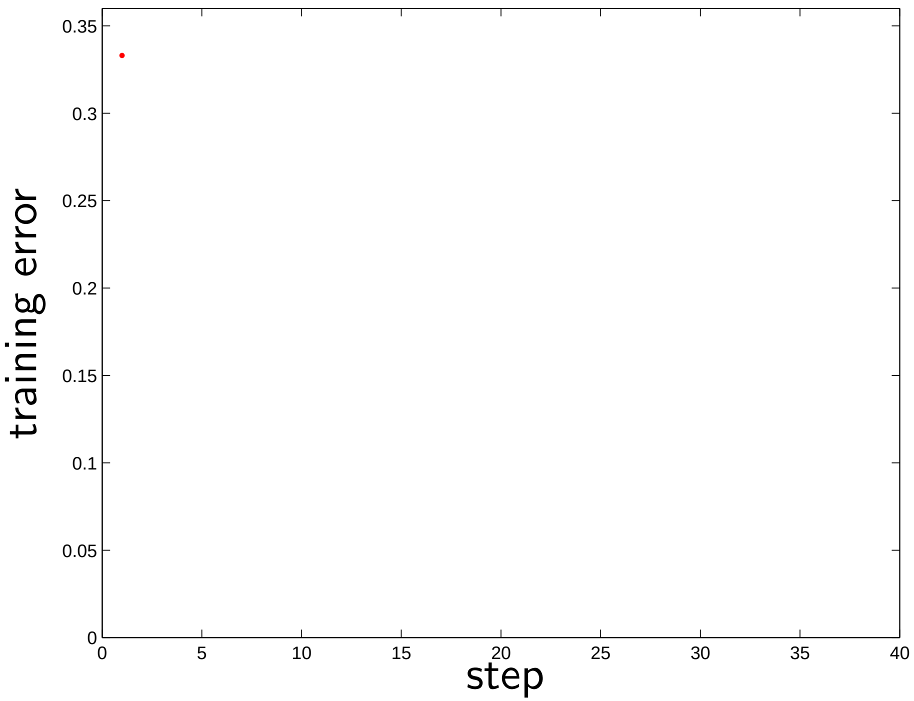
Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 1$





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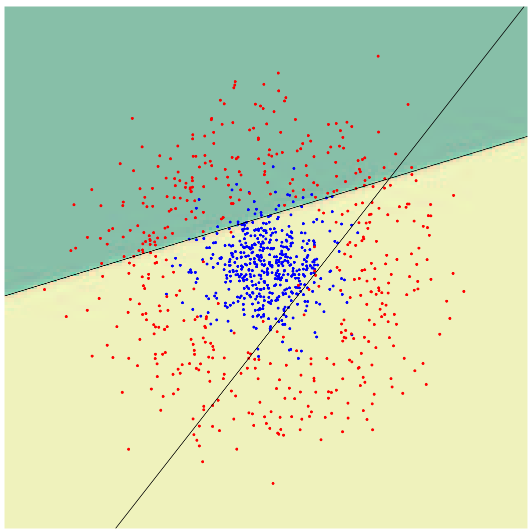
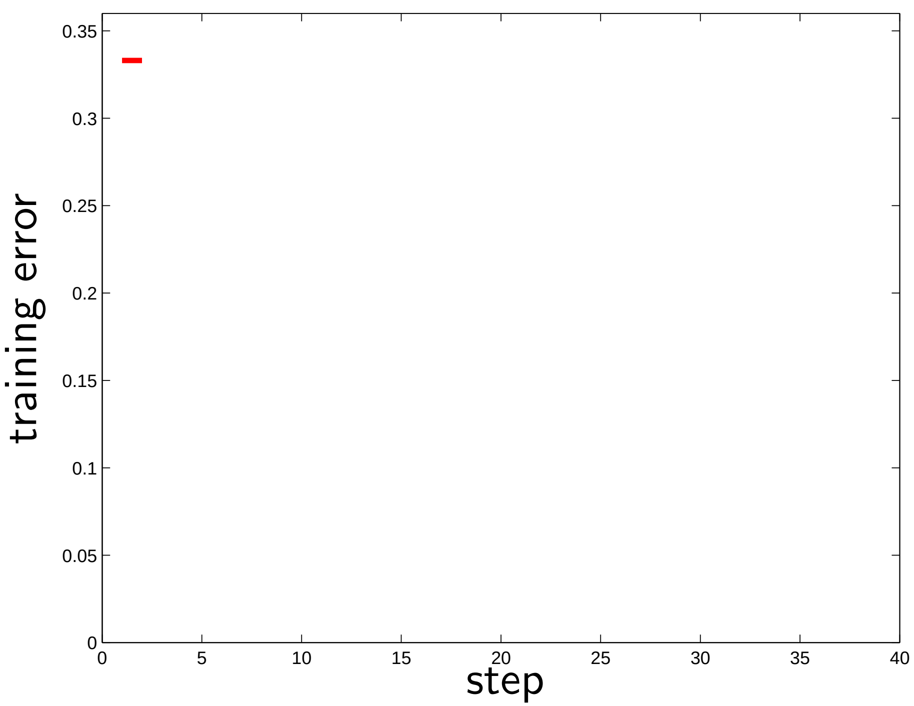
where Z_t is normalisation factor

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 2$





The AdaBoost Algorithm

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$

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- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

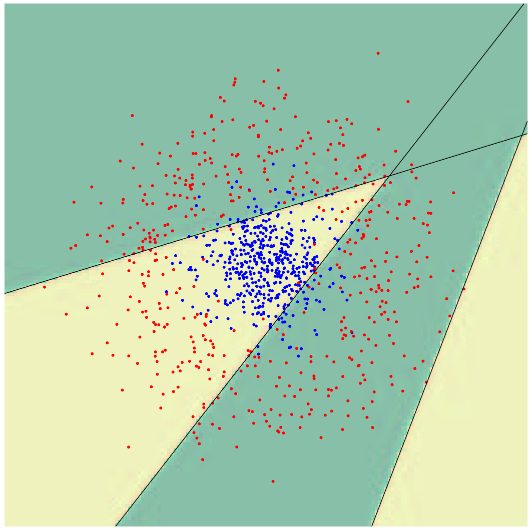
$$D_{t+1}(i) = \frac{D_t(i) exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

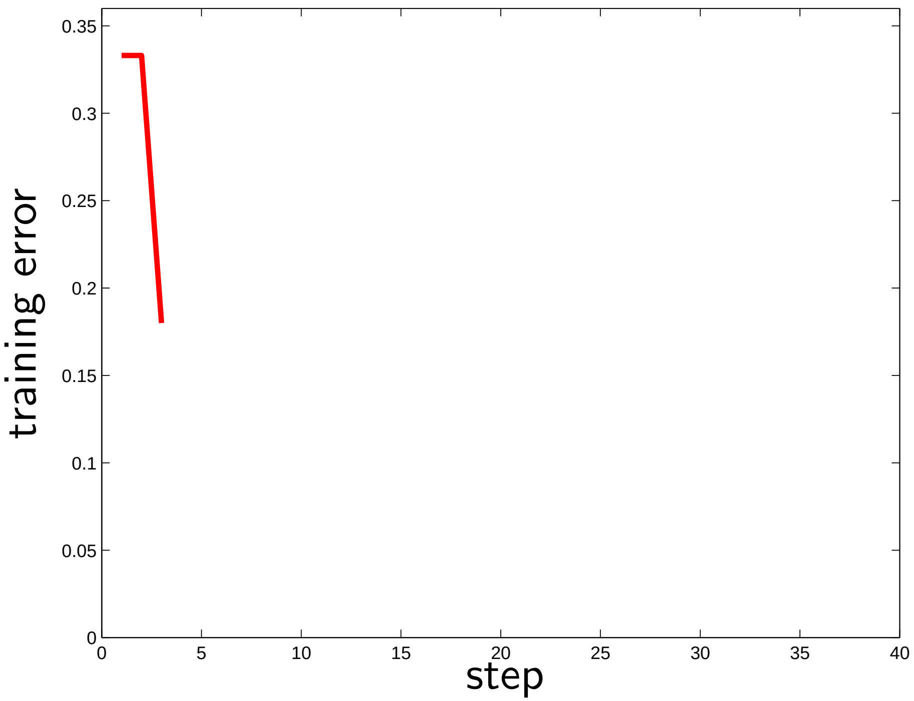
where Z_t is normalisation factor

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 3$





The AdaBoost Algorithm

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
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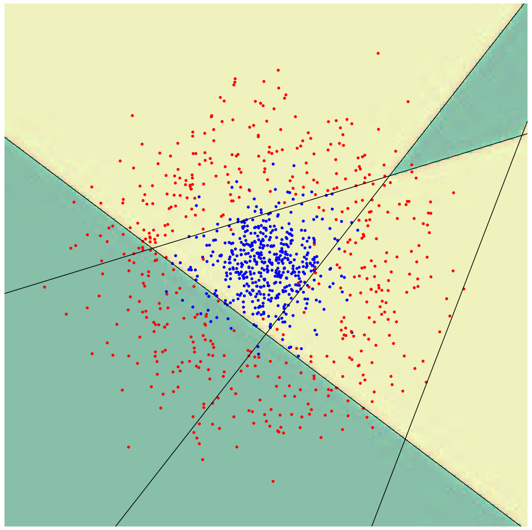
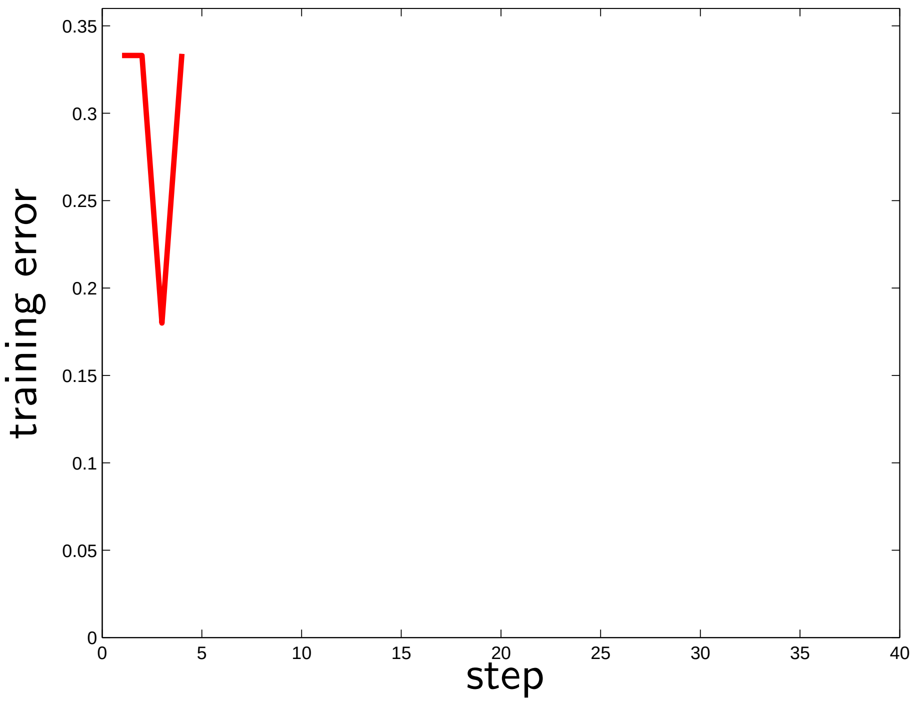
where Z_t is normalisation factor

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 4$





The AdaBoost Algorithm

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$

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Output the final classifier:

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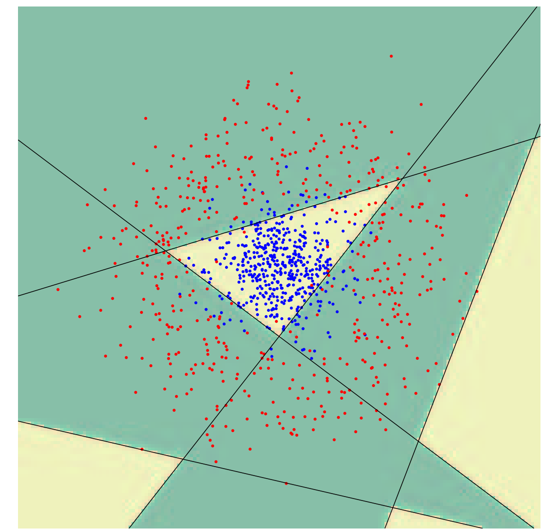
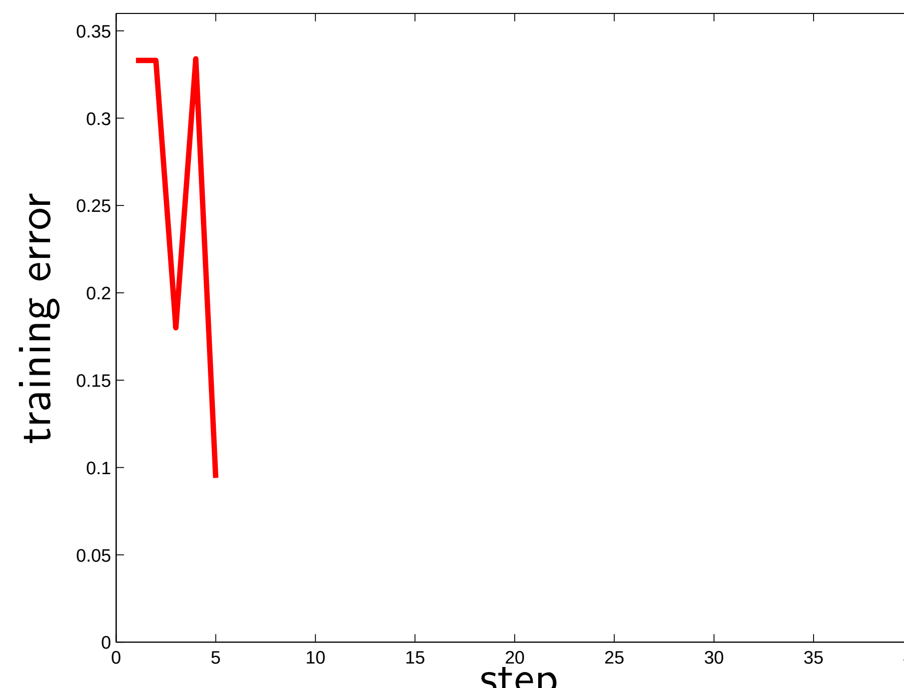
$t = 5$





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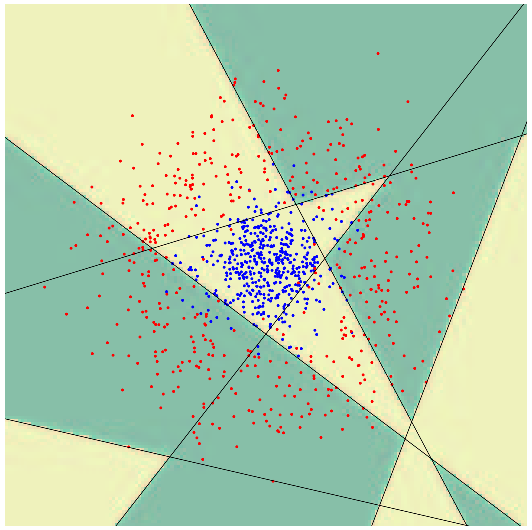
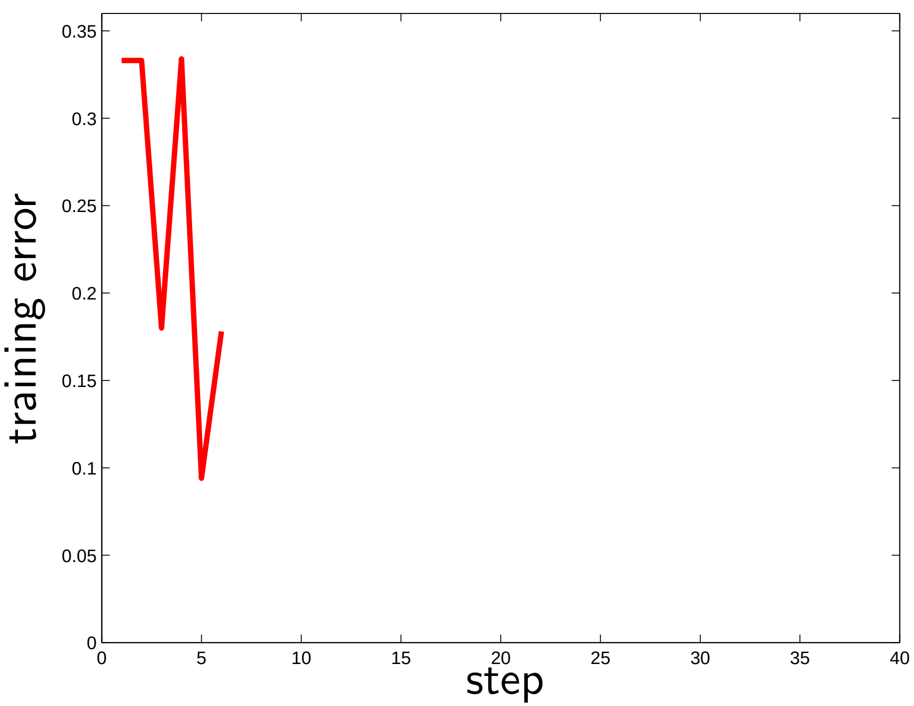
$t = 6$





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Output the final classifier:

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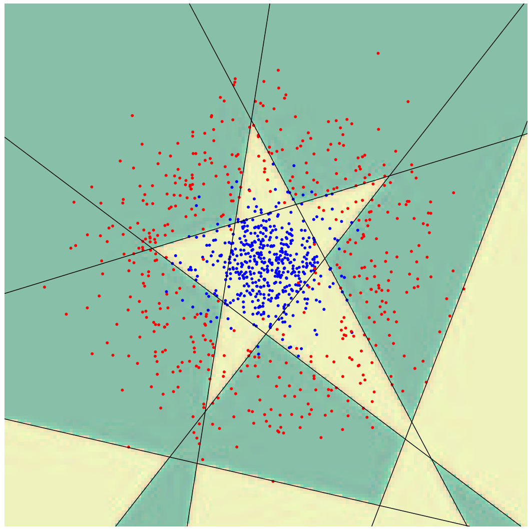
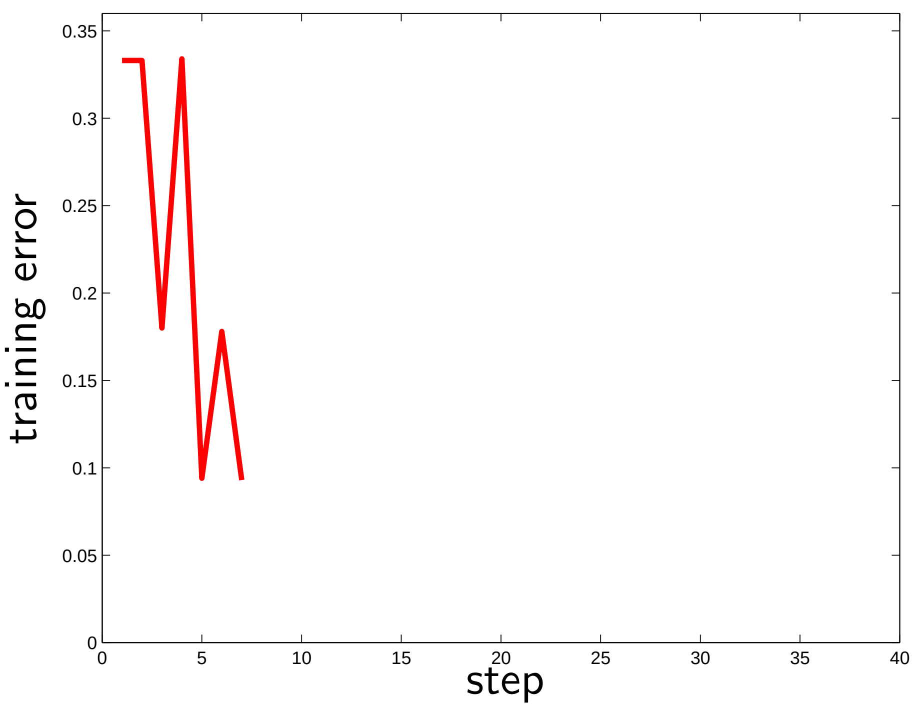
$t = 7$





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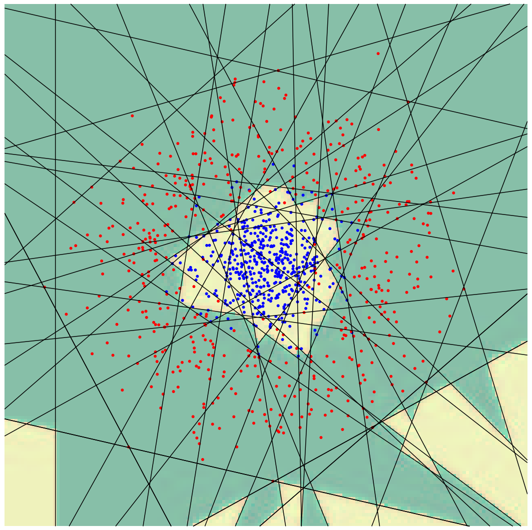
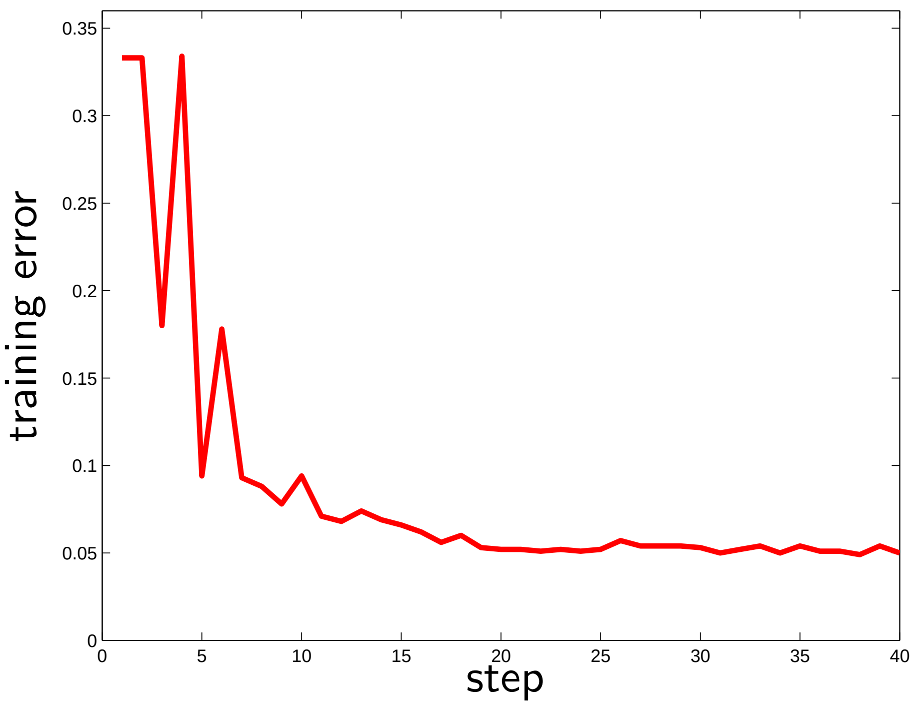
$t = 40$





Reweighting

Effect on the training set

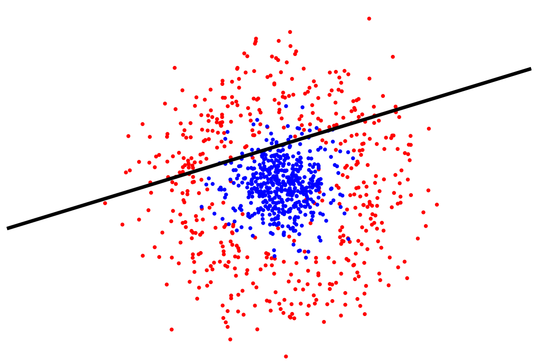
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

$$exp(-\alpha_t y_i h_t(x_i)) \begin{cases} < 1, & y_i = h_t(x_i) \\ > 1, & y_i \neq h_t(x_i) \end{cases}$$

$\Rightarrow$ Increase (decrease) weight of wrongly (correctly) classified examples

$\Rightarrow$ The weight is the upper bound on the error of a given example

$\Rightarrow$ All information about previously selected “features” is captured in D_t



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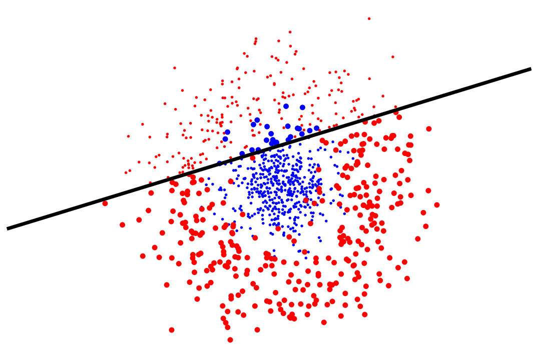

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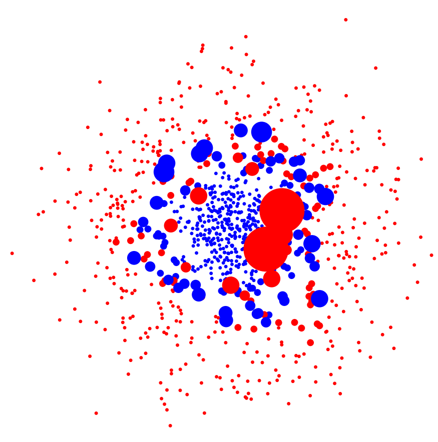


Upper Bound Theorem

Theorem: The following upper bound holds on the training error of H

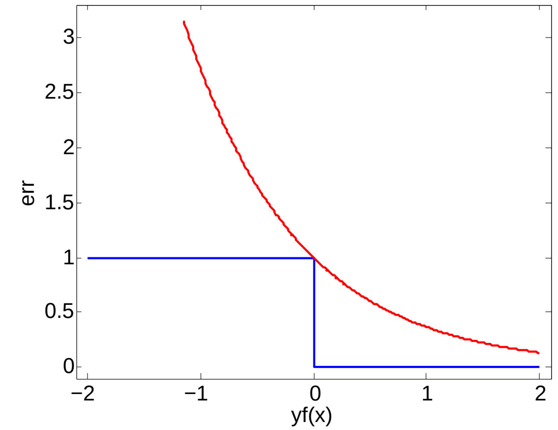
$$\frac{1}{m}|\{i : H(x_i) \neq y_i\}| \leq \prod_{t=1}^{T} Z_t$$

Proof: By unravelling the update rule

$$\begin{aligned} D_{T+1}(i) &= \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t} \\ &= \frac{exp(-\sum_t \alpha_t y_i h_t(x_i))}{m \prod_t Z_t} = \frac{exp(-y_i f(x_i))}{m \prod_t Z_t} \end{aligned}$$

If $H(x_i) \neq y_i$ then $y_i f(x_i) \leq 0$ implying that $exp(-y_i f(x_i)) > 1$, thus

$$\begin{aligned} [\![H(x_i) \neq y_i]\!] &\leq exp(-y_i f(x_i)) \\ \frac{1}{m}\sum_i [\![H(x_i) \neq y_i]\!] &\leq \frac{1}{m}\sum_i exp(-y_i f(x_i)) \\ &= \sum_i (\prod_t Z_t) D_{T+1}(i) = \prod_t Z_t \end{aligned}$$



Consequences of the Theorem

- Instead of minimising the training error, its upper bound can be minimised
- This can be done by minimising Z_t in each training round by:
  - Choosing optimal h_t, and
  - Finding optimal α_t
- AdaBoost can be proved to maximise margin
- AdaBoost iteratively fits an additive logistic regression model

Choosing α_t

We attempt to minimise $Z_t = \sum_i D_t(i) exp(-\alpha_t y_i h_t(x_i))$:

$$
\begin{aligned}
\frac{dZ}{d\alpha} = -\sum_{i=1}^{m} D(i) y_i h(x_i) e^{-y_i \alpha_i h(x_i)} &= 0 \\
-\sum_{i:y_i = h(x_i)} D(i) e^{-\alpha} + \sum_{i:y_i \neq h(x_i)} D(i) e^{\alpha} &= 0 \\
-e^{-\alpha}(1-\epsilon) + e^{\alpha}\epsilon &= 0 \\
\alpha &= \frac{1}{2}\log\frac{1-\epsilon}{\epsilon}
\end{aligned}
$$

$\Rightarrow$ The minimisator of the upper bound is $\alpha_t = \frac{1}{2}\log\frac{1-\epsilon_t}{\epsilon_t}$

Choosing h_t

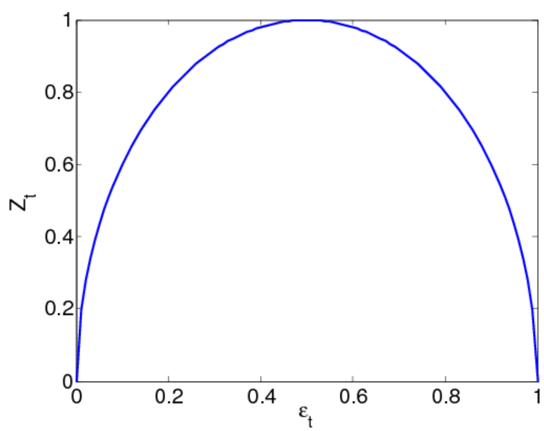
Weak classifier examples

- Decision tree (or stump), Perceptron – $\mathcal{H}$ *infinite*
- Selecting the best one from given *finite* set $\mathcal{H}$

Justification of the weighted error minimisation

Having $\alpha_t = \frac{1}{2} \log \frac{1-\epsilon_t}{\epsilon_t}$

$$\begin{aligned} Z_t &= \sum_{i=1}^{m} D_t(i) e^{-y_i \alpha_i h_t(x_i)} \\ &= \sum_{i:y_i=h_t(x_i)} D_t(i) e^{-\alpha_t} + \sum_{i:y_i \neq h_t(x_i)} D_t(i) e^{\alpha_t} \\ &= (1-\epsilon_t) e^{-\alpha_t} + \epsilon_t e^{\alpha_t} \\ &= 2\sqrt{\epsilon_t(1-\epsilon_t)} \end{aligned}$$



$\Rightarrow$ Z_t is minimised by selecting h_t with minimal weighted error ϵ_t

Generalisation (Schapire & Singer 1999)

Maximising margins in AdaBoost

$$P_{(x,y)\sim S}[yf(x) \leq \theta] \leq 2^T \prod_{t=1}^{T} \sqrt{\epsilon_t^{1-\theta}(1-\epsilon_t)^{1+\theta}} \qquad \text{where } f(x) = \frac{\vec{\alpha} \cdot \vec{h}(x)}{\|\vec{\alpha}\|_1}$$

- Choosing $h_t(x)$ with minimal ϵ_t in each step one minimises the margin
- Margin in SVM use the L_2 norm instead: $(\vec{\alpha} \cdot \vec{h}(x))/\|\vec{\alpha}\|_2$

Upper bounds based on margin

With probability $1-\delta$ over the random choice of the training set S

$$P_{(x,y)\sim \mathcal{D}}[yf(x) \leq 0] \leq P_{(x,y)\sim S}[yf(x) \leq \theta] + \mathcal{O}\left(\frac{1}{\sqrt{m}}\left(\frac{d\log^2(m/d)}{\theta^2} + \log(1/\delta)\right)^{1/2}\right)$$

where $\mathcal{D}$ is a distribution over $\mathcal{X} \times \{+1,-1\}$, and d is pseudodimension of $\mathcal{H}$.

Problem: The upper bound is very loose. In practice AdaBoost works much better.

Convergence (Friedman et al. 1998)

Proposition 1 The discrete AdaBoost algorithm minimises $J(f(x)) = E(e^{-yf(x)})$ by adaptive Newton updates

Lemma $J(f(x))$ is minimised at

$$f(x) = \sum_{t=1}^{T} \alpha_t h_t(x) = \frac{1}{2} \log \frac{P(y=1|x)}{P(y=-1|x)}$$

Hence

$$P(y=1|x) = \frac{e^{f(x)}}{e^{-f(x)} + e^{f(x)}} \qquad \text{and} \qquad P(y=-1|x) = \frac{e^{-f(x)}}{e^{-f(x)} + e^{f(x)}}$$

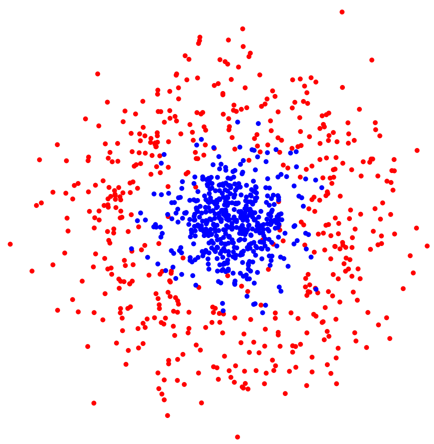
Additive logistic regression model

$$\sum_{t=1}^{T} a_t(x) = \log \frac{P(y=1|x)}{P(y=-1|x)}$$

Proposition 2 By minimising $J(f(x))$ the discrete AdaBoost fits (up to a factor 2) an additive logistic regression model

The Algorithm Recapitulation

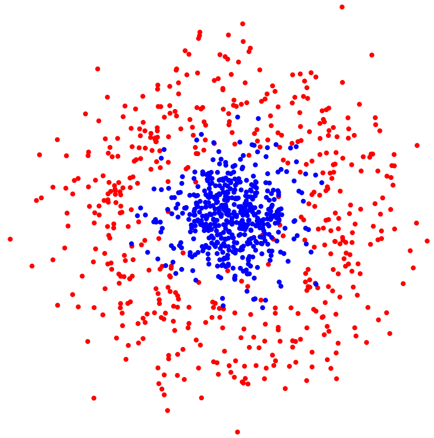
Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$



The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$

Initialise weights $D_1(i) = 1/m$



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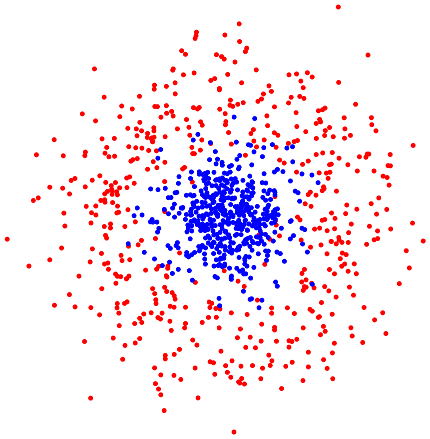
Initialise weights $D_1(i) = 1/m$

For $t = 1, ..., T$:

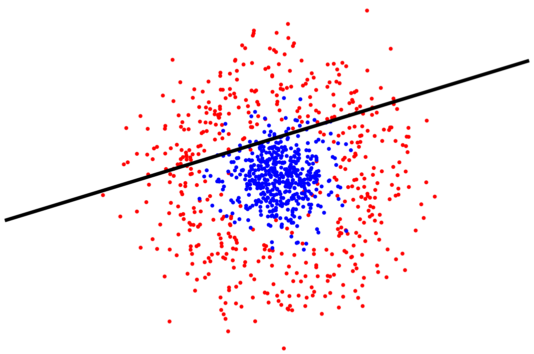


The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$
For $t = 1, ..., T$:

- Find $h_t = \arg\min_{h_j \in \mathcal{H}} \epsilon_j = \sum_{i=1}^{m} D_t(i)[\![y_i \neq h_j(x_i)]\!]$

$t = 1$



The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$

Initialise weights $D_1(i) = 1/m$

For $t = 1, ..., T$:

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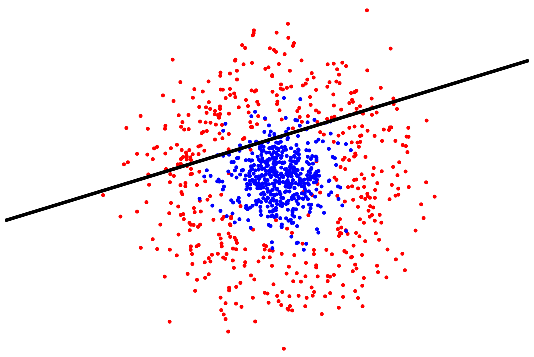
- If $\epsilon_t \geq 1/2$ then stop

$t = 1$



The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
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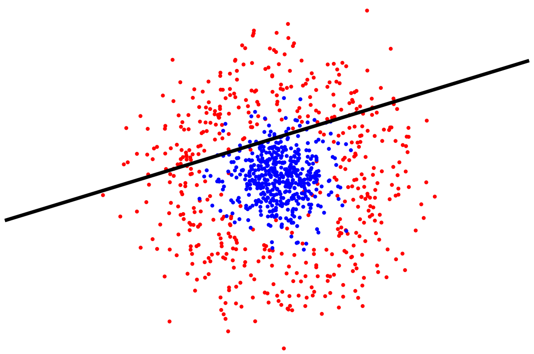
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2} \log(\frac{1-\epsilon_t}{\epsilon_t})$

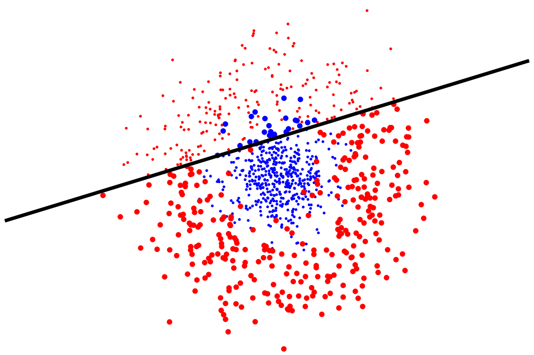
$t = 1$



The Algorithm Recapitulation

Given: $(x_1, y_1), \dots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$
For $t = 1, ..., T$:

$t = 1$



- Find $h_t = \arg\min_{h_j \in \mathcal{H}} \epsilon_j = \sum_{i=1}^{m} D_t(i)[\![y_i \neq h_j(x_i)]\!]$
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
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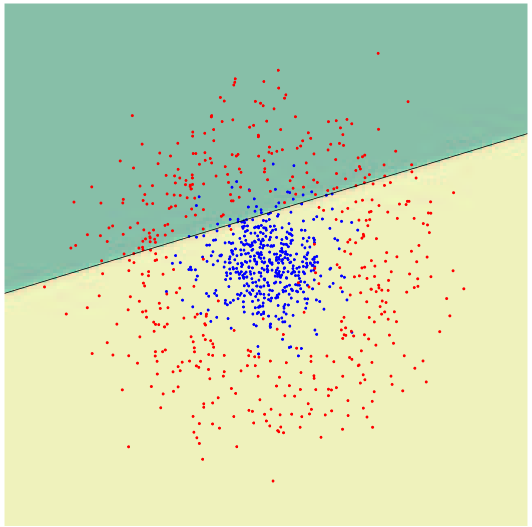
- Update

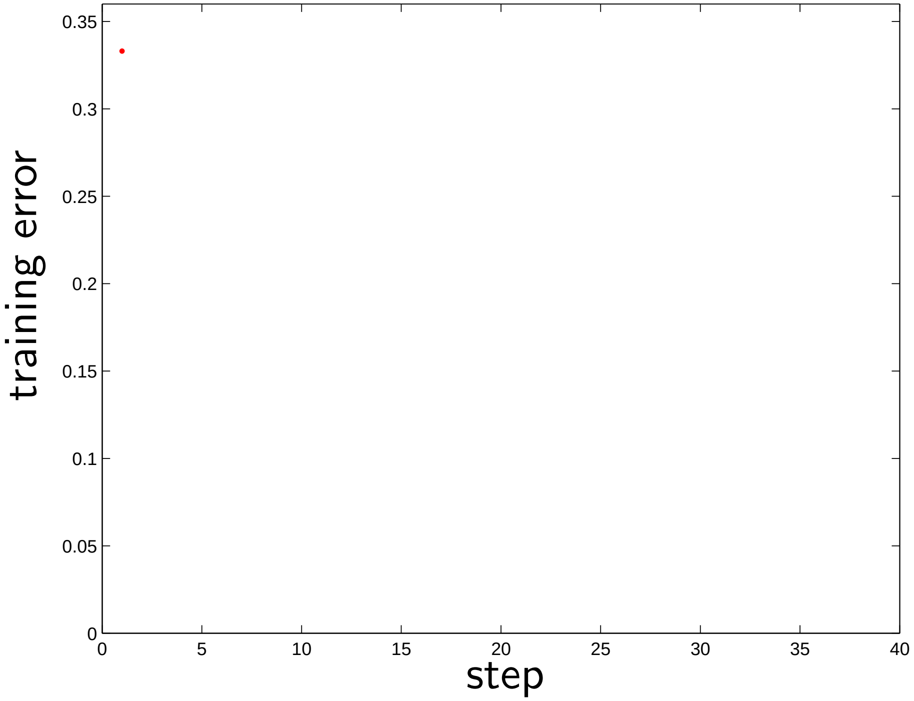
$$D_{t+1}(i) = \frac{D_t(i) exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign \left(\sum_{t=1}^{T} \alpha_t h_t(x) \right)$$

$t = 1$





The Algorithm Recapitulation

Given: $(x_1, y_1), \dots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$
For $t = 1, ..., T$:

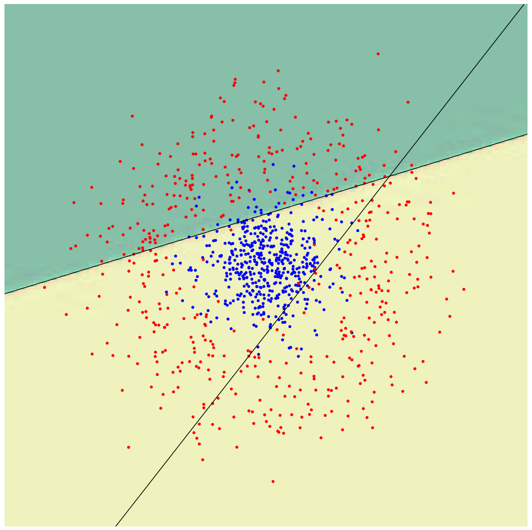
- Find $h_t = \arg\min_{h_j \in \mathcal{H}} \epsilon_j = \sum_{i=1}^{m} D_t(i) [\![y_i \neq h_j(x_i)]\!]$
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2} \log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

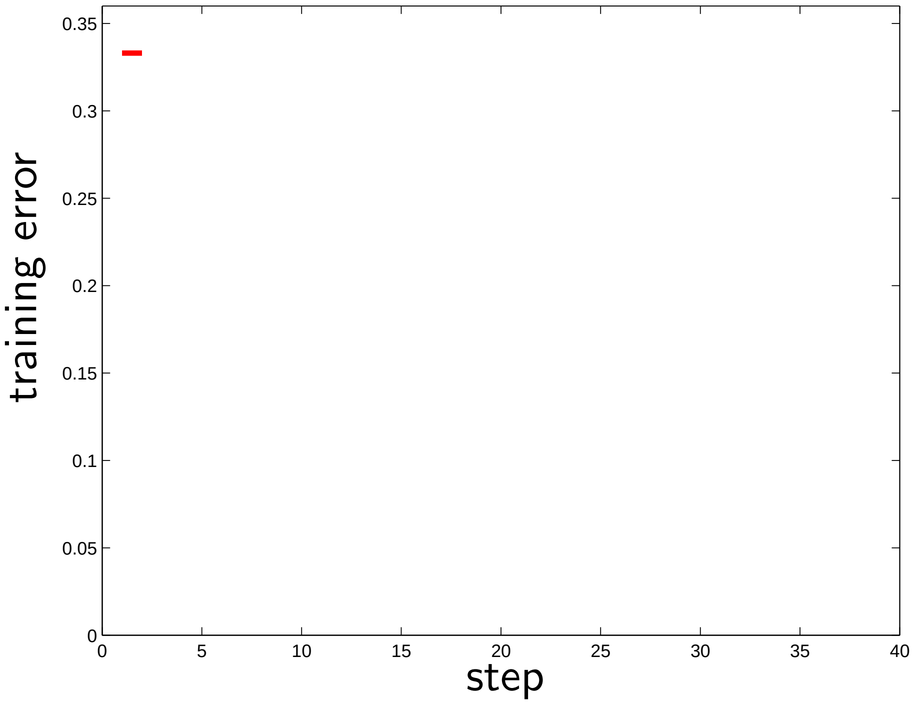
$$D_{t+1}(i) = \frac{D_t(i) exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 2$





The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$
For $t = 1, ..., T$:

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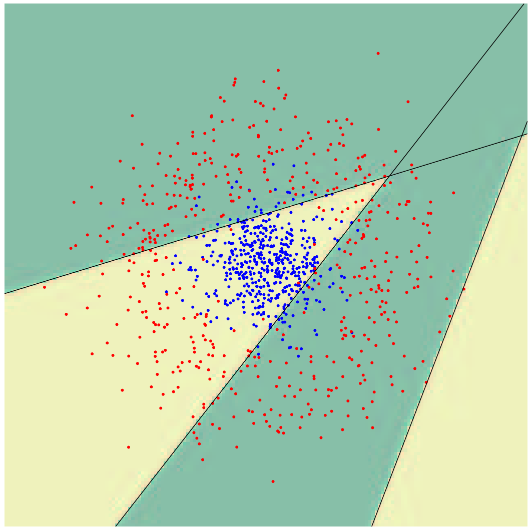
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

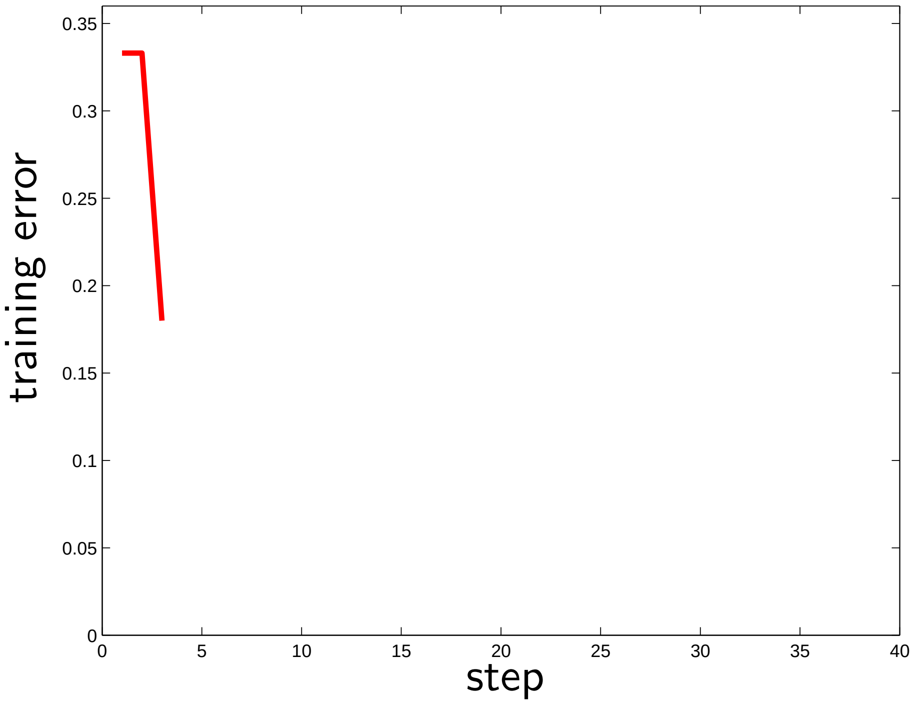
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 3$





The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$
For $t = 1, ..., T$:

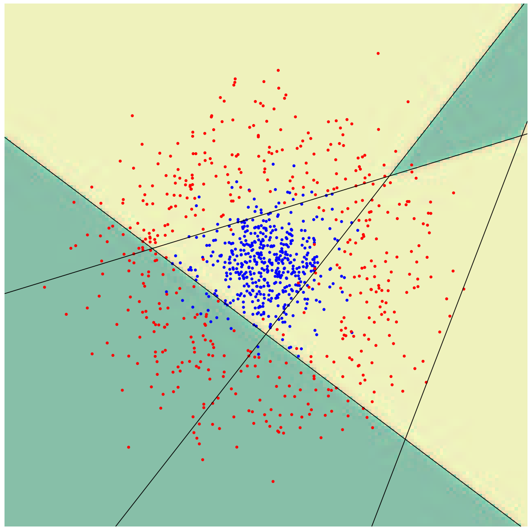
- Find $h_t = \arg\min_{h_j \in \mathcal{H}} \epsilon_j = \sum_{i=1}^{m} D_t(i)[\![y_i \neq h_j(x_i)]\!]$
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

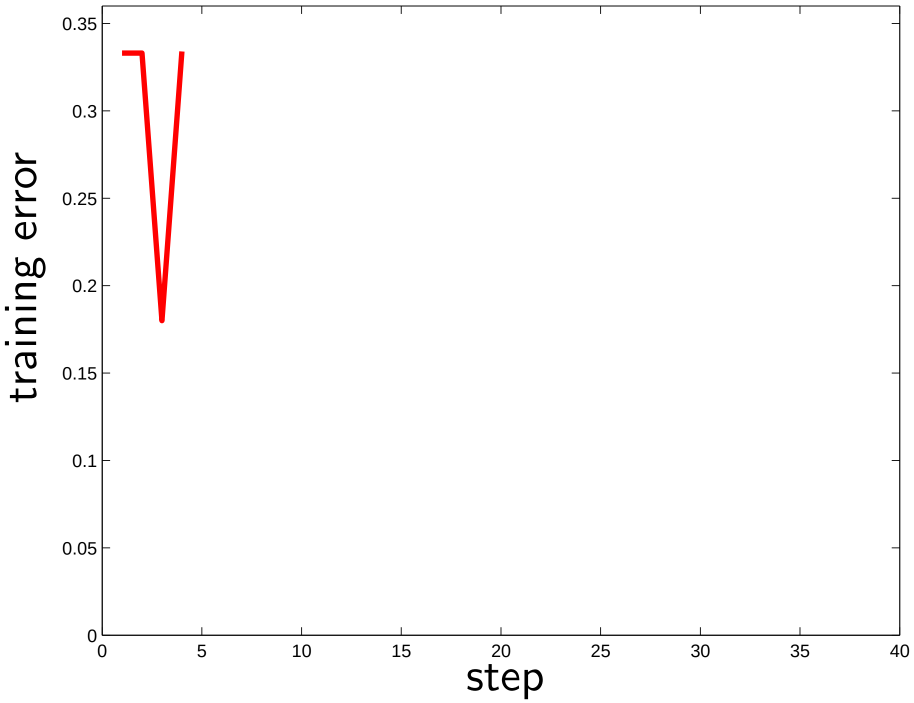
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 4$





The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$
For $t = 1, ..., T$:

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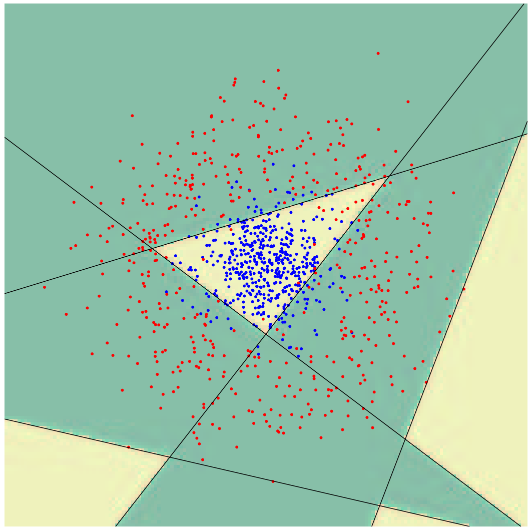
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

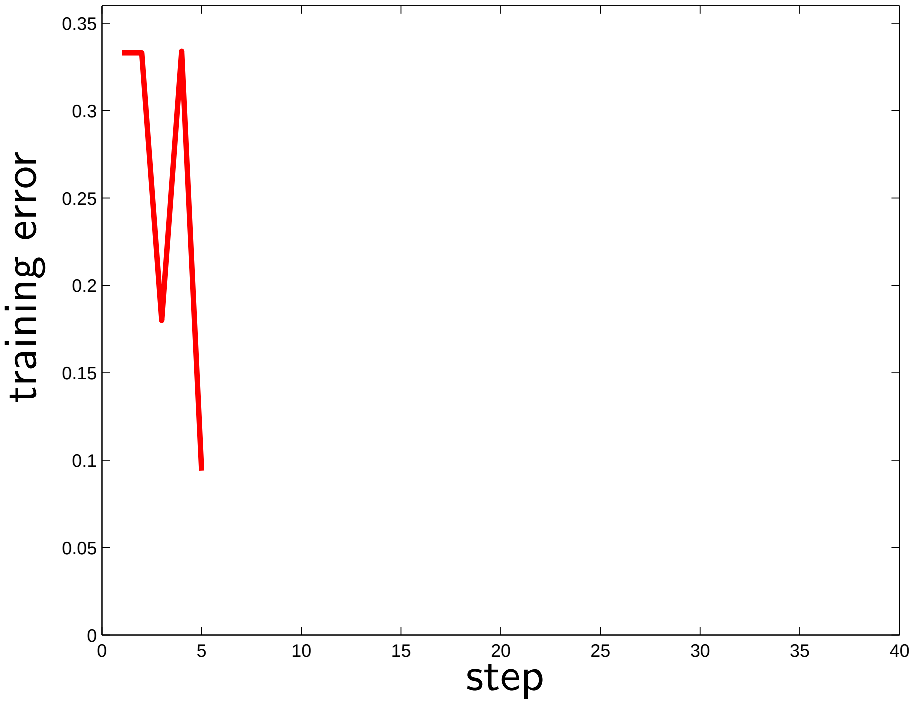
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 5$





The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
Initialise weights $D_1(i) = 1/m$
For $t = 1, ..., T$:

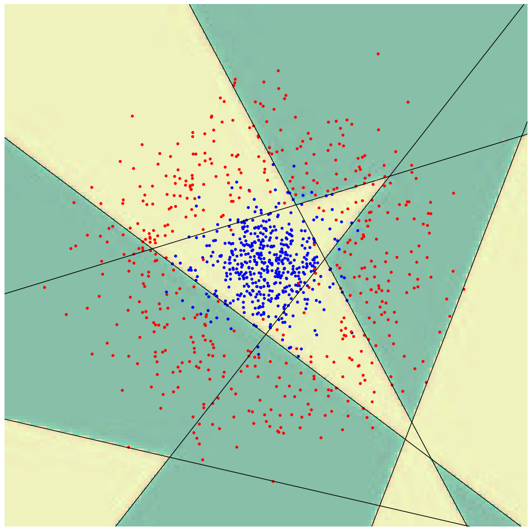
- Find $h_t = \arg\min_{h_j \in \mathcal{H}} \epsilon_j = \sum_{i=1}^{m} D_t(i)[\![y_i \neq h_j(x_i)]\!]$
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

