

Problem solving by search

Tomáš Svoboda and Matěj Hoffmann

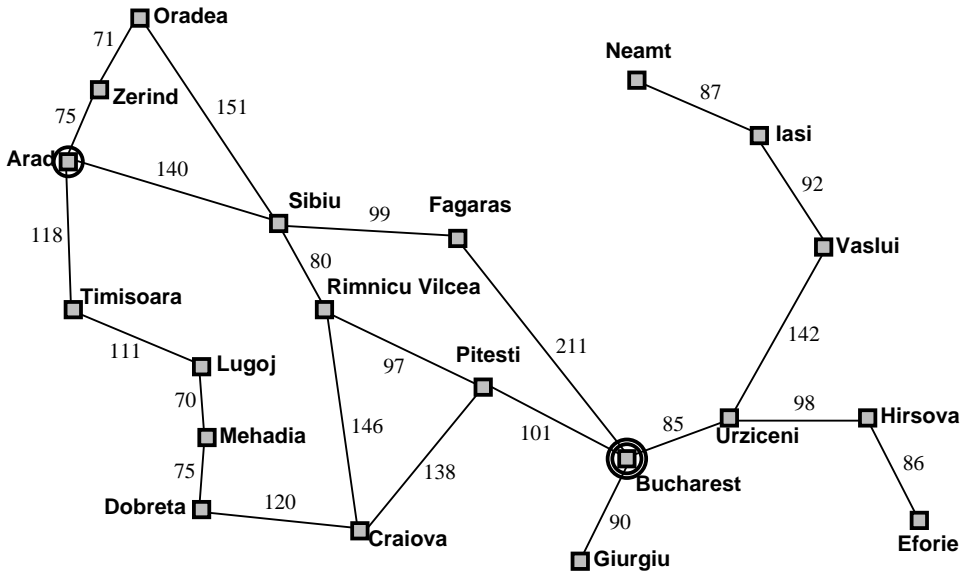
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Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University in Prague

May 23, 2022

Outline

- ▶ Search problem.
- ▶ State space graphs.
- ▶ Search trees.
- ▶ Strategies: which tree branches to choose?
- ▶ Strategy/Algorithm properties.
- ▶ Programming infrastructure.

Example: Traveling in Romania



Notes

Ok, start with a simple one, almost everybody knows about the navigation - path planning problem. Waze, Garmin, ...

Can you think about more problems?

For example:

- Touring problems. Special case: Traveling salesperson problem – each city must be visited exactly once.
- Planning robot movements – mobile robot or manipulator.
- VLSI (chip) layout.
- ...

Example: Map of Romania

Goal:

be in Bucharest

Problem formulation:

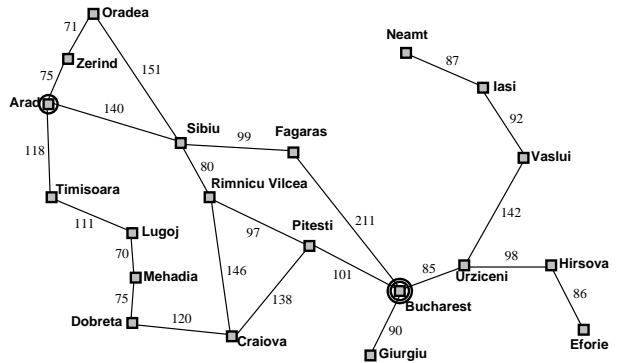
states: position in a city (cities)

actions: drive between cities

Solution:

Sequence of cities (path)

(action sequence [2])



Notes

Classical problem from the Book [2], we use it, too.

states and **actions** will be frequently discussed in several lectures and algorithms. It is important to fully understand them.