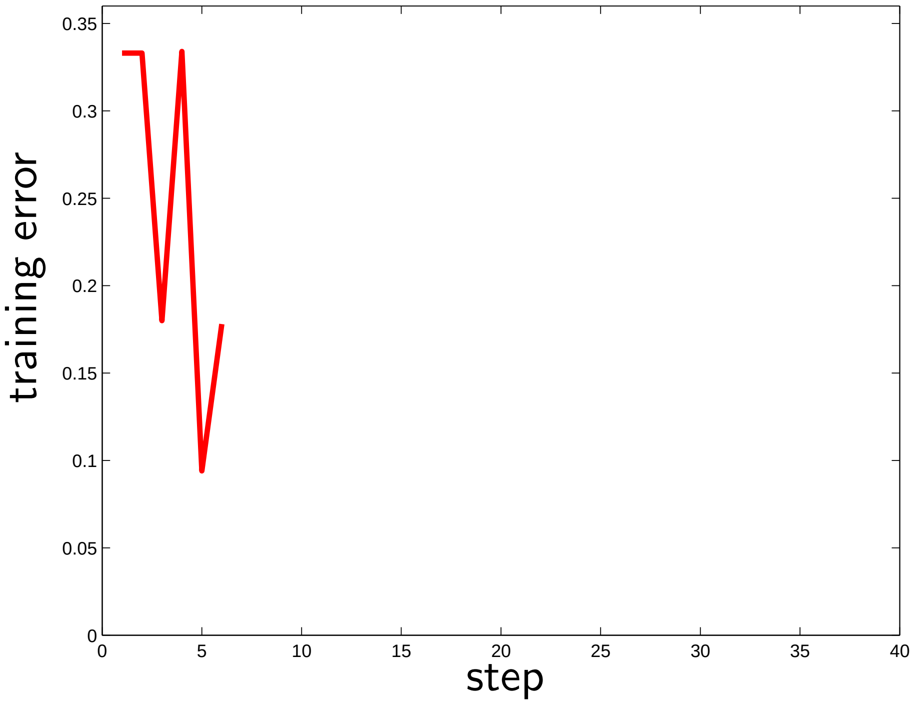
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 6$





The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$

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For $t = 1, ..., T$:

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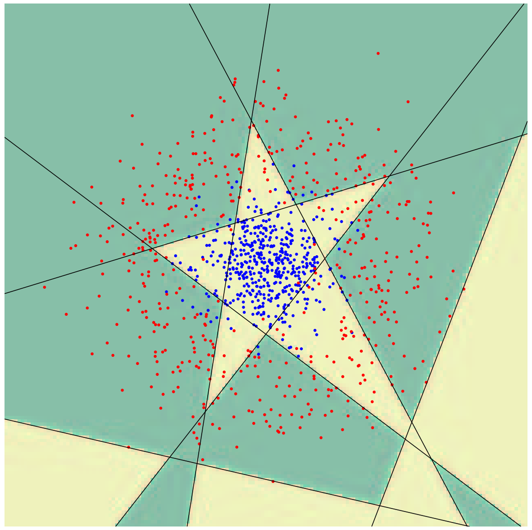
- If $\epsilon_t \geq 1/2$ then stop
- Set $\alpha_t = \frac{1}{2}\log(\frac{1-\epsilon_t}{\epsilon_t})$
- Update

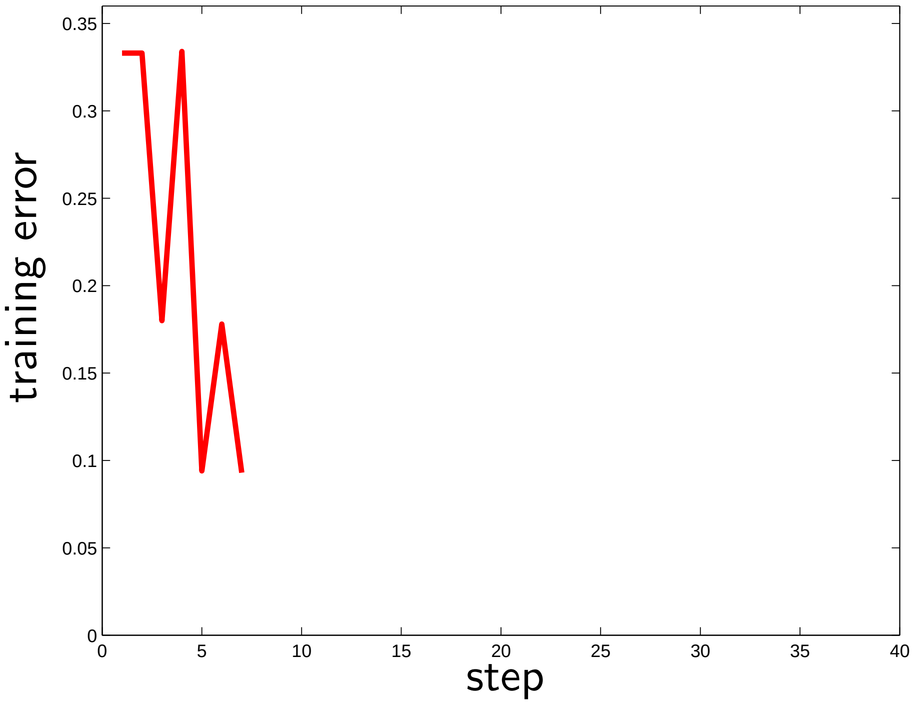
$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 7$





The Algorithm Recapitulation

Given: $(x_1, y_1), \ldots, (x_m, y_m); x_i \in \mathcal{X}, y_i \in \{-1, +1\}$
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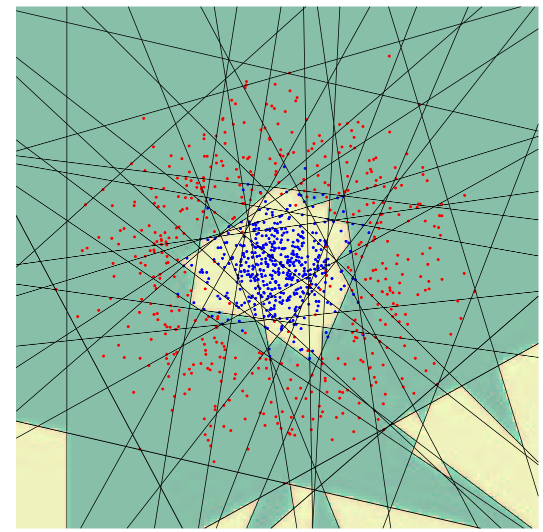
- Update

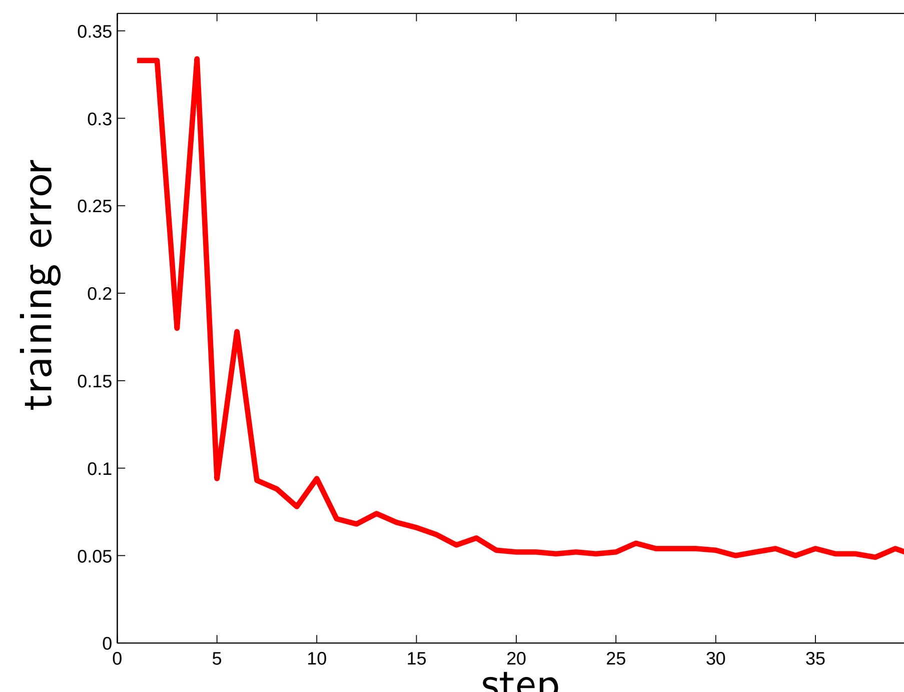
$$D_{t+1}(i) = \frac{D_t(i) exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

Output the final classifier:

$$H(x) = sign\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right)$$

$t = 40$





AdaBoost Variants

Freund & Schapire 1995

- Discrete ($h : \mathcal{X} \rightarrow \{0, 1\}$)
- Multiclass AdaBoost.M1 ($h : \mathcal{X} \rightarrow \{0, 1, ..., k\}$)
- Multiclass AdaBoost.M2 ($h : \mathcal{X} \rightarrow [0, 1]^k$)
- Real valued AdaBoost.R ($Y = [0, 1]$, $h : \mathcal{X} \rightarrow [0, 1]$)

Schapire & Singer 1999

- Confidence rated prediction ($h : \mathcal{X} \rightarrow R$, two-class)
- Multilabel AdaBoost.MR, AdaBoost.MH (different formulation of minimised loss)

Oza 2001

- Online AdaBoost

Many other modifications since then: cascaded AB, WaldBoost, probabilistic boosting tree, ...

Online AdaBoost

Offline

Given:

- Set of labeled training samples
  $\mathcal{X} = \{(x_1, y_1), ..., (x_m, y_m) | y = \pm 1\}$
- Weight distribution over $\mathcal{X}$
  $D_0 = 1/m$

For $t = 1, \ldots, T$

- Train a weak classifier using samples and weight distribution

$$h_t(x) = \mathcal{L}(\mathcal{X}, D_{t-1})$$

- Calculate error ϵ_t
- Calculate coeficient α_t from ϵ_t
- Update weight distribution D_t

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online

Given:

For $t = 1, \ldots, T$

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online AdaBoost

Offline

Given:

- Set of labeled training samples $\mathcal{X} = \{(x_1, y_1), ..., (x_m, y_m) | y = \pm 1\}$
- Weight distribution over $\mathcal{X}$ $D_0 = 1/m$

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$$h_t(x) = \mathcal{L}(\mathcal{X}, D_{t-1})$$

- Calculate error ϵ_t
- Calculate coeficient α_t from ϵ_t
- Update weight distribution D_t

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online

Given:

- *One* labeled training sample $(x, y) | y = \pm 1$
- Strong classifier to update

For $t = 1, \ldots, T$

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online AdaBoost

Offline

Given:

- Set of labeled training samples $\mathcal{X} = \{(x_1, y_1), ..., (x_m, y_m) | y = \pm 1\}$
- $\boxed{\text{Weight distribution over } \mathcal{X} \quad D_0 = 1/m}$

For $t = 1, \ldots, T$

- Train a weak classifier using samples and weight distribution

$$h_t(x) = \mathcal{L}(\mathcal{X}, D_{t-1})$$

- Calculate error ϵ_t
- Calculate coeficient α_t from ϵ_t
- Update weight distribution D_t

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online

Given:

- *One* labeled training sample $(x, y) | y = \pm 1$
- Strong classifier to update
- $\boxed{\text{Initial importance } \lambda = 1}$

For $t = 1, \ldots, T$

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online AdaBoost

Offline

Given:

- Set of labeled training samples
  $\mathcal{X} = \{(x_1, y_1), ..., (x_m, y_m) | y = \pm 1\}$
- Weight distribution over $\mathcal{X}$
  $D_0 = 1/m$

For $t = 1, \ldots, T$

- Train a weak classifier using samples and weight distribution

$$h_t(x) = \mathcal{L}(\mathcal{X}, D_{t-1})$$

- Calculate error ϵ_t
- Calculate coeficient α_t from ϵ_t
- Update weight distribution D_t

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online

Given:

- *One* labeled training sample
  $(x, y) | y = \pm 1$
- Strong classifier to update
- Initial importance $\lambda = 1$

For $t = 1, \ldots, T$

- Update the weak classifier using the sample and the importance

$$h_t(x) = \mathcal{L}(h_t, (x, y), \lambda)$$

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online AdaBoost

Offline

Given:

- Set of labeled training samples $\mathcal{X} = \{(x_1, y_1), ..., (x_m, y_m) | y = \pm 1\}$
- Weight distribution over $\mathcal{X}$ $D_0 = 1/m$

For $t = 1, \ldots, T$

- Train a weak classifier using samples and weight distribution

$$h_t(x) = \mathcal{L}(\mathcal{X}, D_{t-1})$$

- Calculate error ϵ_t
- Calculate coeficient α_t from ϵ_t
- Update weight distribution D_t

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online

Given:

- *One* labeled training sample $(x, y) | y = \pm 1$
- Strong classifier to update
- Initial importance $\lambda = 1$

For $t = 1, \ldots, T$

- Update the weak classifier using the sample and the importance

$$h_t(x) = \mathcal{L}(h_t, (x, y), \lambda)$$

- Update error estimation ϵ_t
- Update weight α_t based on ϵ_t

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online AdaBoost

Offline

Given:

- Set of labeled training samples
  $\mathcal{X} = \{(x_1, y_1), ..., (x_m, y_m) | y = \pm 1\}$
- Weight distribution over $\mathcal{X}$
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$$h_t(x) = \mathcal{L}(\mathcal{X}, D_{t-1})$$

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- Calculate coeficient α_t from ϵ_t
- Update weight distribution D_t

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online

Given:

- *One* labeled training sample
  $(x, y) | y = \pm 1$
- Strong classifier to update
- Initial importance $\lambda = 1$

For $t = 1, \ldots, T$

- Update the weak classifier using the sample and the importance

$$h_t(x) = \mathcal{L}(h_t, (x, y), \lambda)$$

- Update error estimation ϵ_t
- Update weight α_t based on ϵ_t
- Update importance weight λ

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online AdaBoost

Offline

Given:

- Set of labeled training samples
  $\mathcal{X} = \{(x_1, y_1), ..., (x_m, y_m) | y = \pm 1\}$
- Weight distribution over $\mathcal{X}$
  $D_0 = 1/m$

For $t = 1, \ldots, T$

- Train a weak classifier using samples and weight distribution

$$h_t(x) = \mathcal{L}(\mathcal{X}, D_{t-1})$$

- Calculate error ϵ_t
- Calculate coeficient α_t from ϵ_t
- Update weight distribution D_t

Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online

Given:

- *One* labeled training sample
  $(x, y) | y = \pm 1$
- Strong classifier to update
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Output:

$$F(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

Online AdaBoost

Converges to offline results given the same training set and the number of iterations $N \to \infty$

N. Oza and S. Russell. Online Bagging and Boosting.
Artificial Inteligence and Statistics, 2001.

Pros and Cons of AdaBoost

Advantages

- Very simple to implement
- General learning scheme - can be used for various learning tasks
- Feature selection on very large sets of features
- Good generalisation
- Seems not to overfit in practice (probably due to margin maximisation)

Disadvantages

- Suboptimal solution (greedy learning)

Selected references

- Y. Freund, R.E. Schapire. **A Decision-theoretic Generalization of On-line Learning and an Application to Boosting**. Journal of Computer and System Sciences. 1997
- R.E. Schapire, Y. Freund, P. Bartlett, W.S. Lee. **Boosting the Margin: A New Explanation for the Effectiveness of Voting Methods**. The Annals of Statistics, 1998
- R.E. Schapire, Y. Singer. **Improved Boosting Algorithms Using Confidence-rated Predictions**. Machine Learning. 1999
- J. Friedman, T. Hastie, R. Tibshirani. **Additive Logistic Regression: a Statistical View of Boosting**. Technical report. 1998
- N.C. Oza. **Online Ensemble Learning**. PhD thesis. 2001
- `http://www.boosting.org`

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Thank you for attention

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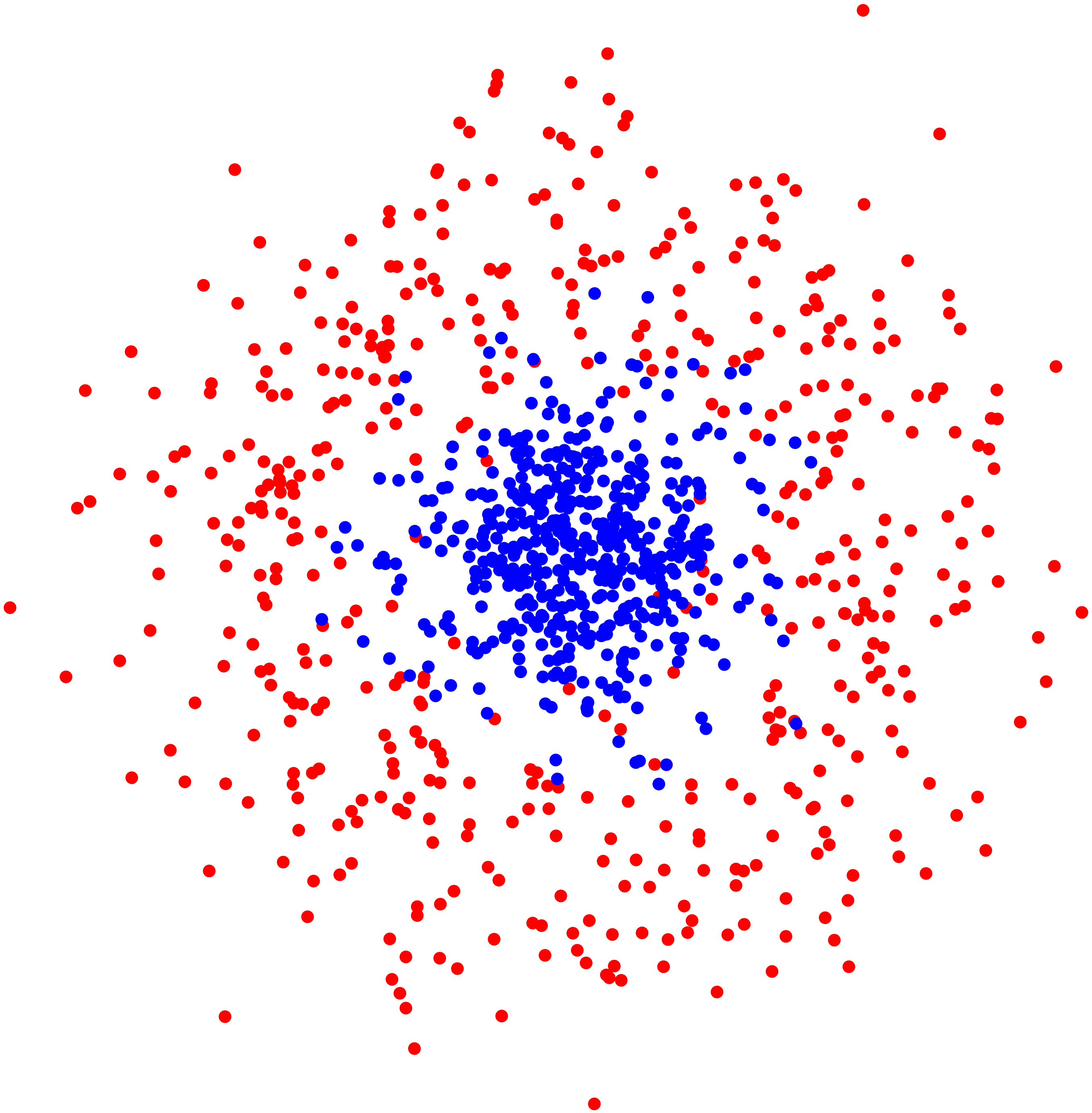

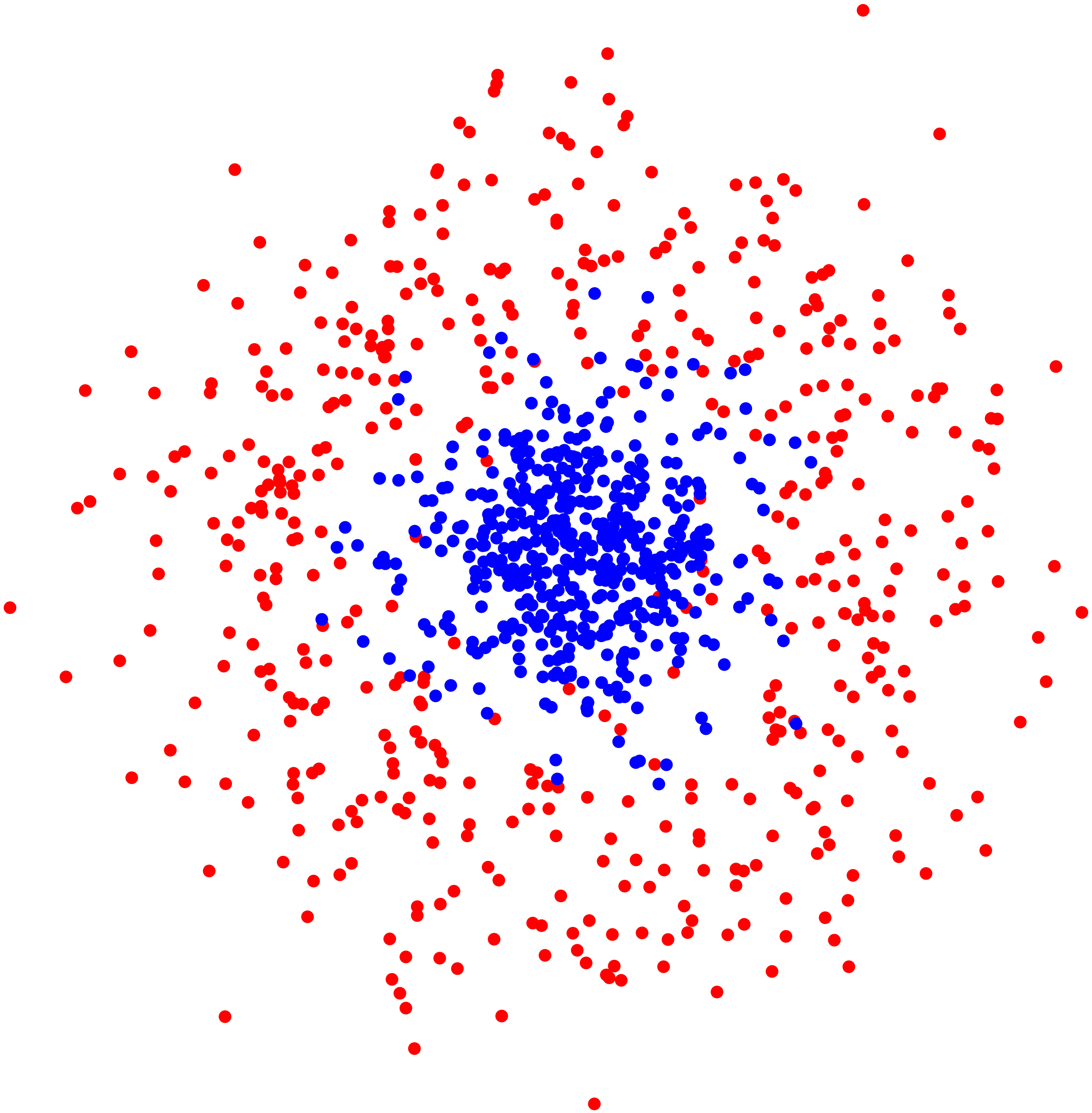

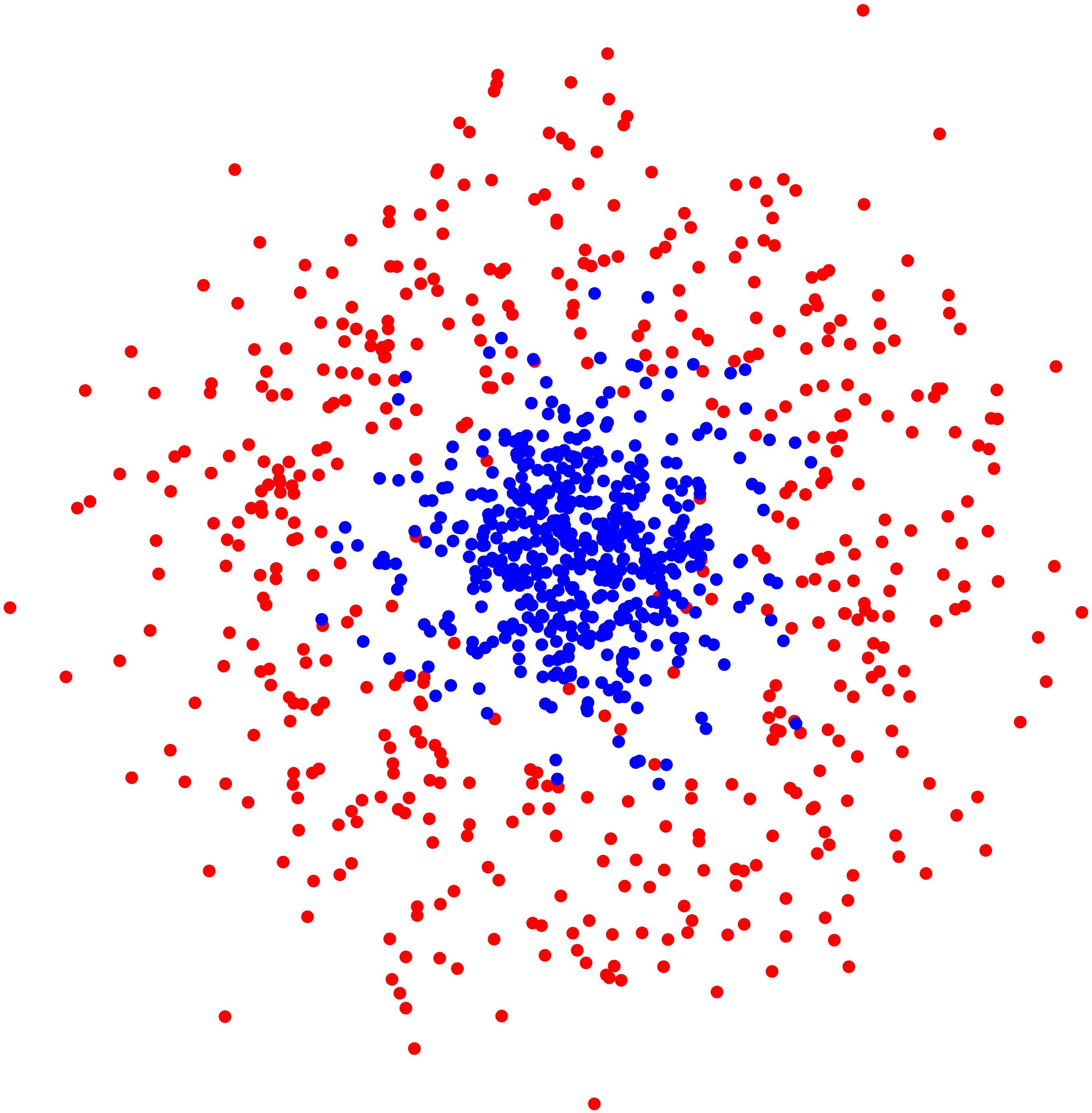

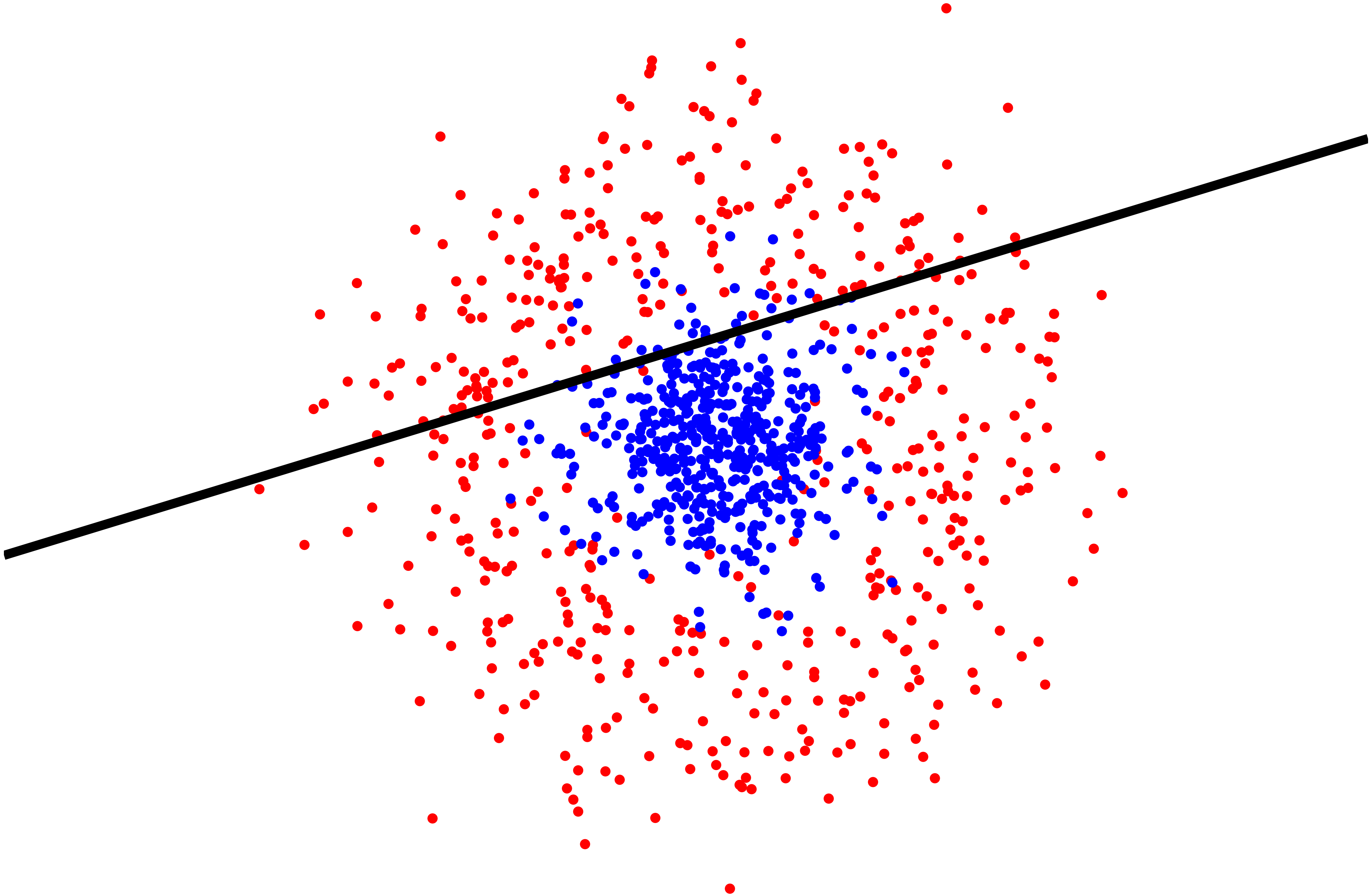

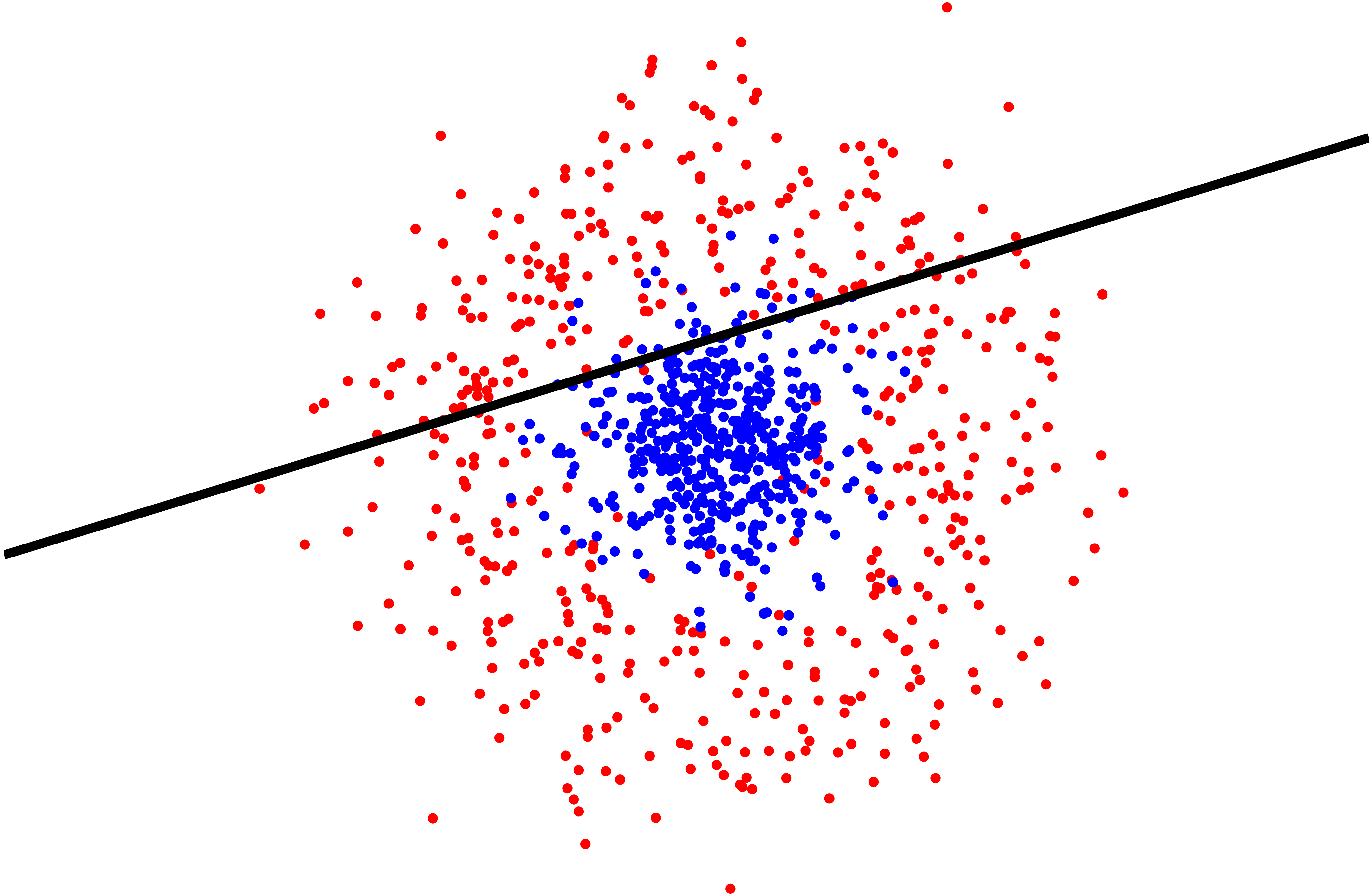

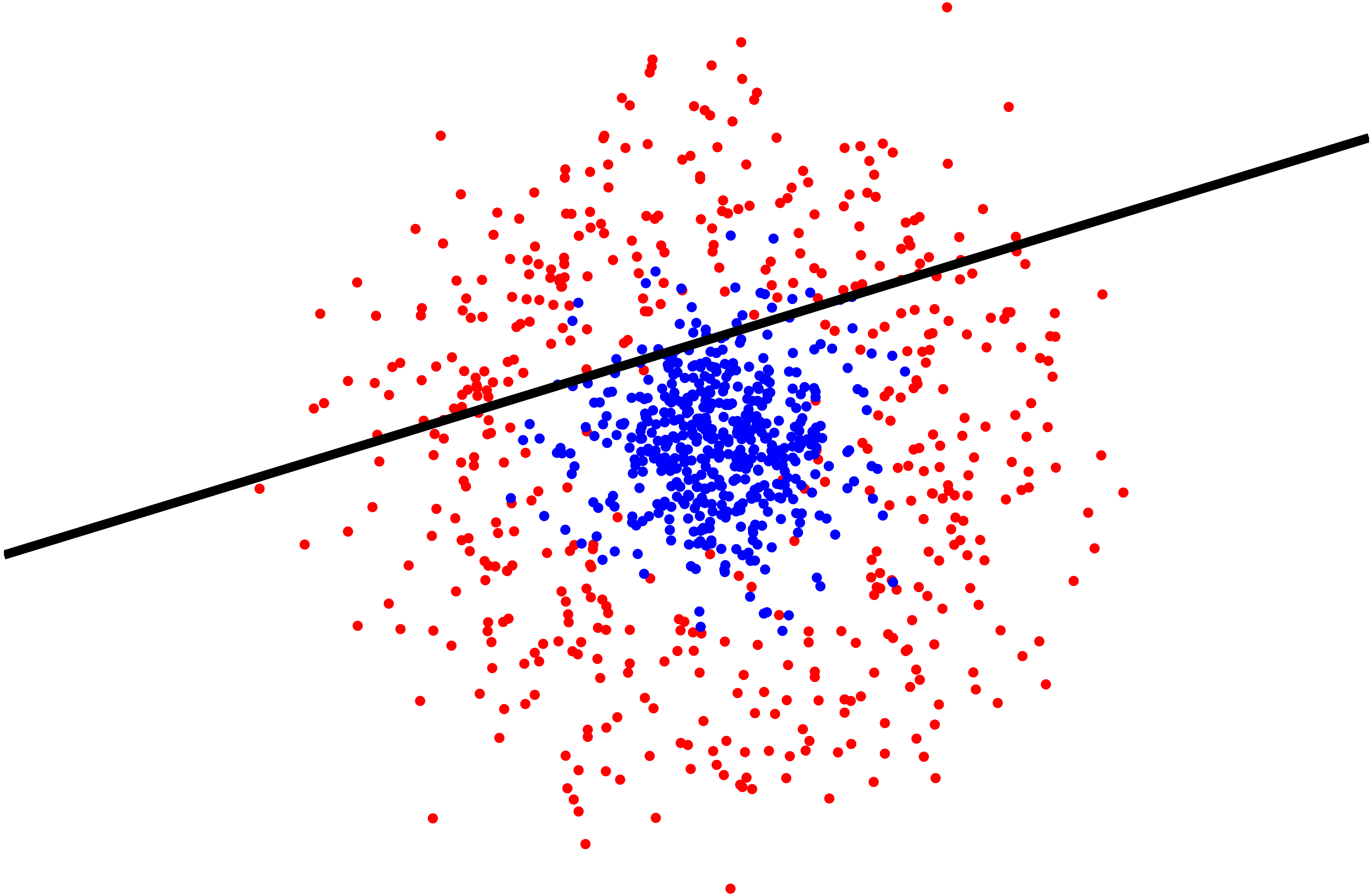

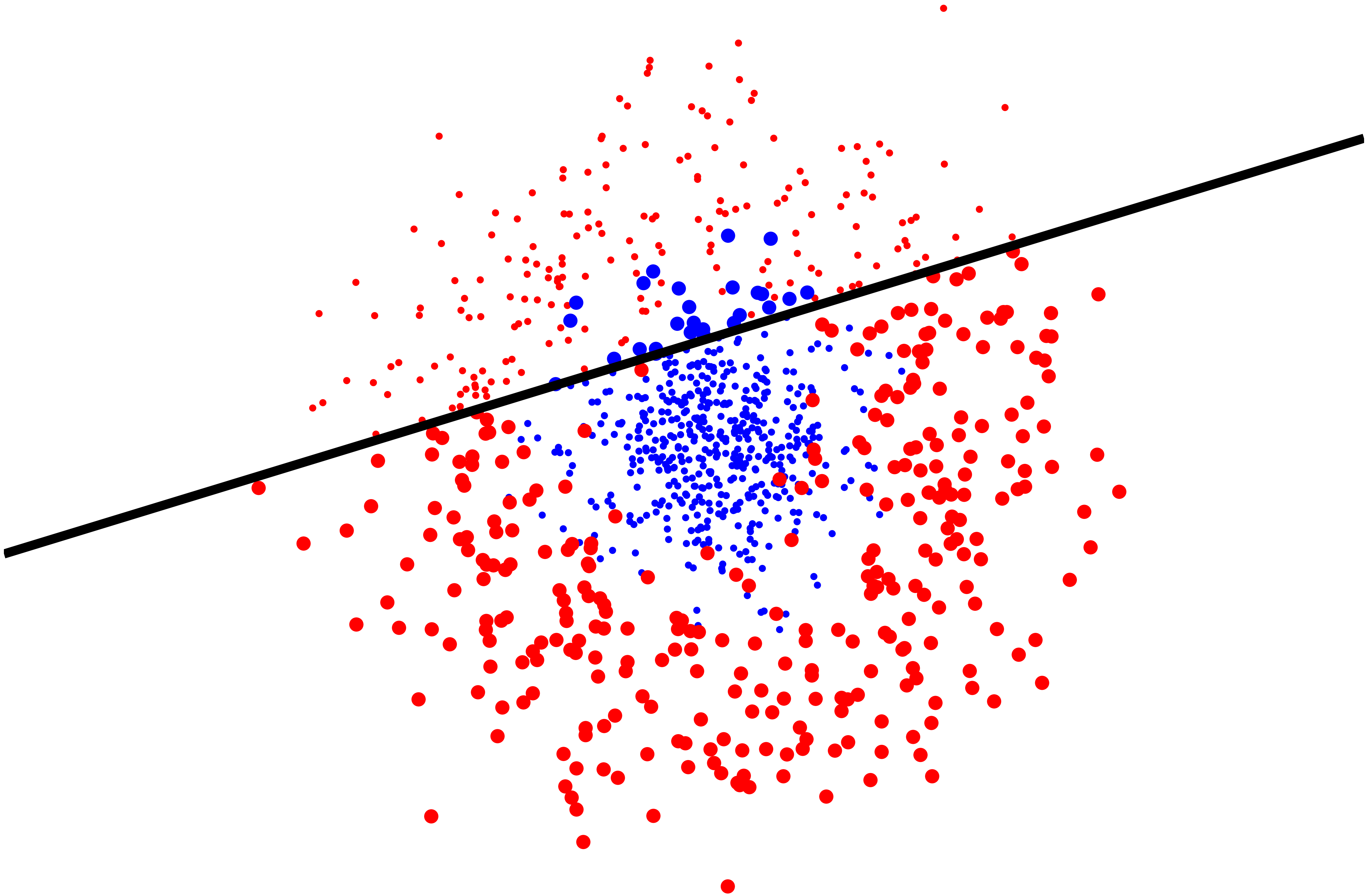

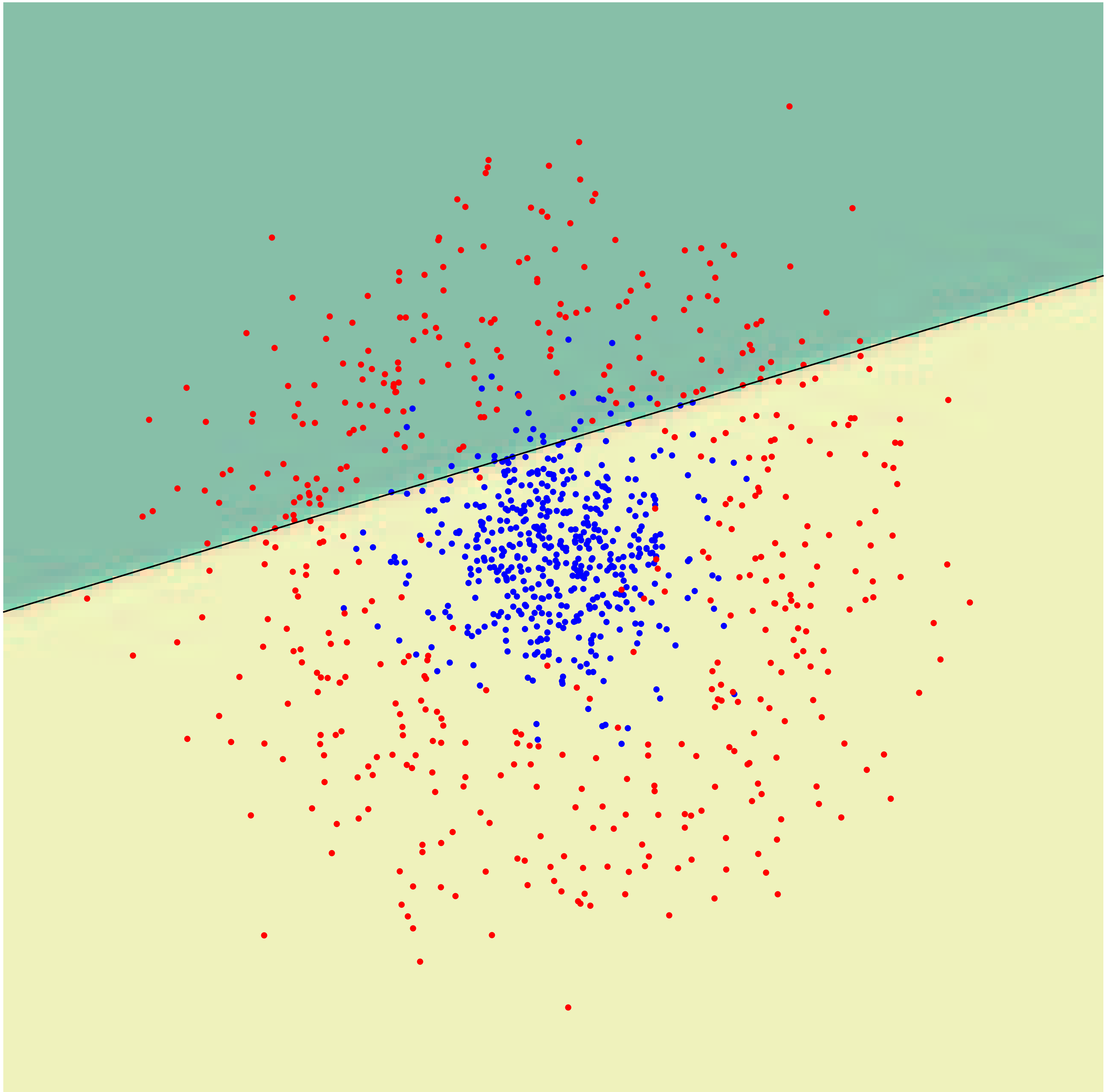

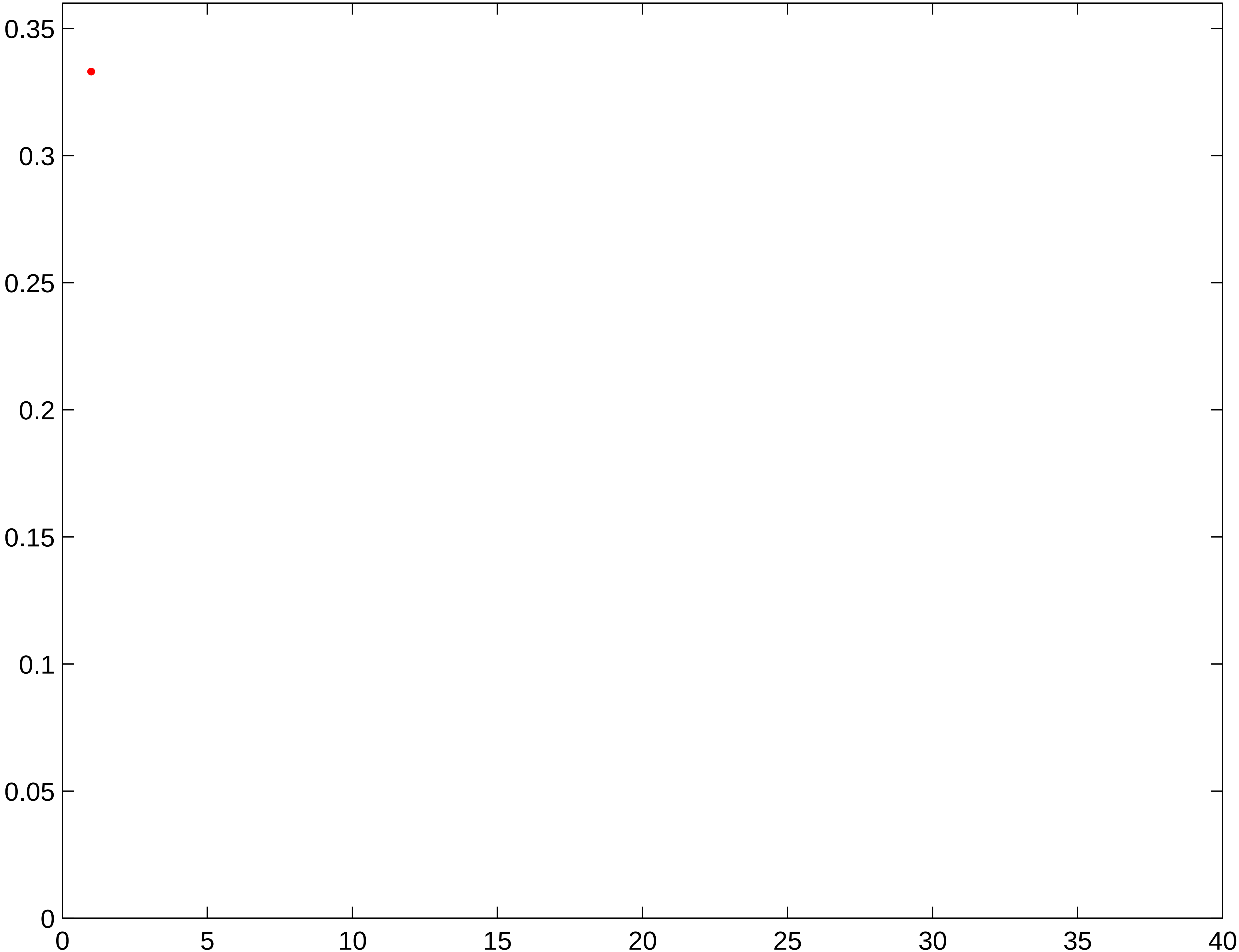
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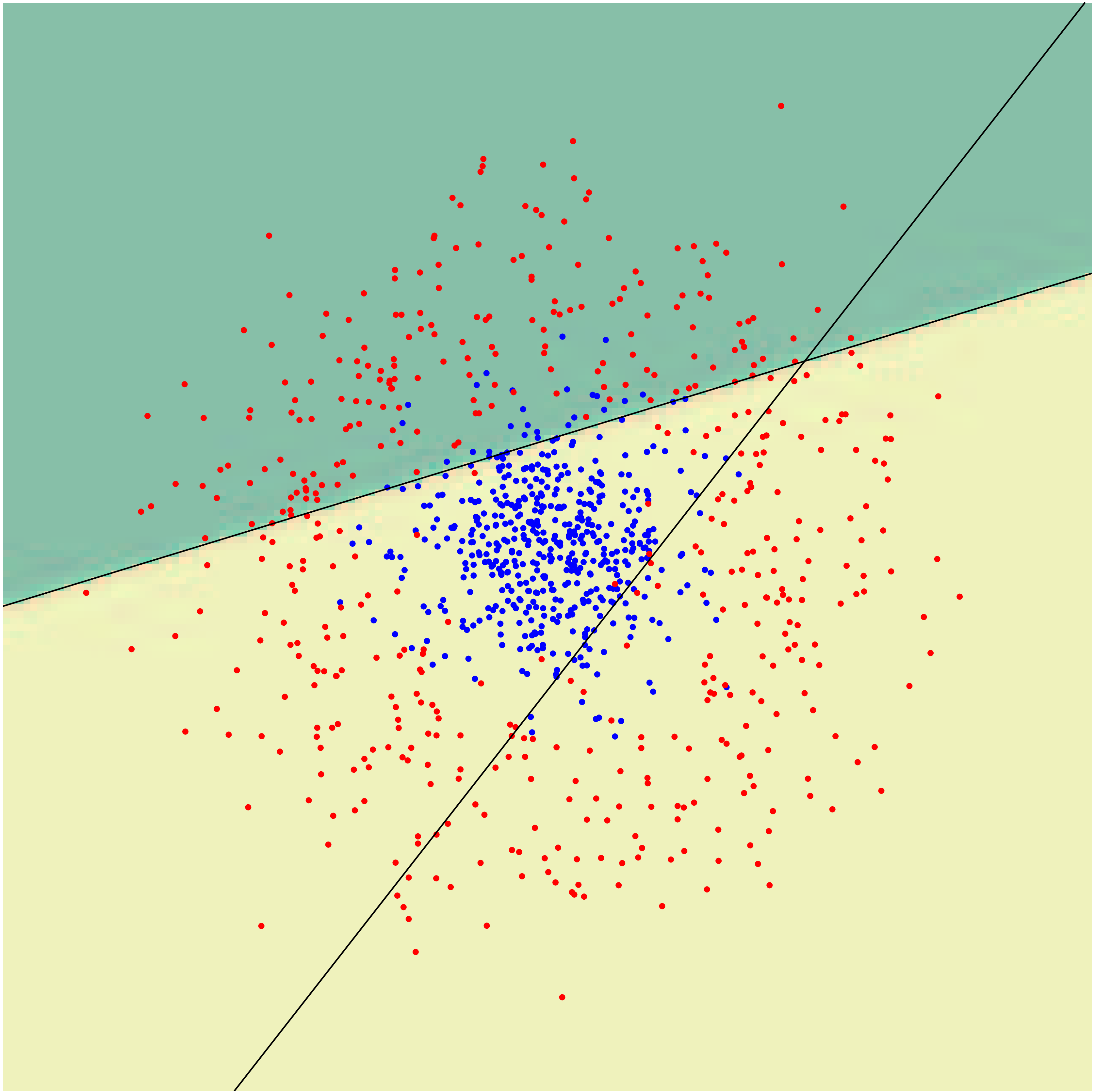