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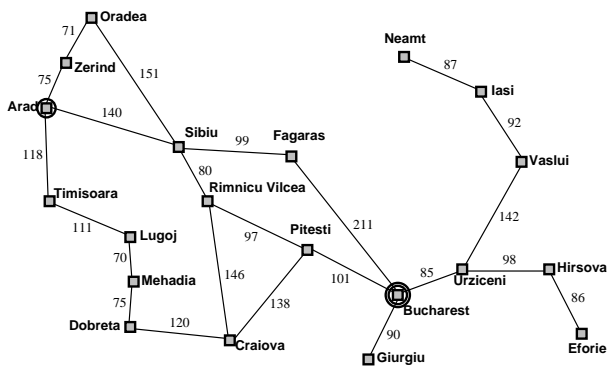
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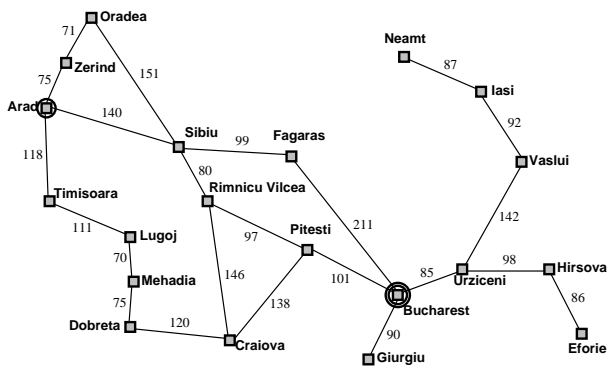
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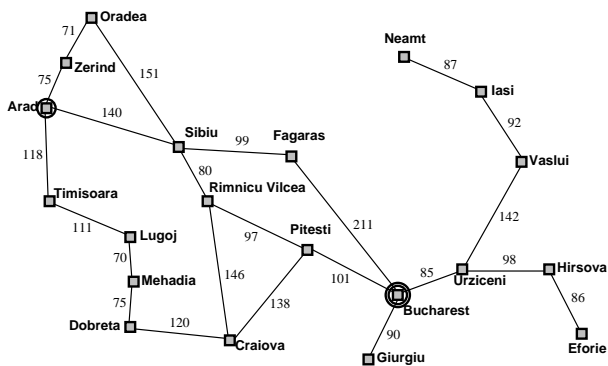
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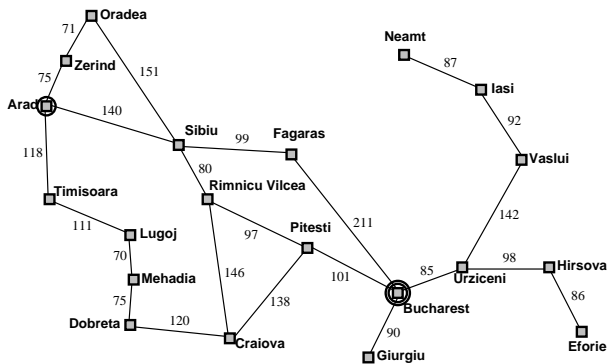
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Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

states?
actions?
solution?
cost?

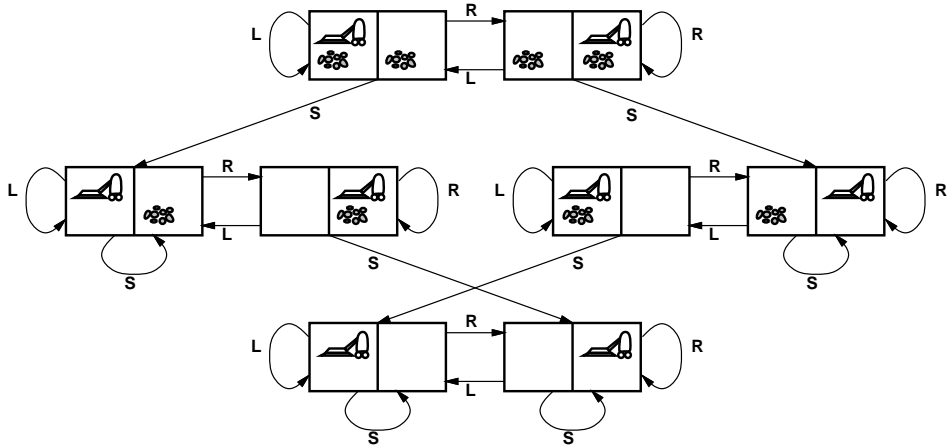
Notes

Also known as $n - 1$ puzzle.

- States: Location of each of the 8 tiles and the blank.
- Number of states: $9!$
- Initial state: any state. (Note that any given goal state can be reached from exactly half of the initial states.)
- Actions: Movements of the blank space: Left, Right, Up, Down (or a subset of these)
- Solution / goal test: Check whether state matches the goal configuration.
- Path cost: nr. steps in the path (each step costs 1)

Toy problem (3.2.1) from [2].