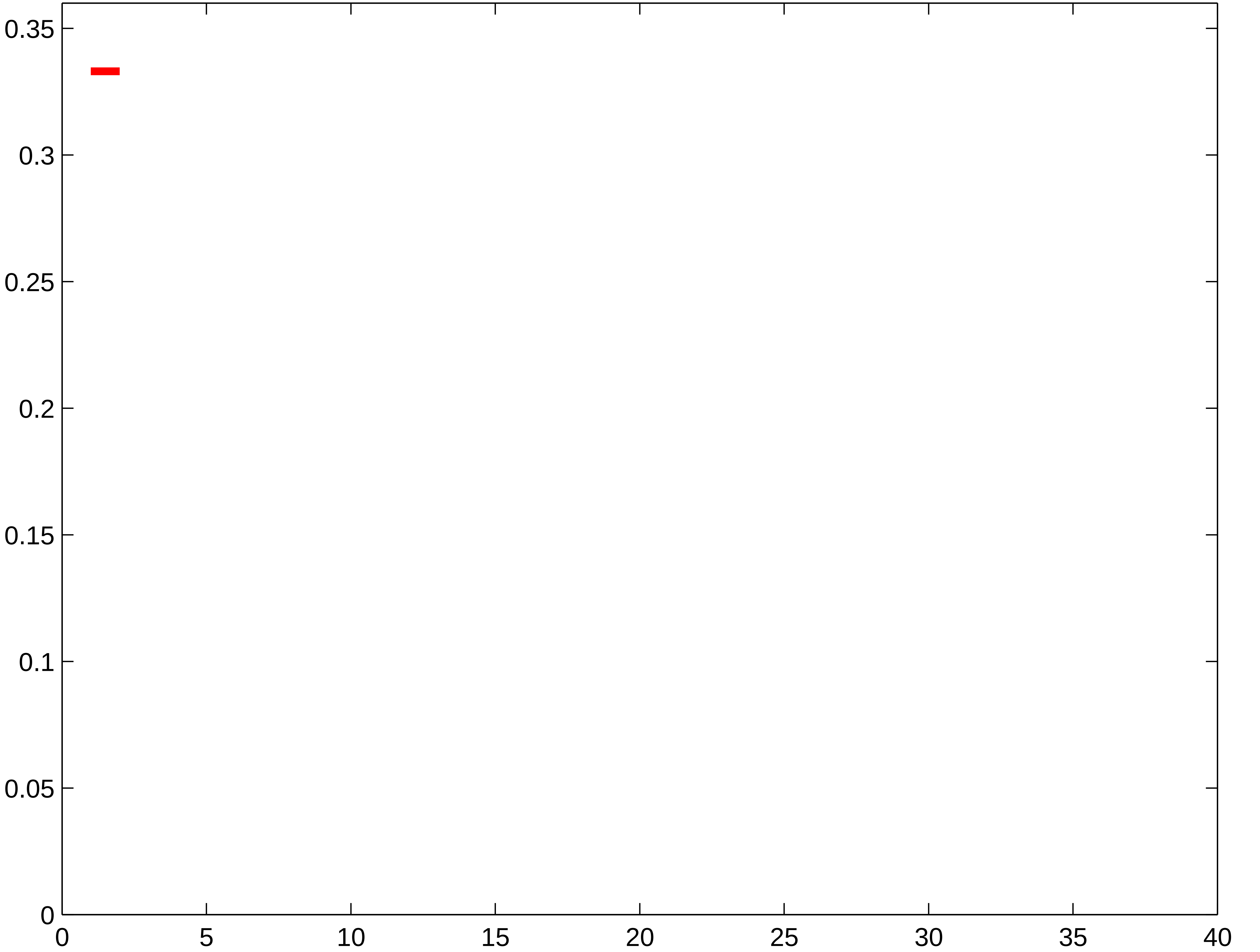
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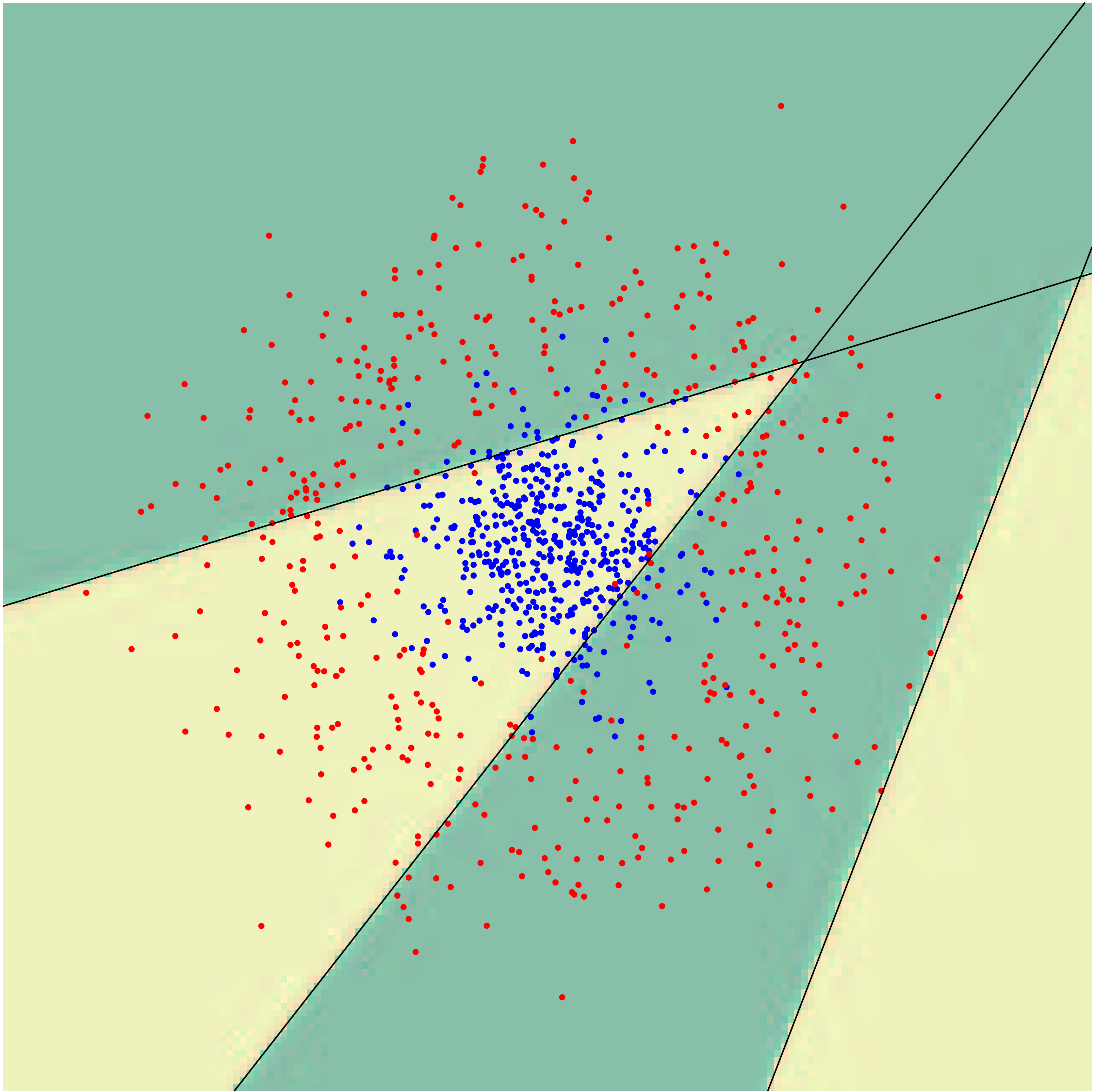

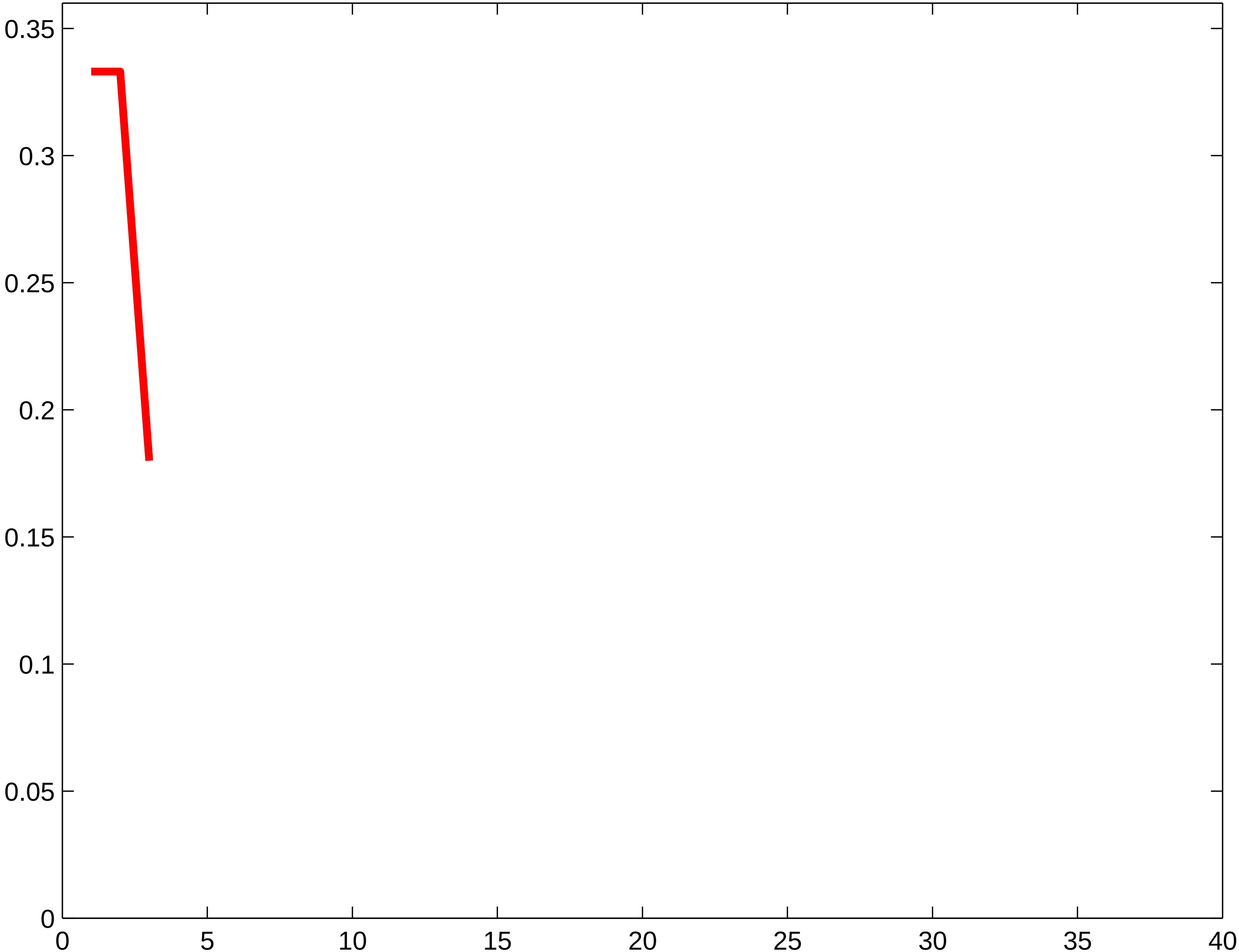
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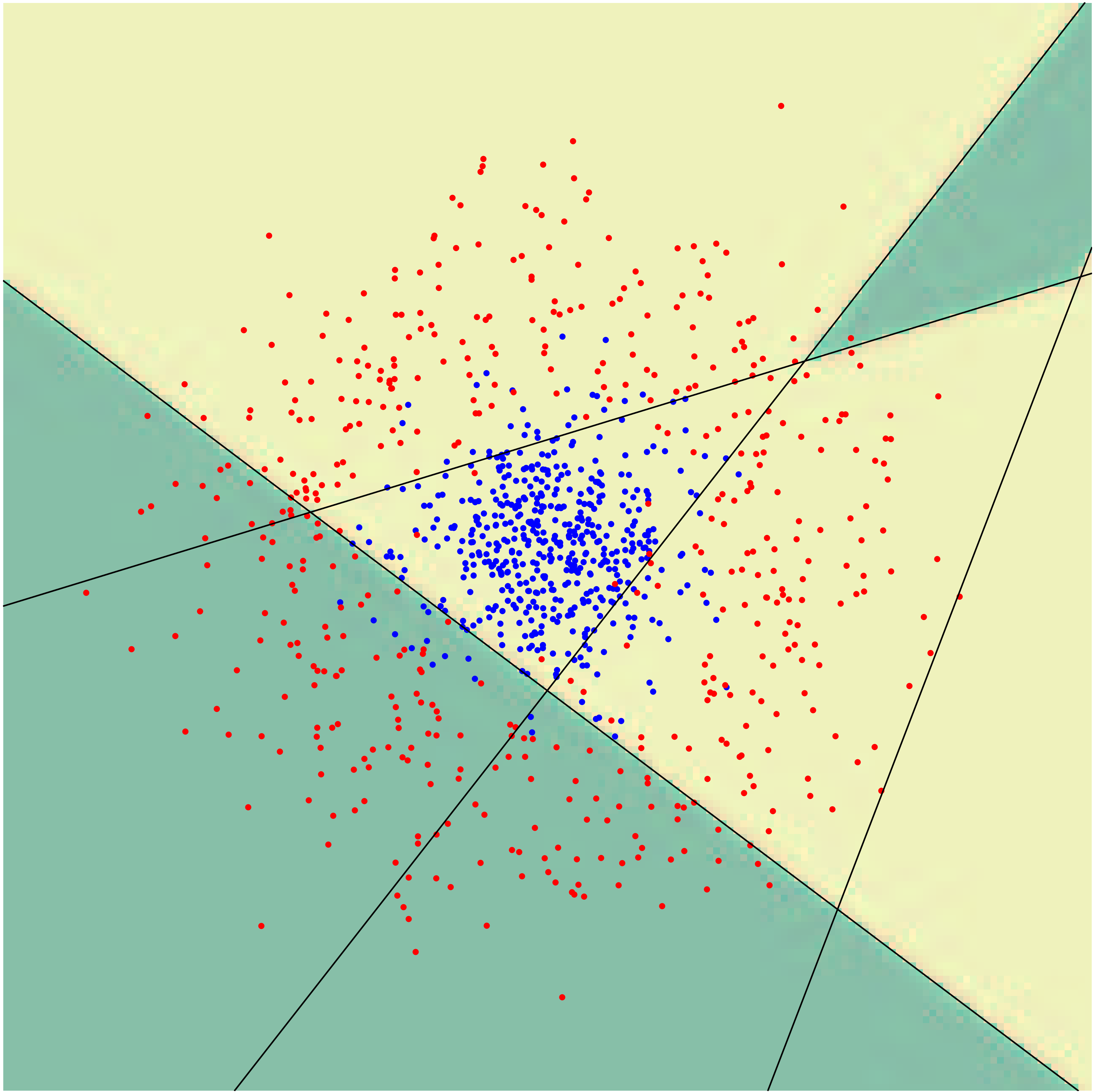

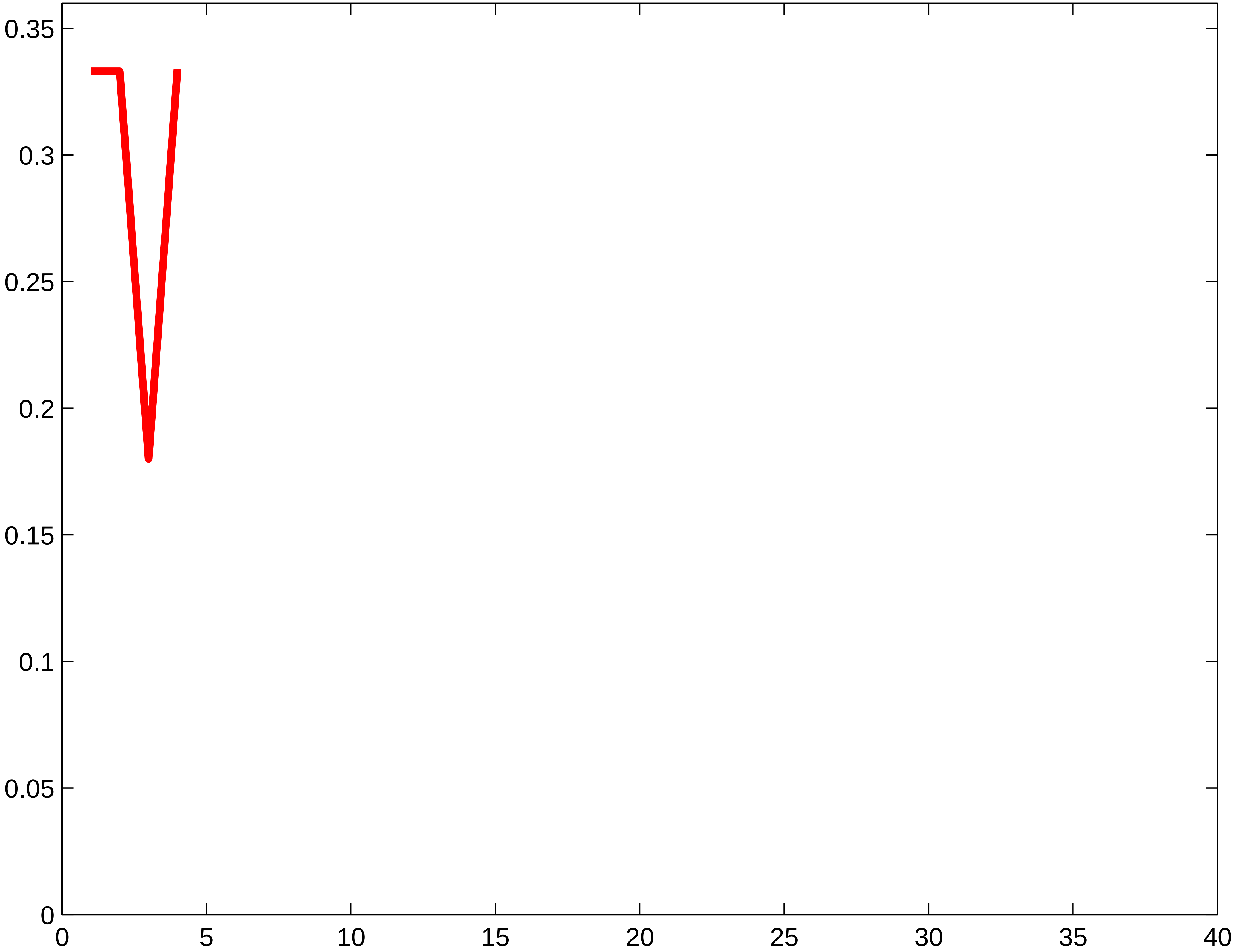
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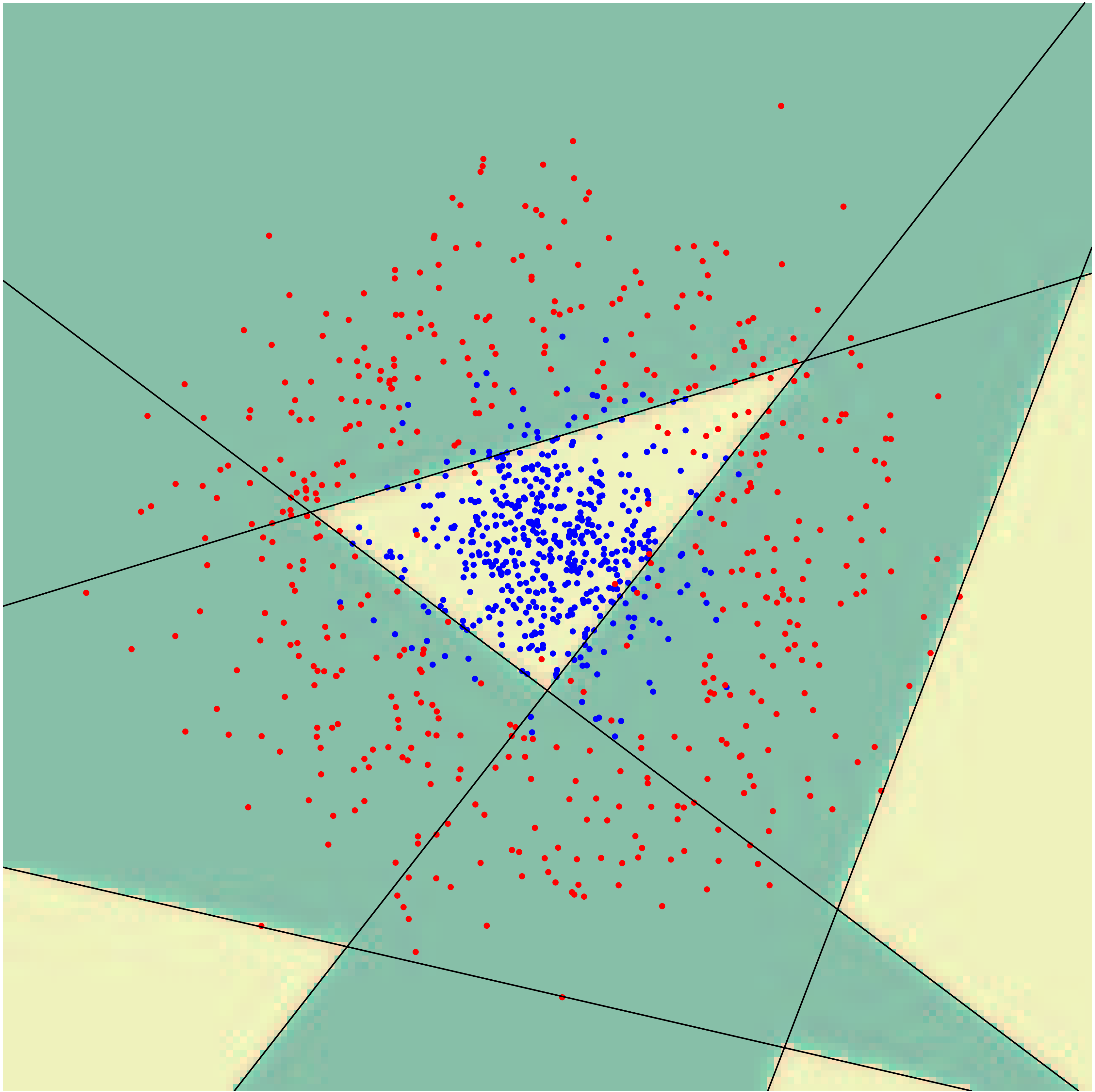

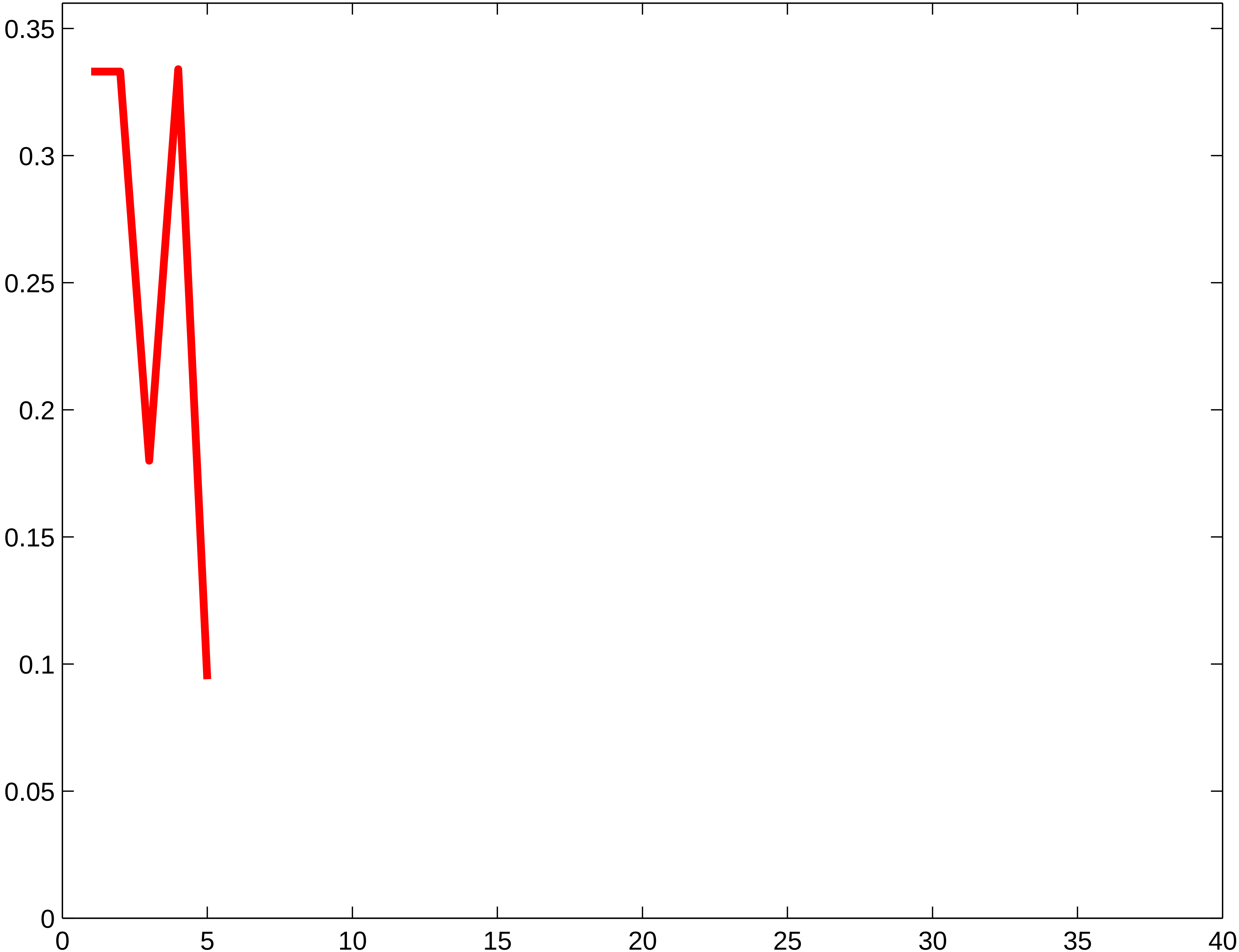

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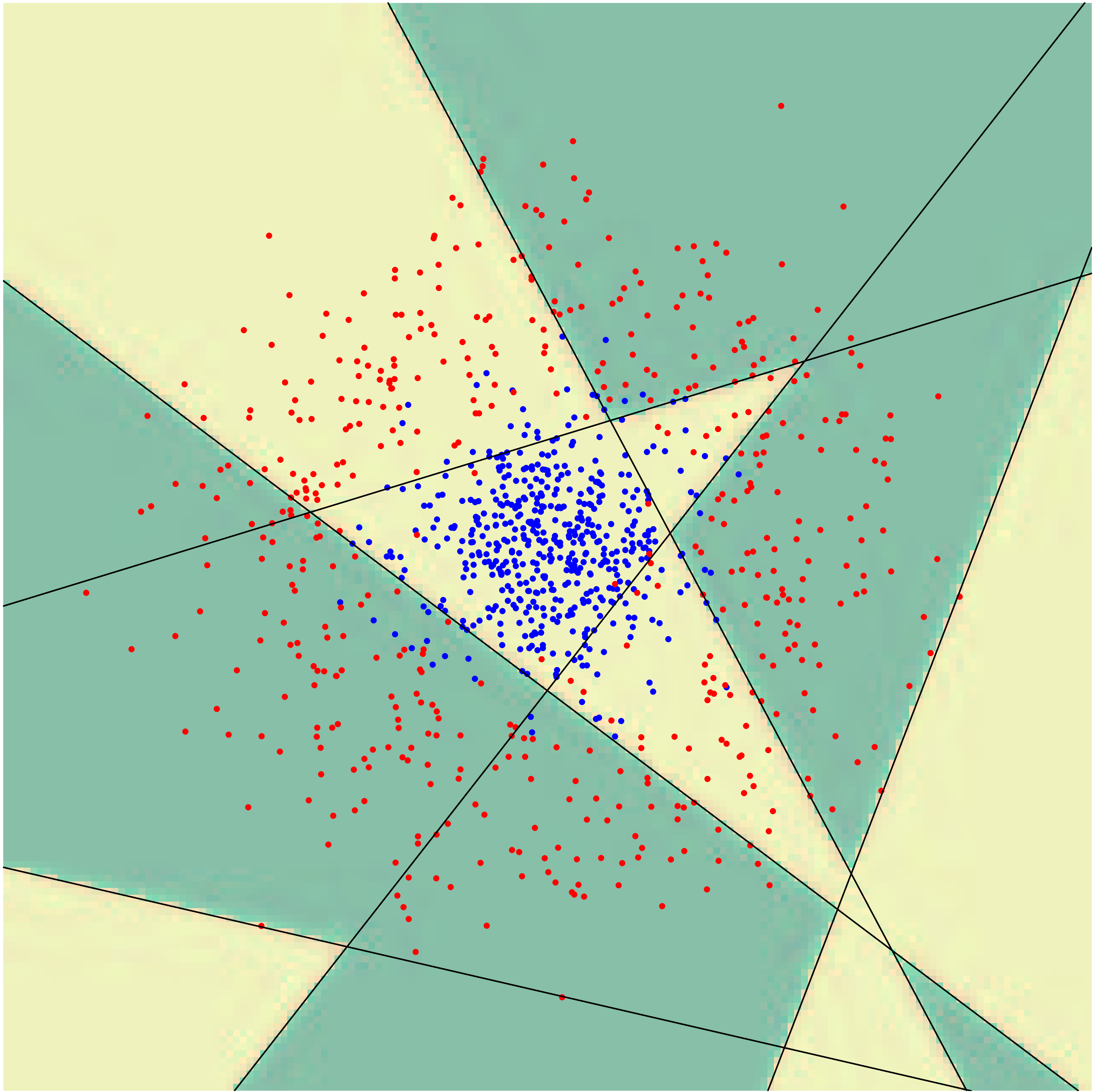

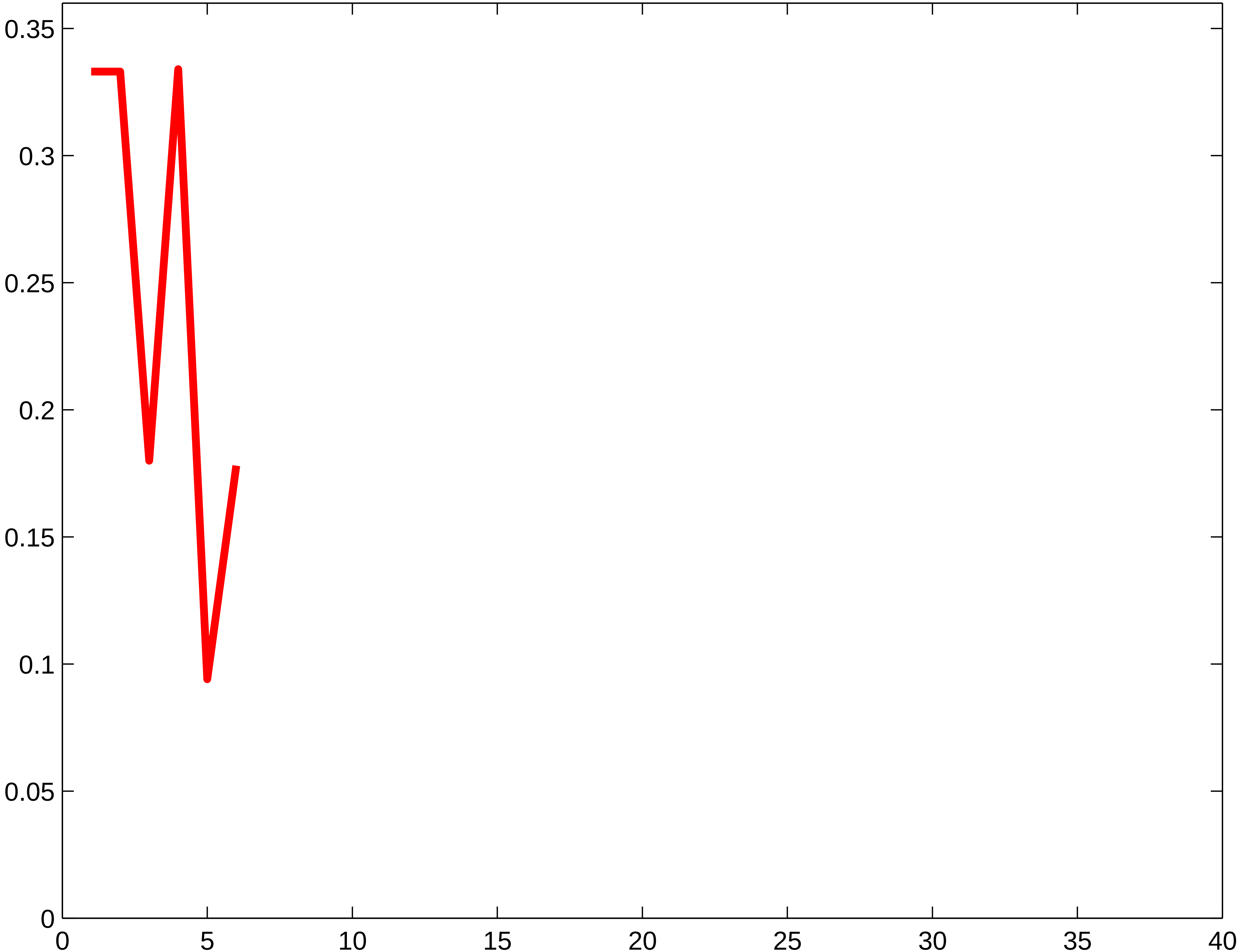
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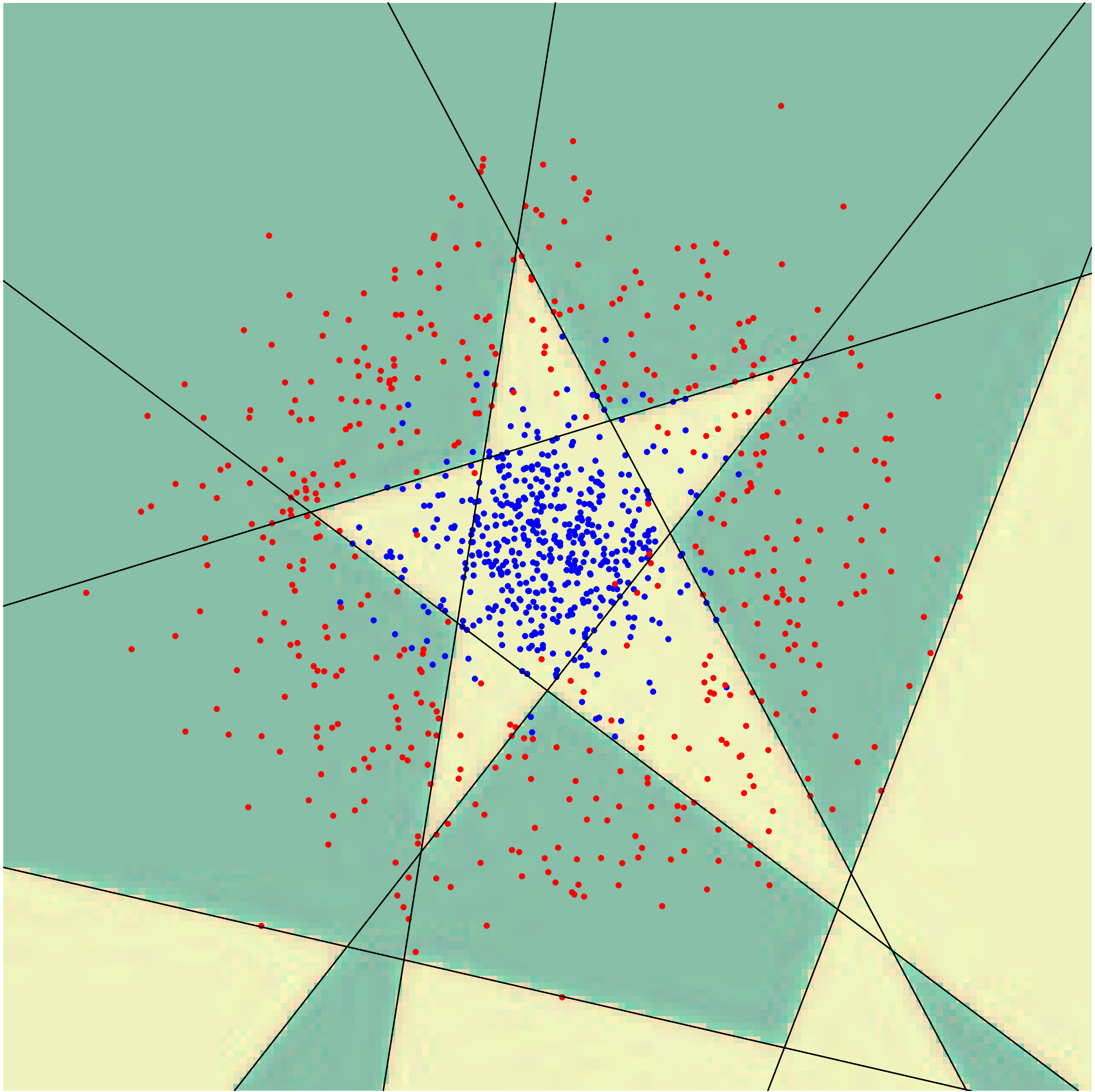

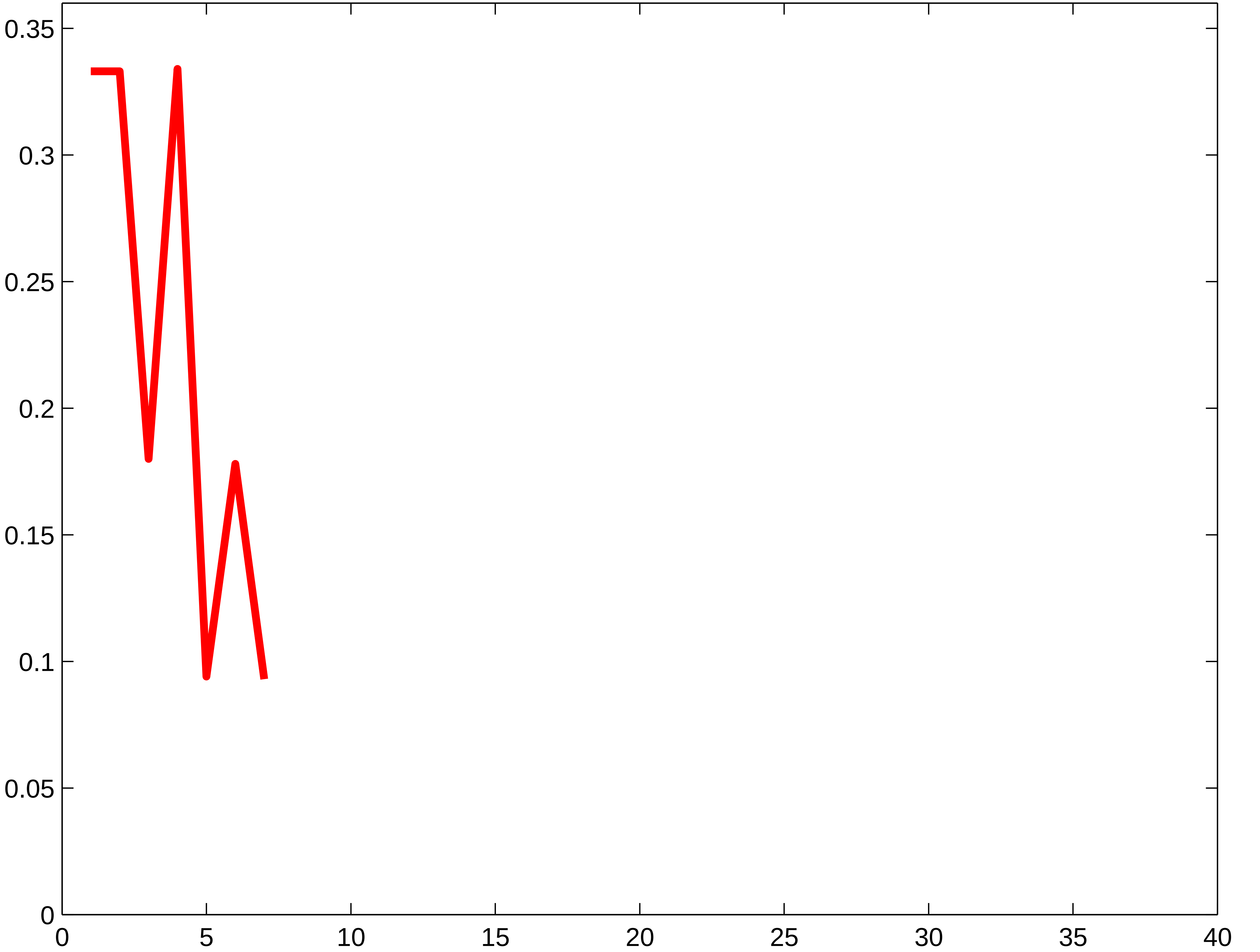
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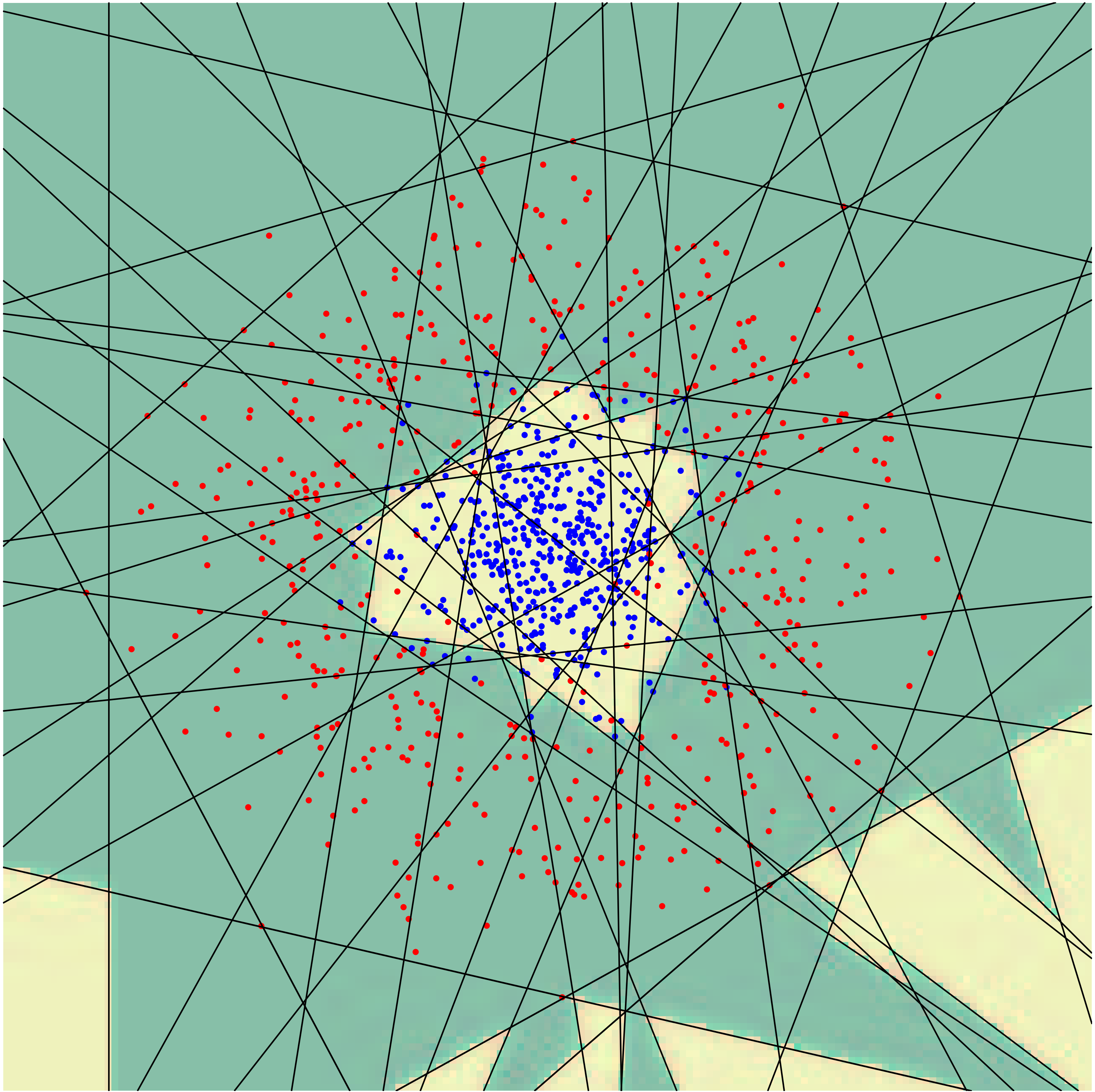

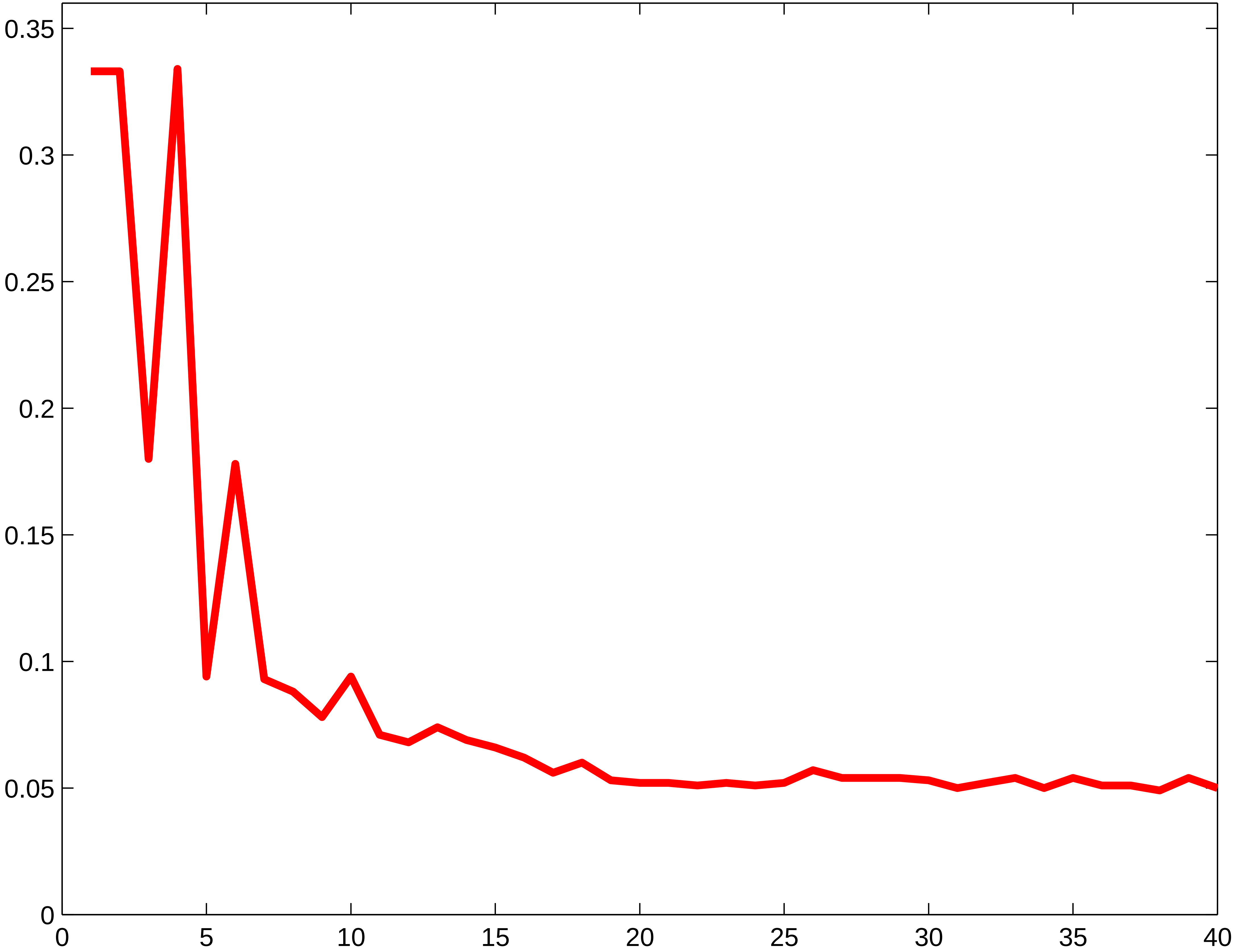
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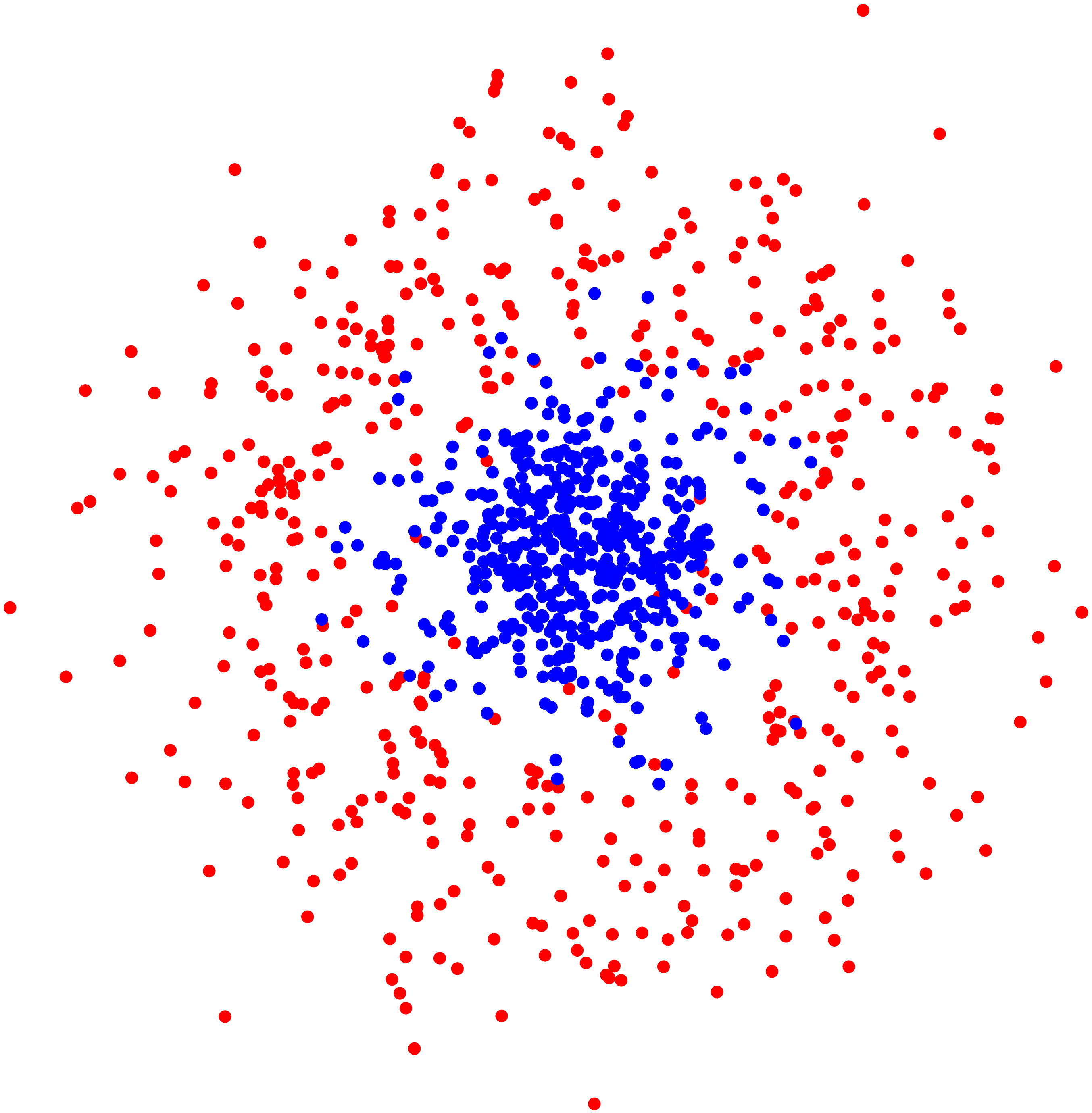

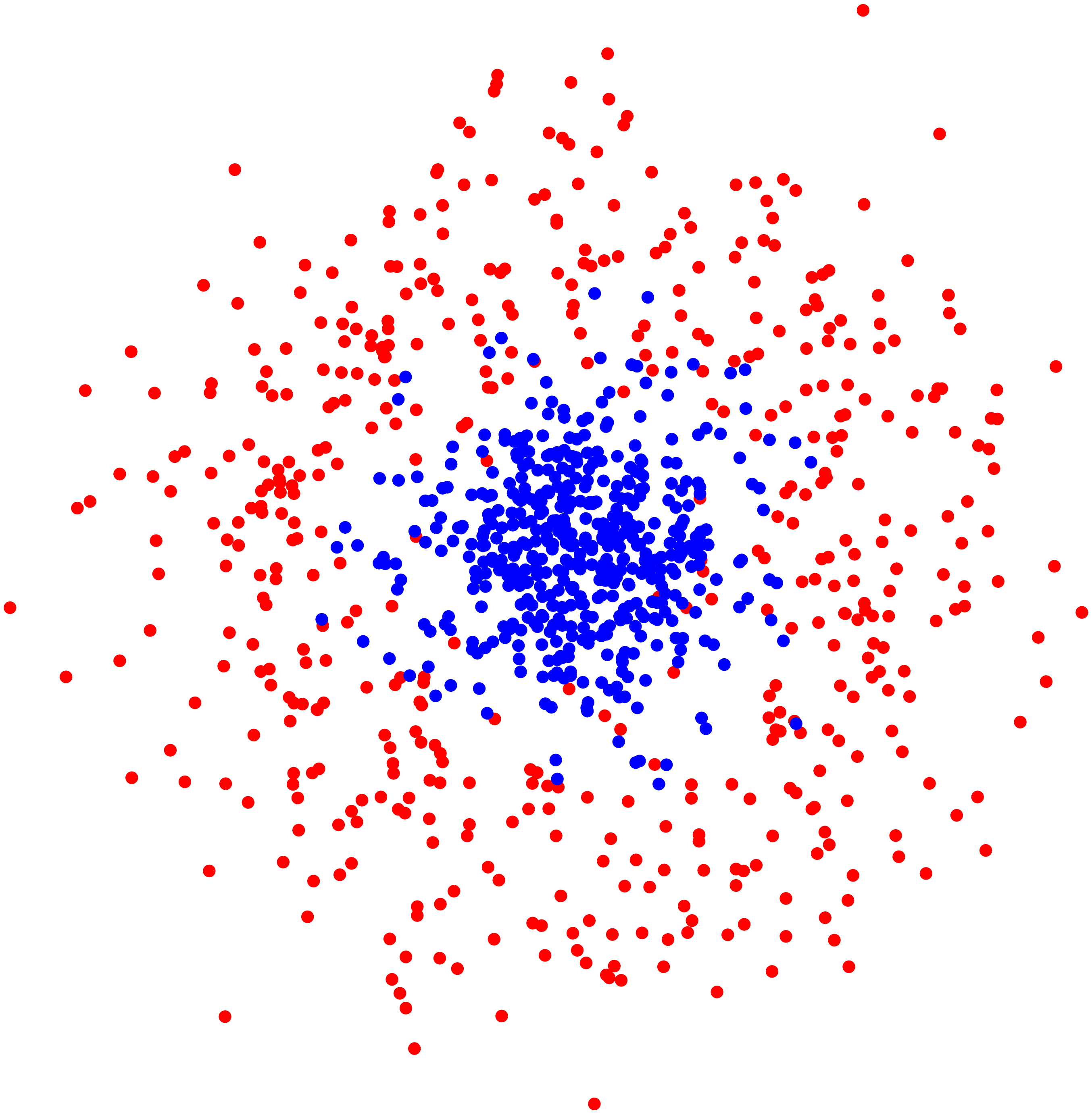

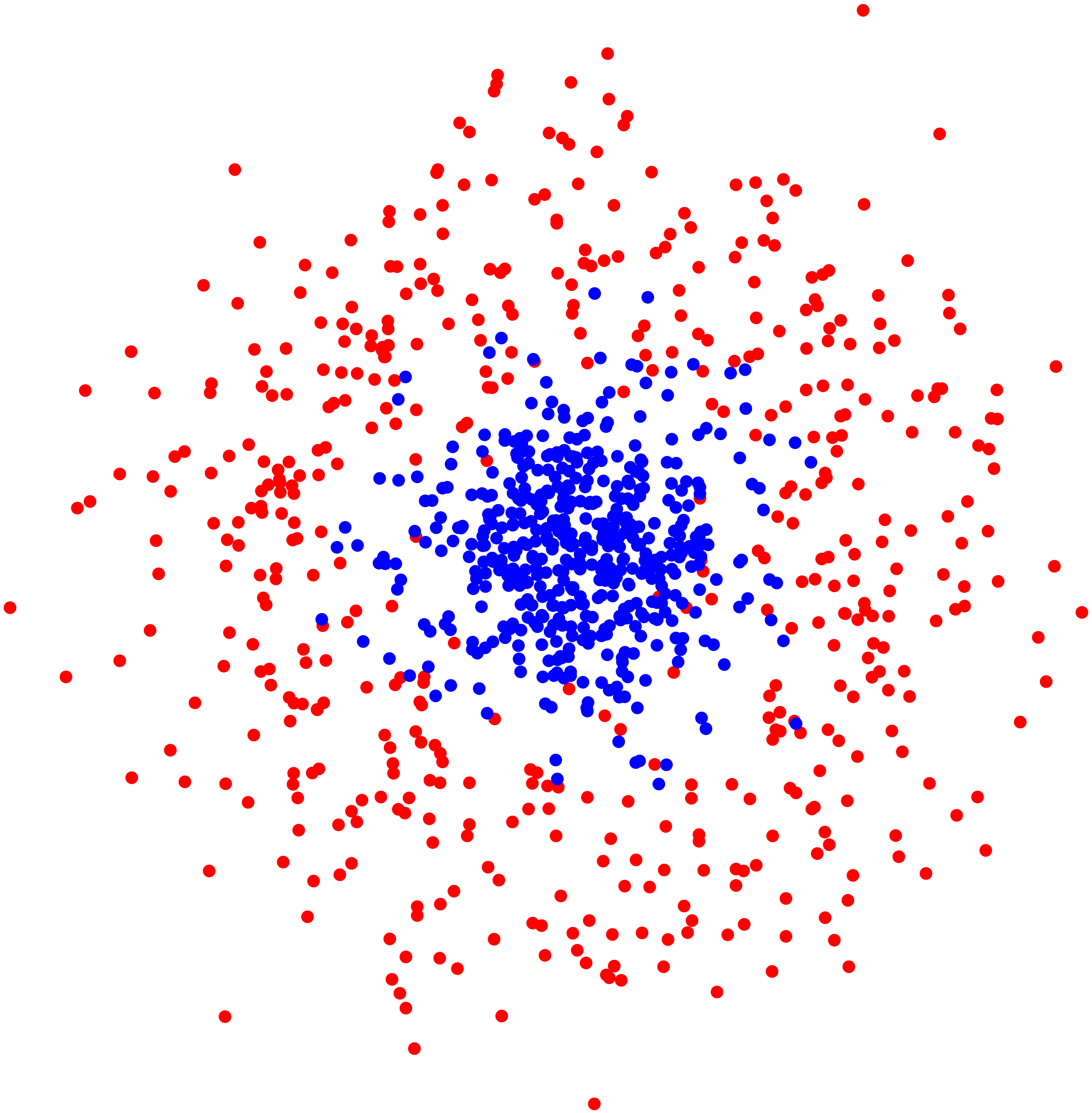

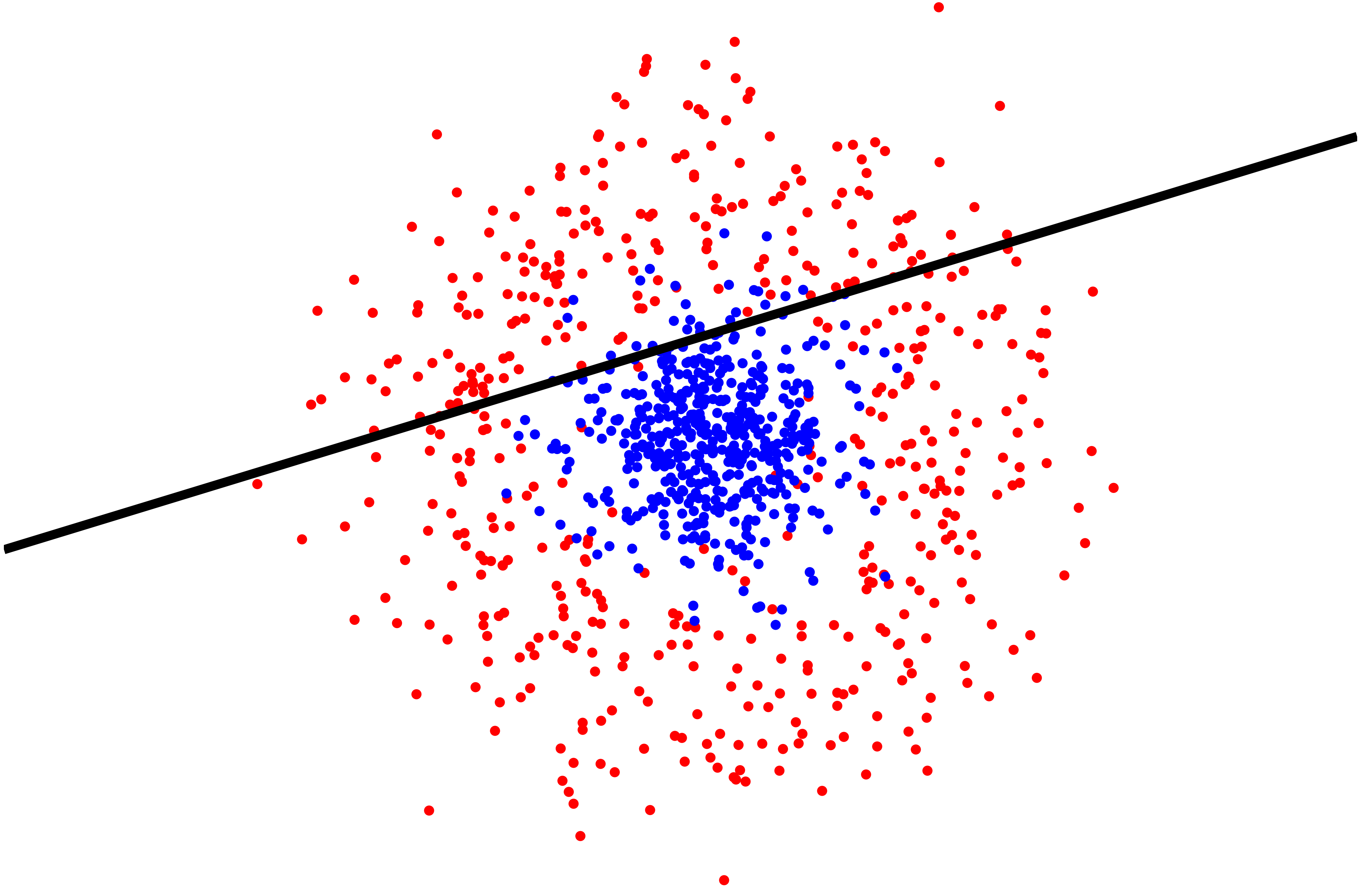

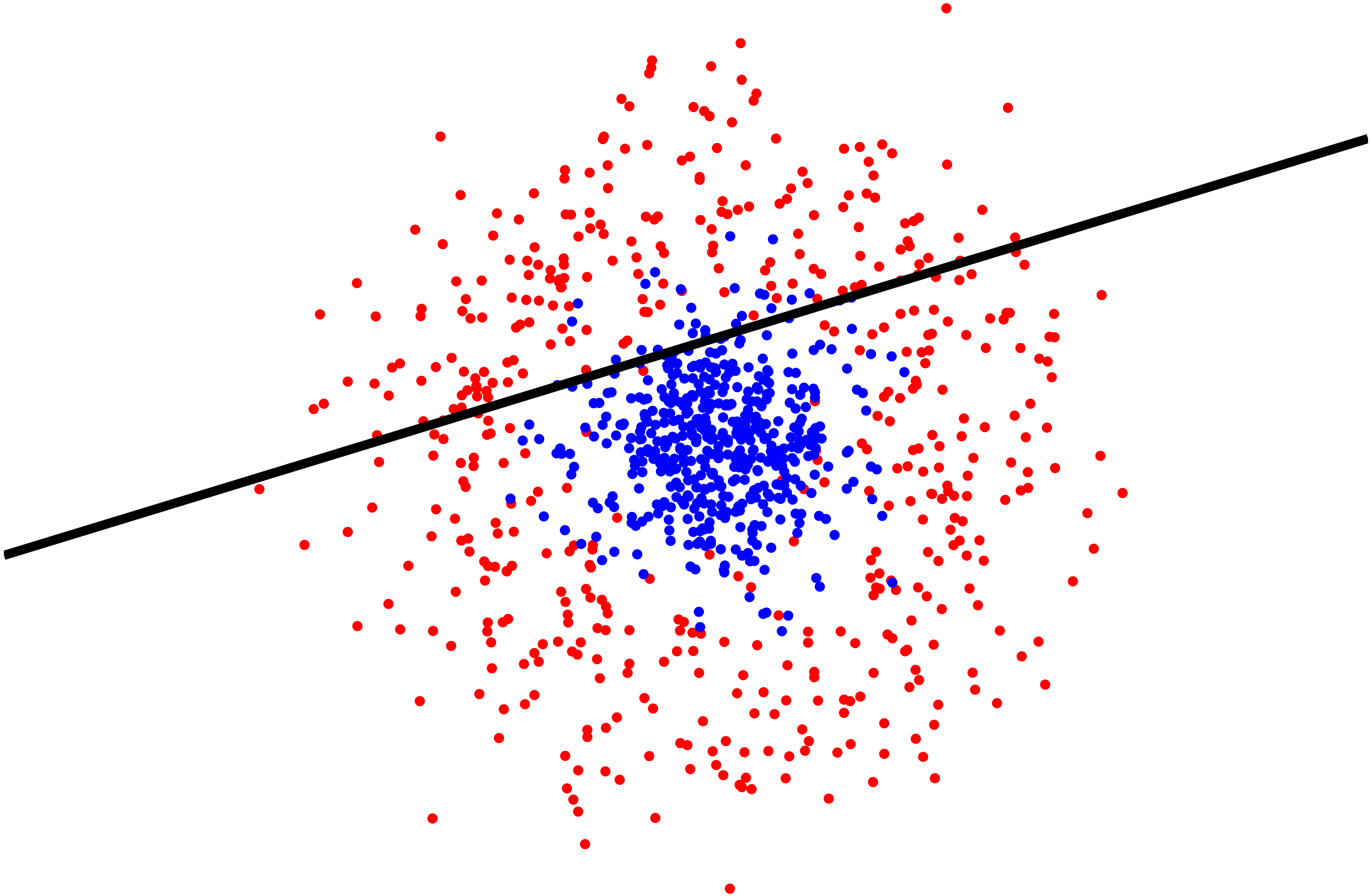

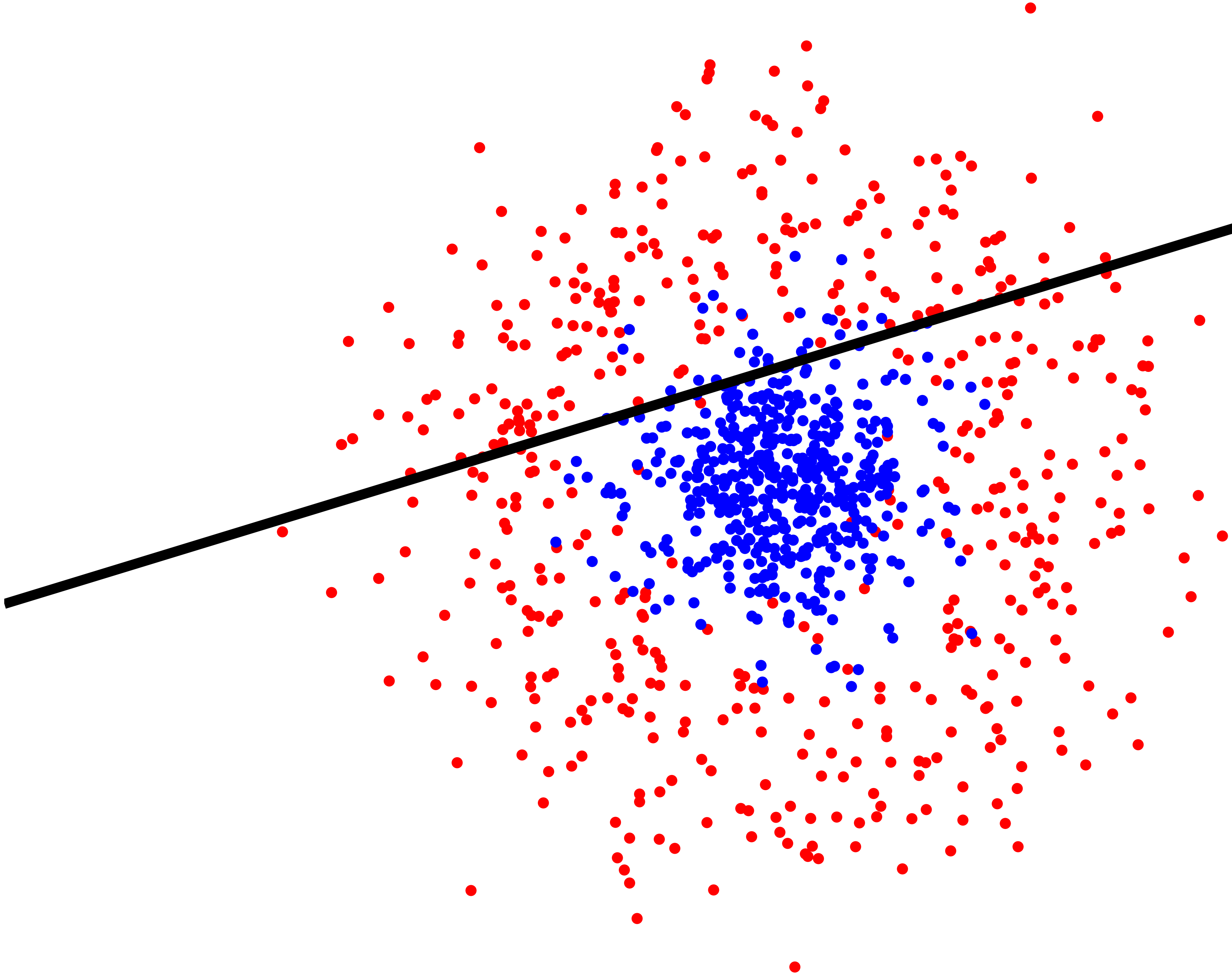

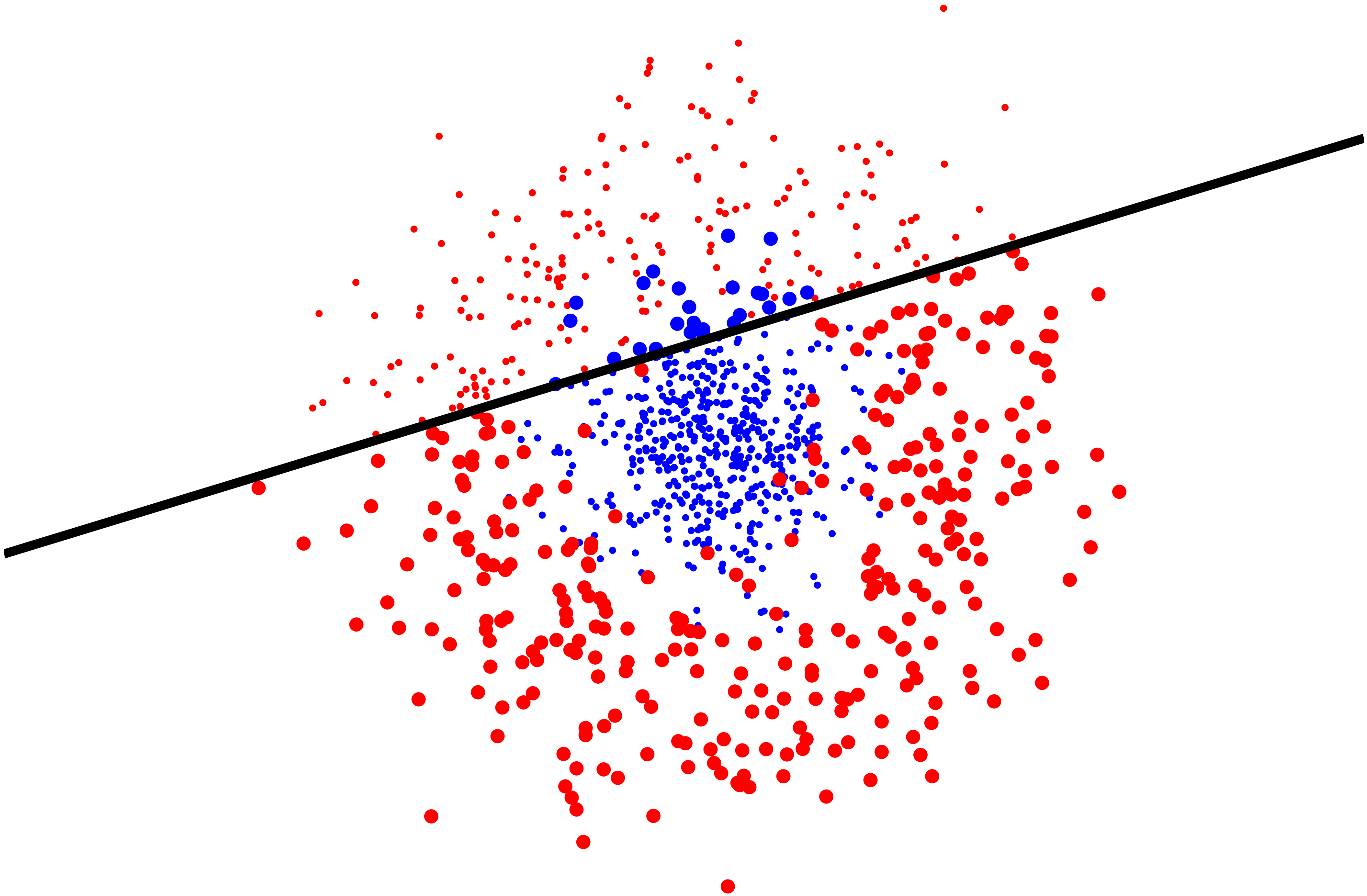

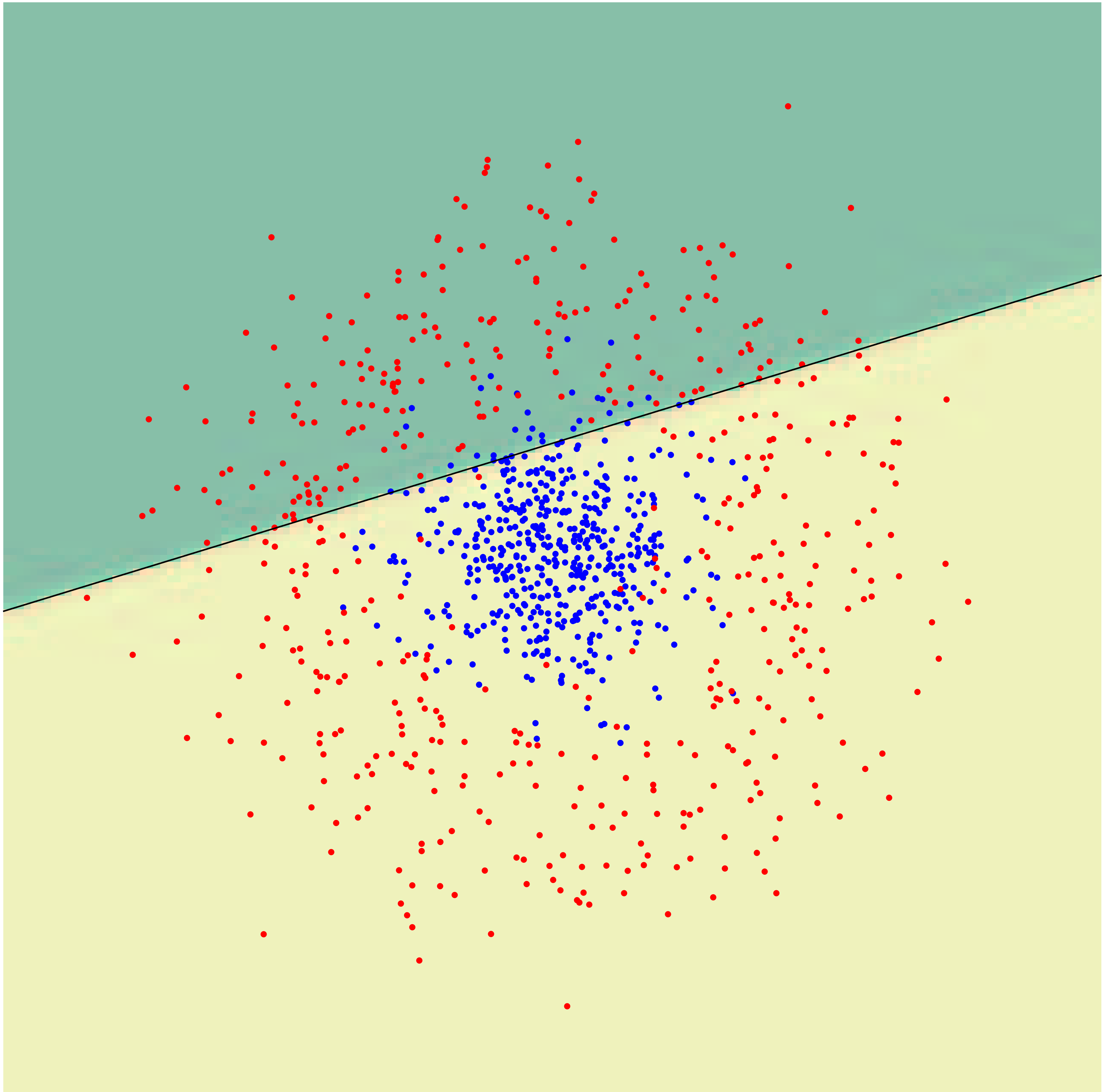

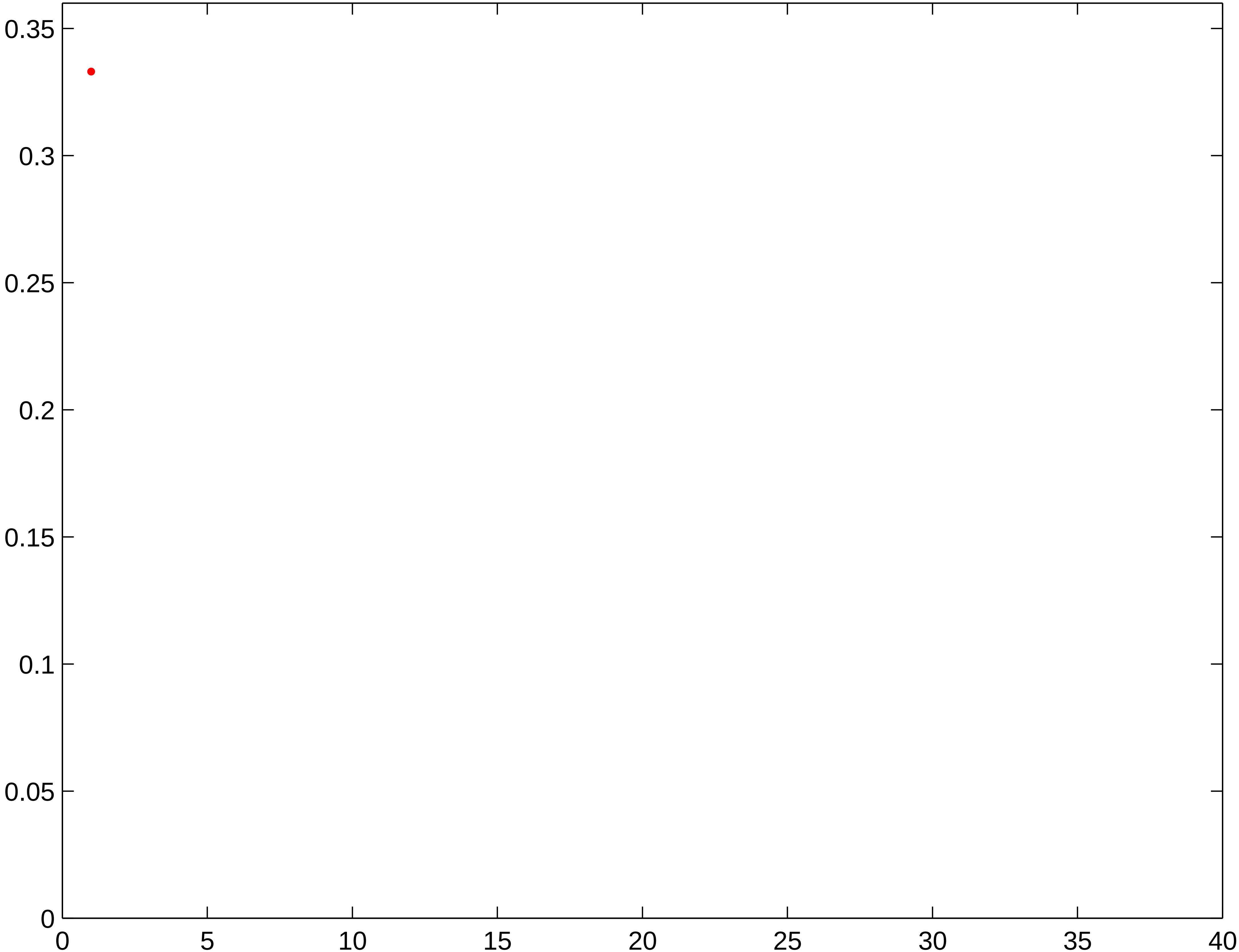
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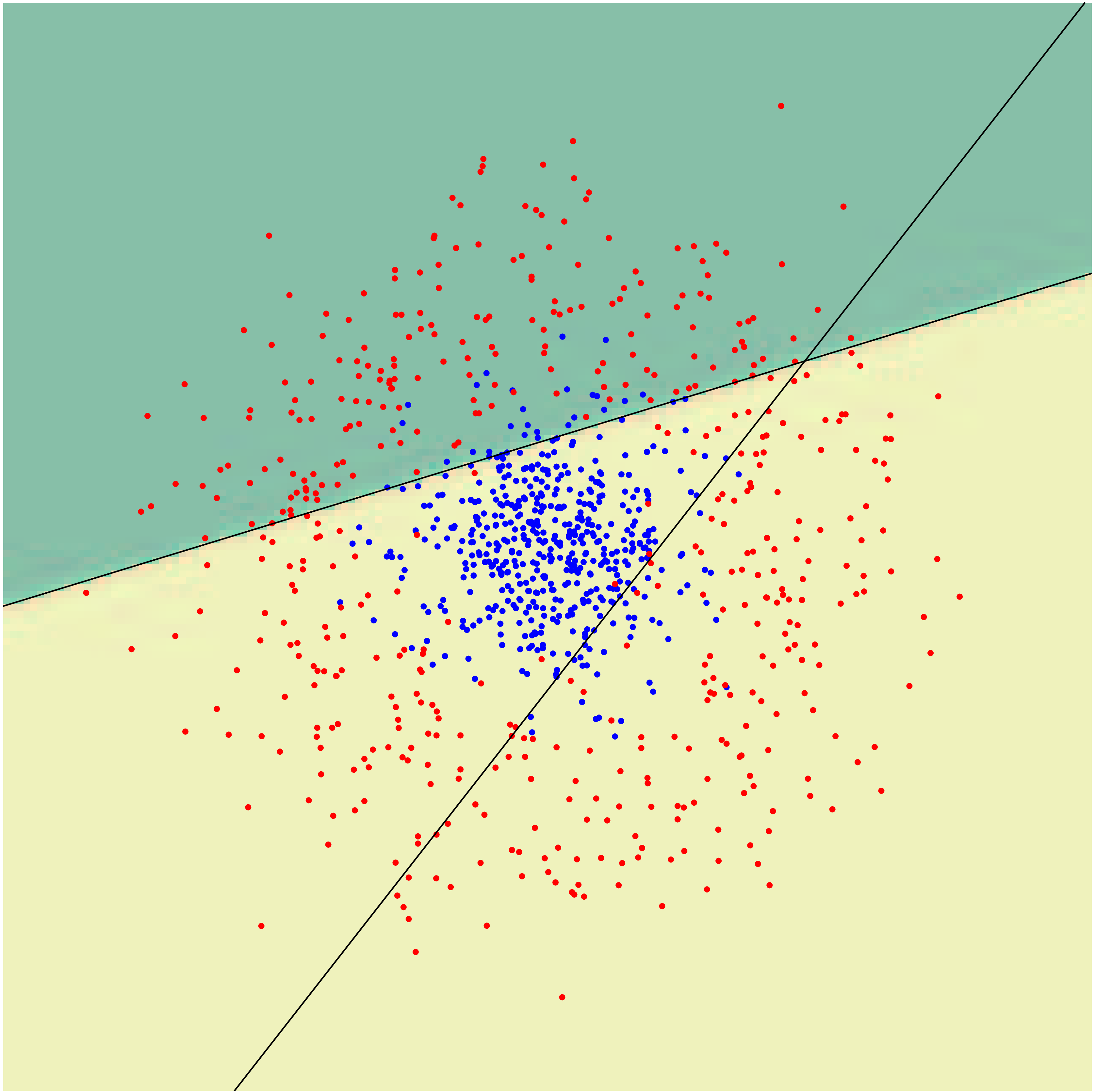

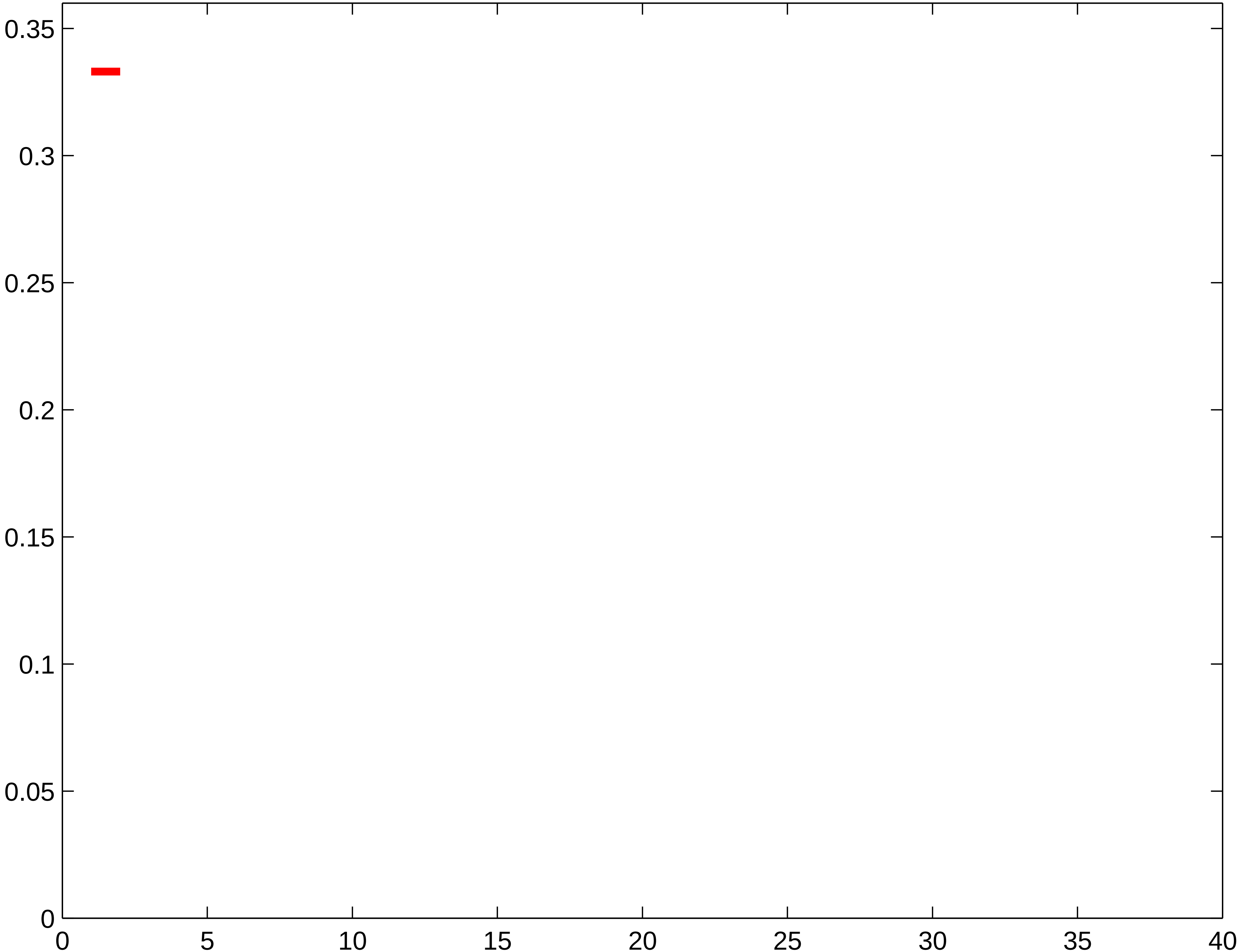
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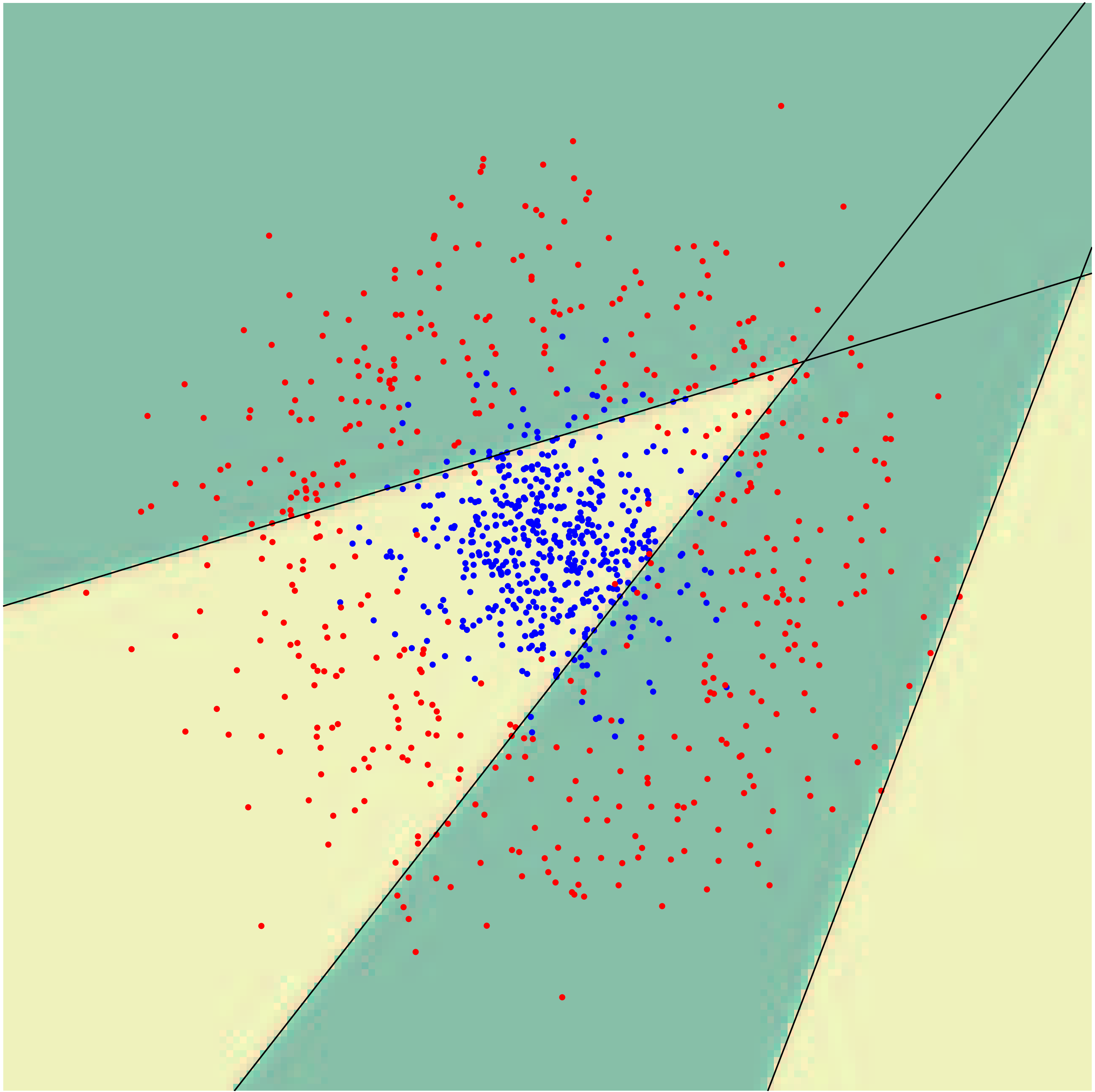

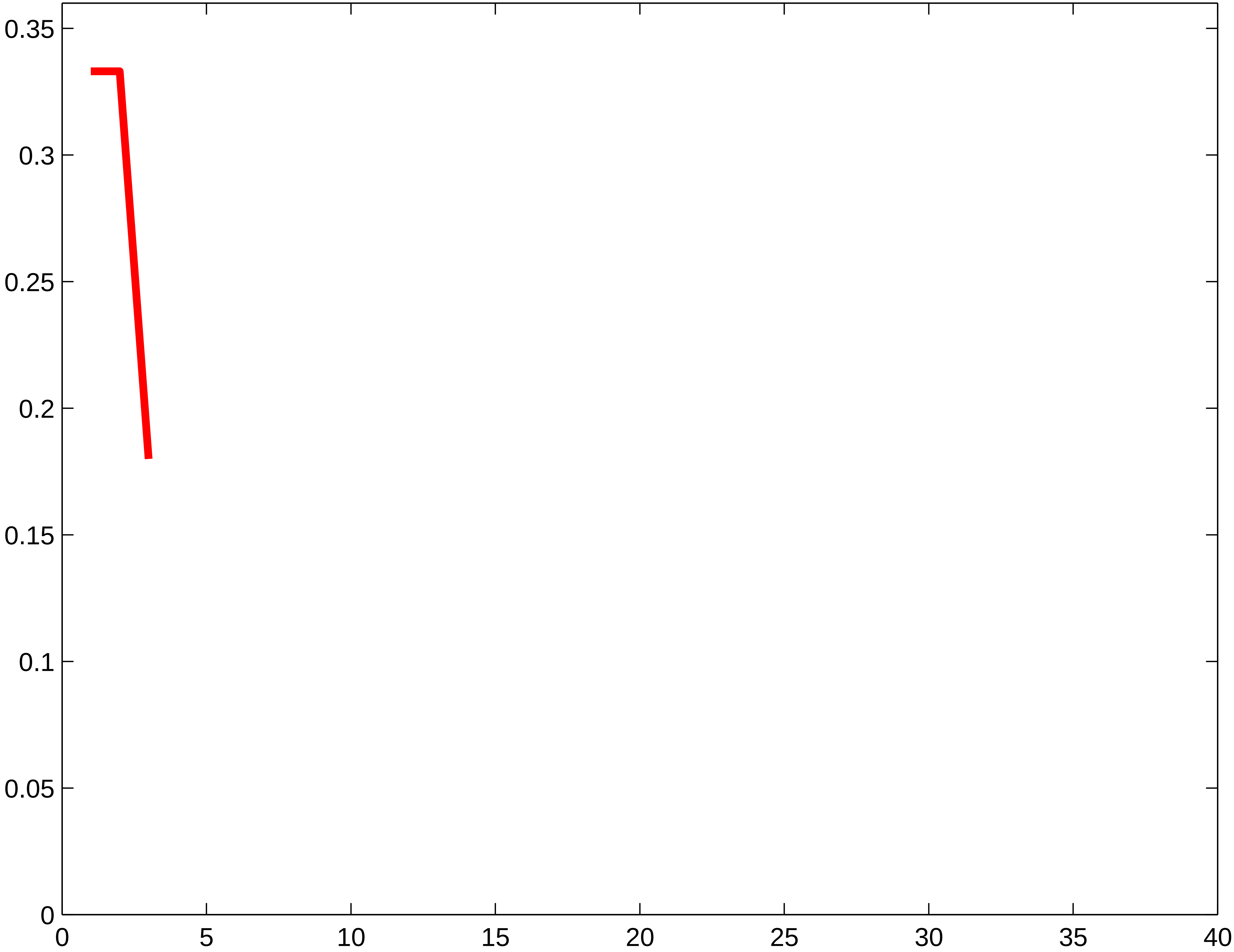

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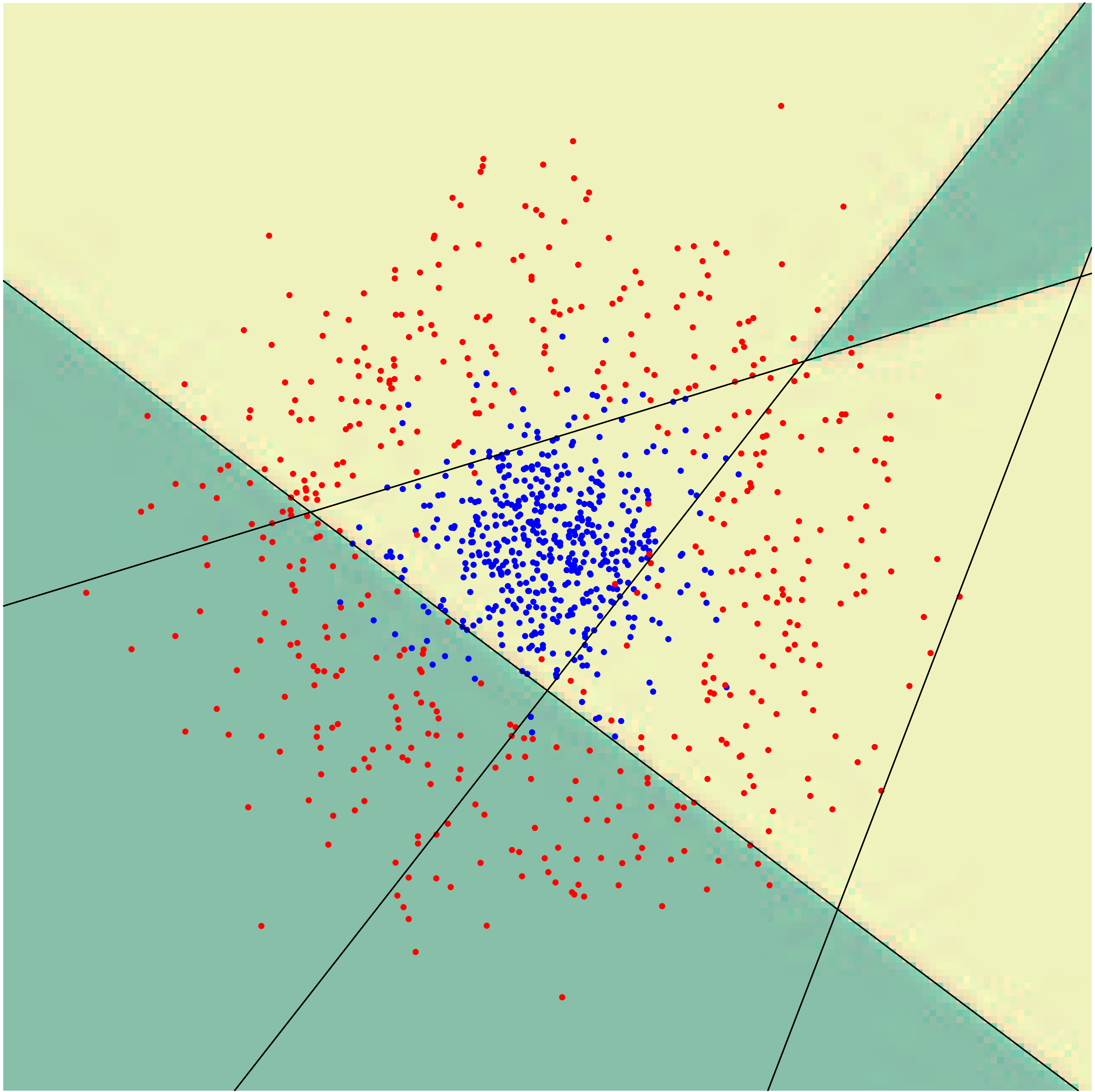

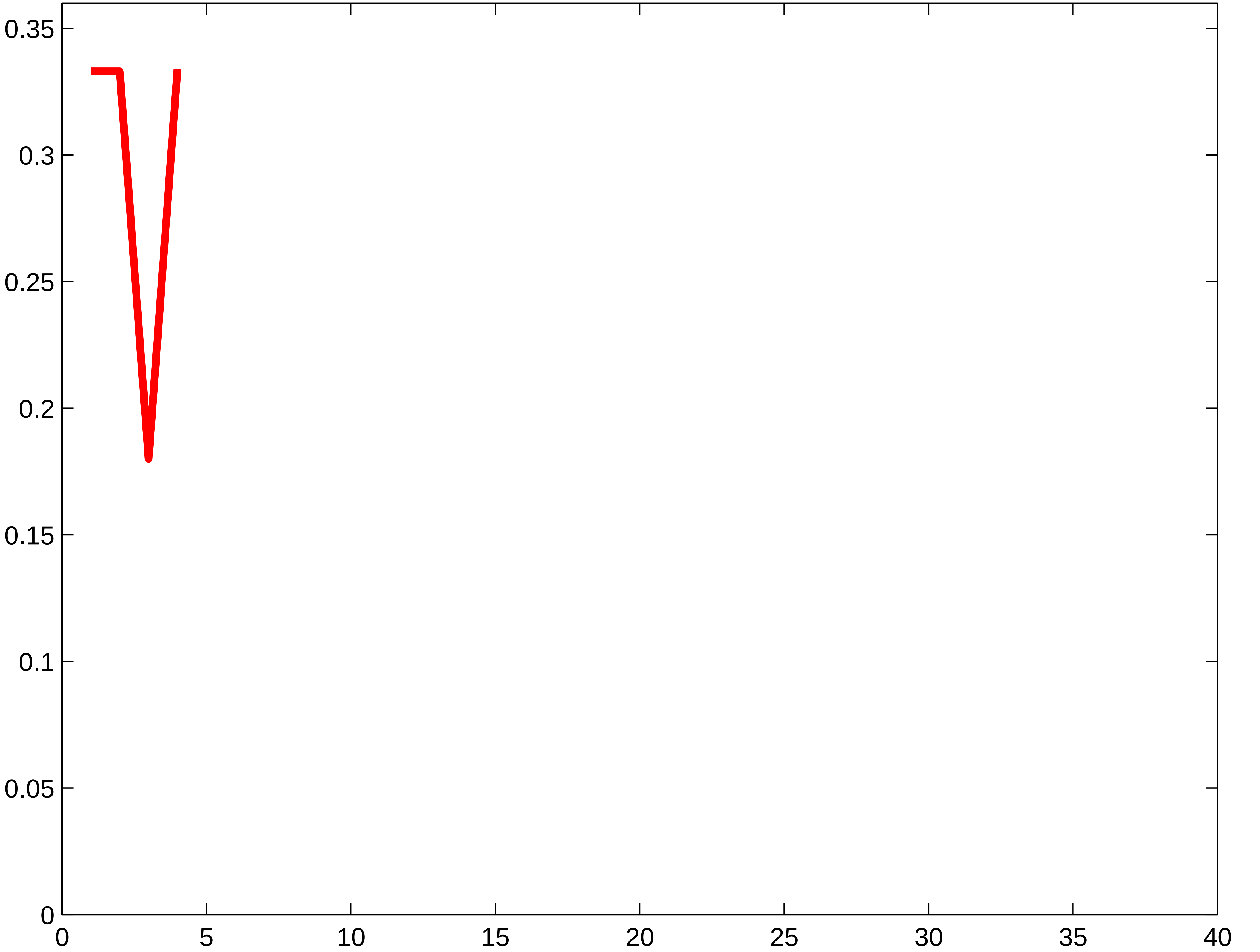
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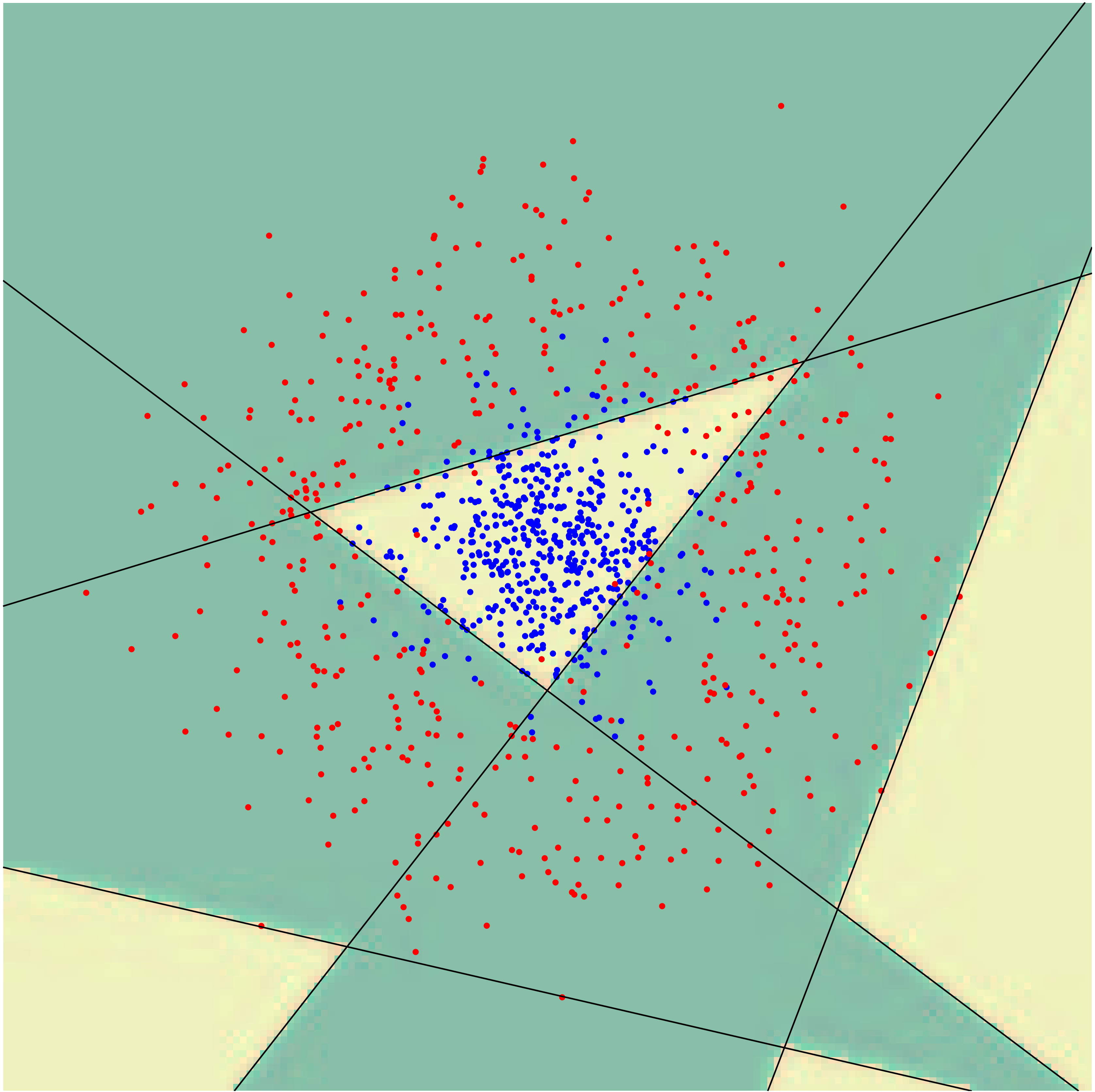

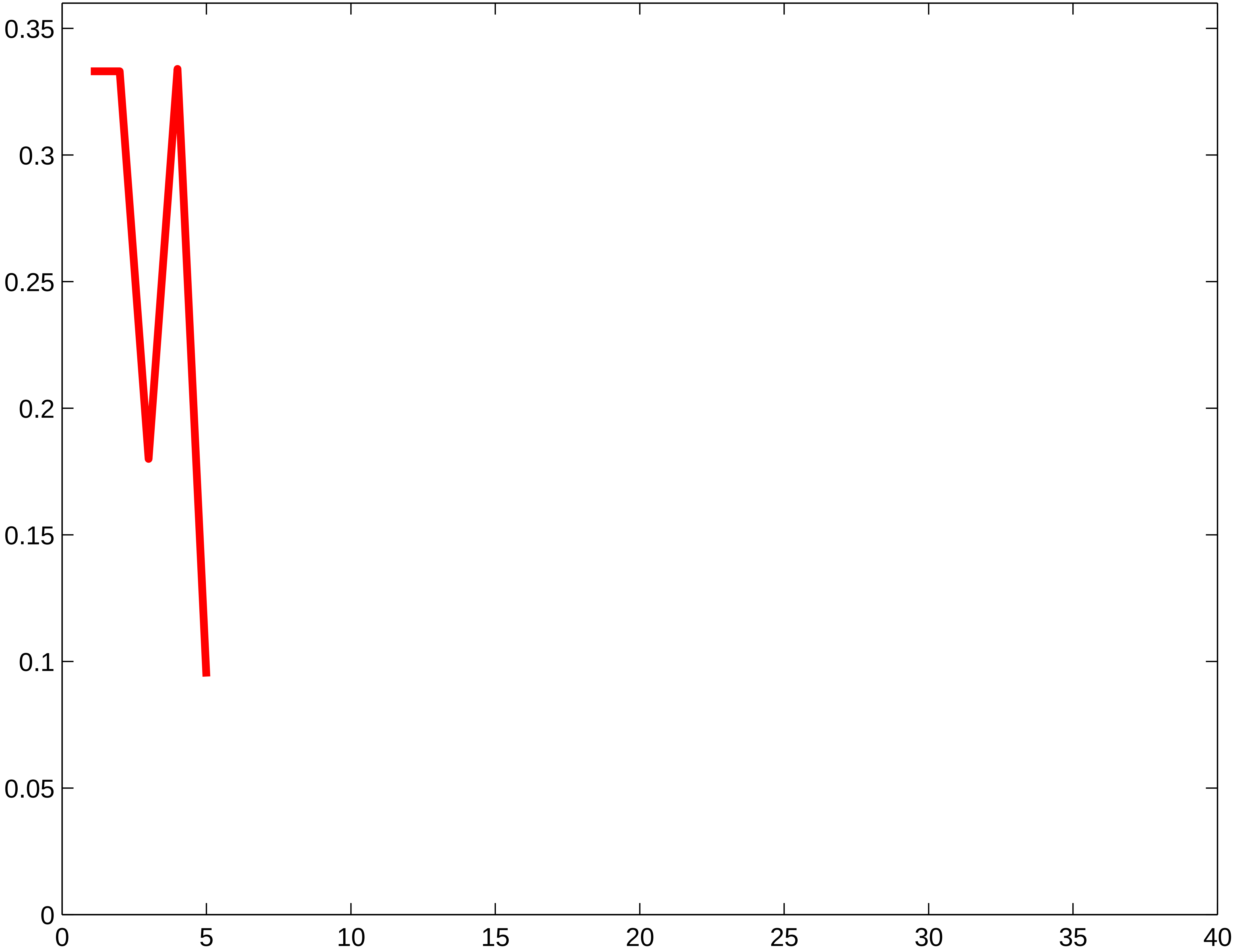
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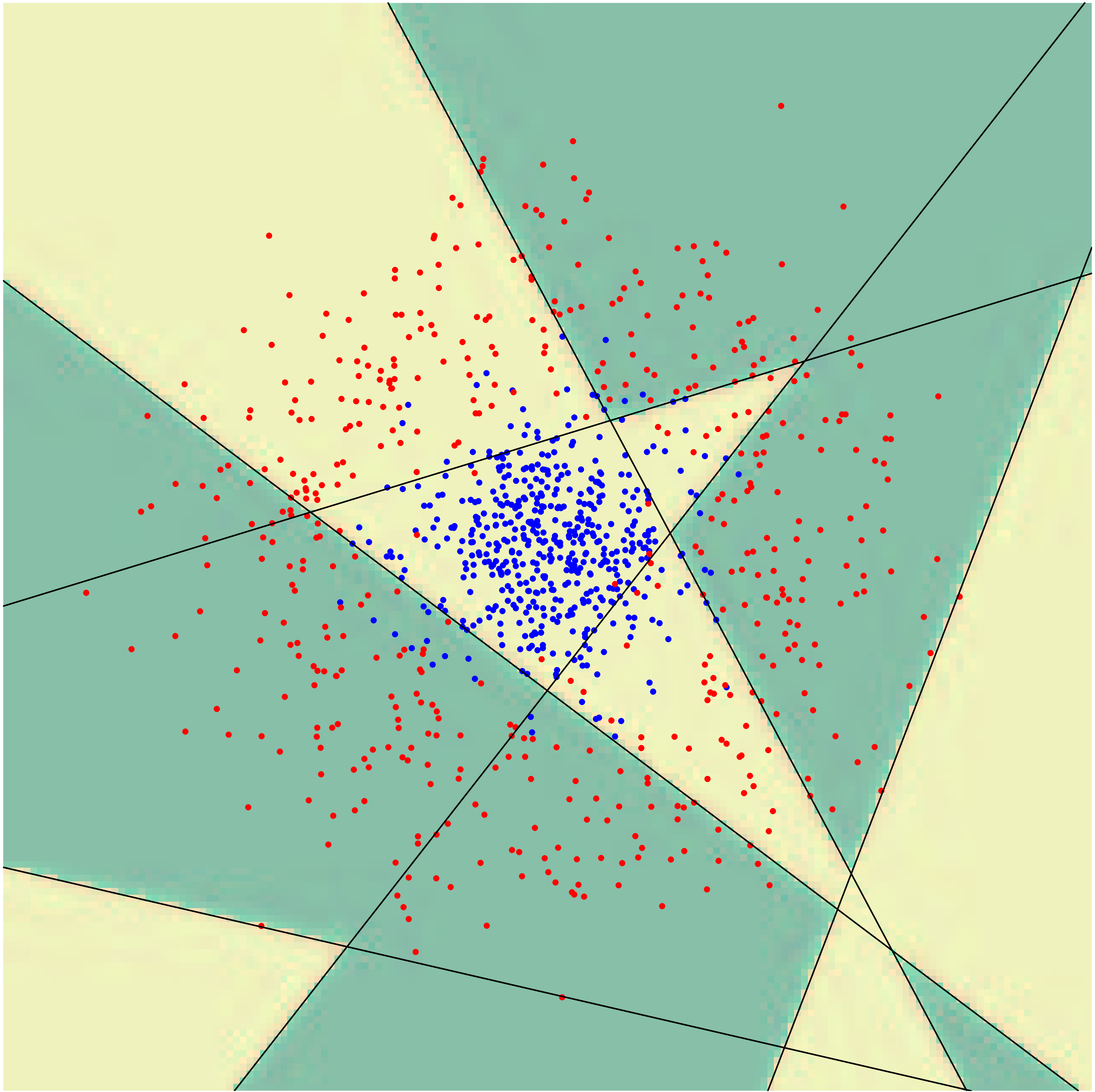