Example: Vacuum cleaner



states?
actions?
solution?
cost?

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Notes

- States: Determined by agent location and dirt location. The agent is in one of two locations, each of which may or may not contain dirt.
- Number of states: 2×2^2 (two possible choices for agent location; for every location, choice dirt vs. no dirt). For n locations: $n \times 2^n$
- Initial state: any state
- Actions: Left, Right, Suck (larger envs. can have also Up and Down)
- Solution / goal test: Are all squares clean?
- Path cost: nr. steps in the path (each step costs 1)

Toy problem (3.2.1) from [2].

A Search Problem

- ▶ **State space** (including Start/Initial state): position, board configuration,
 - ▶ Actions : drive to, Up, Down, Left ...
 - ▶ Transition model : Given state and action return state (and cost)
 - ▶ Goal test : Are we done?

7 / 30

Notes

We will use the terminology through the next 5-6 lectures; also for Markov (Sequential) Decision Processes, Reinforcement Learning.

Make a mental test: You are a robot, going from home to school. What would be **states**, **actions**, **transition model**, **goal test**?

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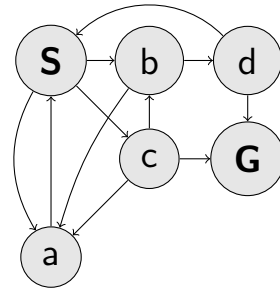
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State Space Graphs

State space graph: a representation of a search problem

- ▶ Graph Nodes – states – are abstracted world configurations
- ▶ Arcs represent action results
- ▶ Goal test – a set of goal nodes

Each state occurs only *once* in a state (search) space.



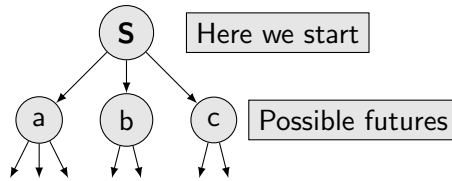
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Formalizing a real world problem – (creating) a state space graph – could be a problem in itself. I put creating into brackets as it may be also infinite.

Close connection to graph algorithms like Dijkstra, Floyd-Warshall.

- Graph algorithms assume complete info about the graphs - the main input.
- For many real-world problems, the graph is not known in advance.
- The state space graph is *revealed during the search*. The graph serves as an abstraction - mental model - rather than as an actual data representation.
- Many real world problems have too many vertices, think about $n - 1$ puzzle or chess, number of possible configurations is enormous.
- A solution can be actually quite shallow.

Search Trees

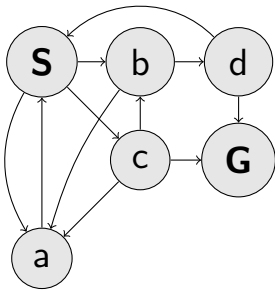


- ▶ A “what if” tree of plans and their outcomes
- ▶ Start node is the root
- ▶ Children are successors
- ▶ Nodes show/contains states, but correspond to *plans* that achieve those states

Notes

- What if decision about an action, repeats ...
- Nodes in the search tree are not the same as the nodes in the state space graph.

State Space Graphs vs. Search Trees

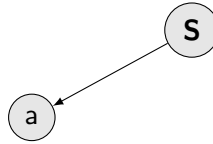
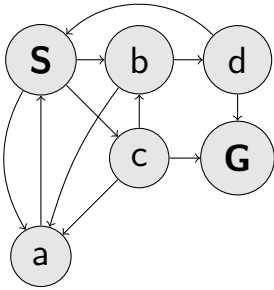


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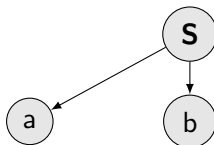
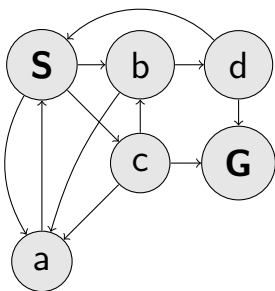