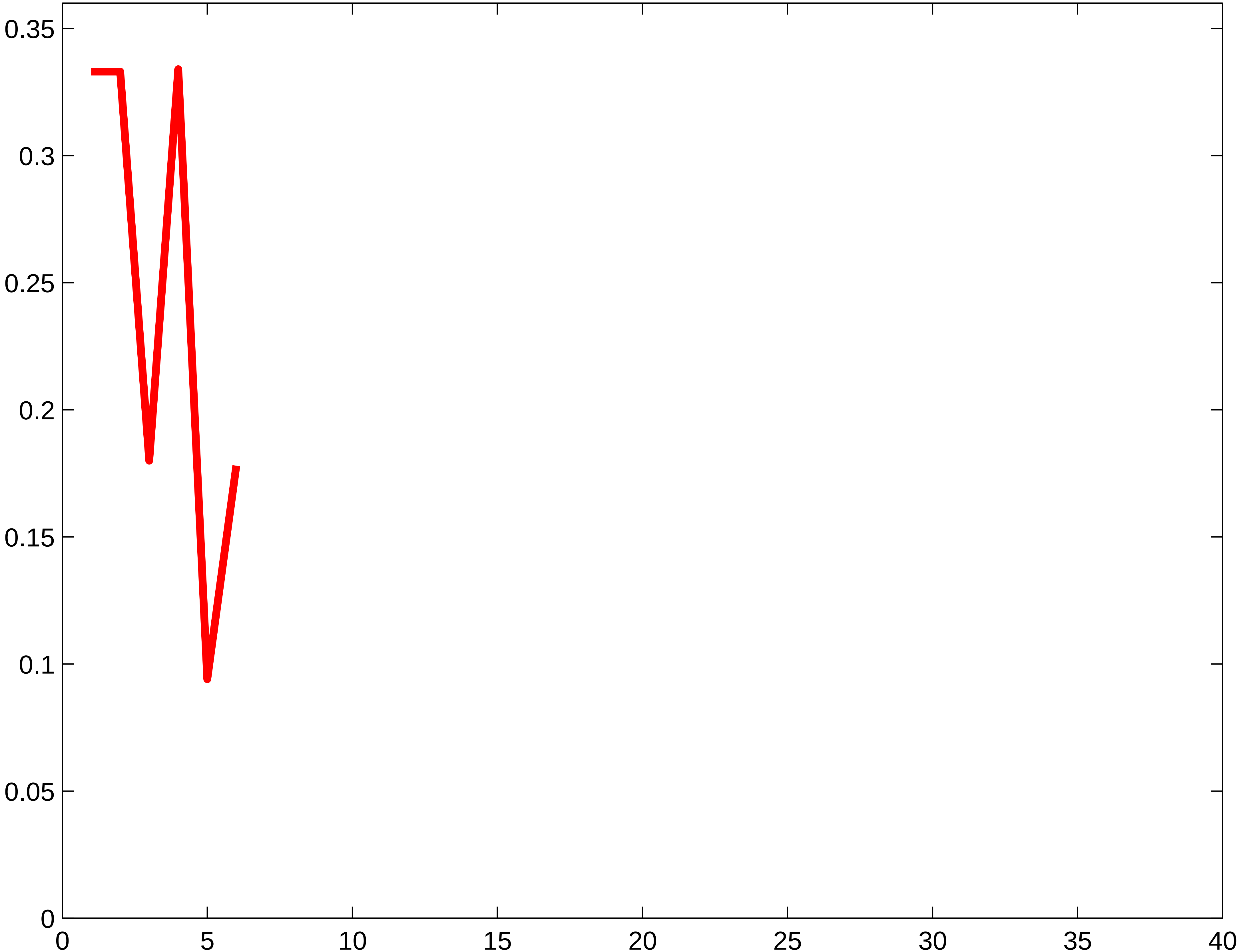
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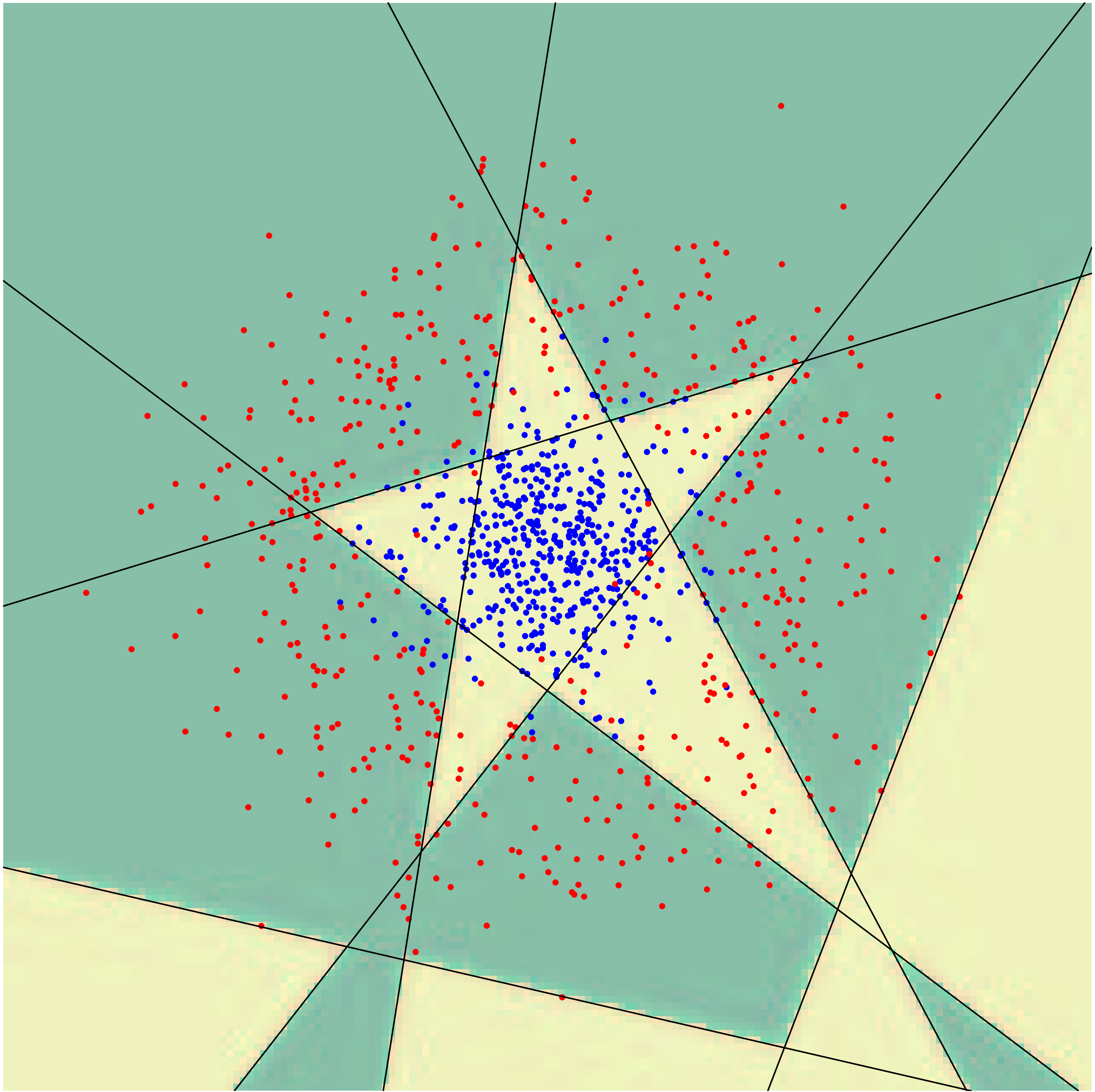

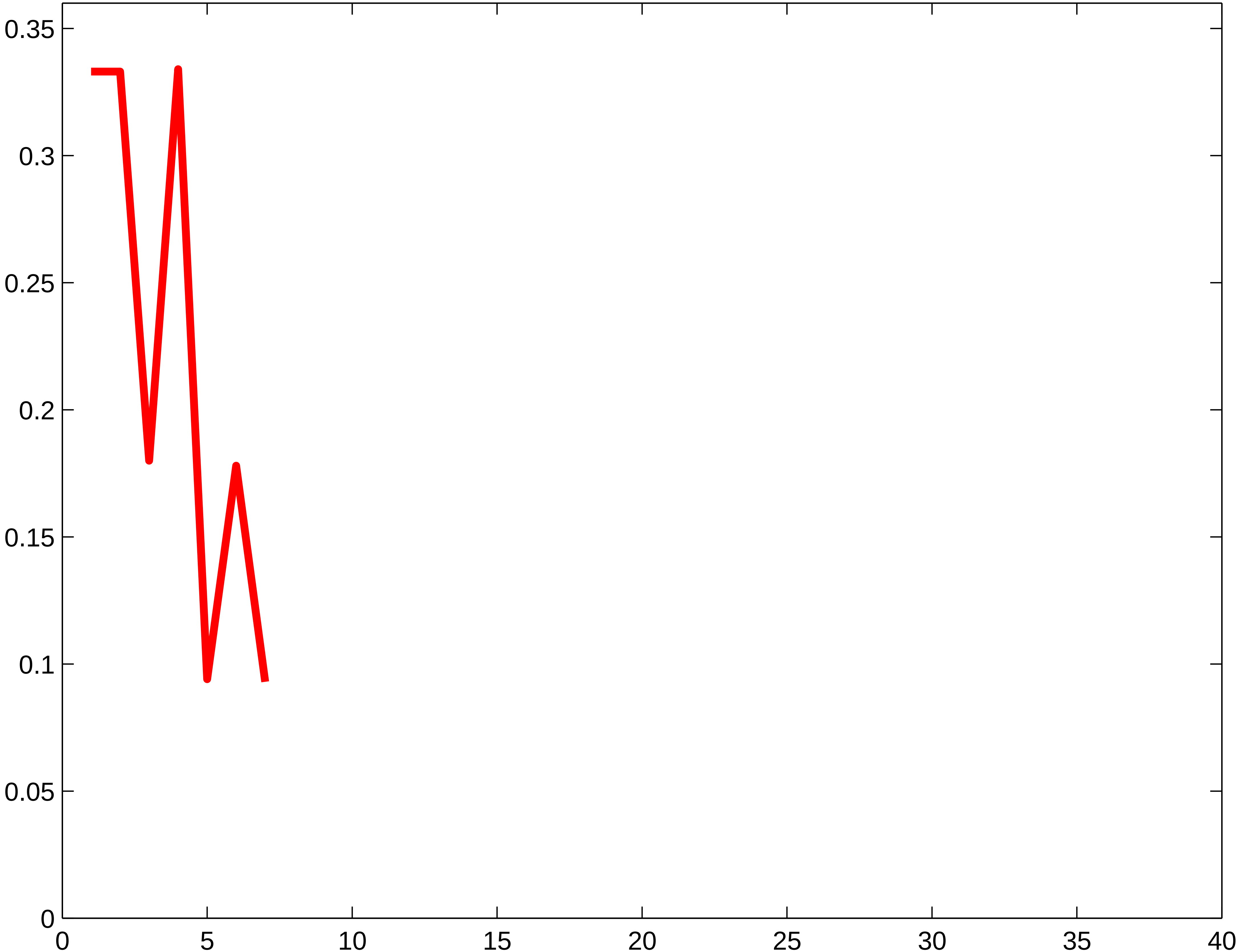
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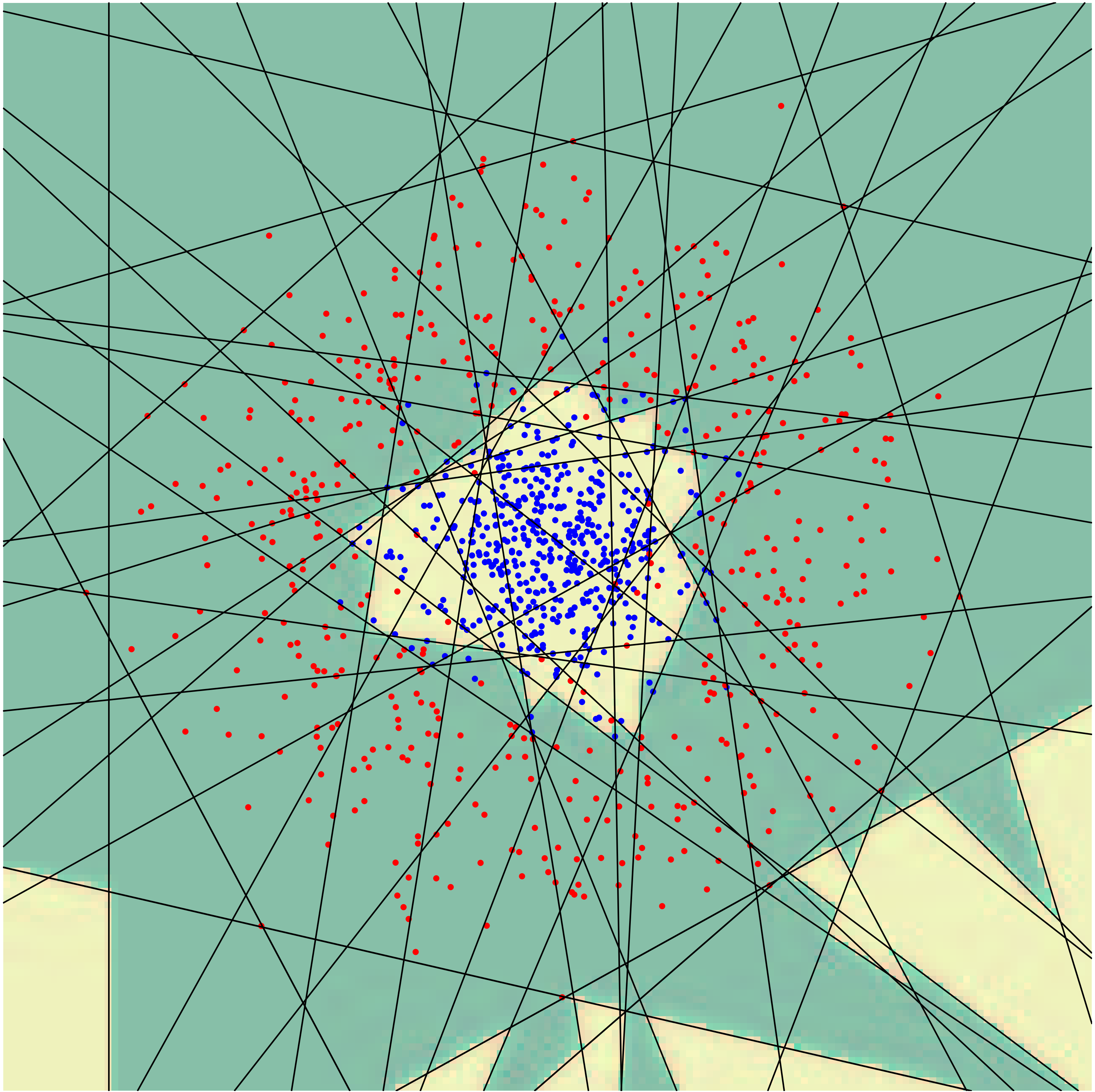

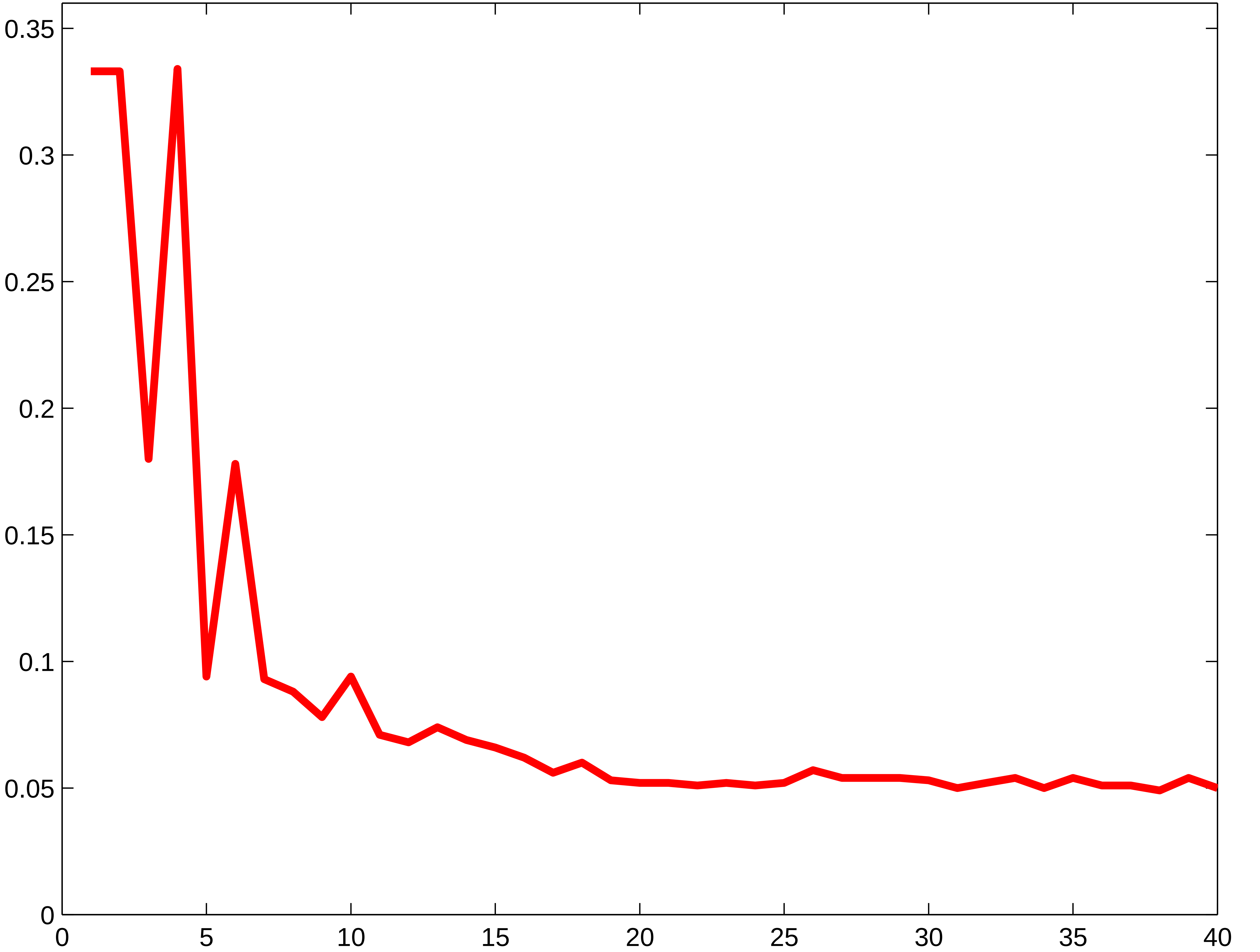
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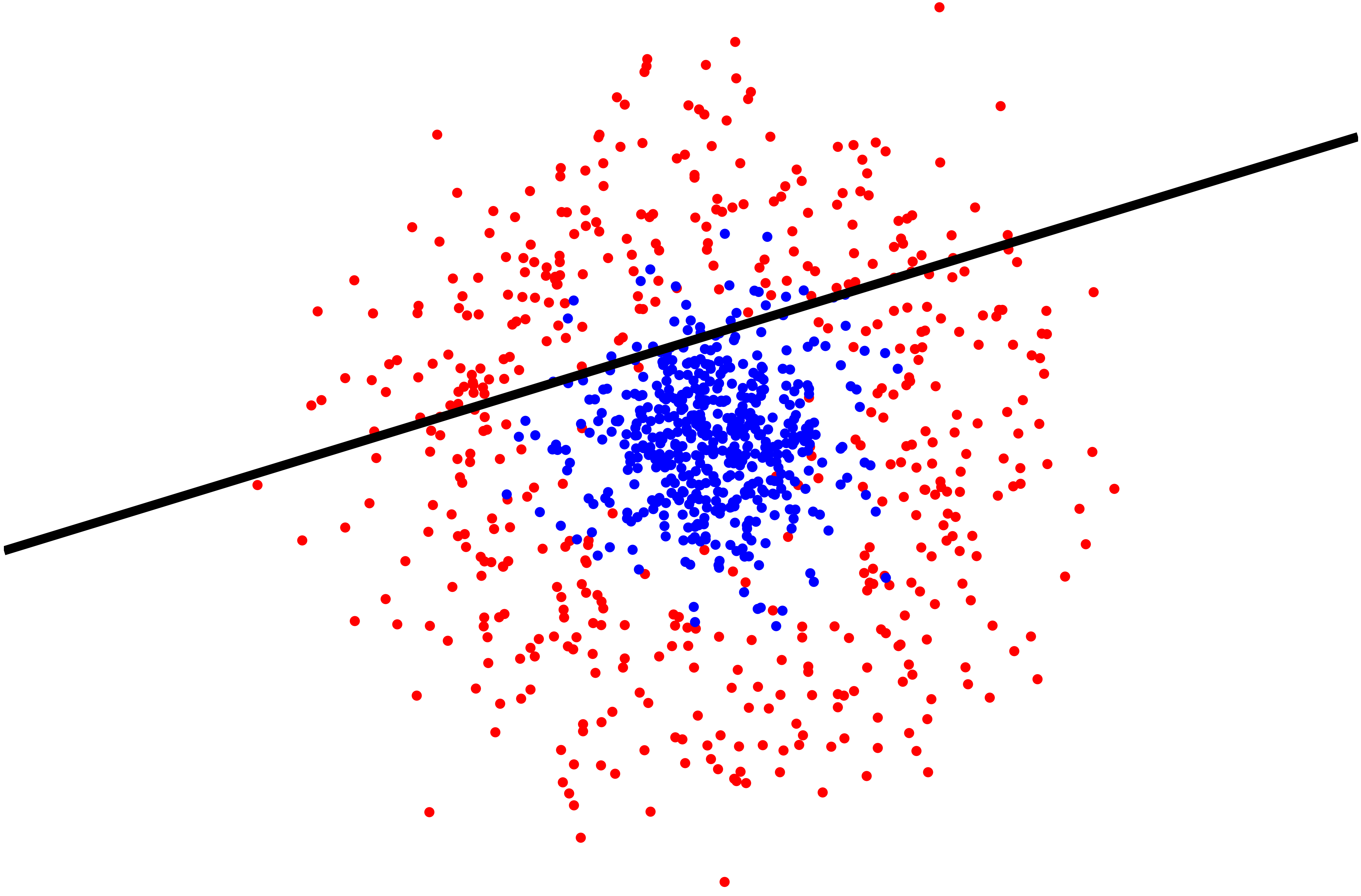

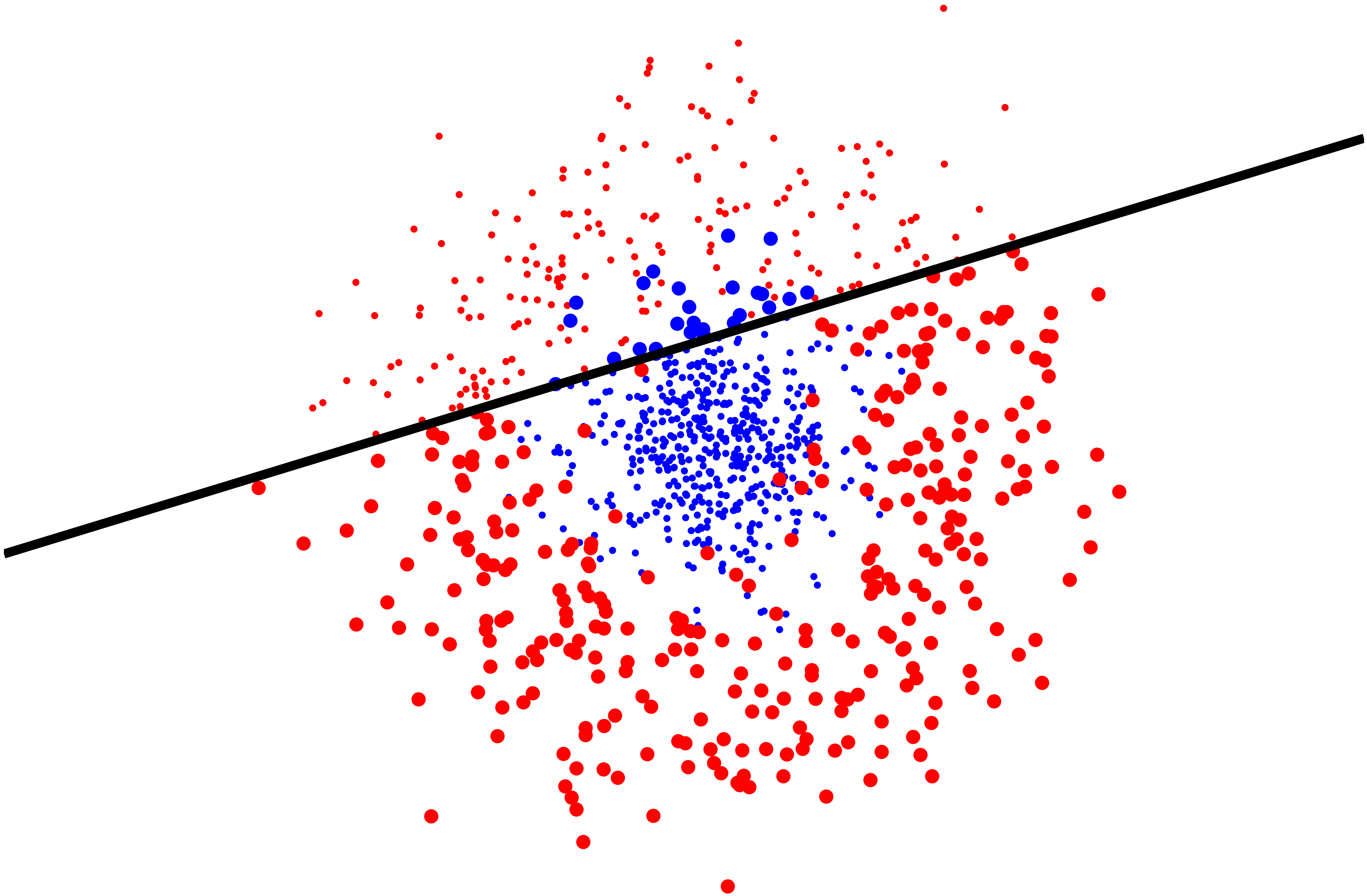

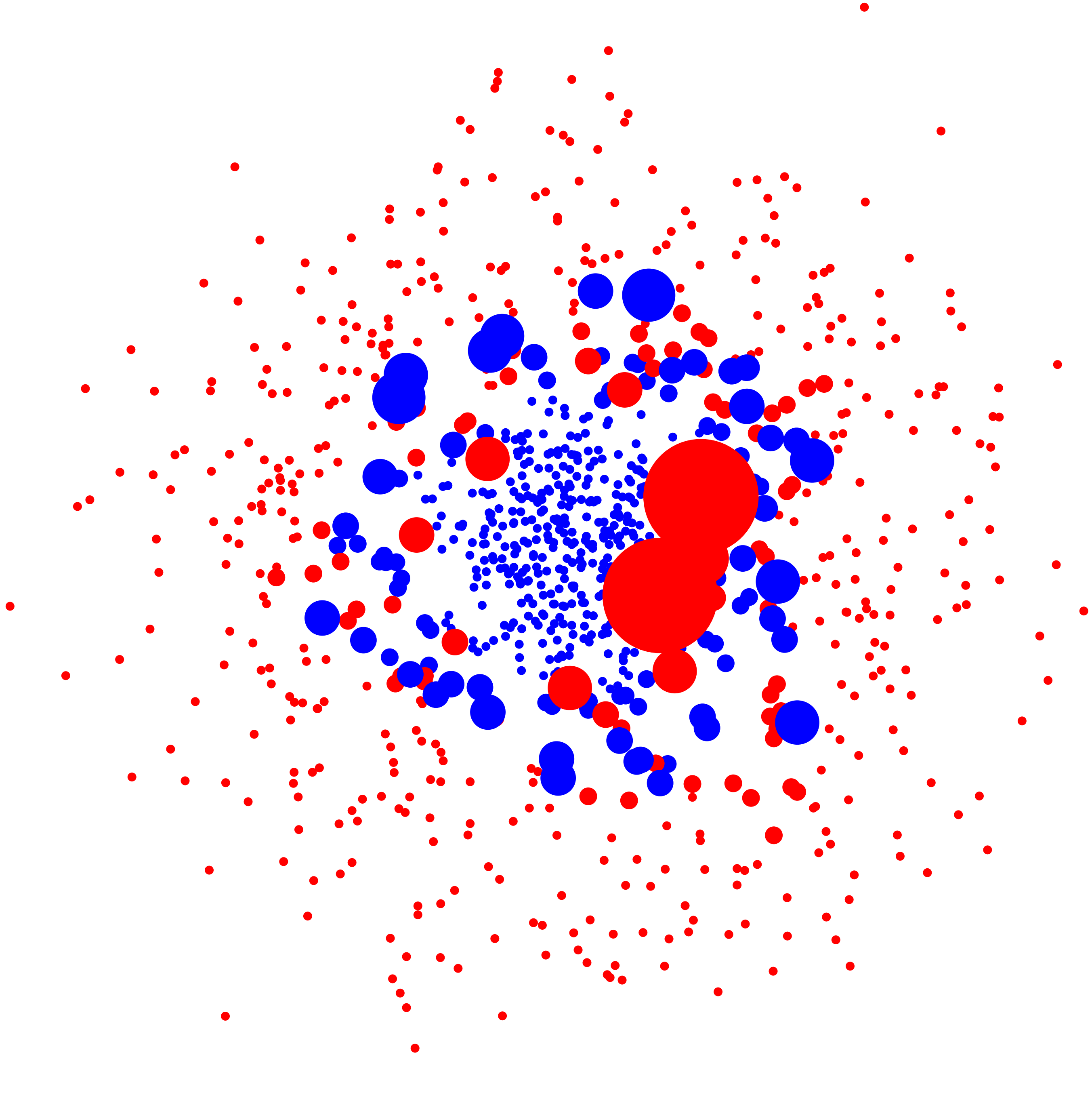

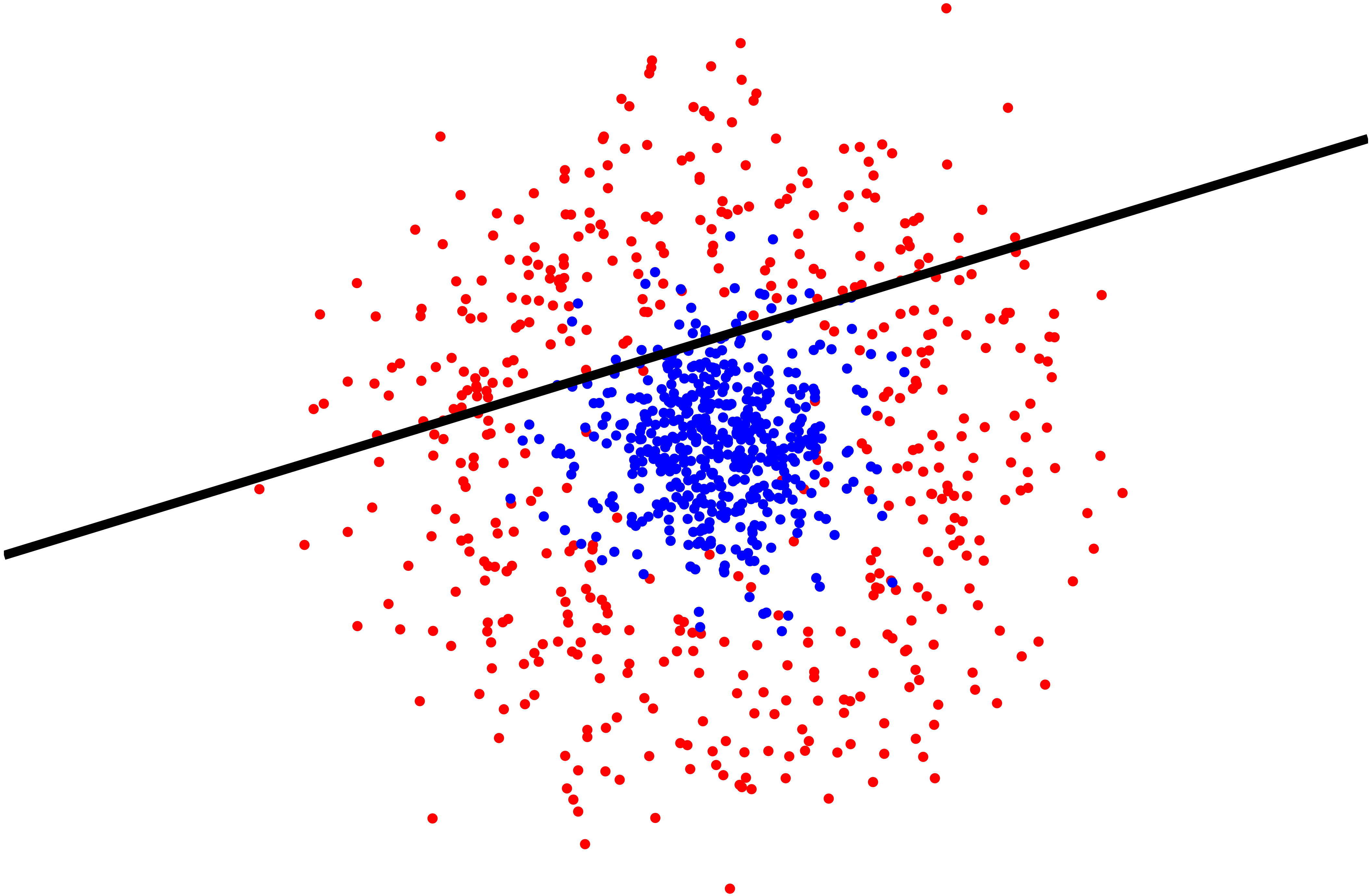

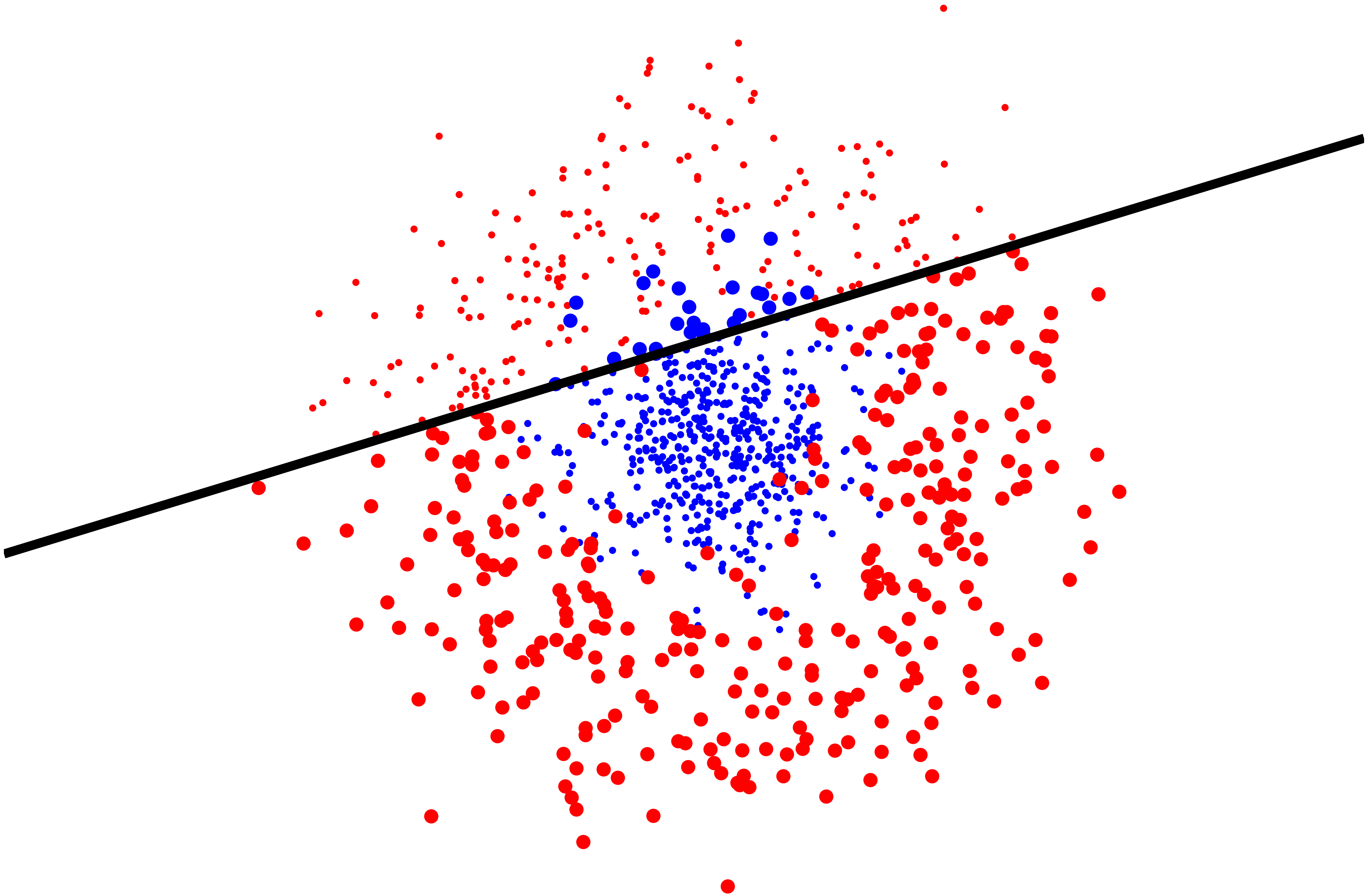

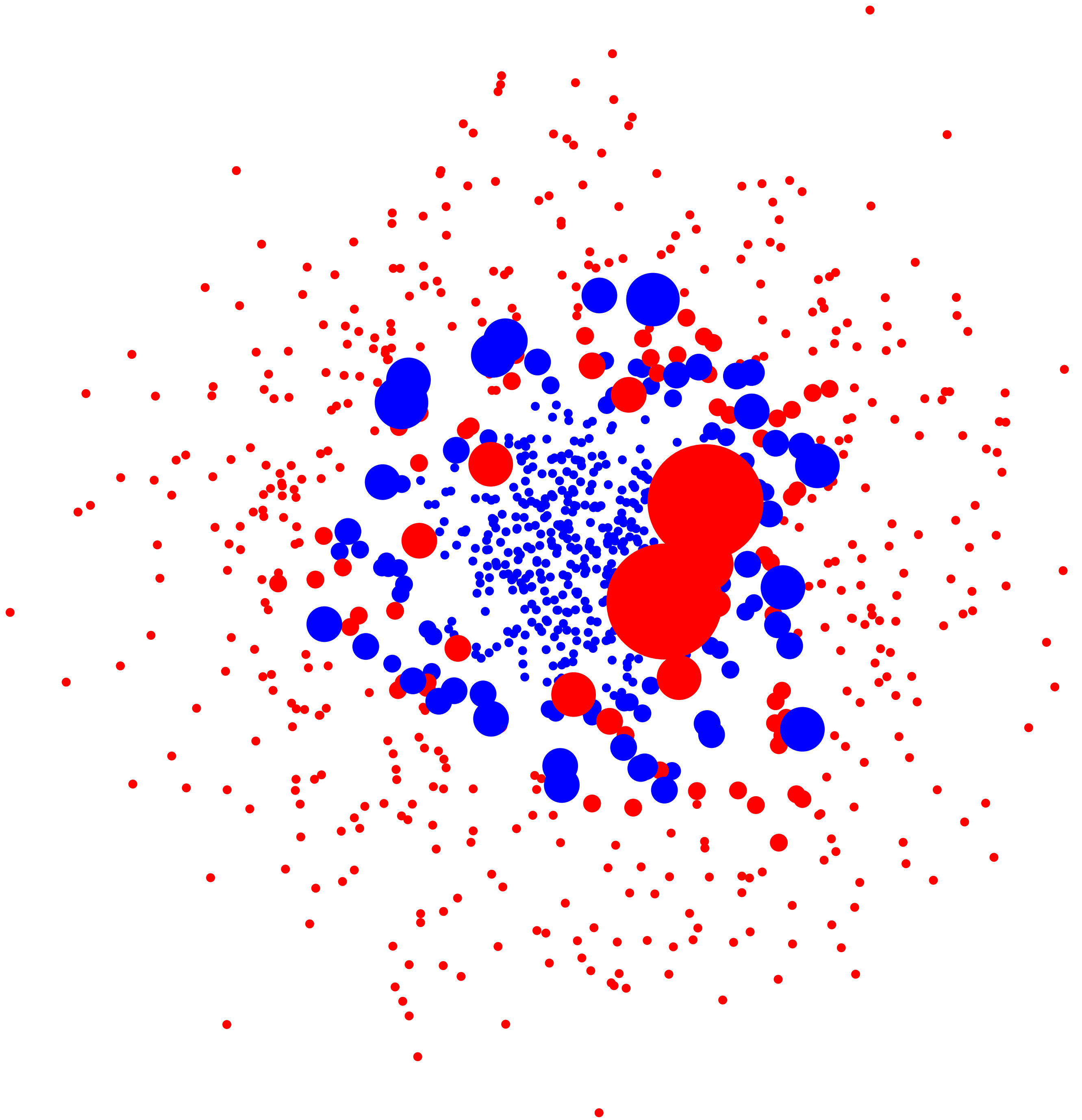

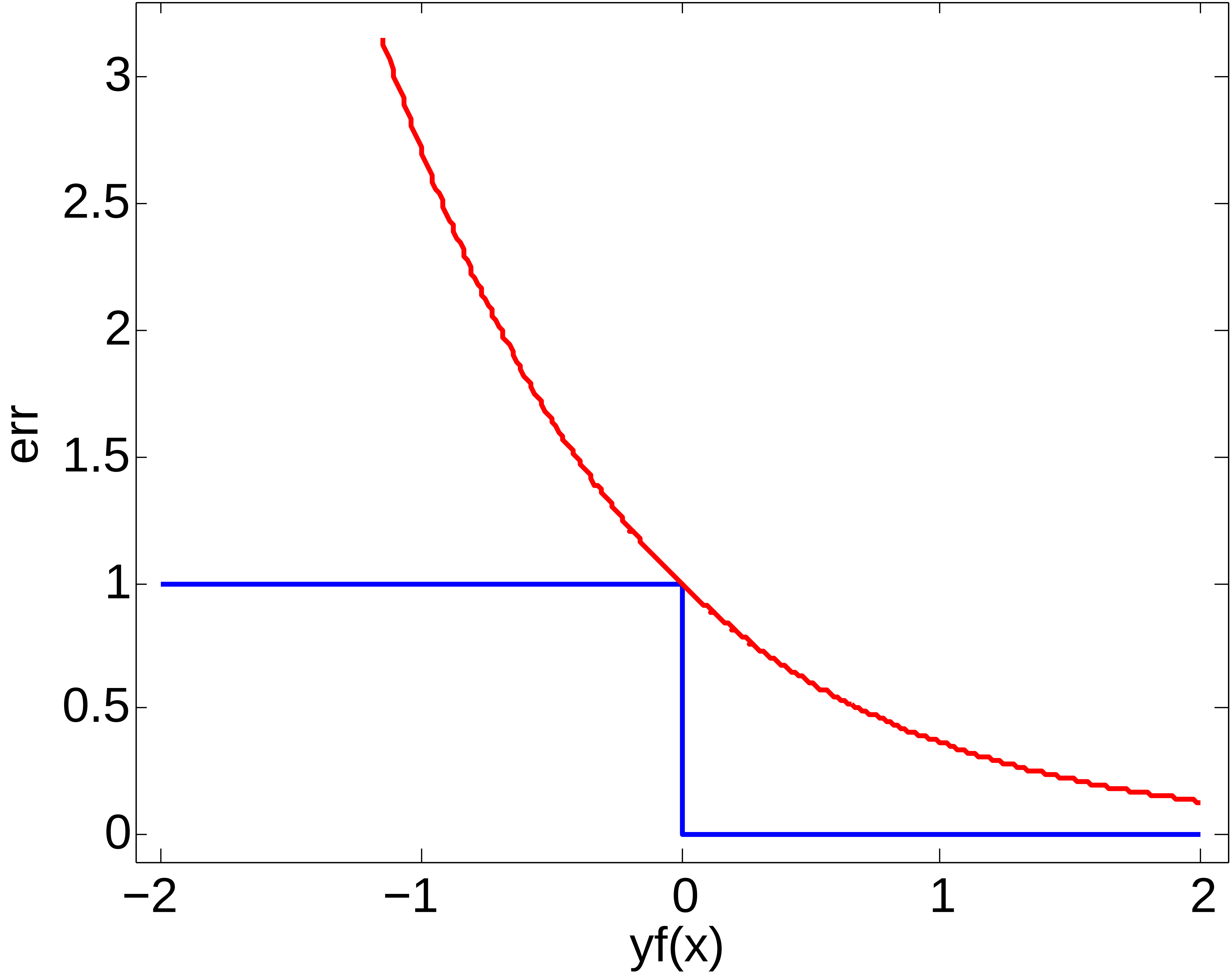

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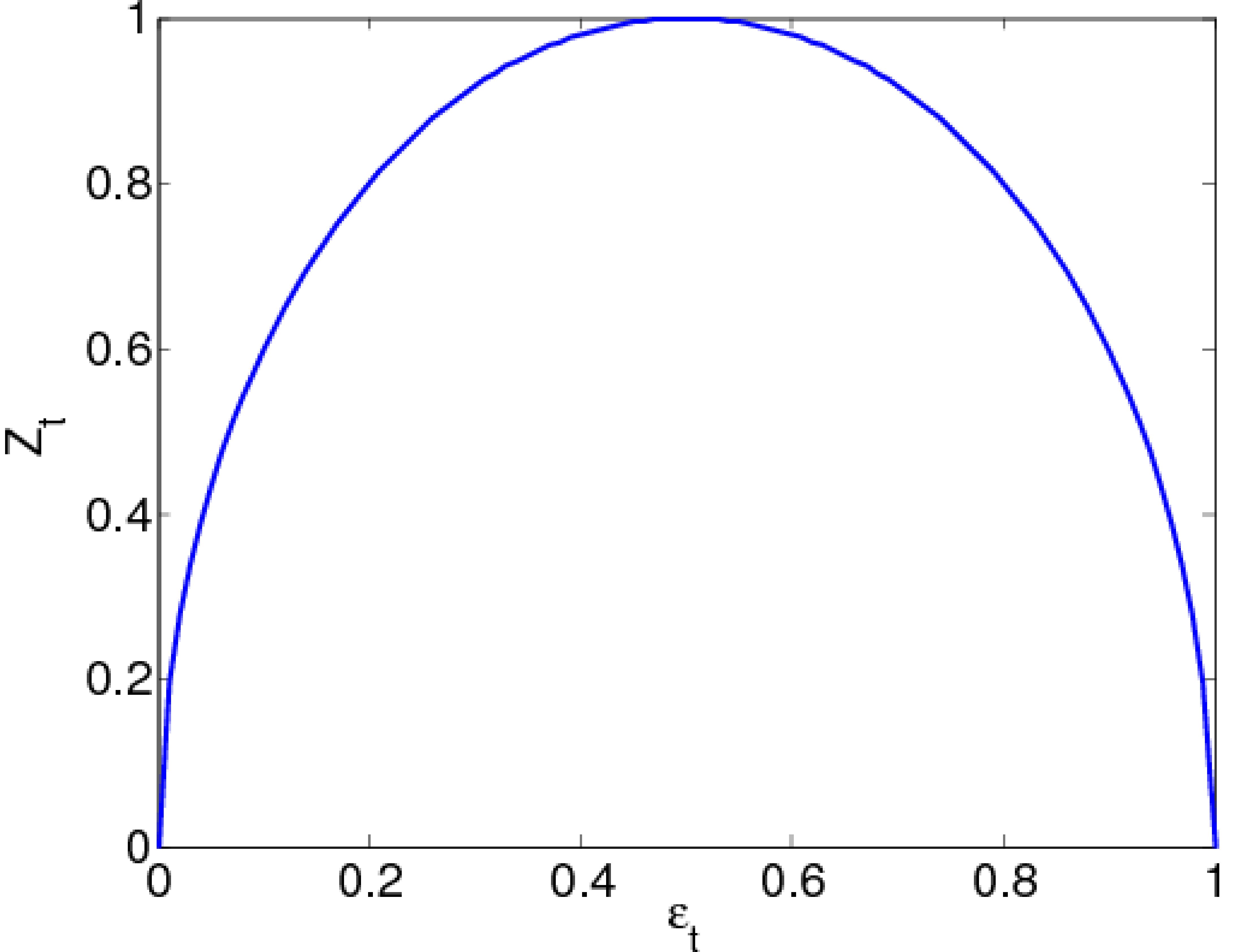

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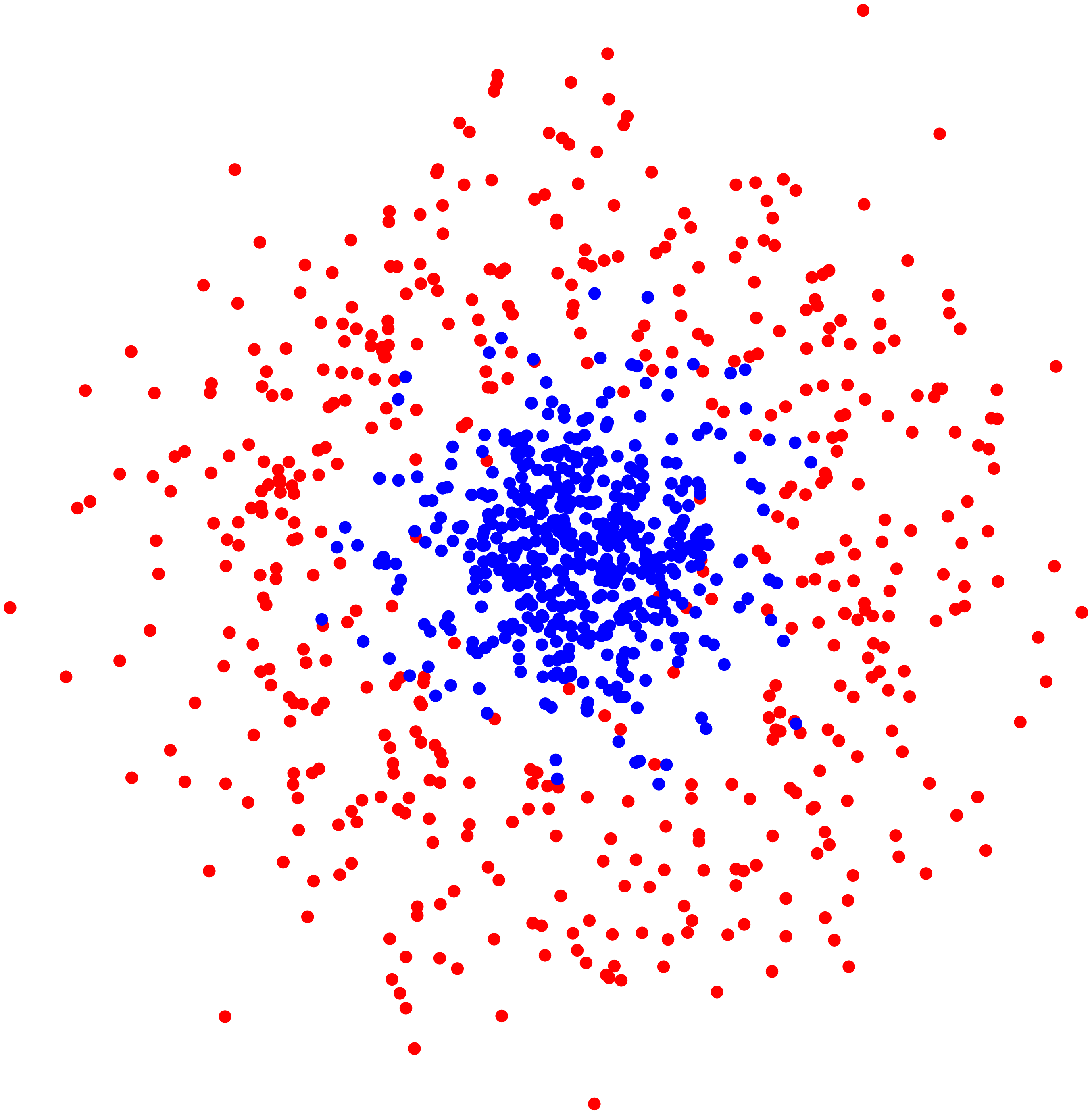