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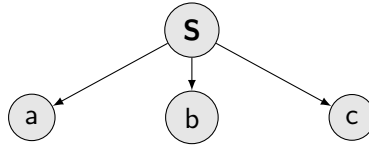
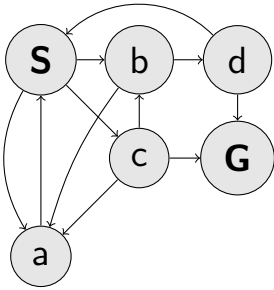


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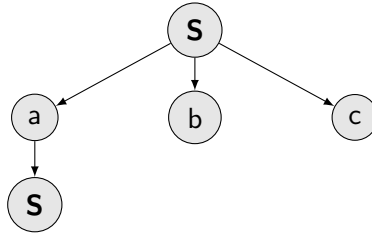
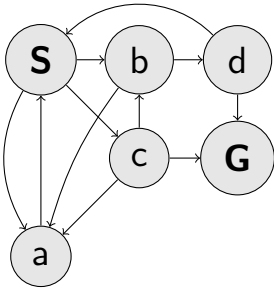


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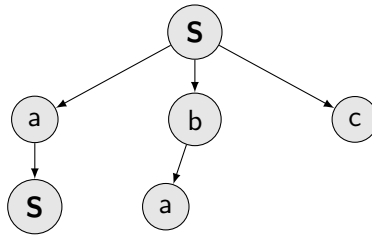
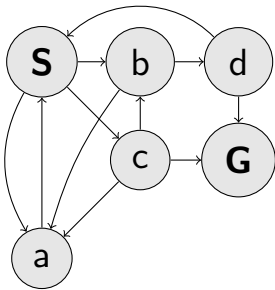


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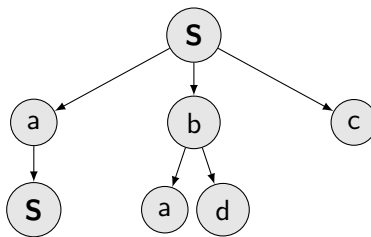
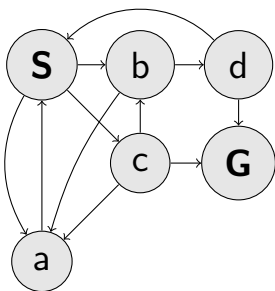


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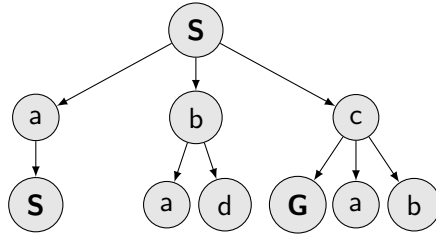
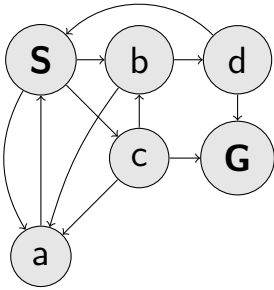


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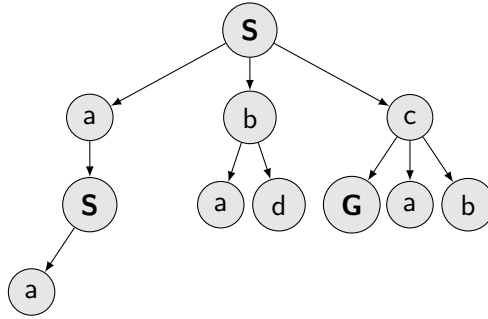
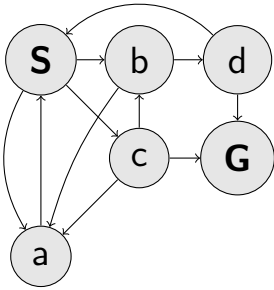


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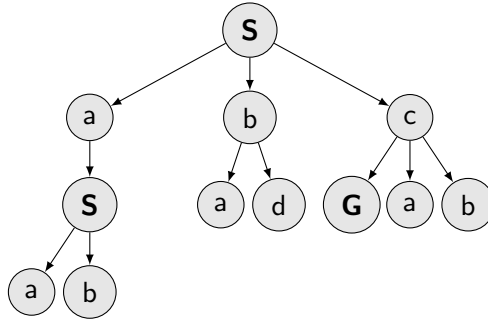
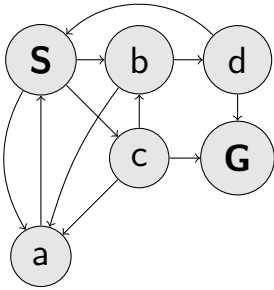


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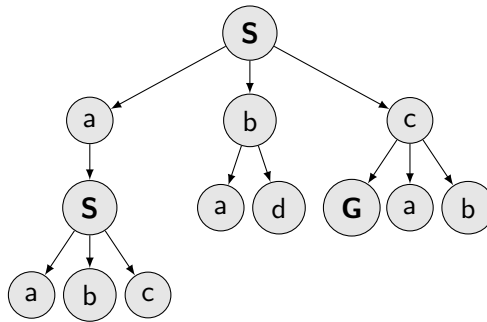
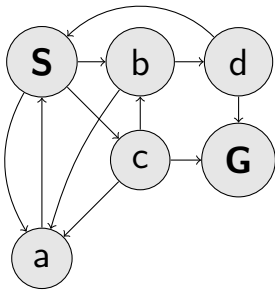


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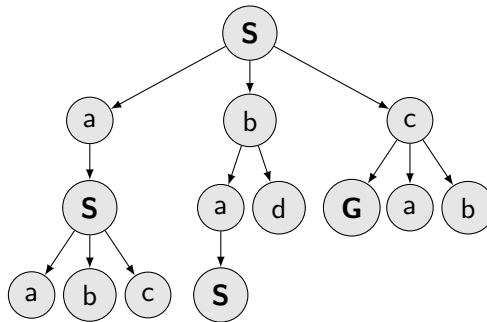
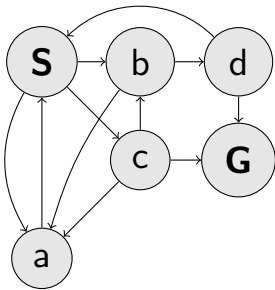


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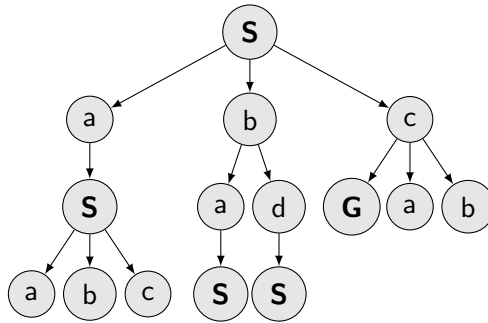
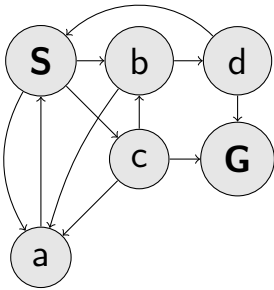


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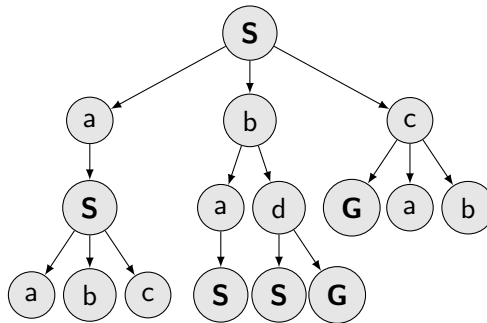
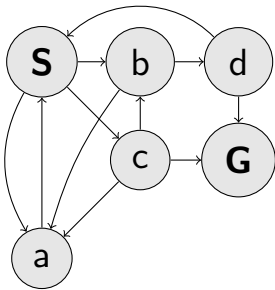


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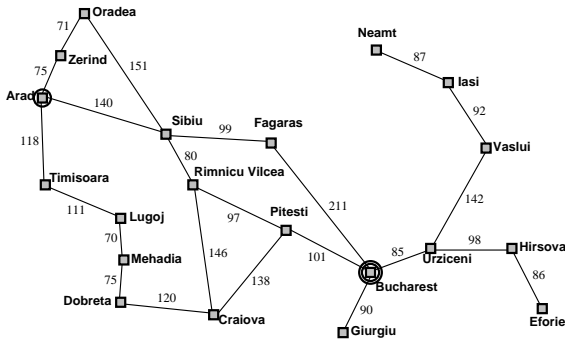