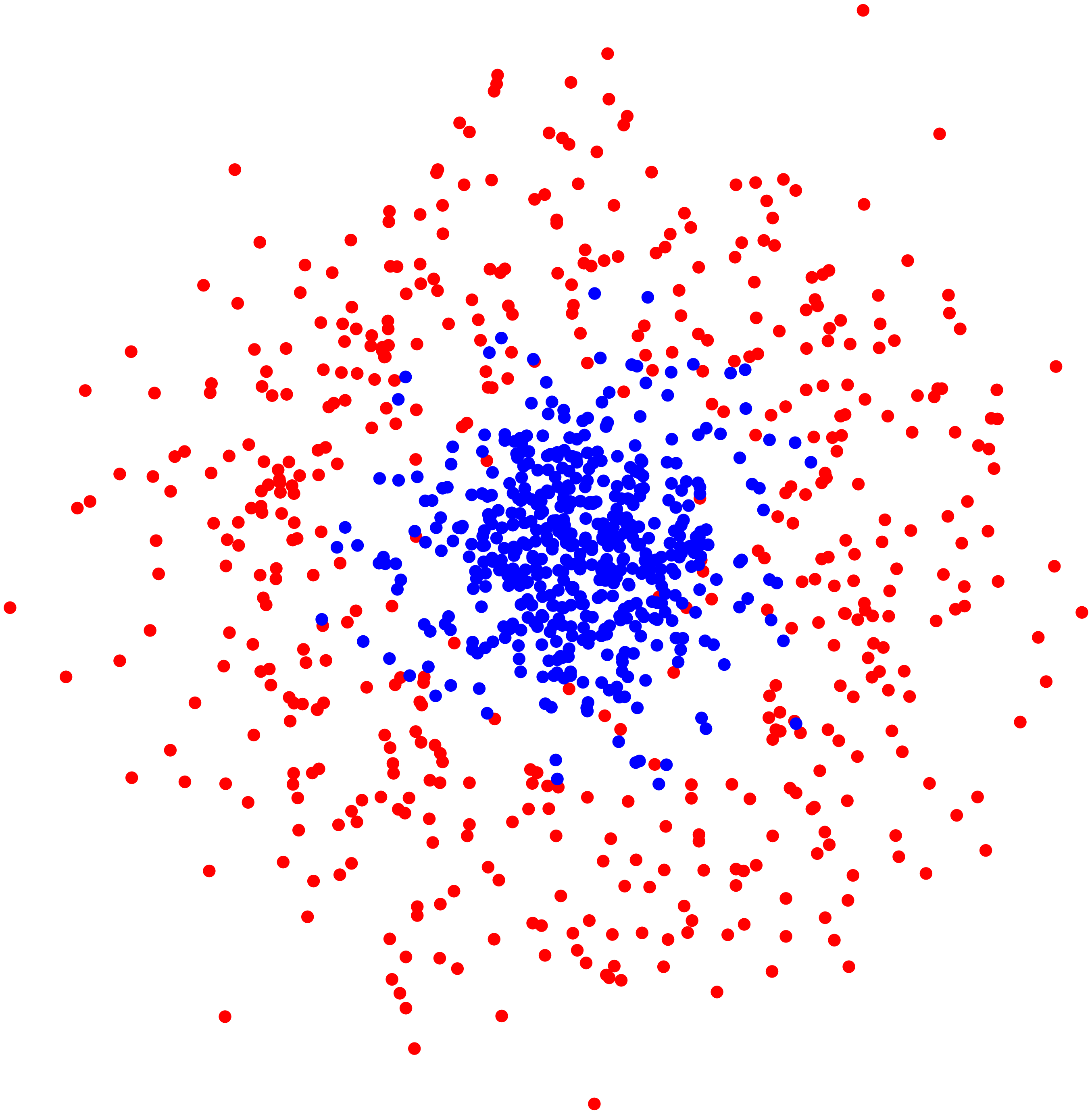

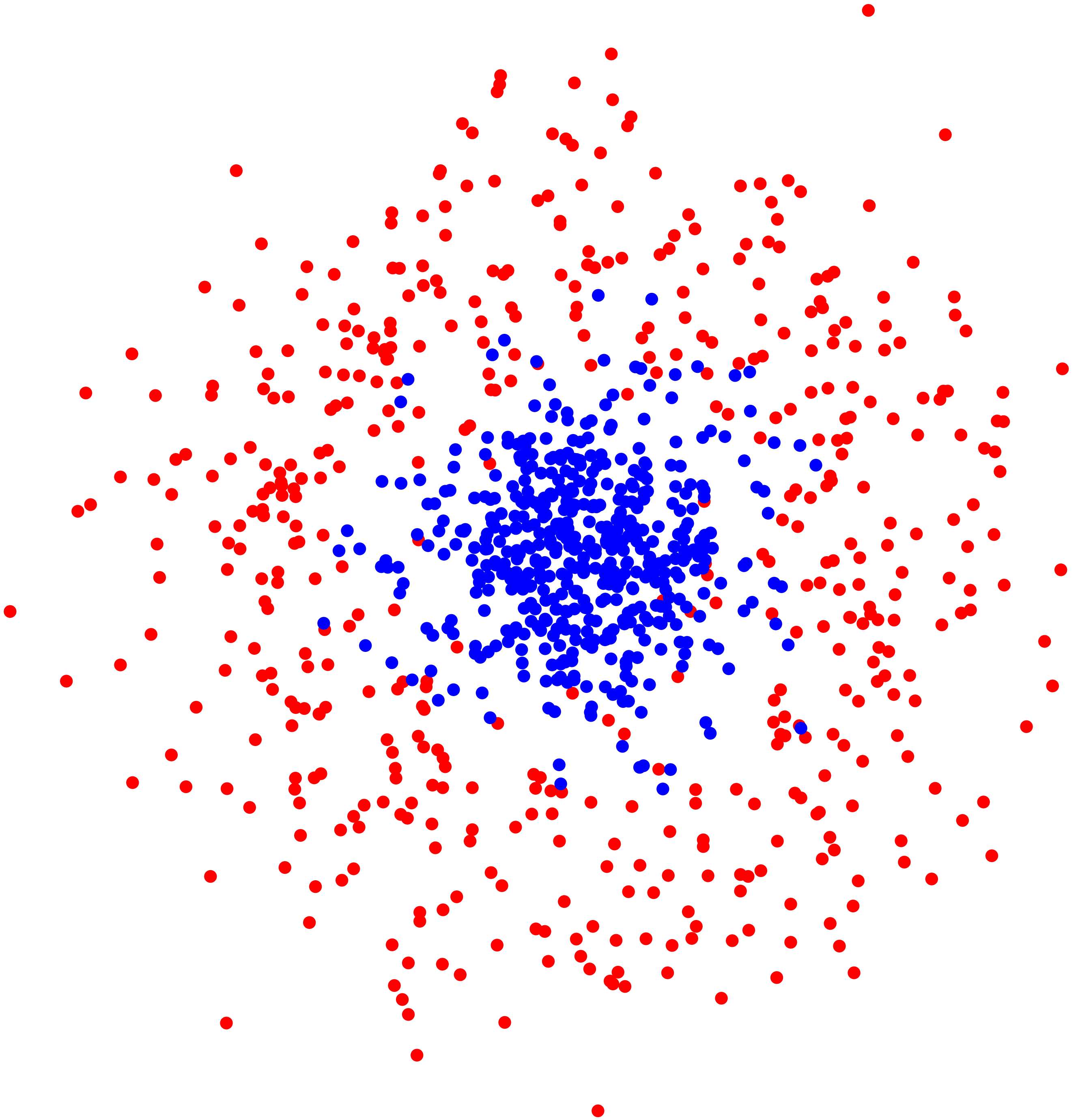

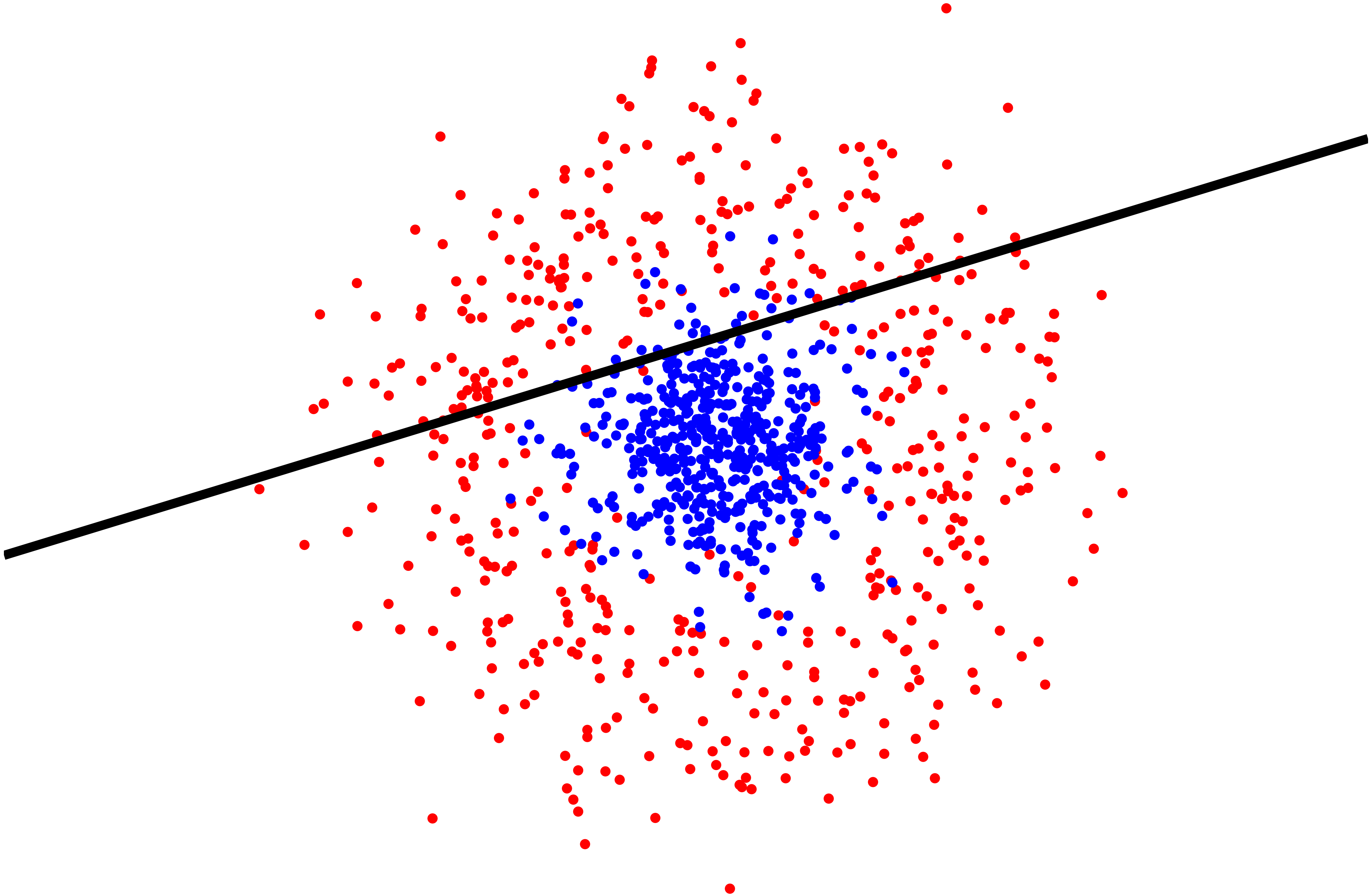

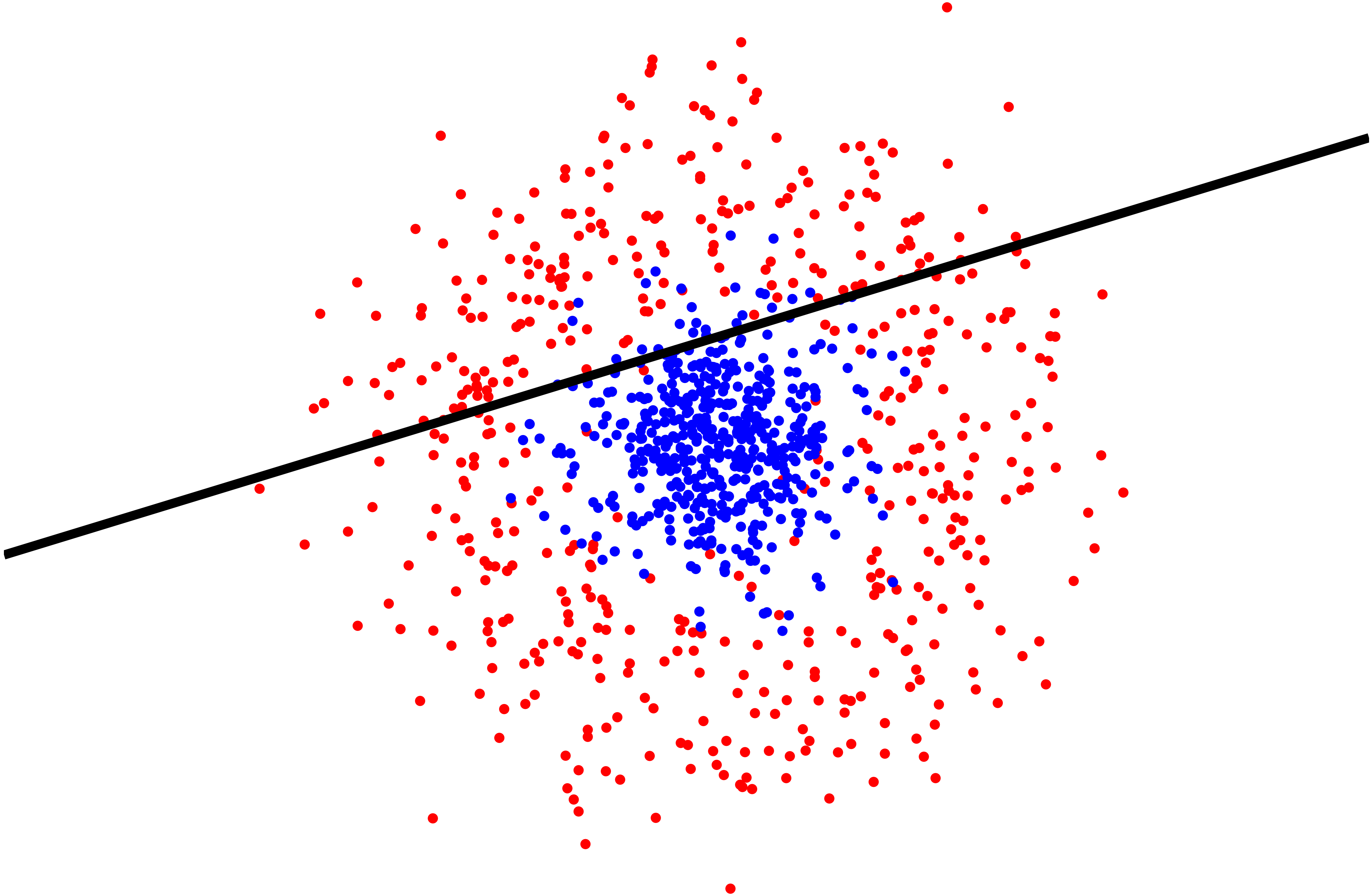

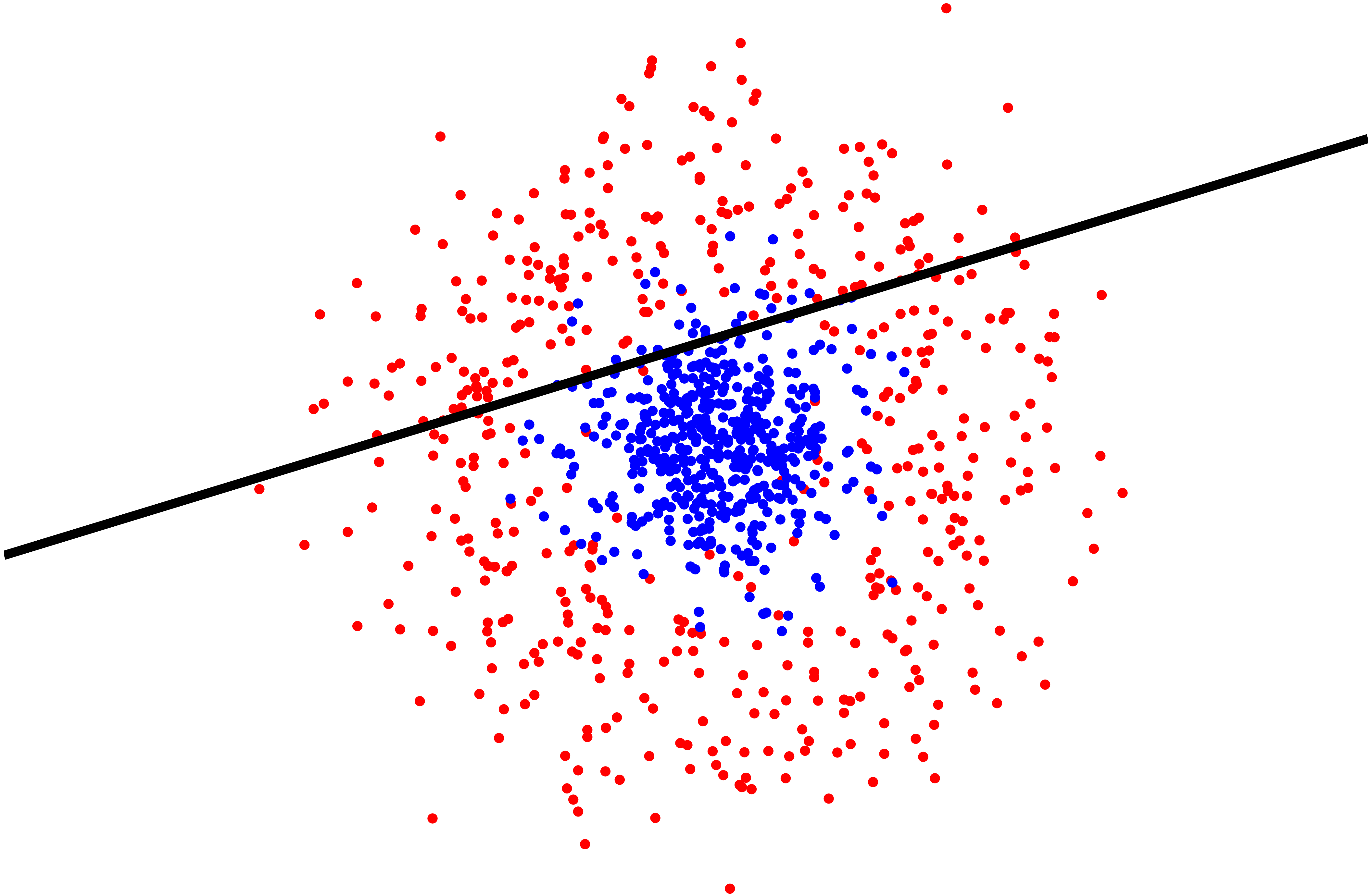

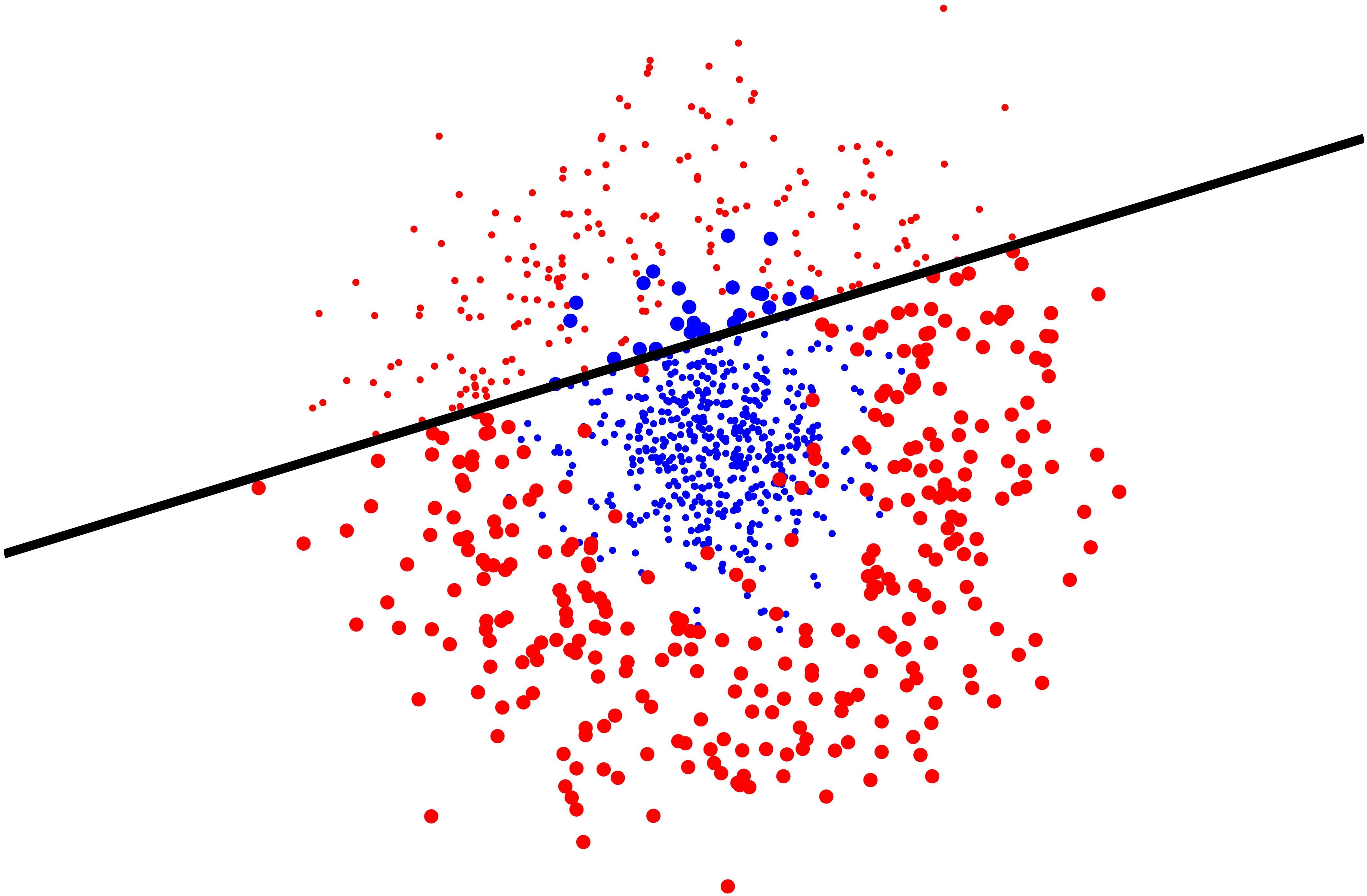

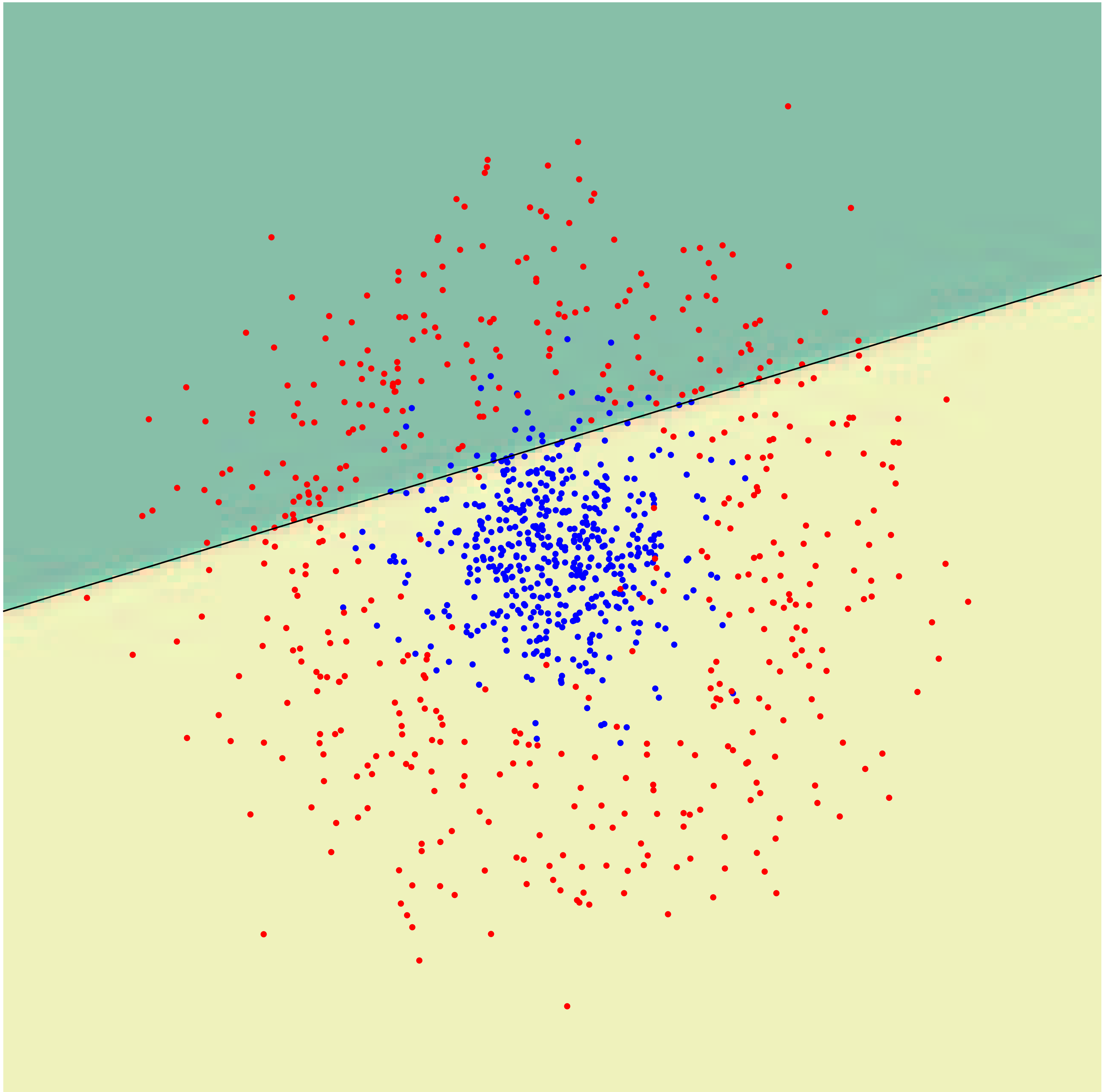

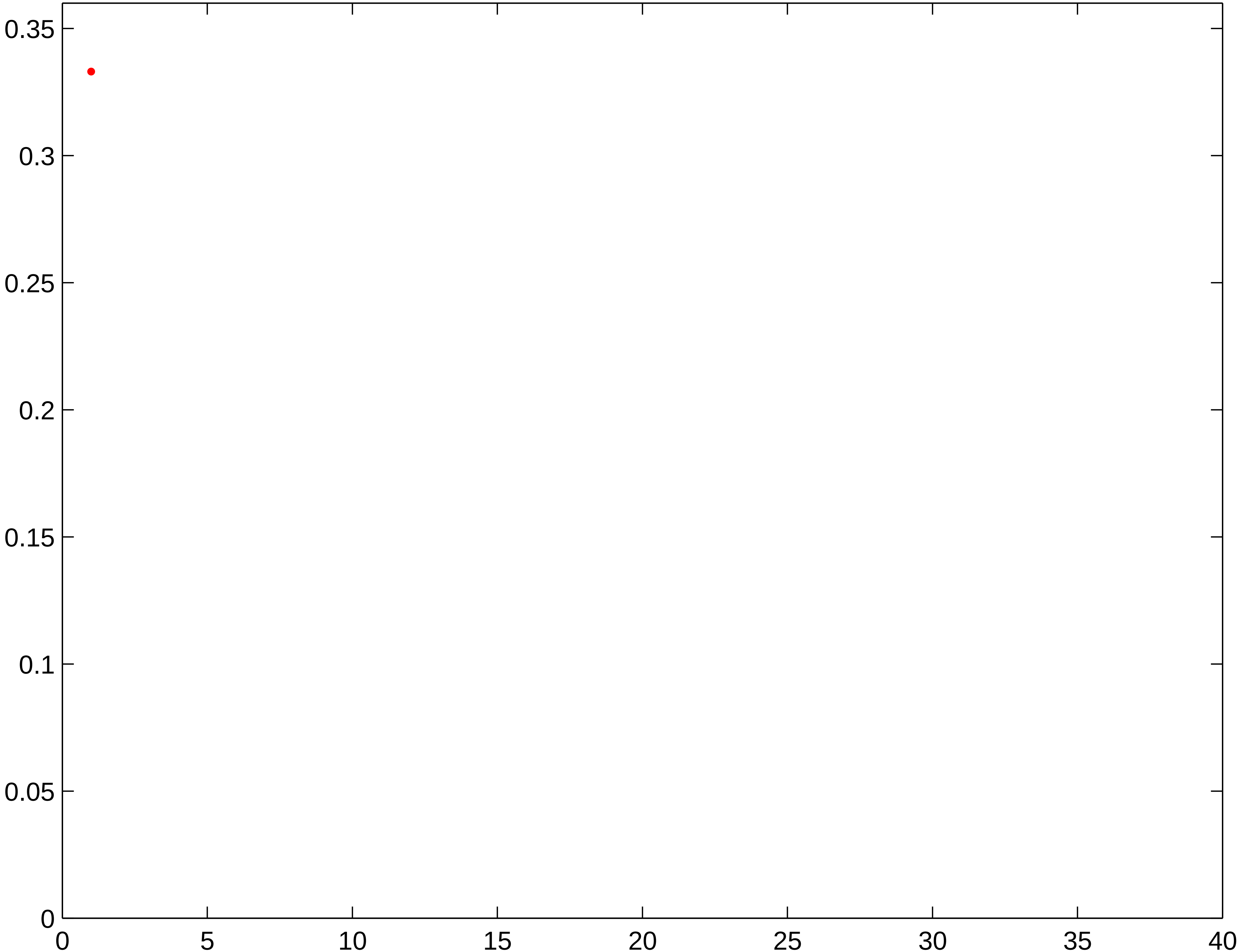
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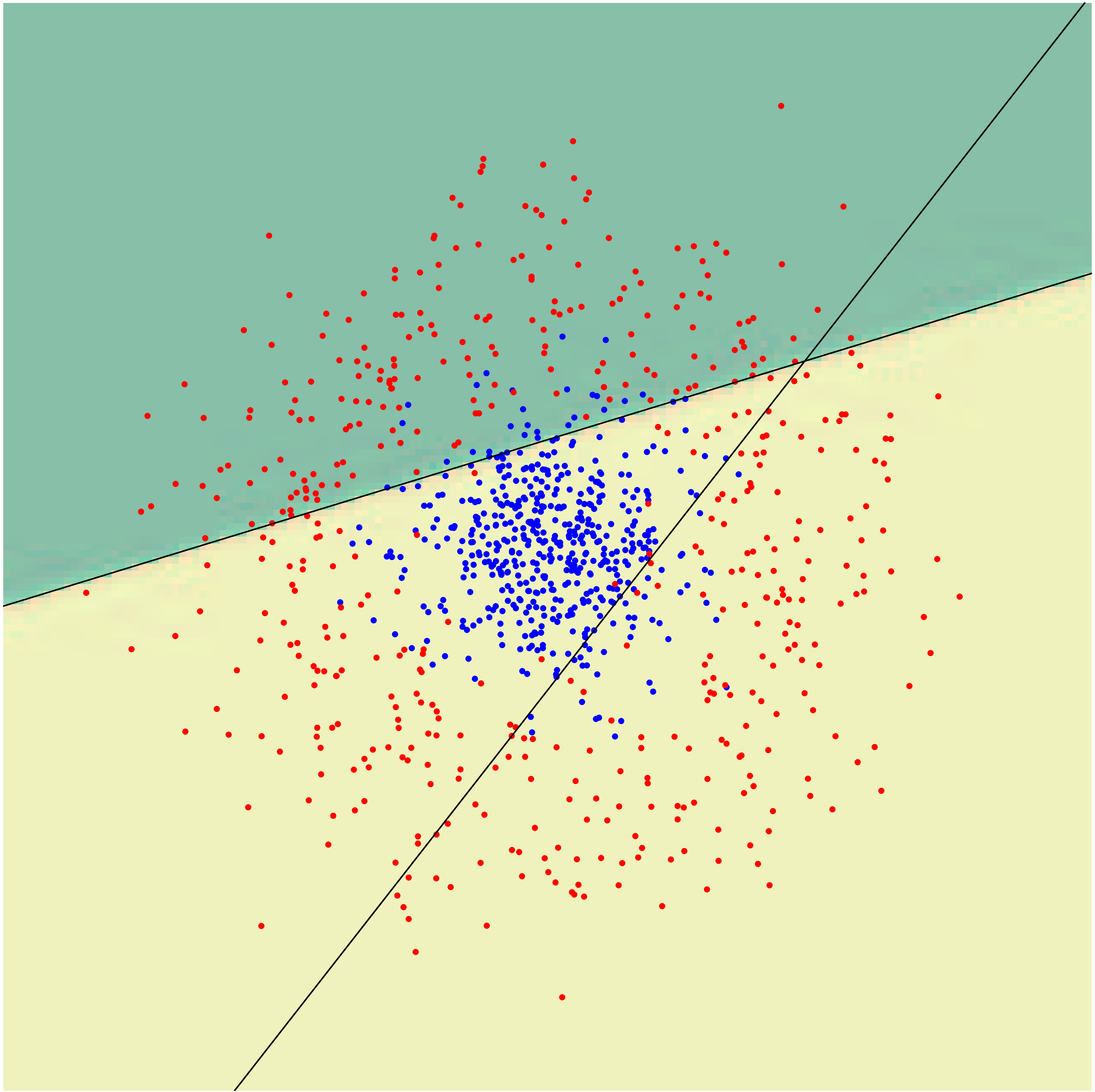

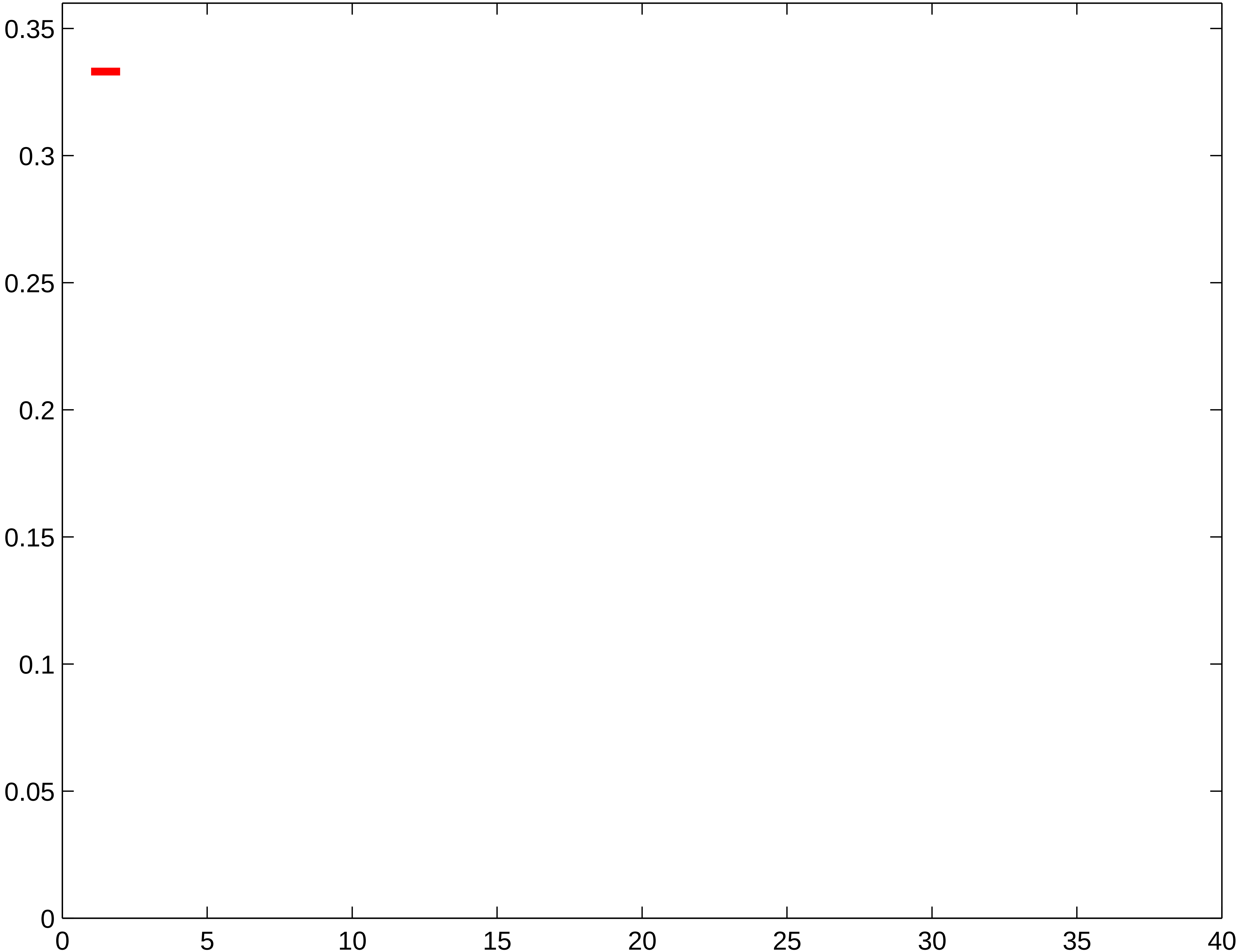

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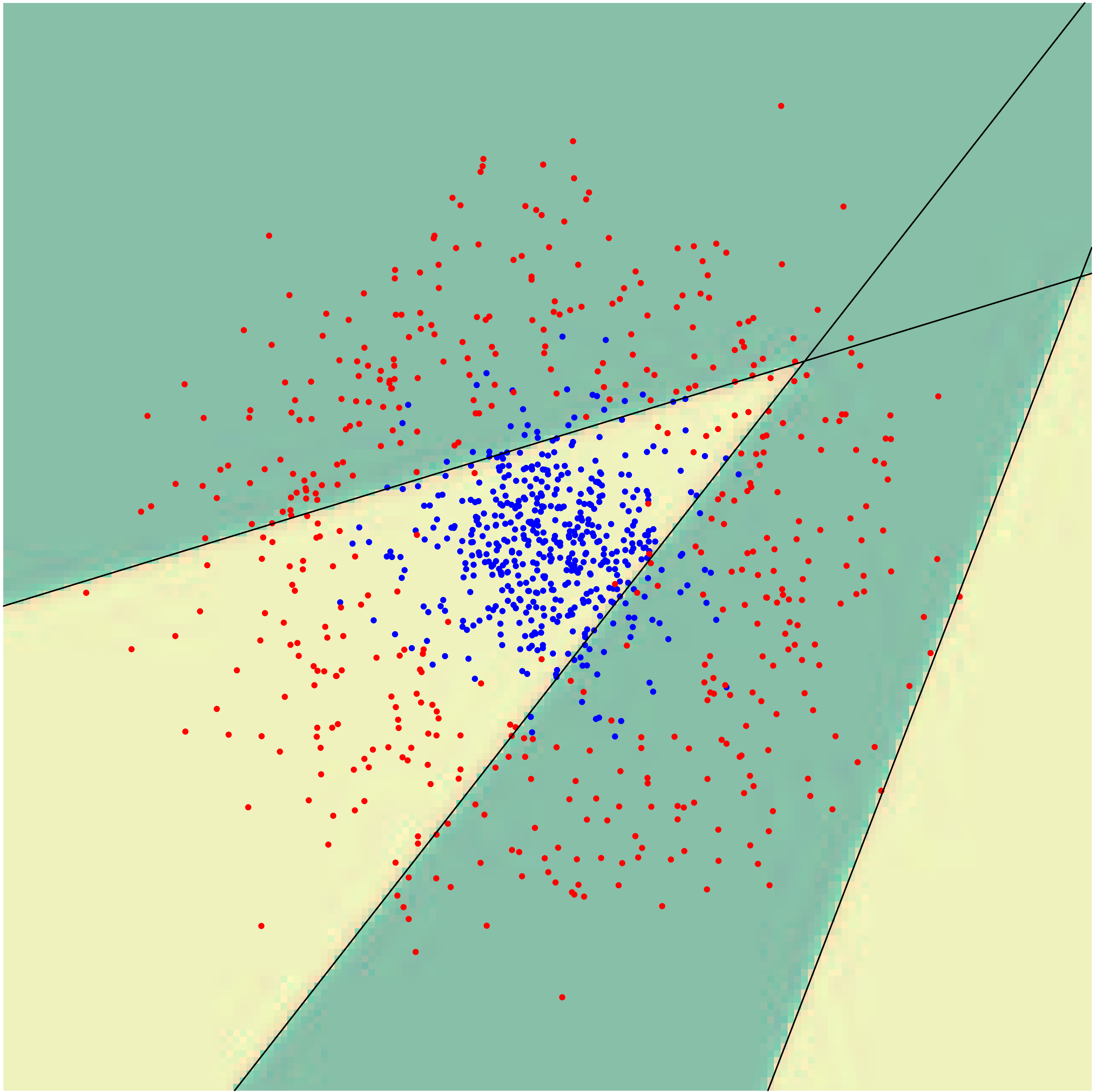

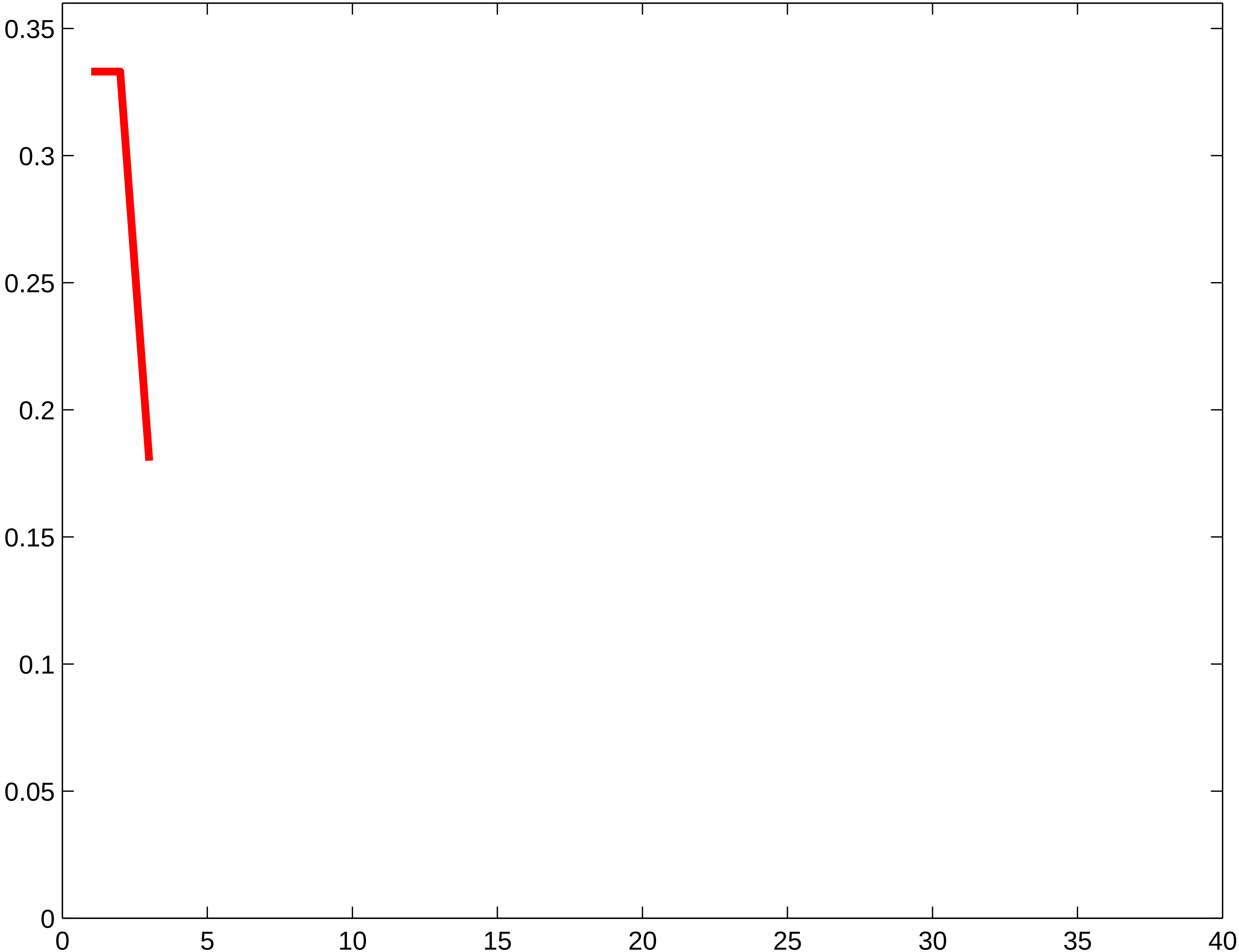
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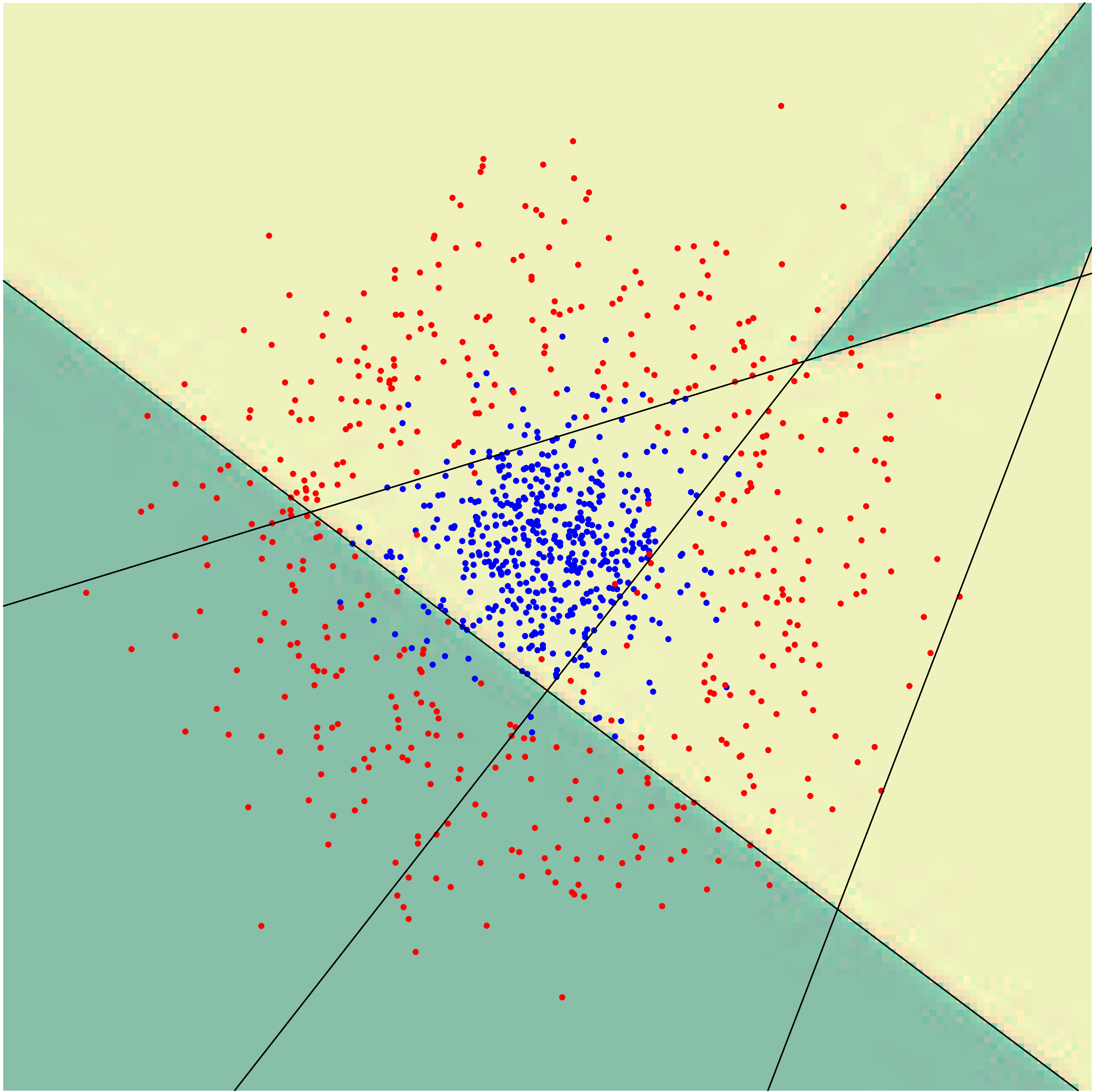

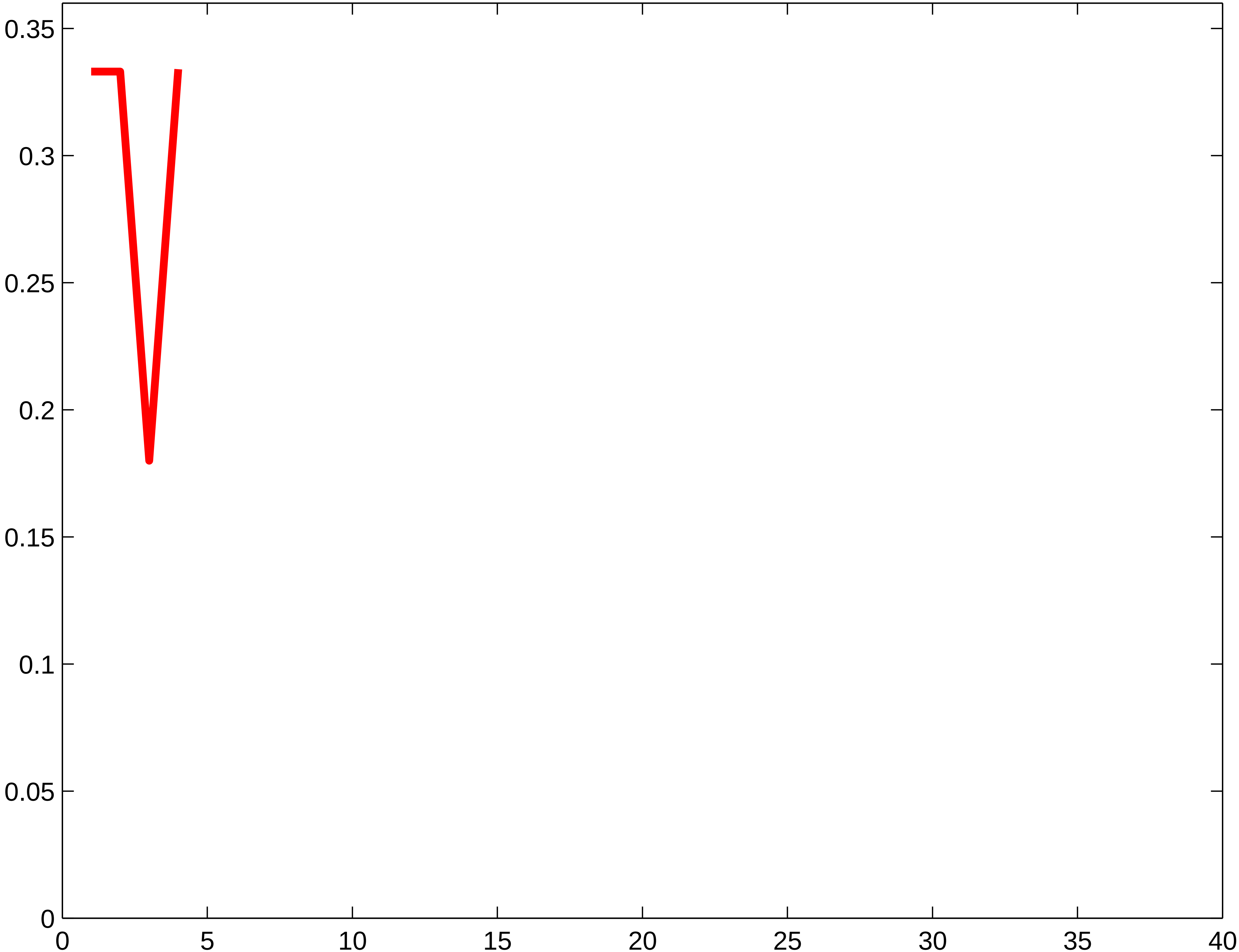

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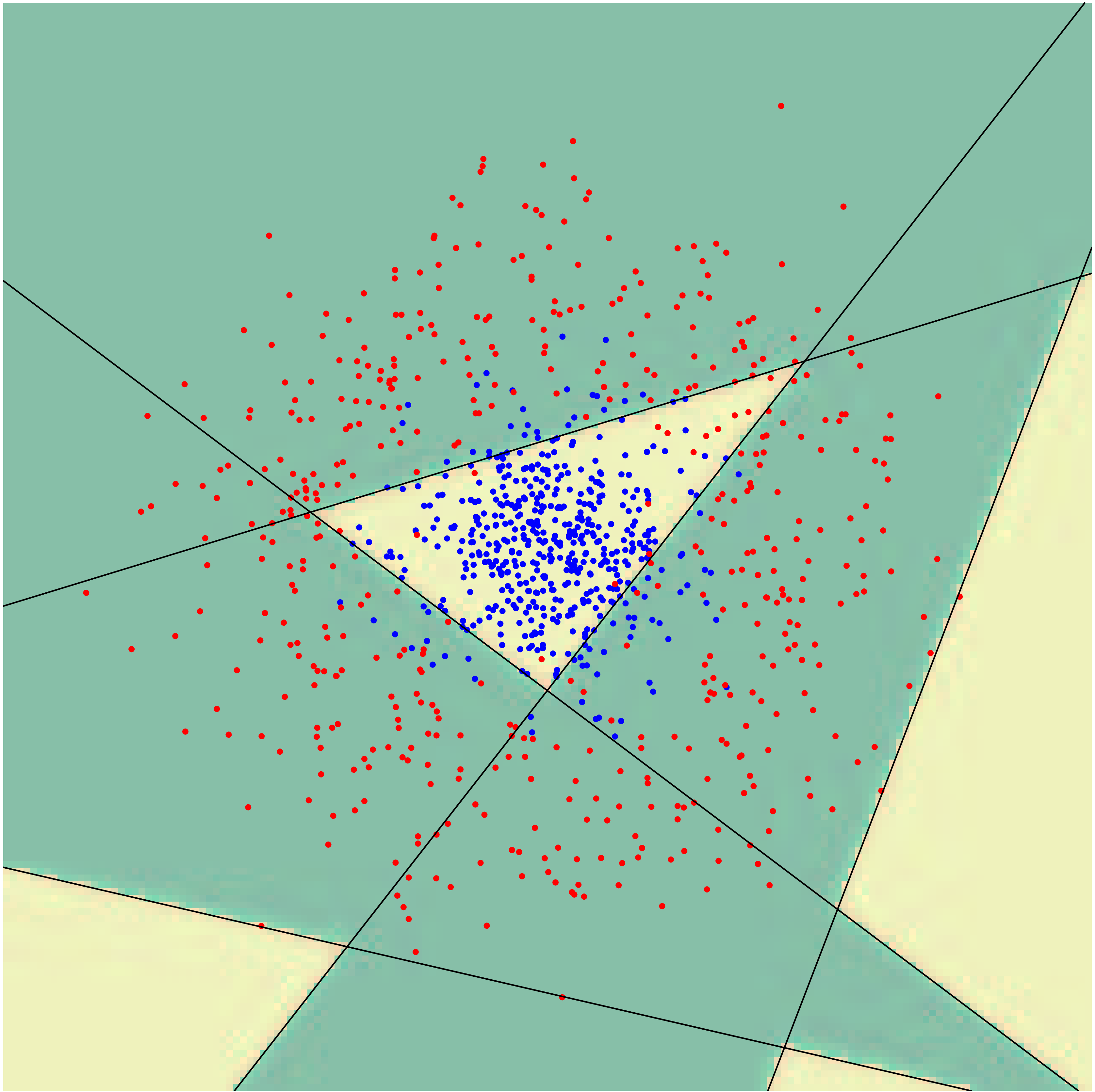