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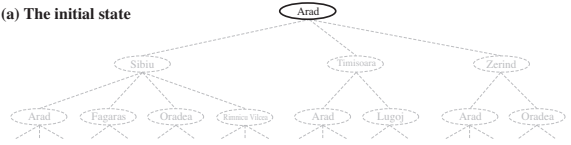
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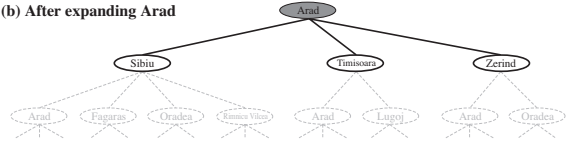
From problem/transition graph to search tree (Romania)



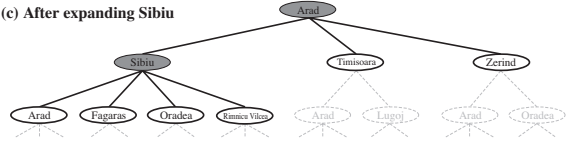
(a) The initial state



(b) After expanding Arad



(c) After expanding Sibiu

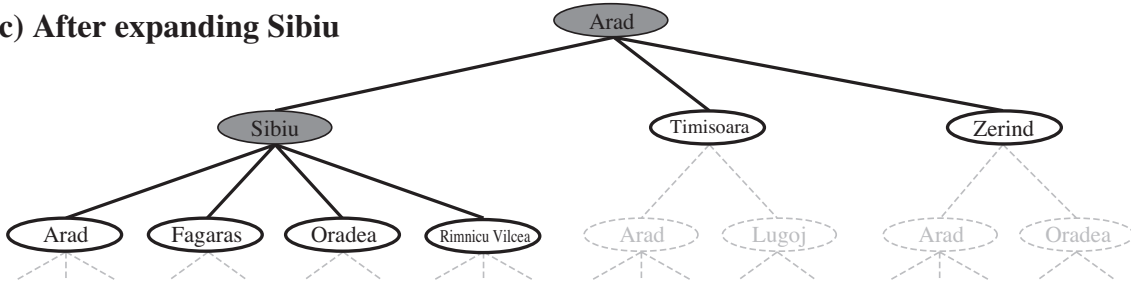


Problem/transition graph is revealed incrementally.
The revealing strategy can be visualized as a search tree.

Notes

Images from [2].

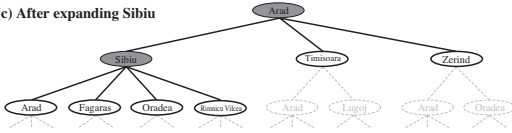
(c) After expanding Sibiu



- ▶ Expand **plans** - possible ways (**tree nodes**).
- ▶ Manage/Maintain **fringe** (or **frontier**) of plans under consideration.
- ▶ Expand new nodes *wisely(?)*.

Tree search algorithm

(c) After expanding Sibiu



function TREE_SEARCH(problem) **return** a solution or failure

initialize by using the initial state of the problem

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Expand the node and add the resulting nodes to the tree

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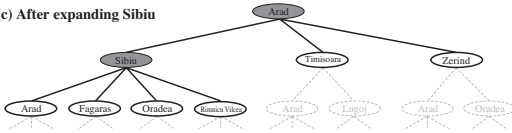
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A *general* tree search algorithm. Individual search algorithms vary primarily in how they choose which state to expand next – the “search strategy”.