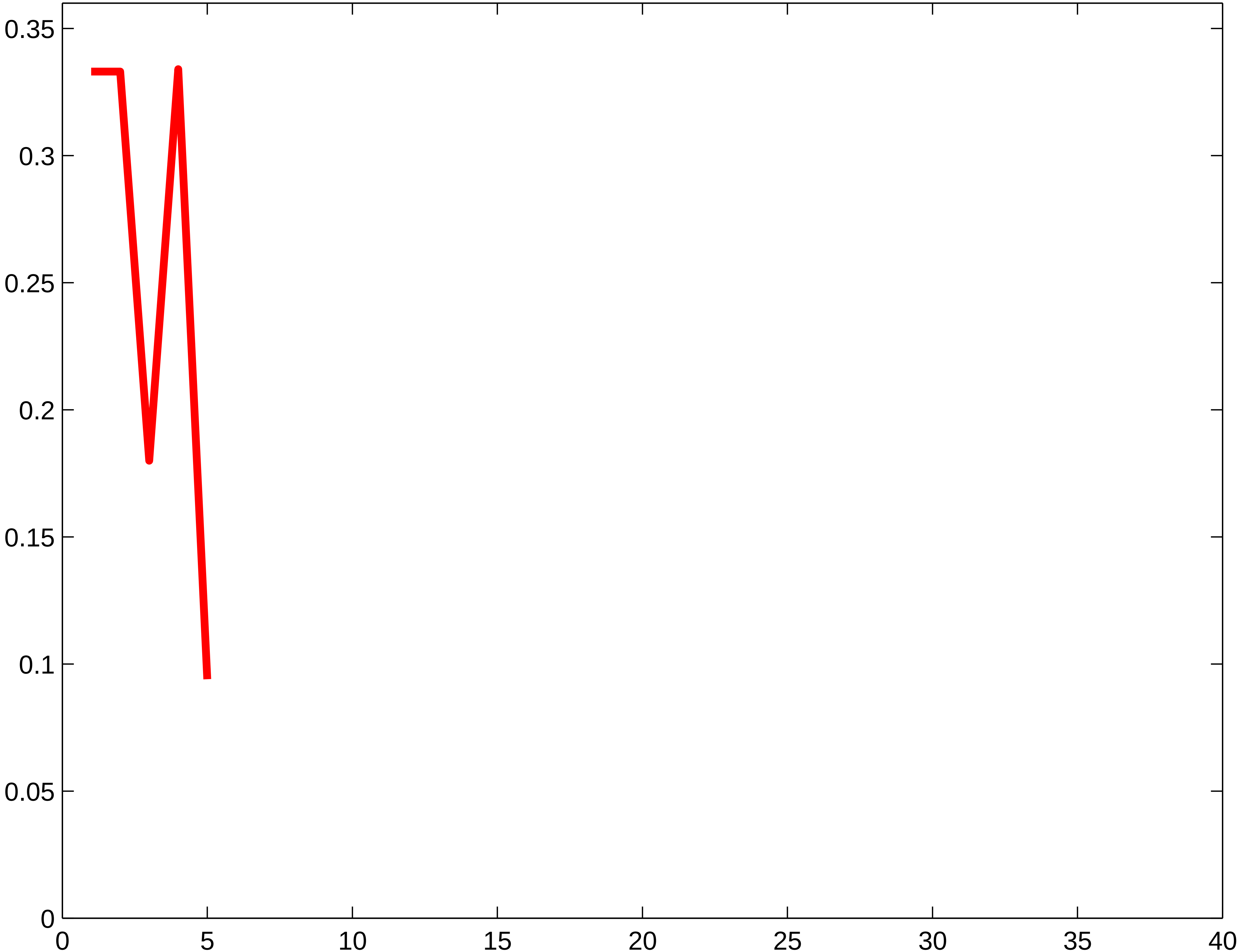
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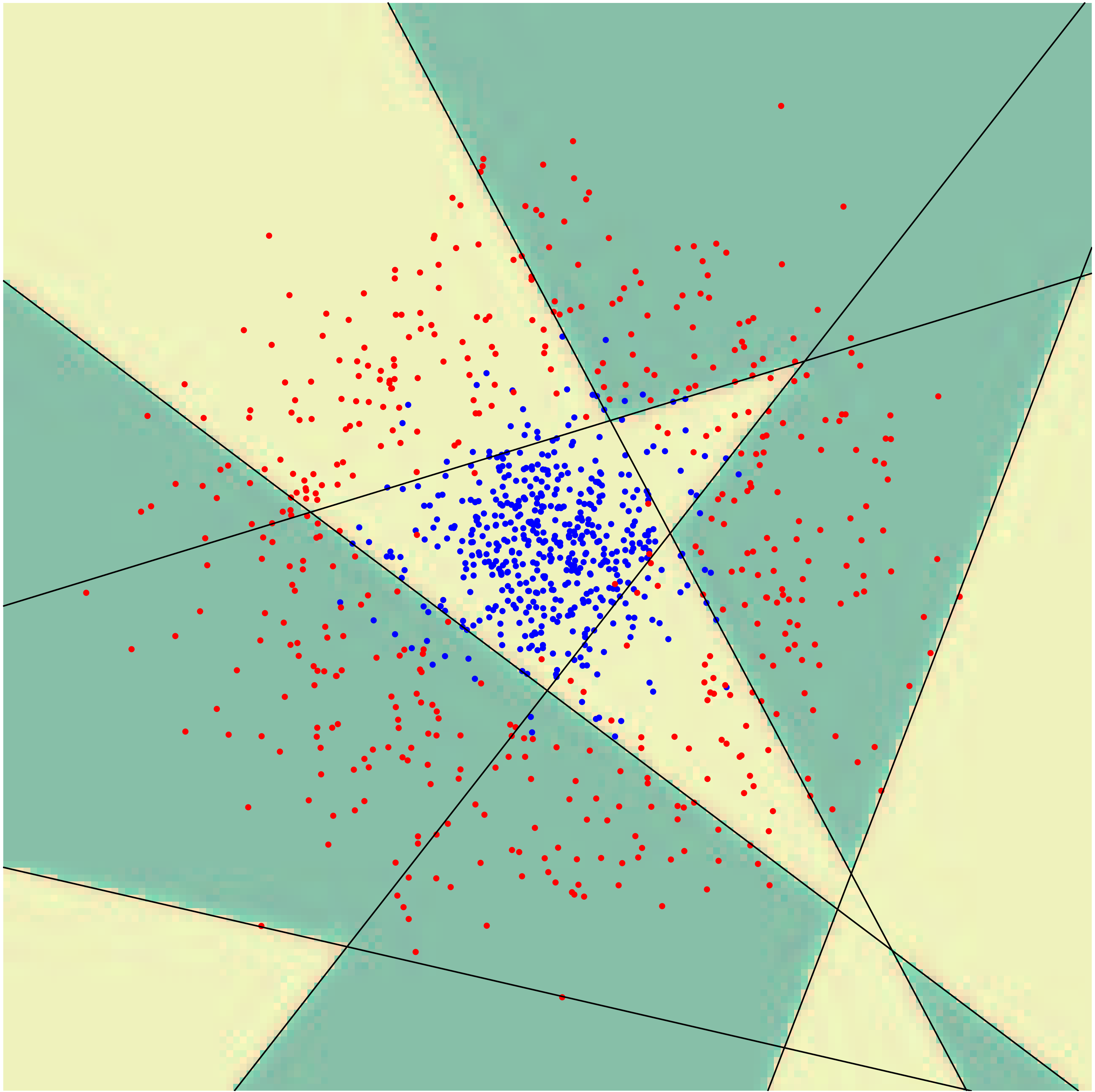

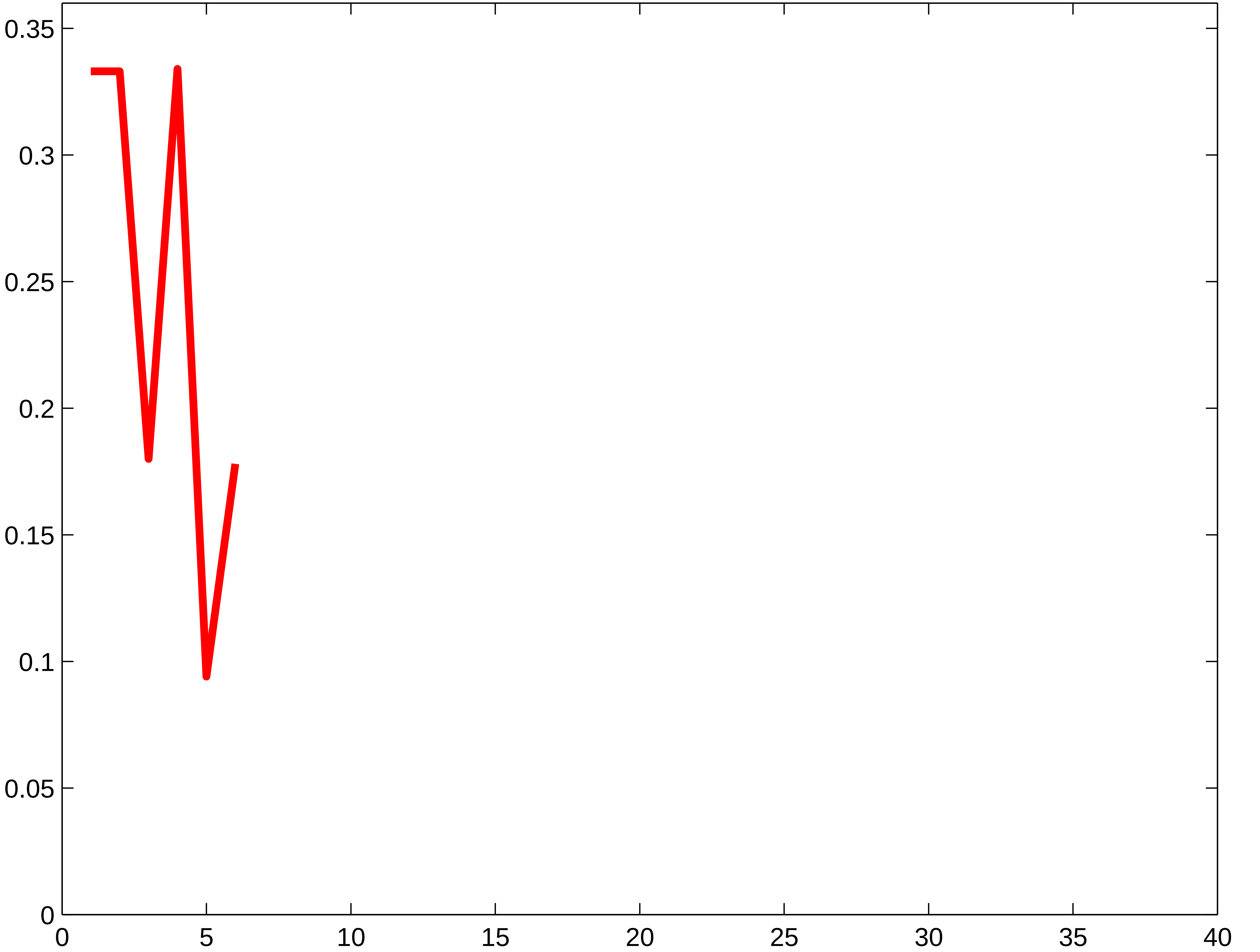
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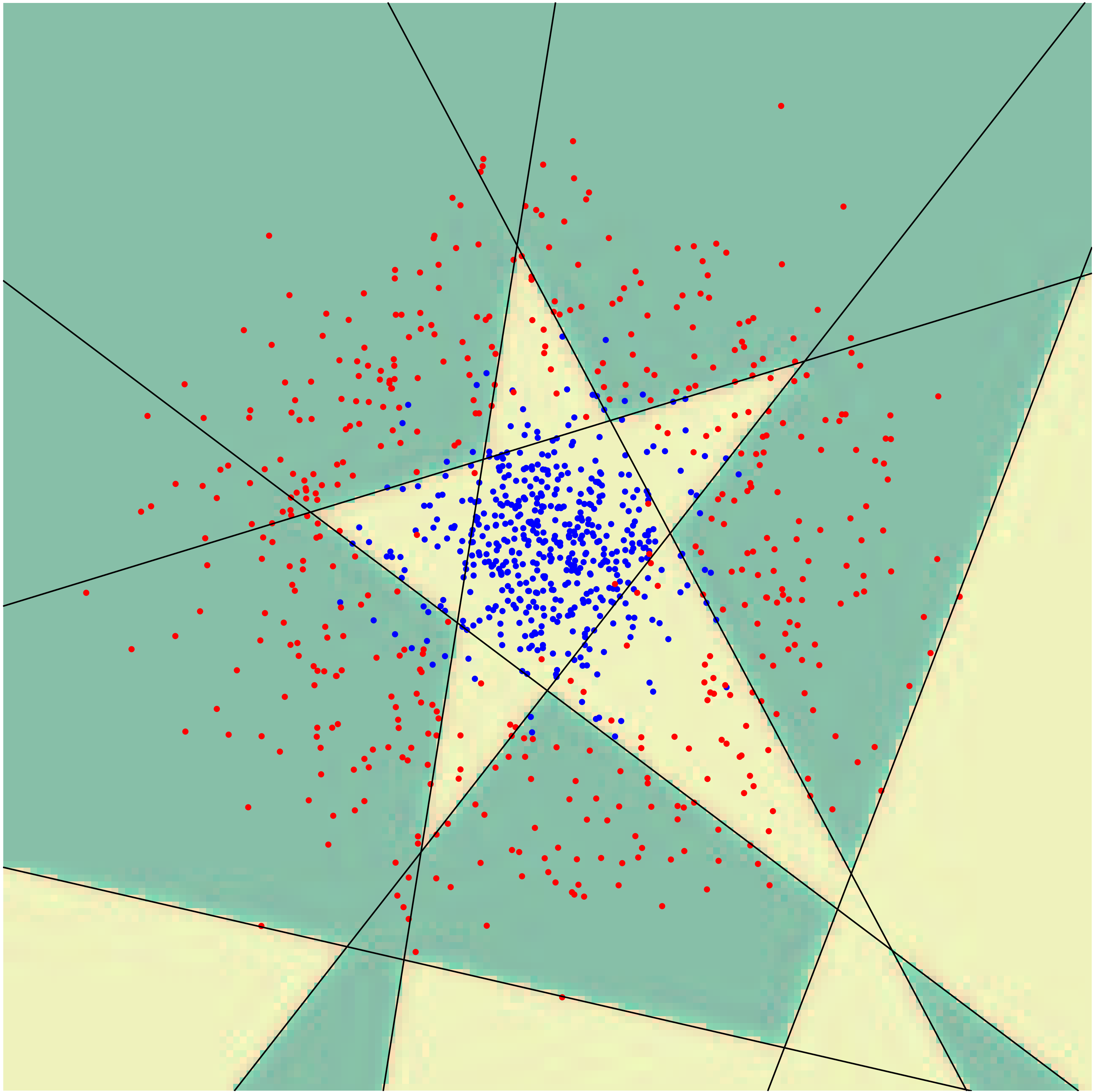

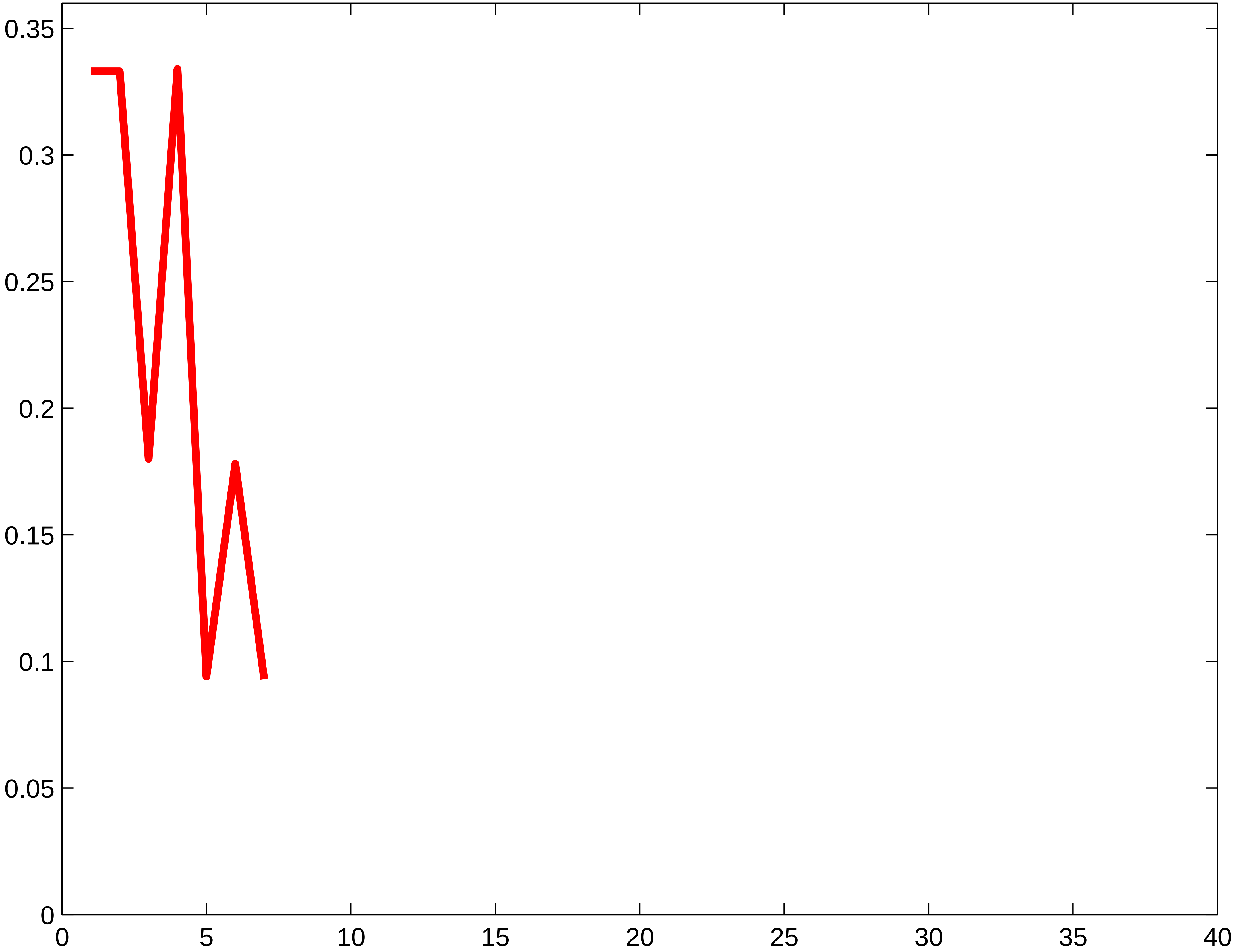

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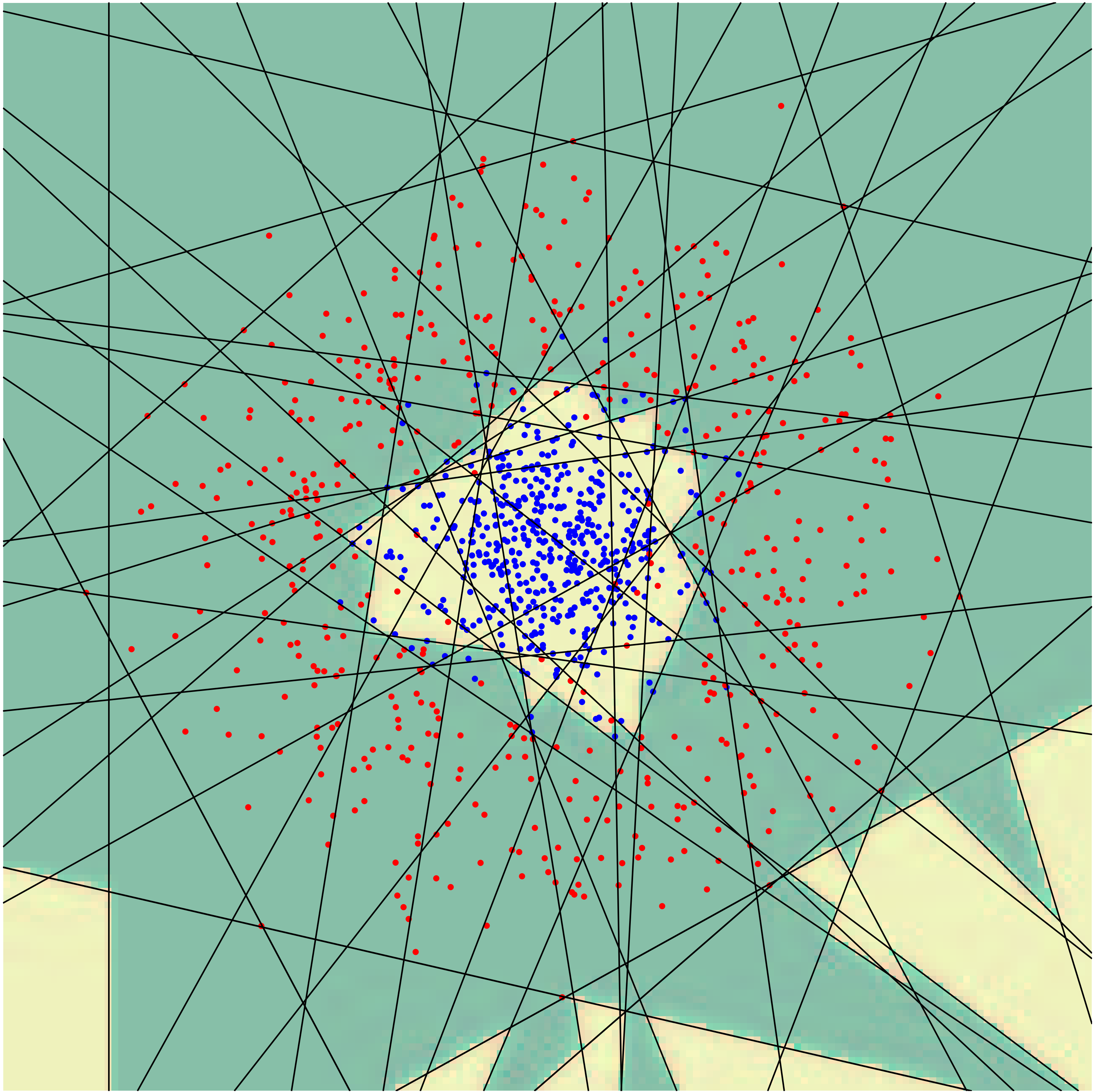

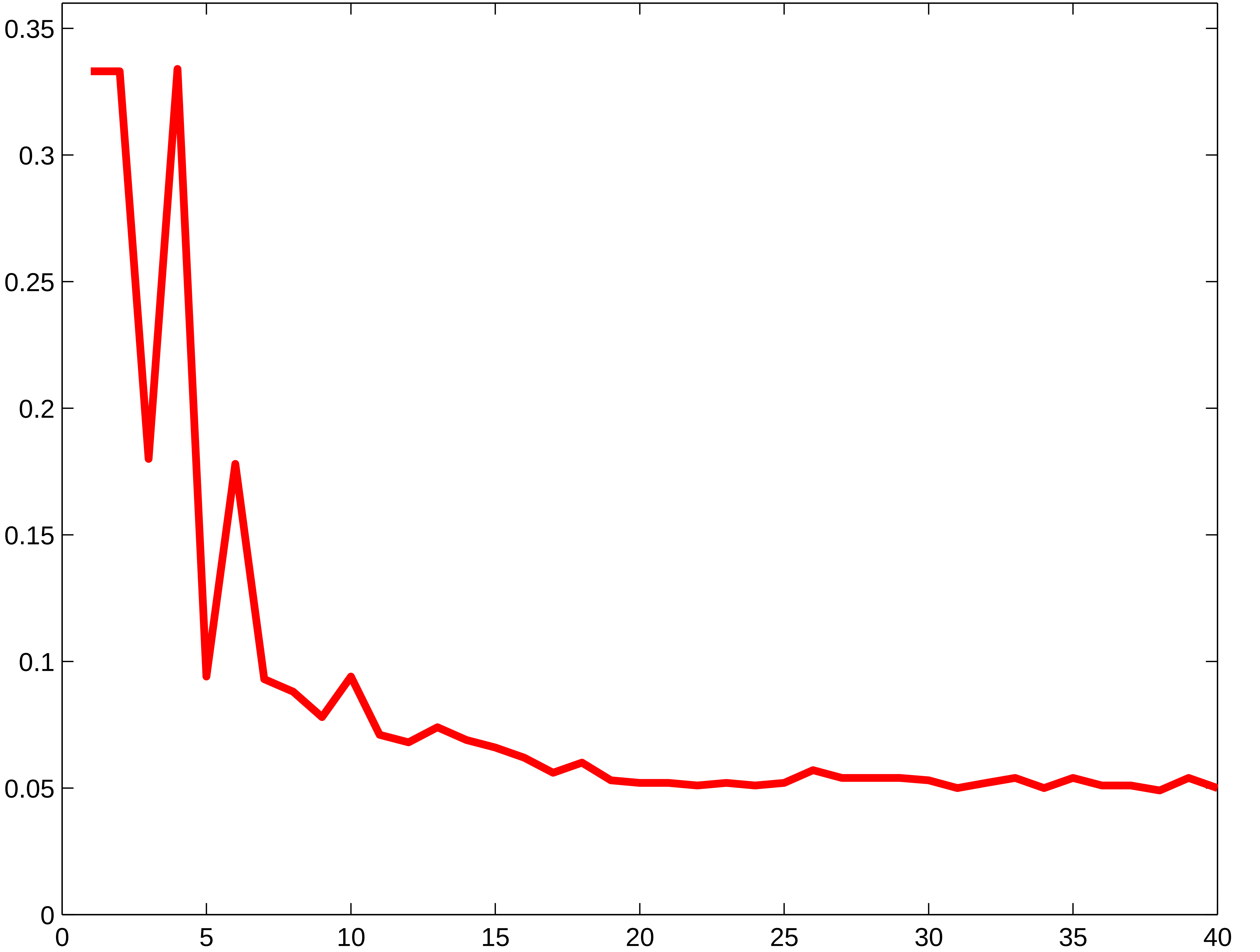
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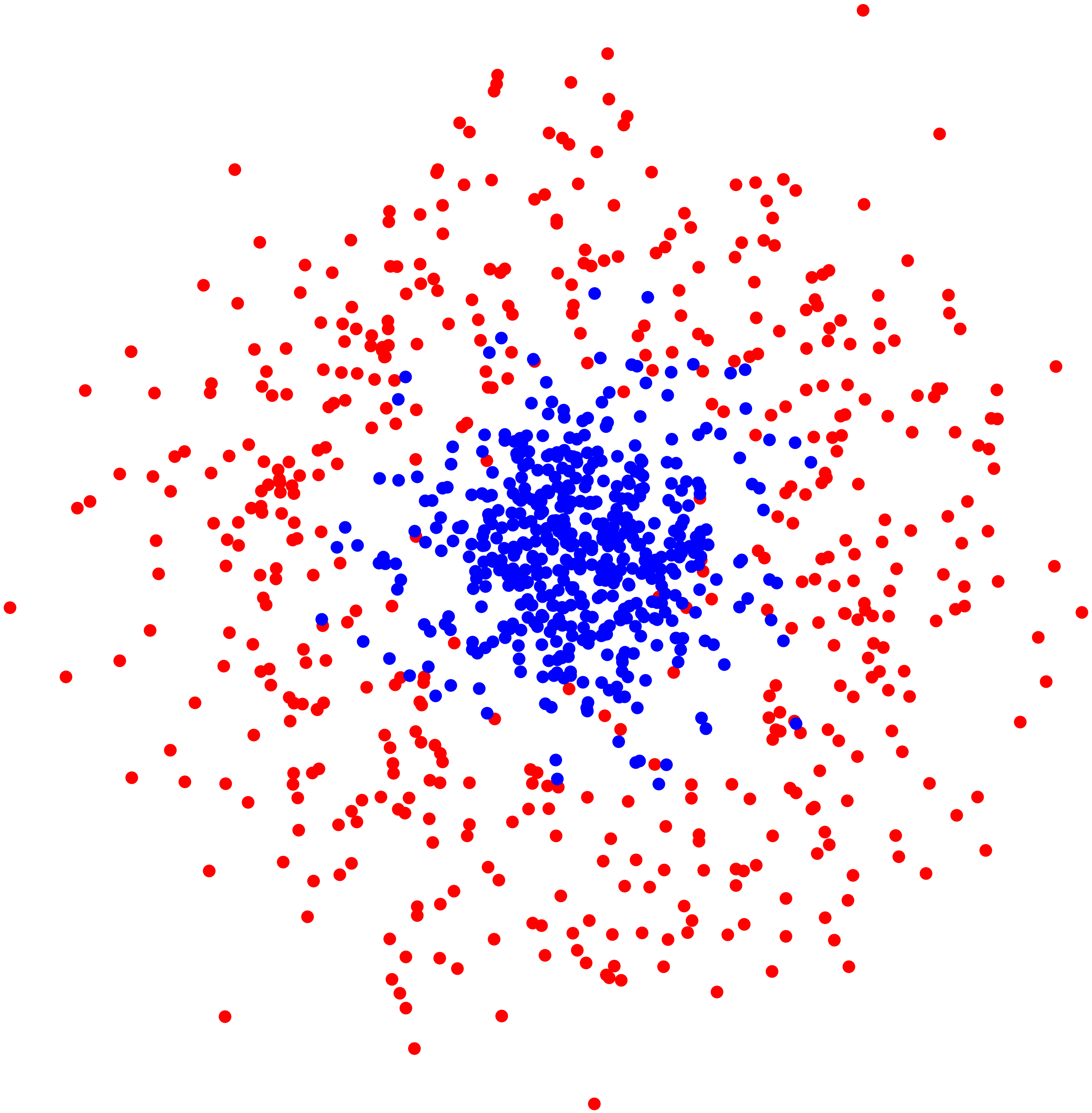

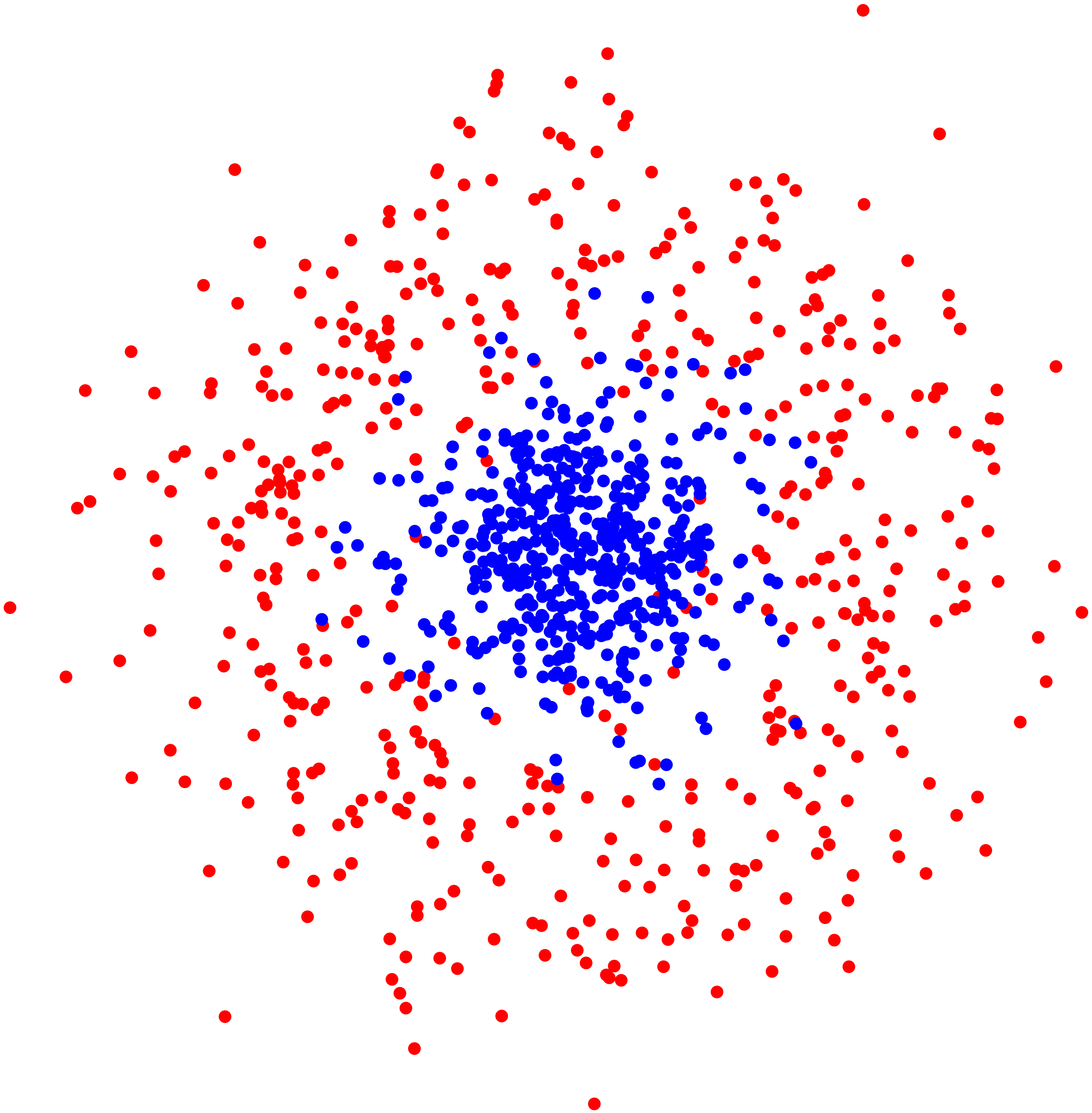

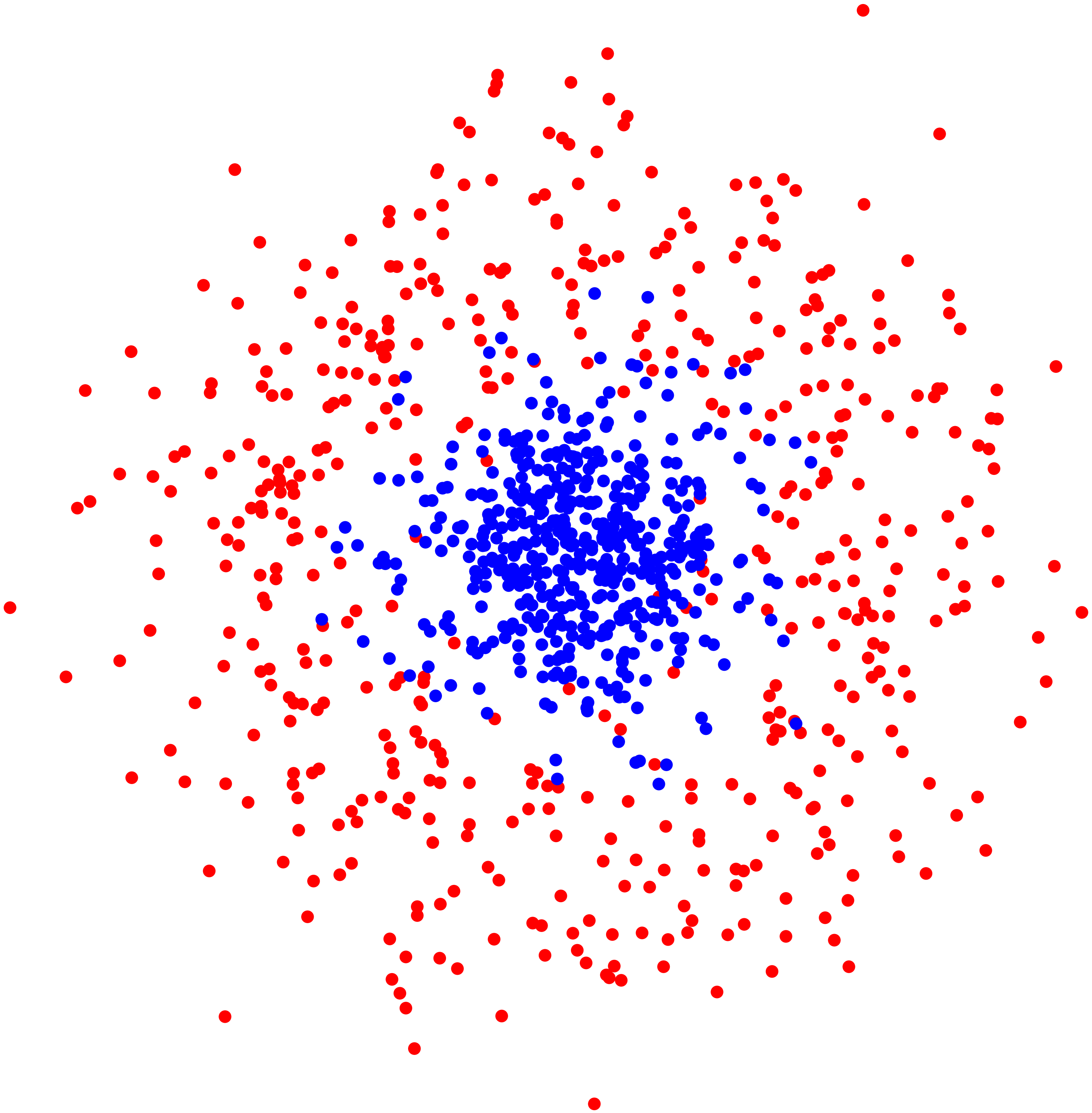

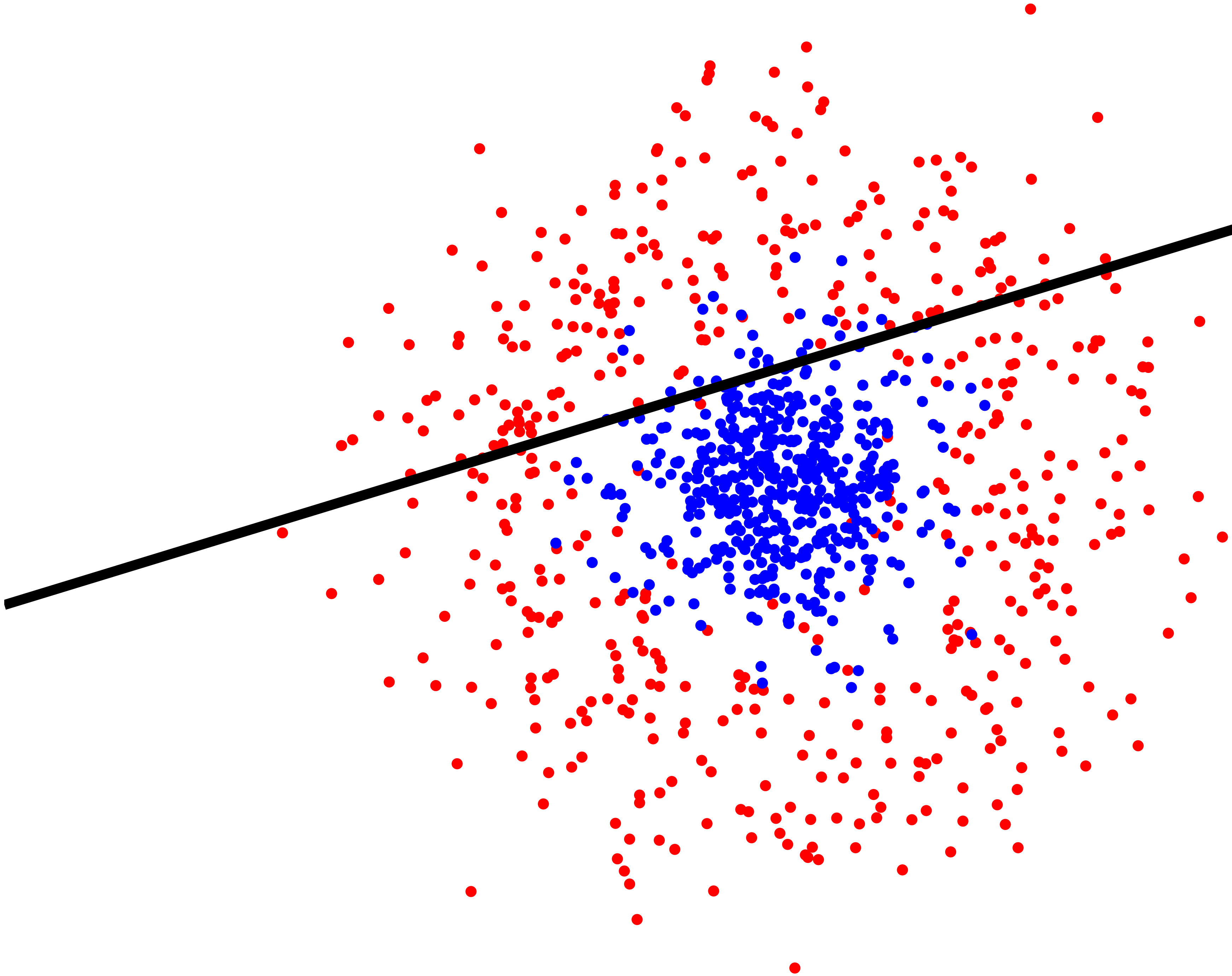

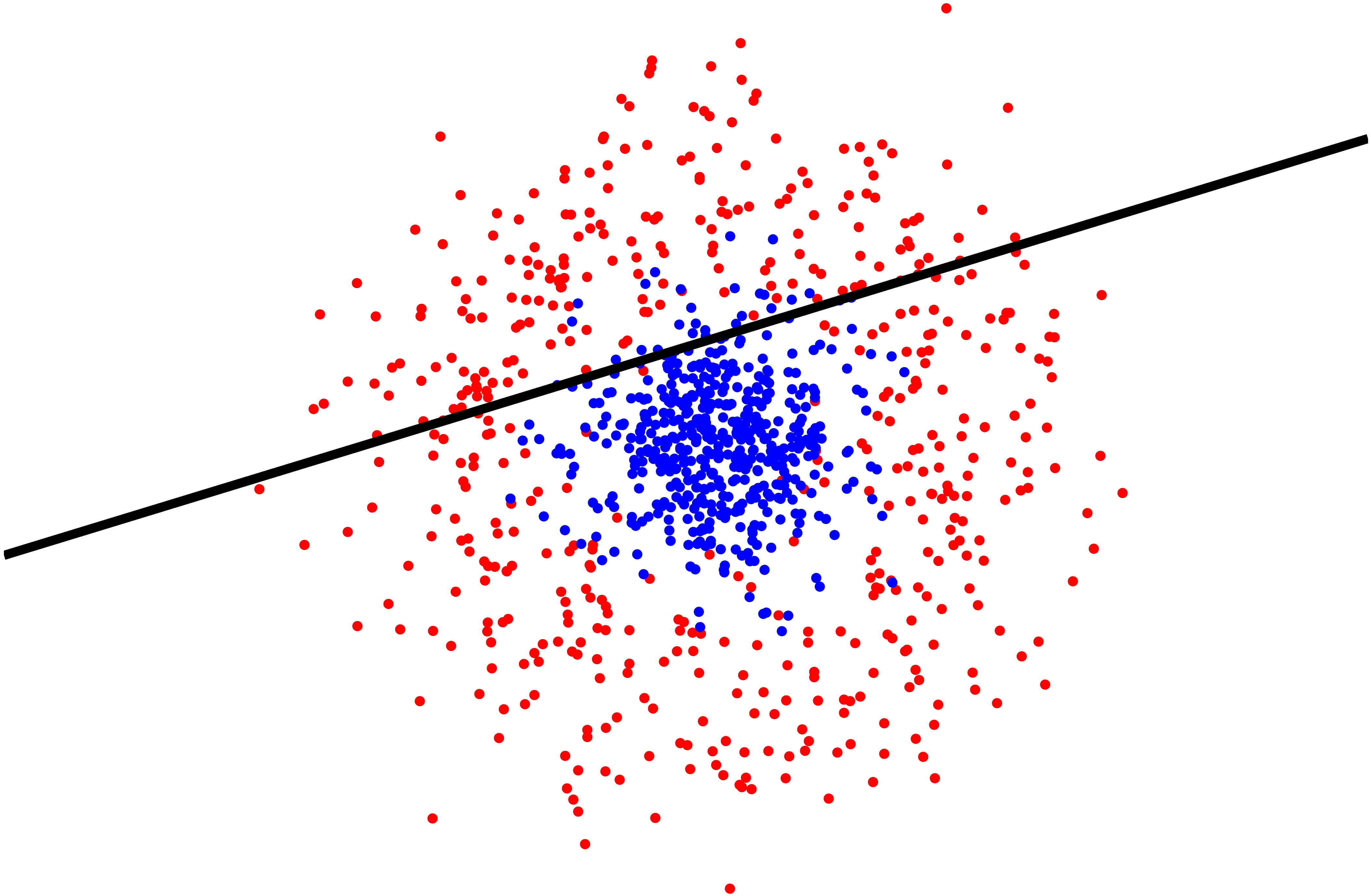

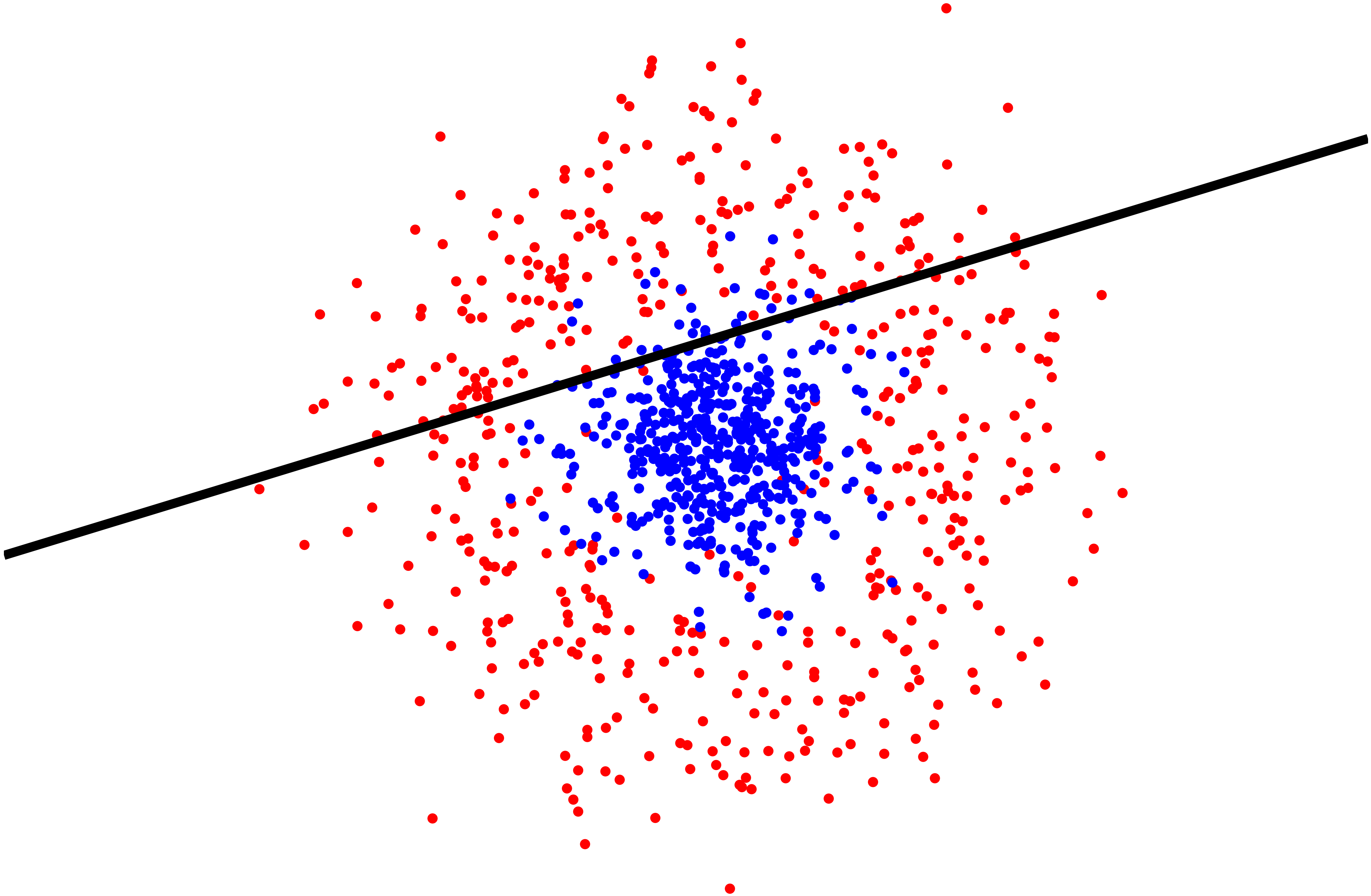

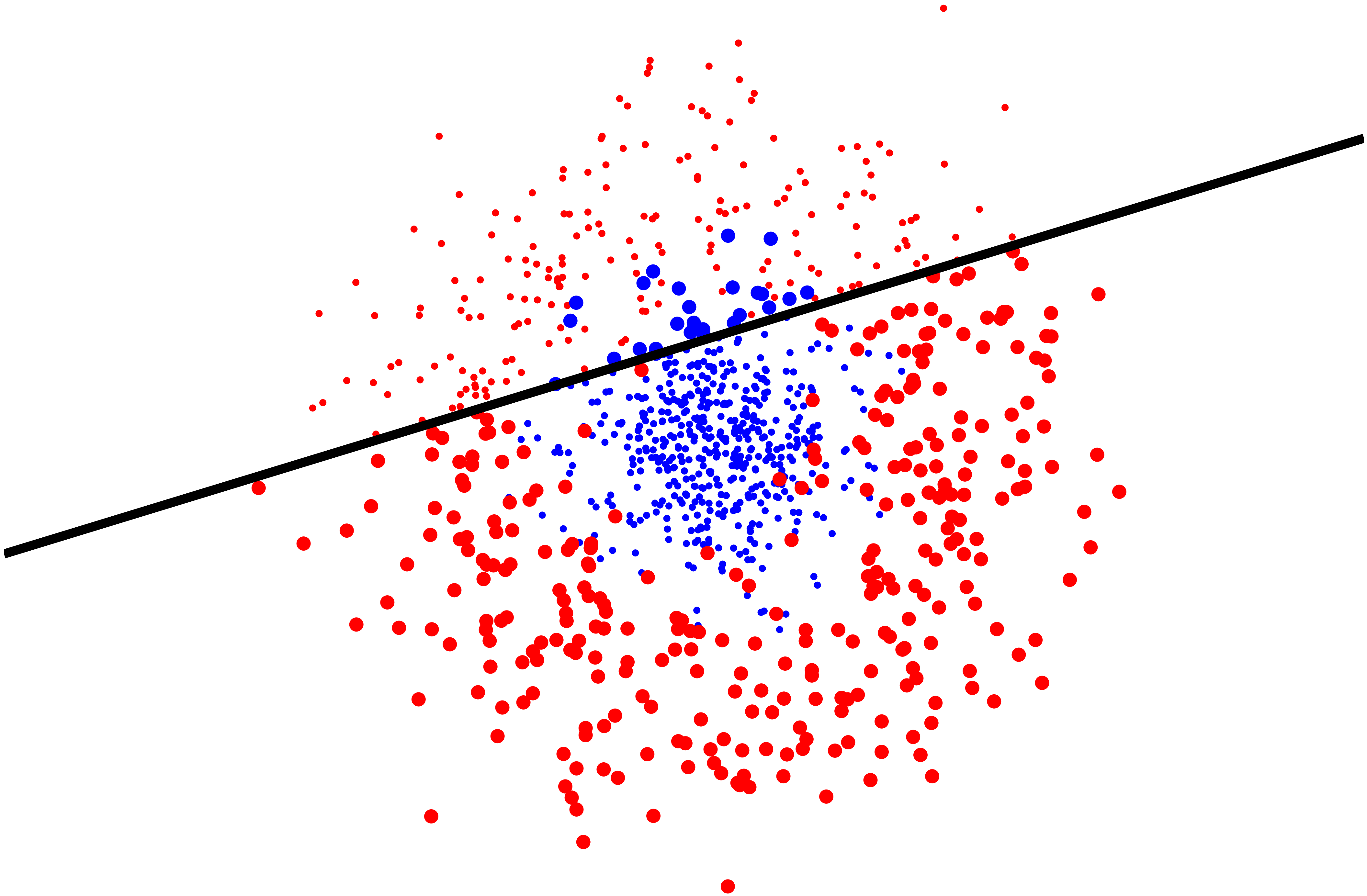

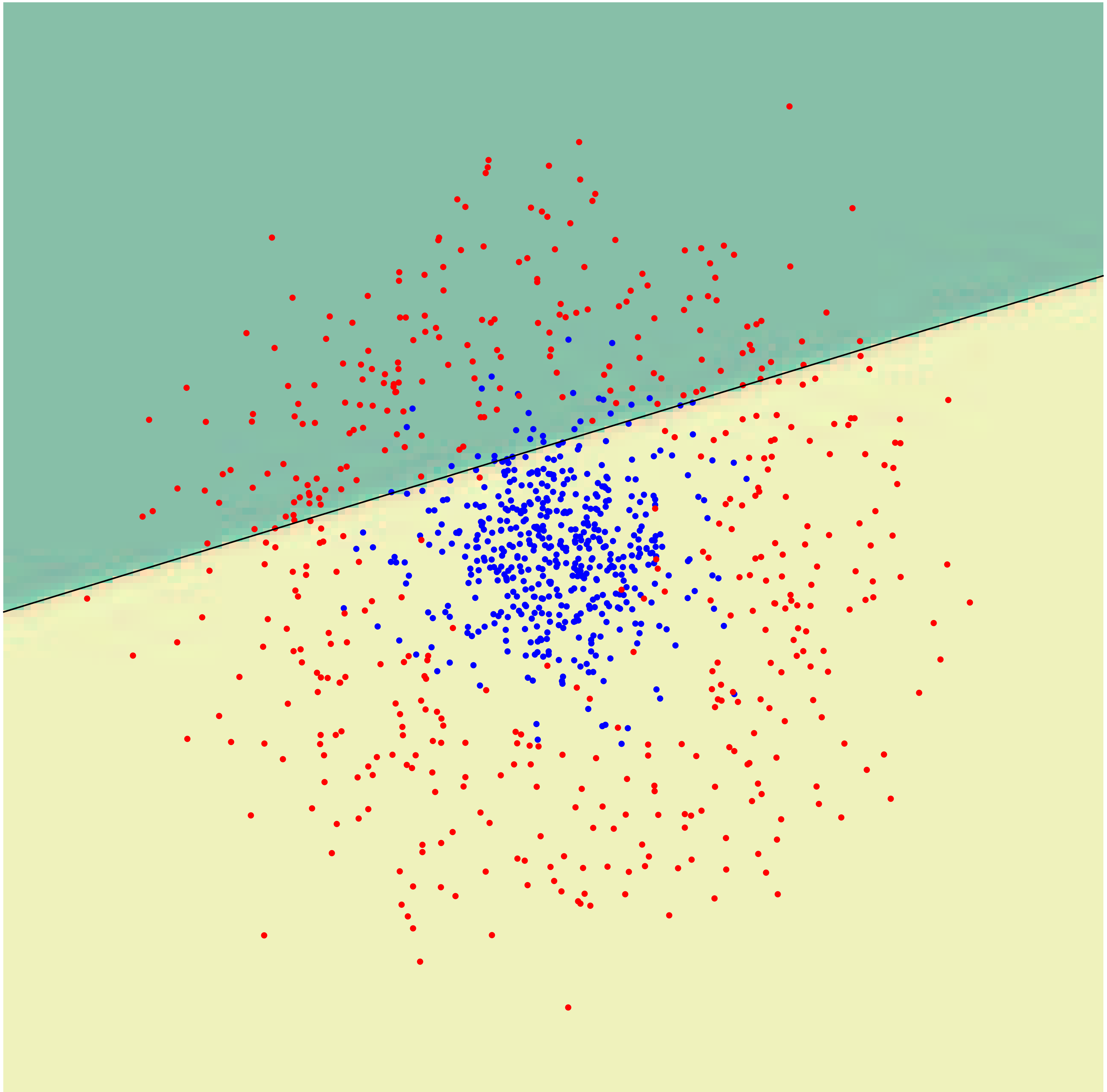

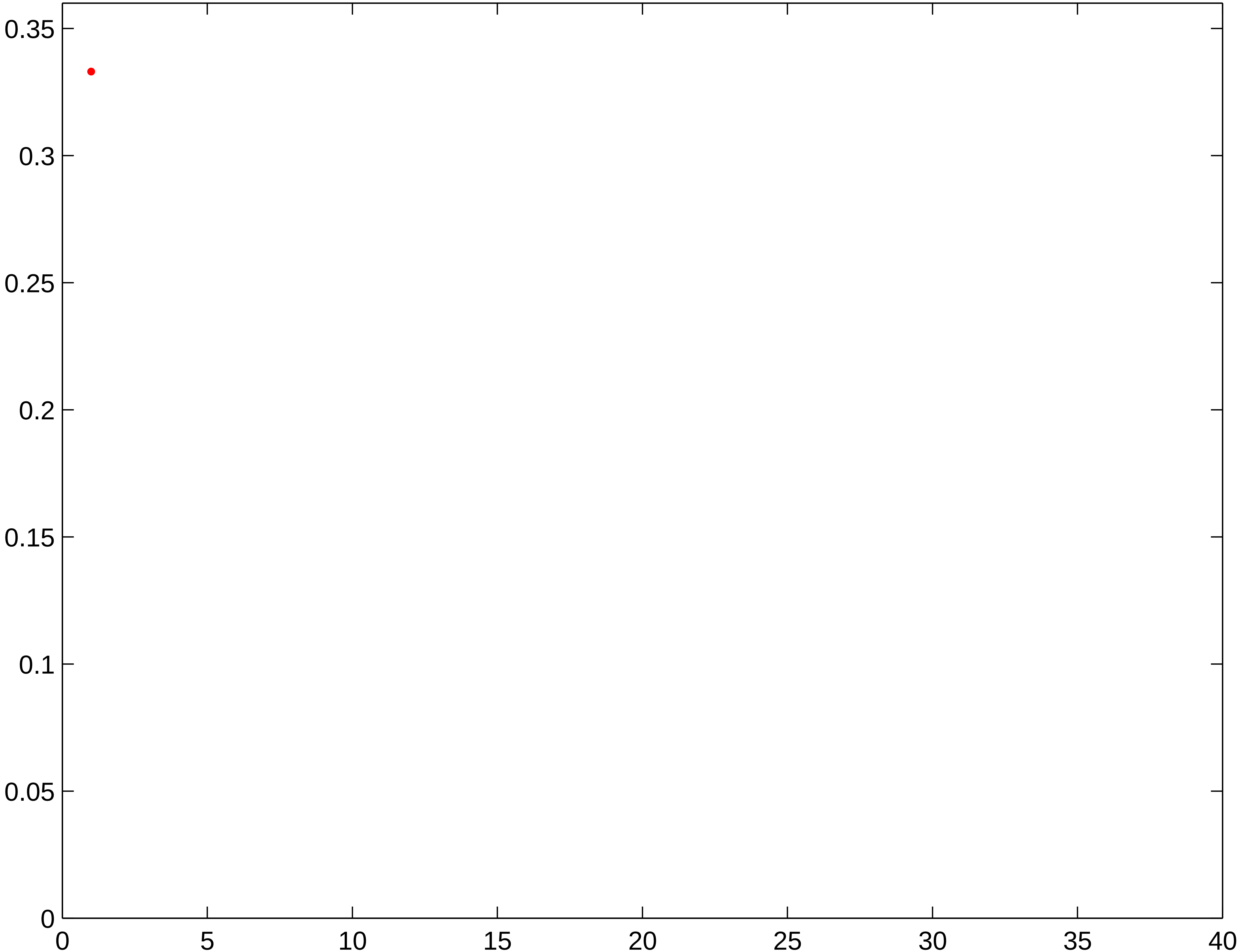
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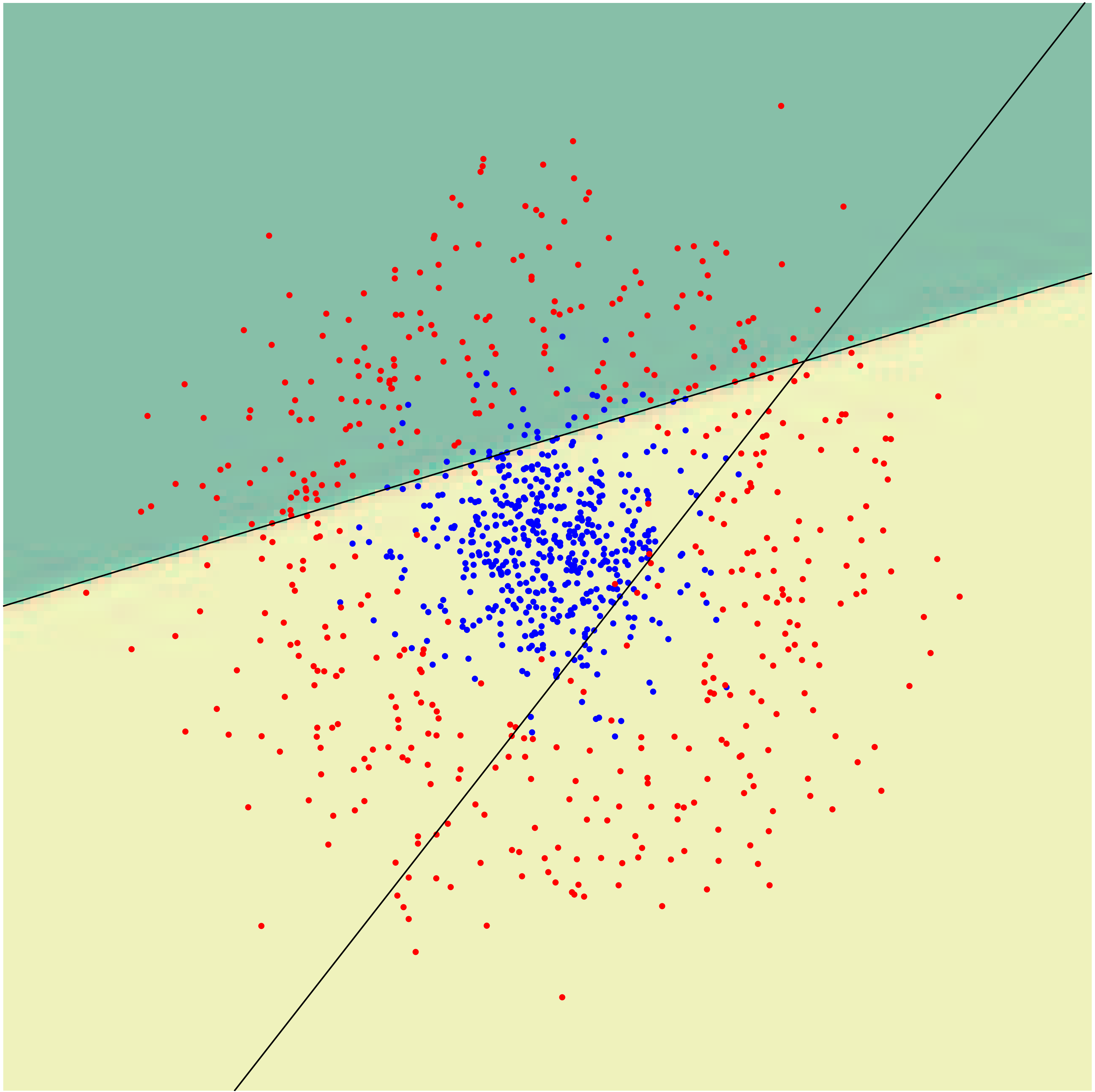