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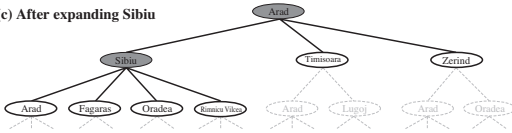
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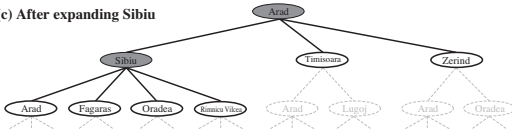
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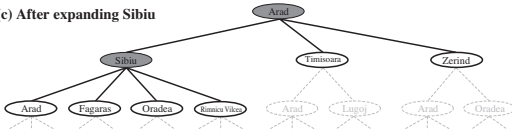
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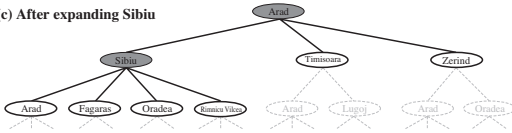
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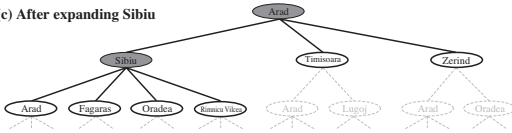
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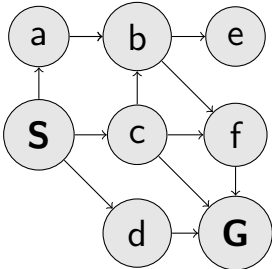
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Example of a tree search



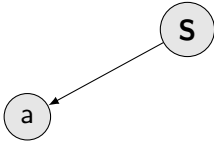
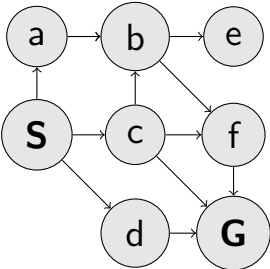
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What are the properties of a strategy/algorithm?

Notes

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Example of a tree search



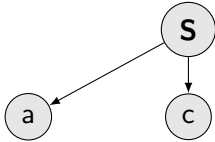
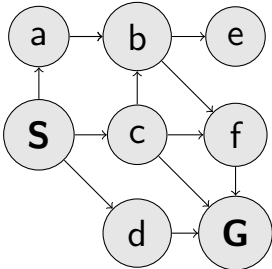
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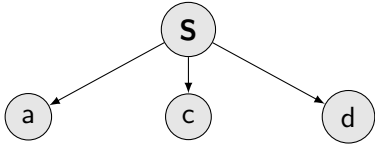
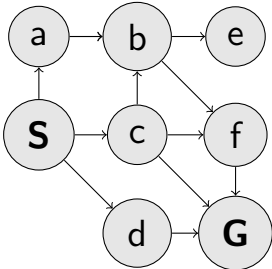
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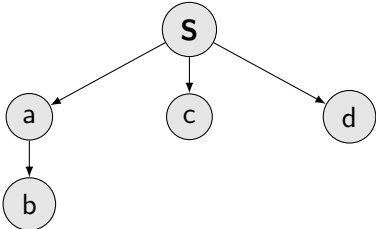
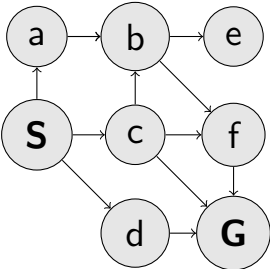
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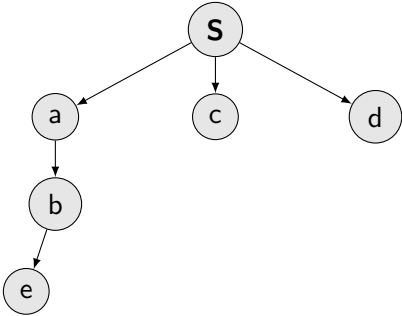
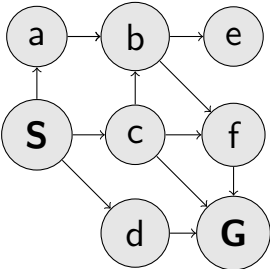
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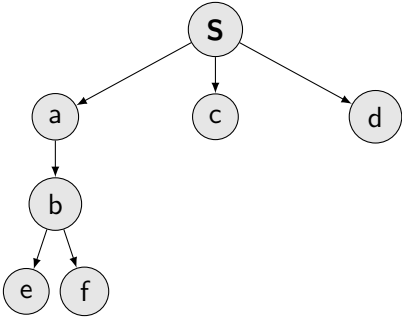
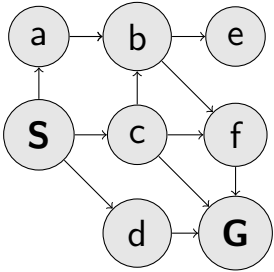
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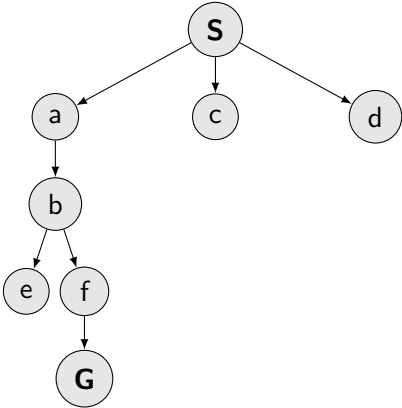
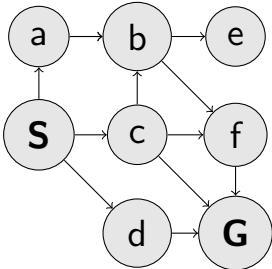