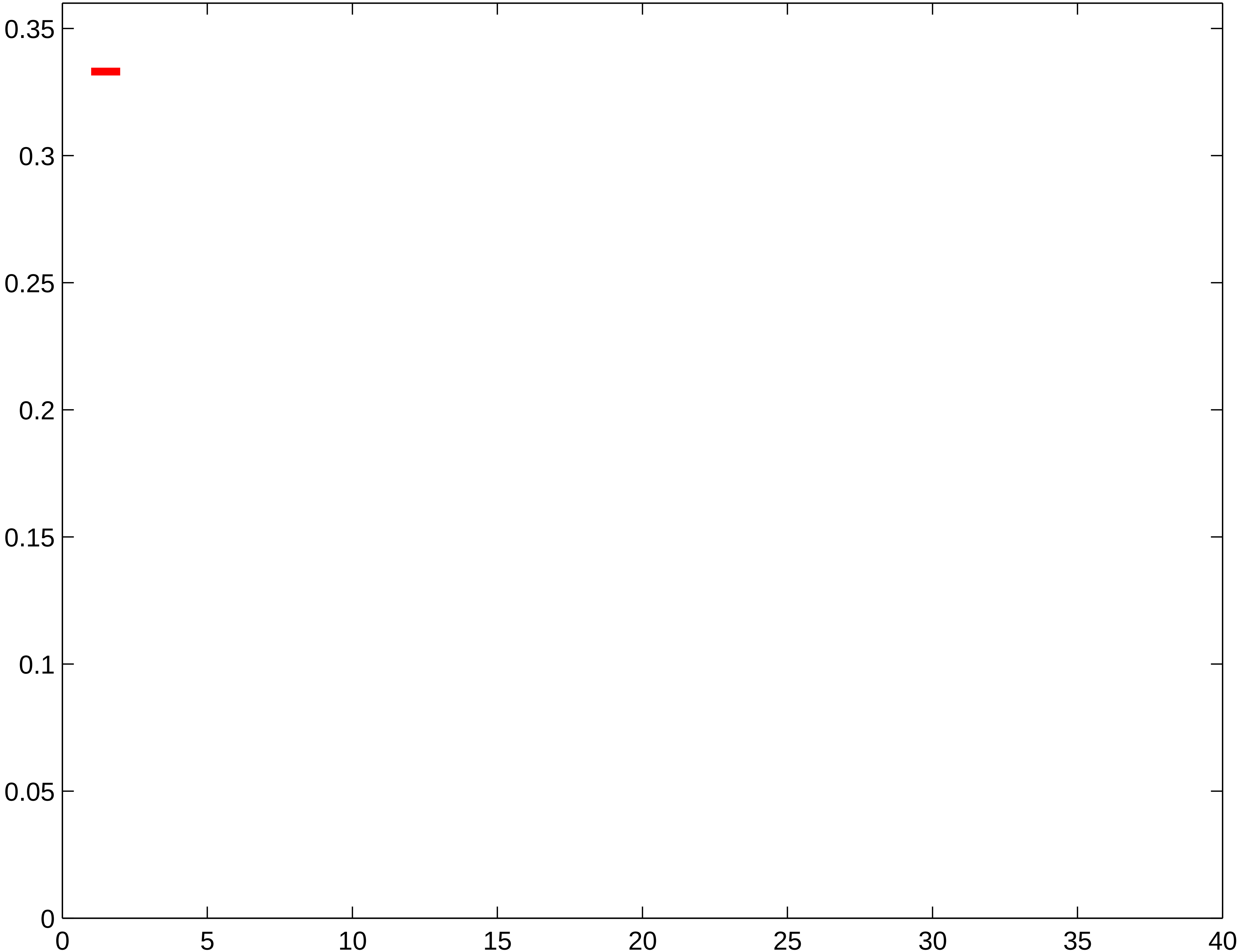
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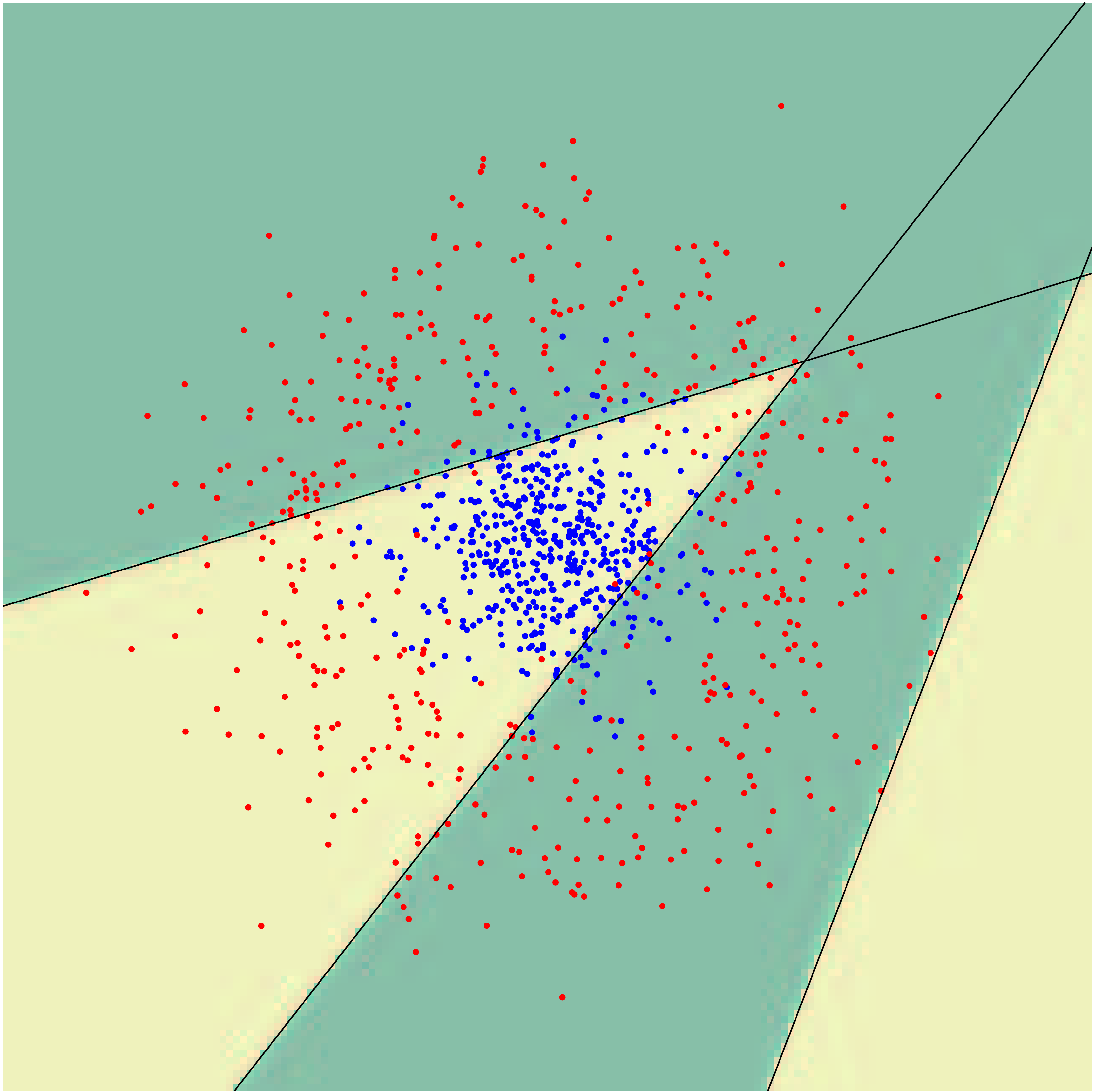

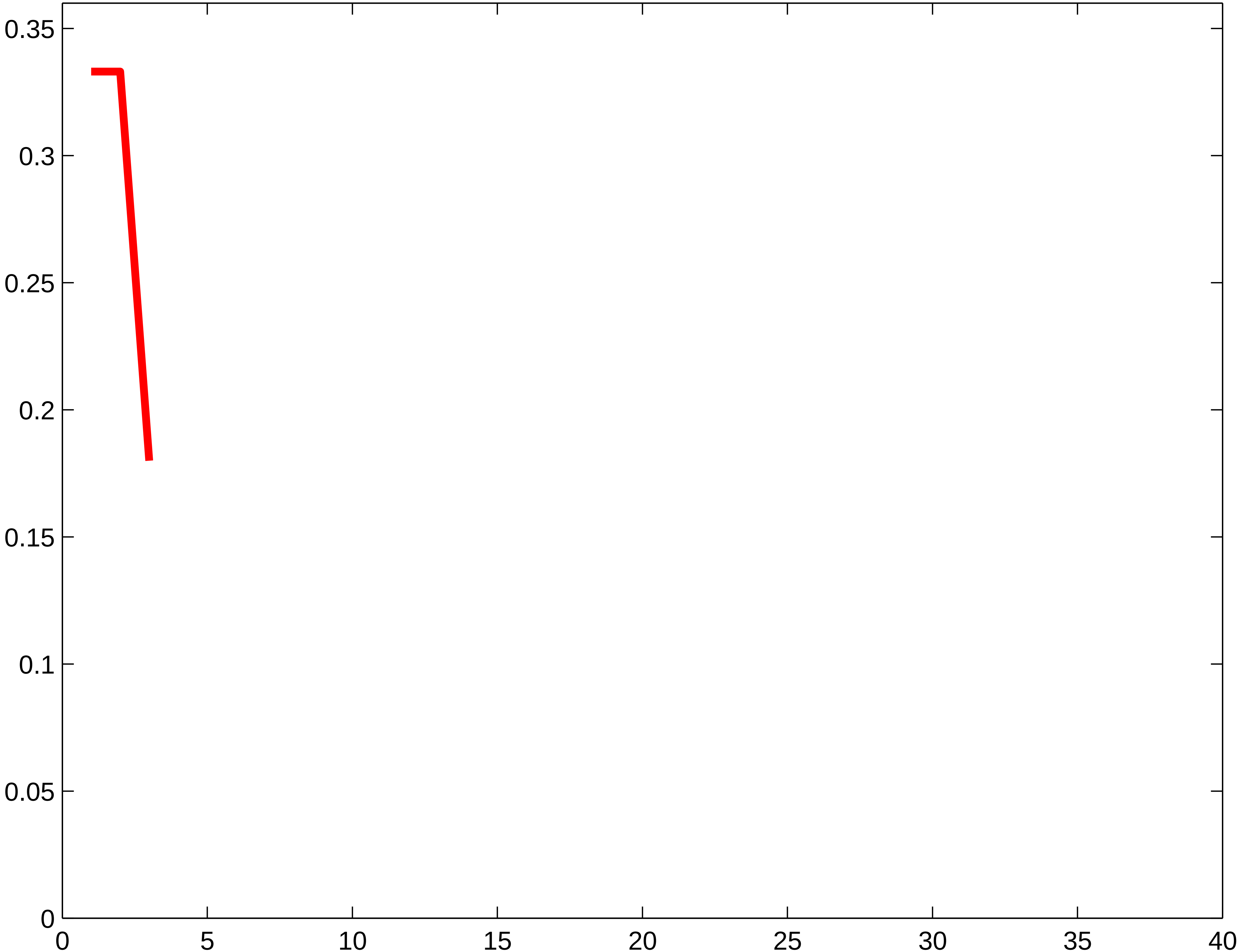
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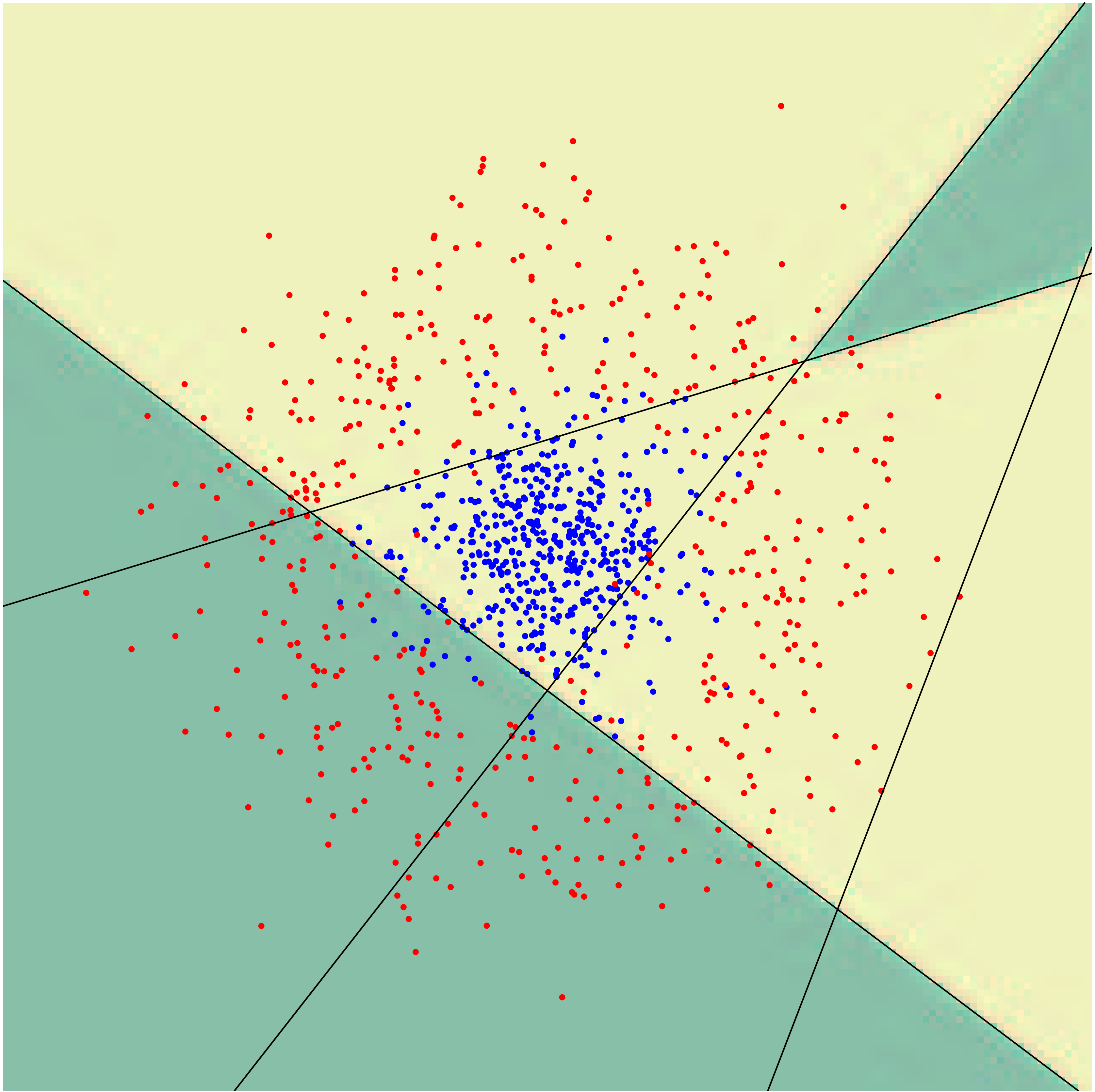

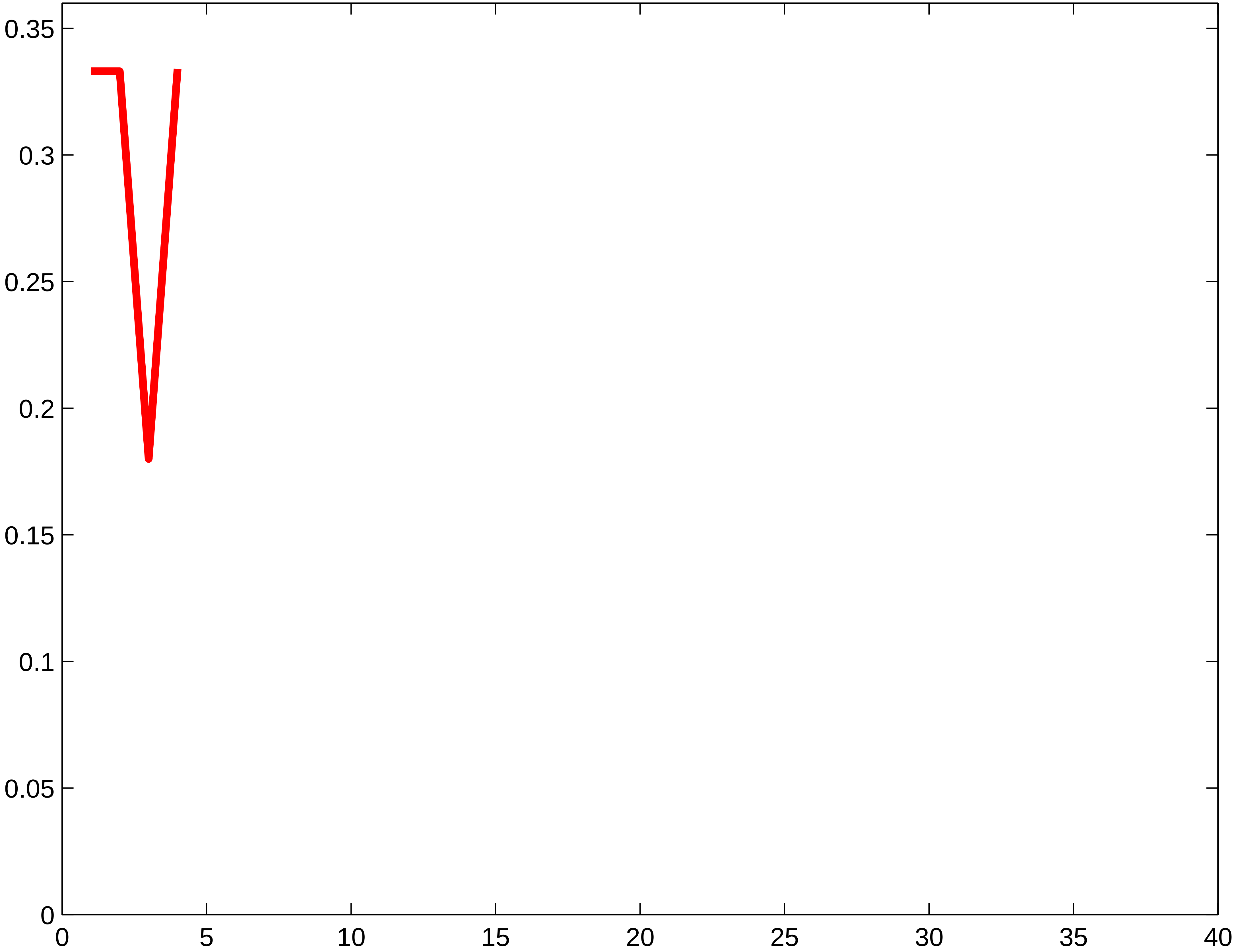
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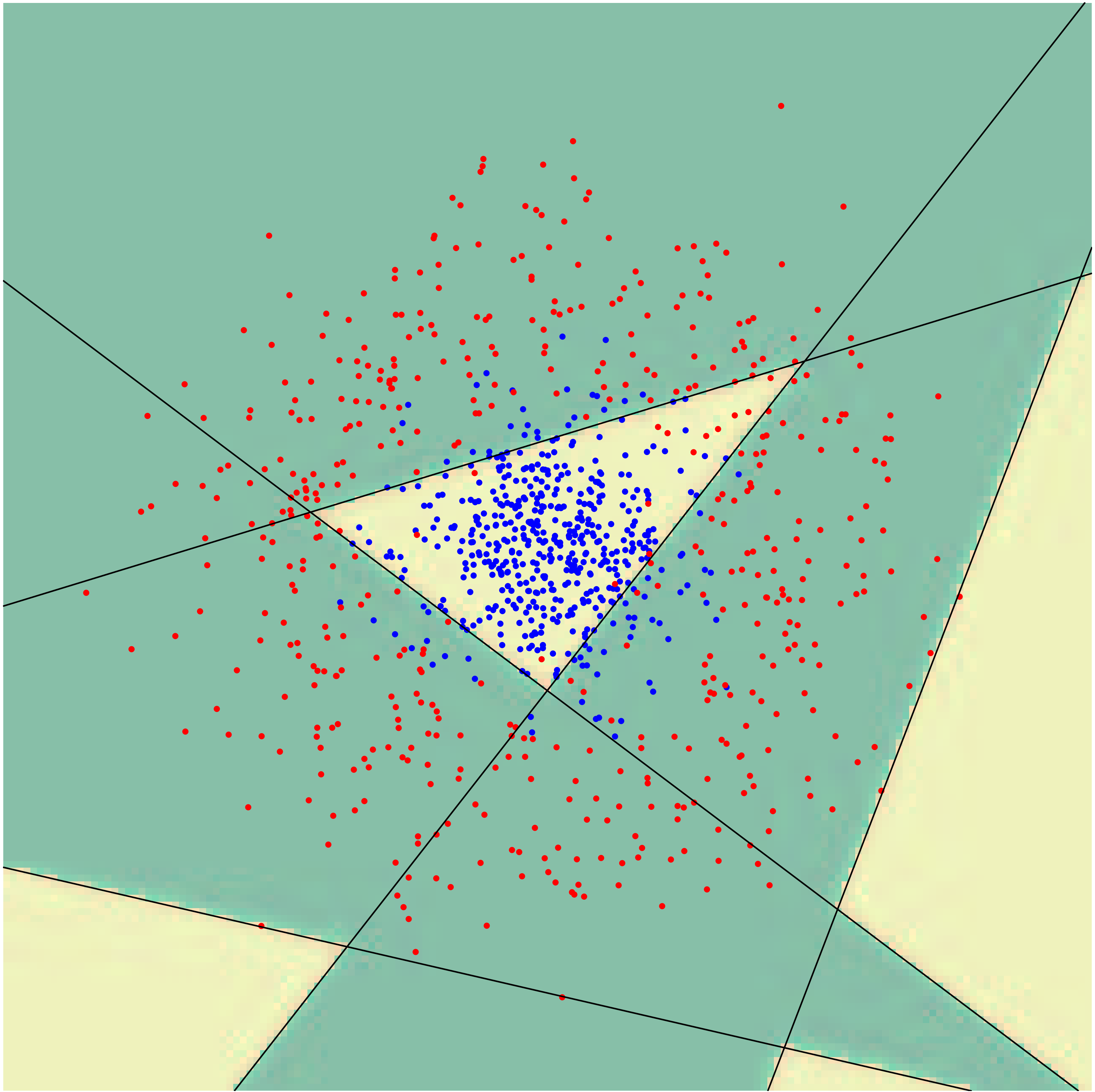

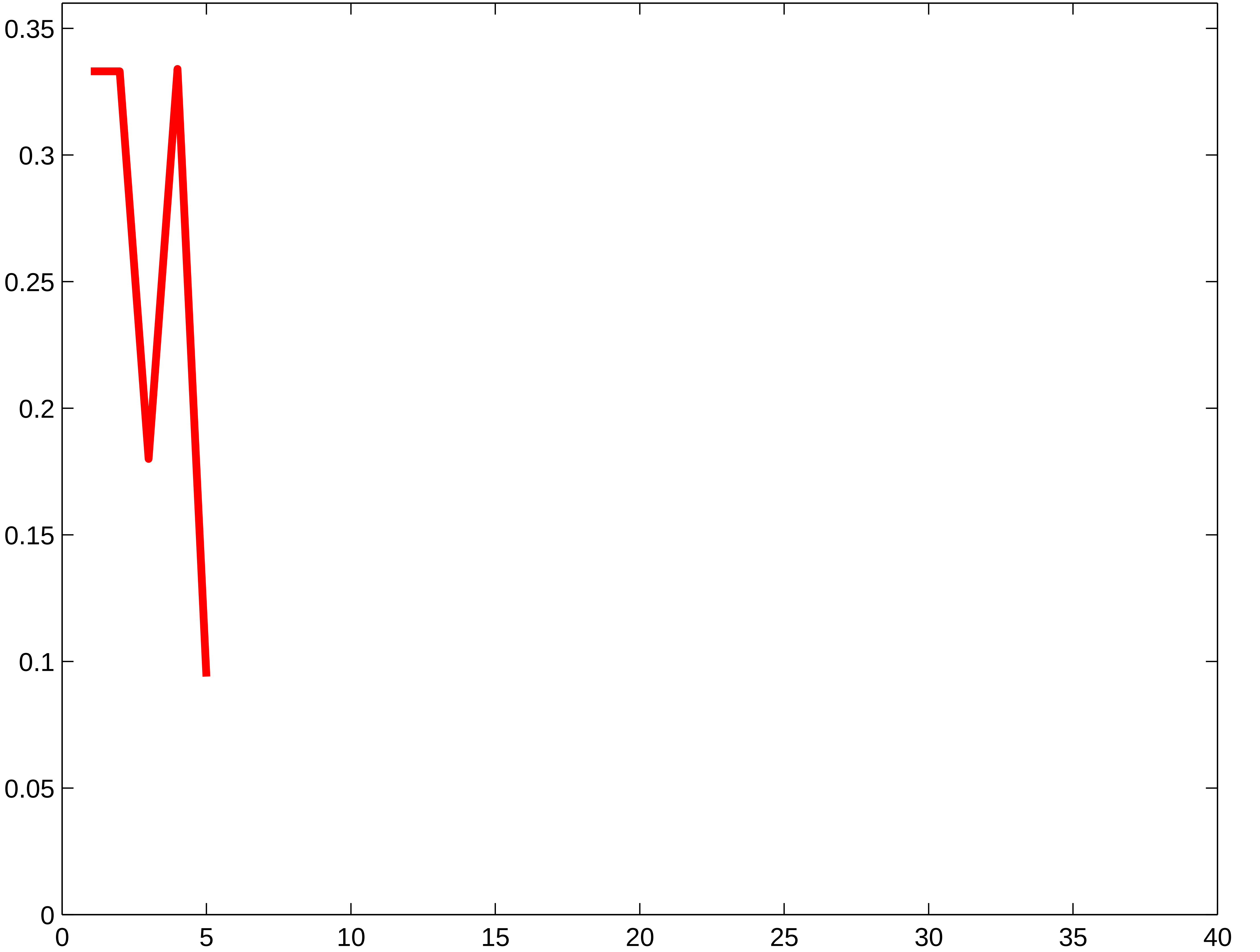

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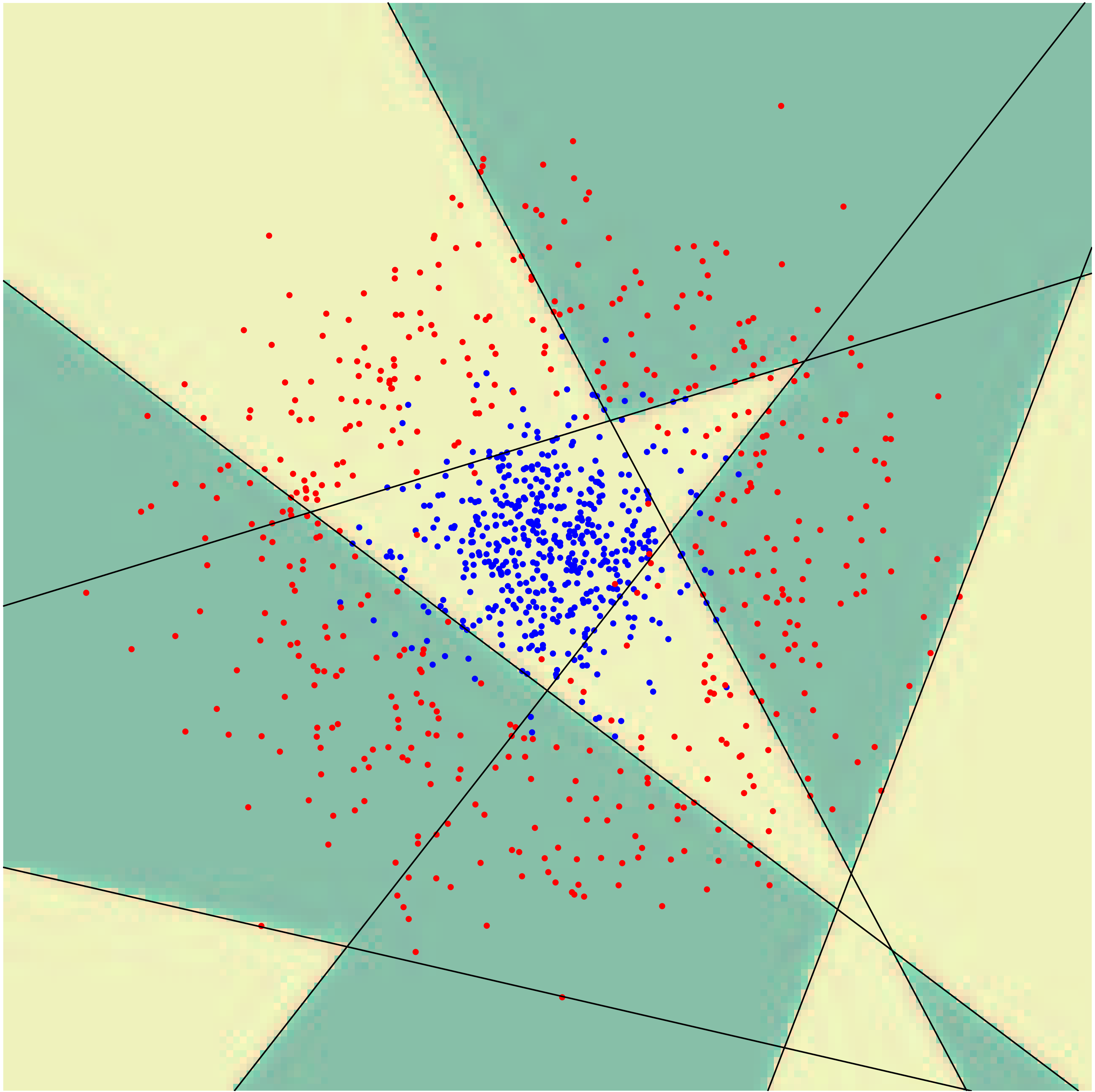

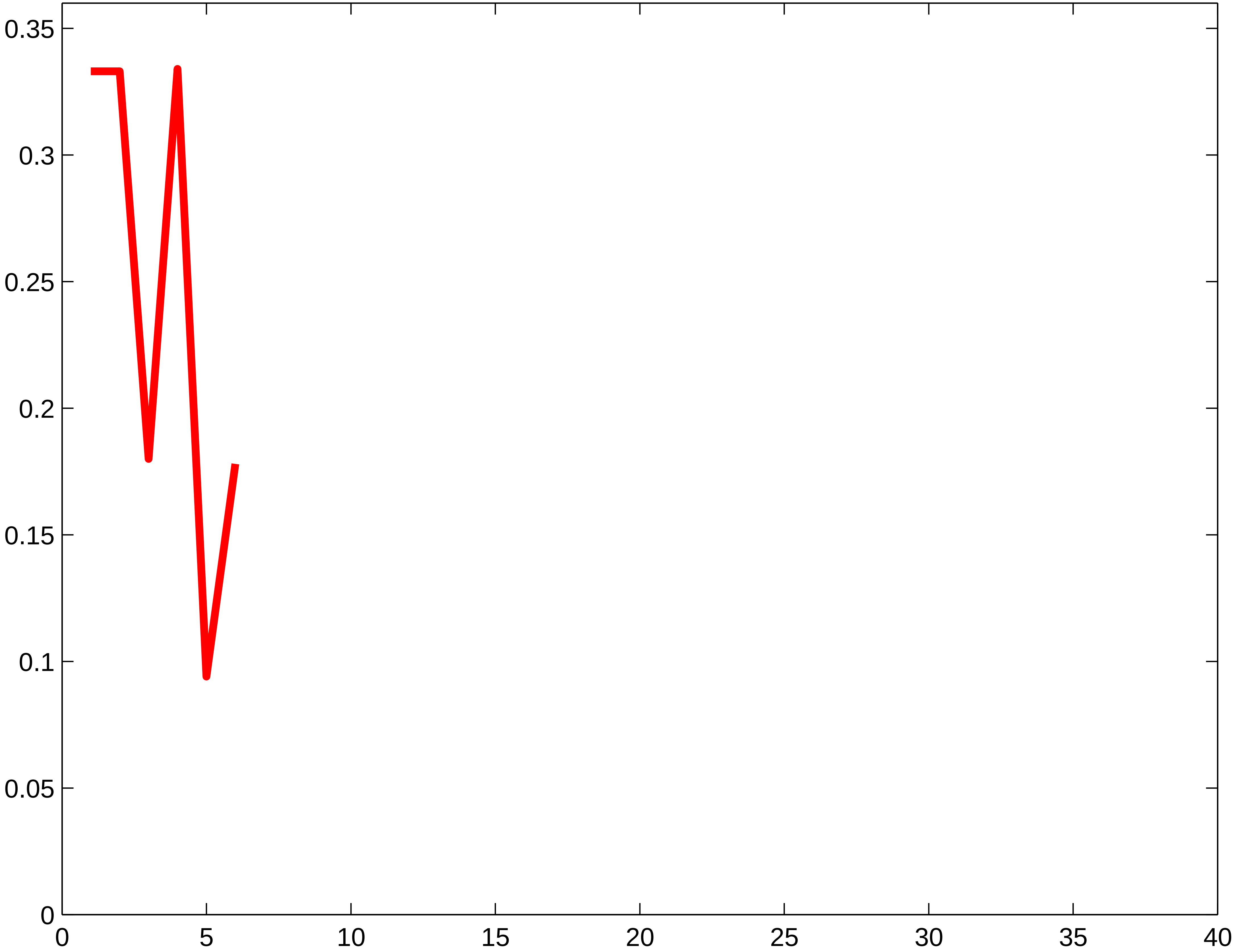

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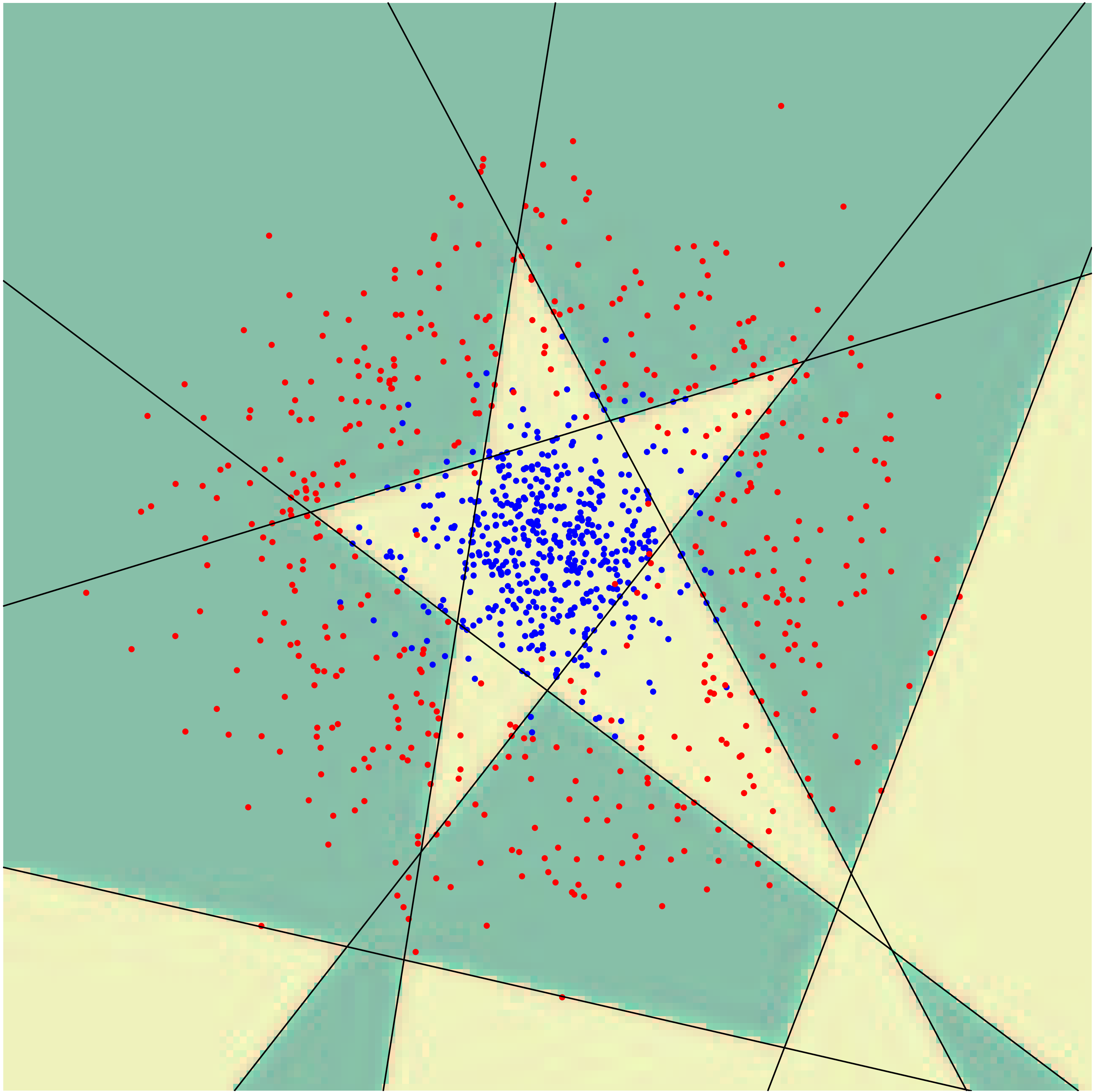

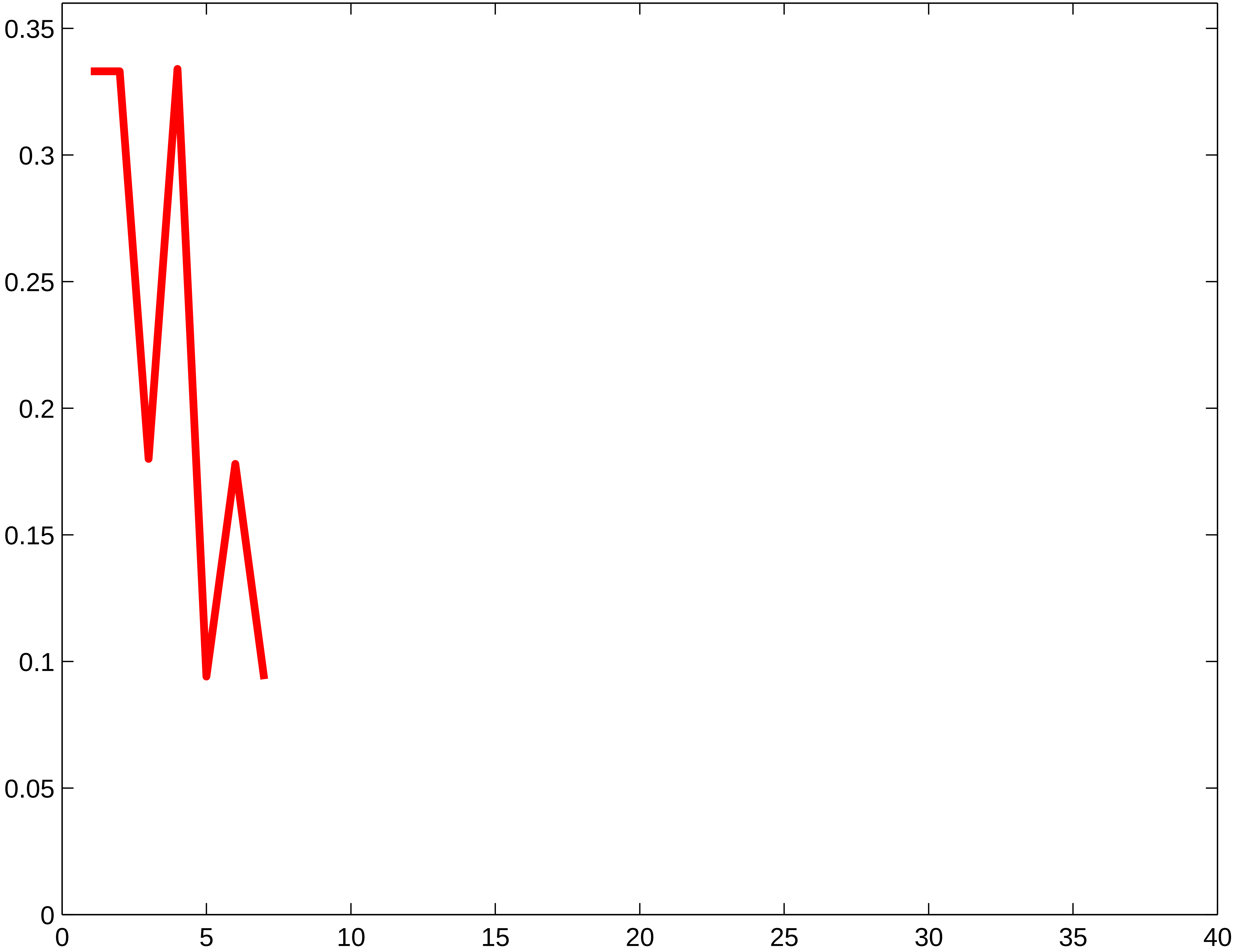
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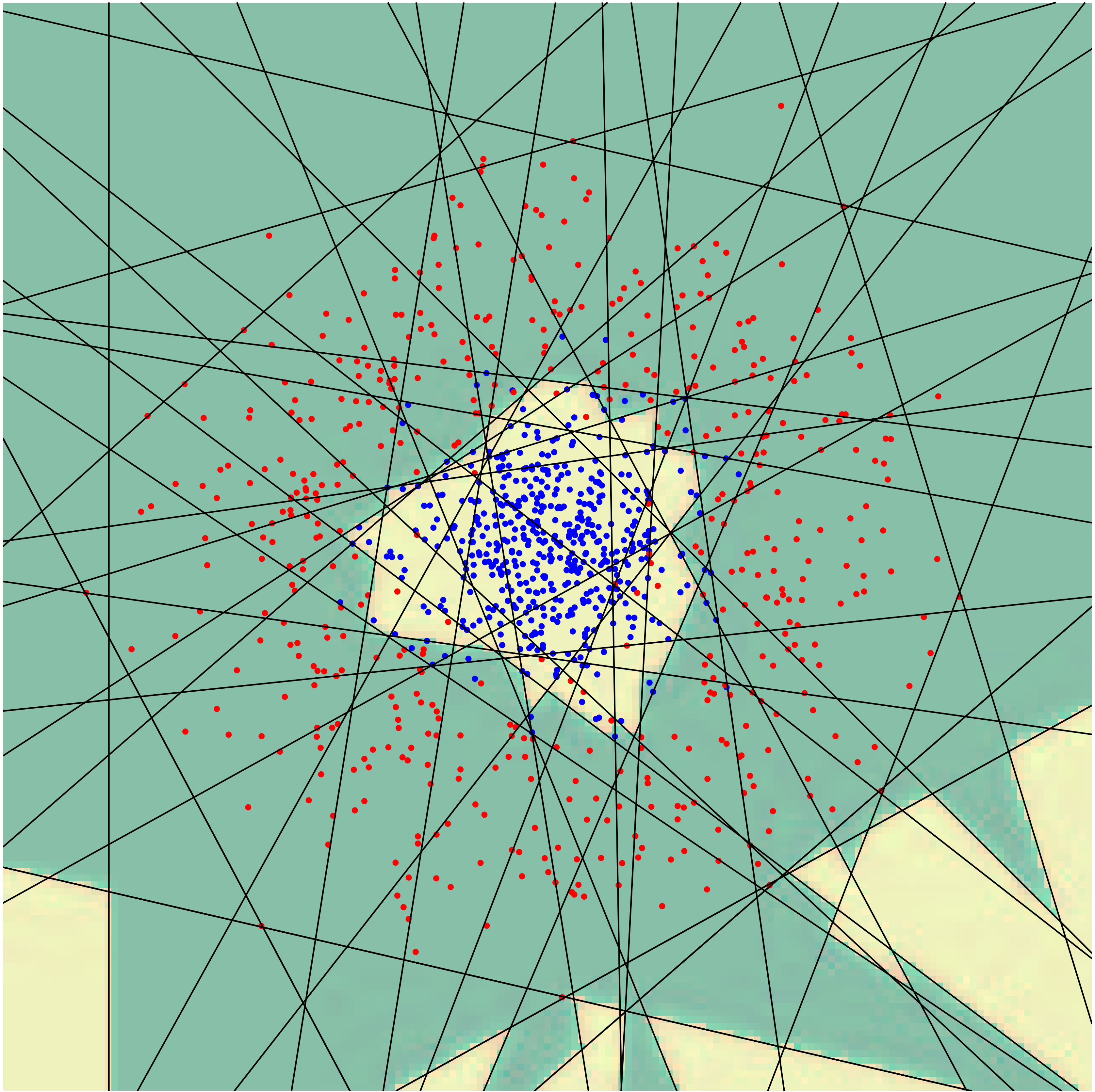

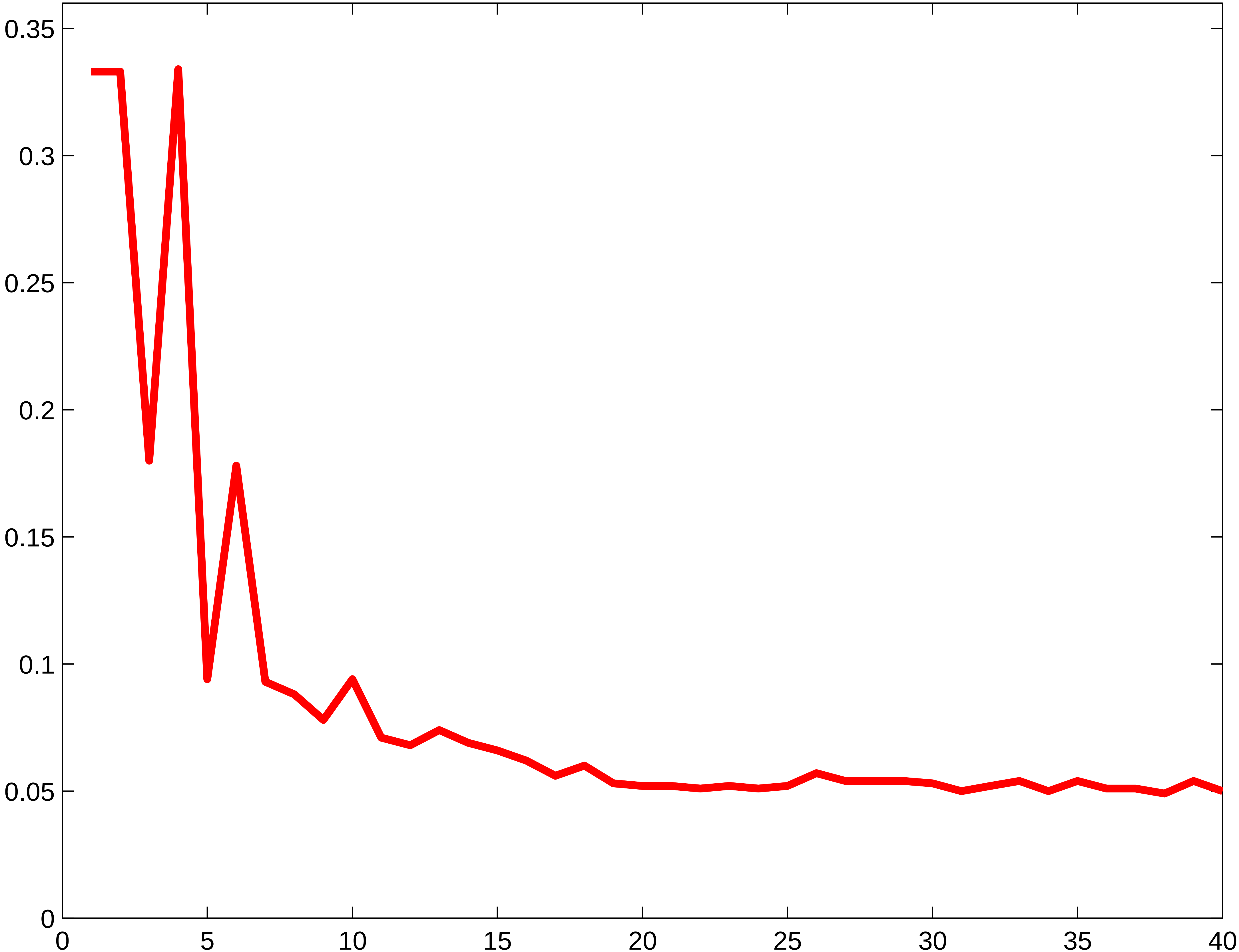
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