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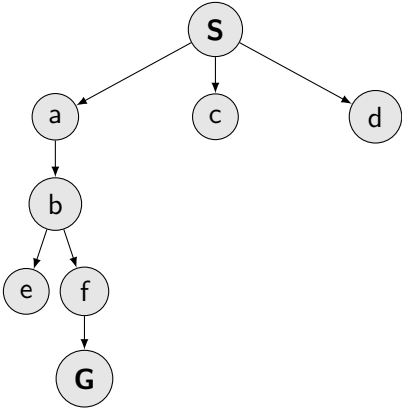
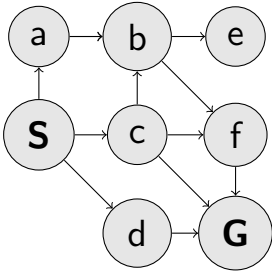
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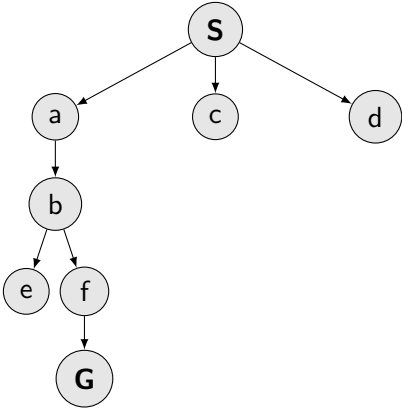
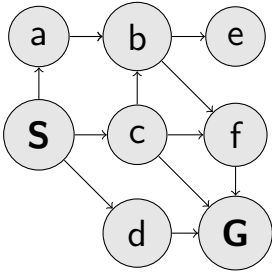
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Search (algorithm) properties

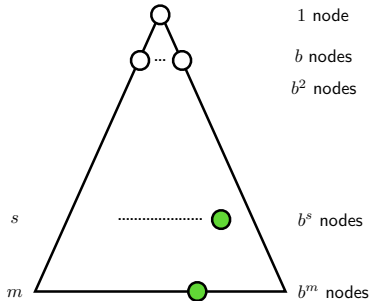
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How many nodes in a (search) tree? What are tree parameters?

Notes

Draw a (symbolic—think about a triangle) sketch of a (search) tree. It may grow upwards or downwards. How would you characterize/parametrize size of a tree.

- Depth of the tree d .
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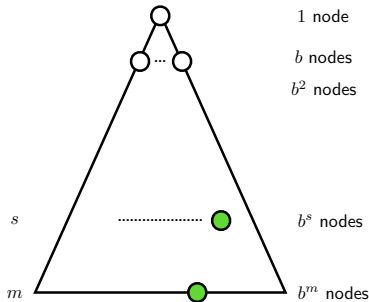
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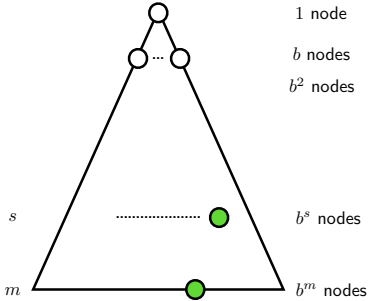
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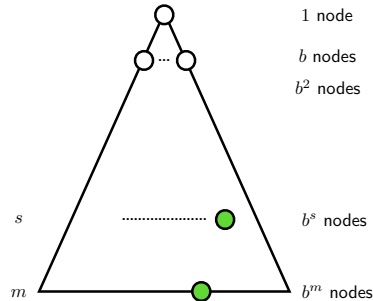
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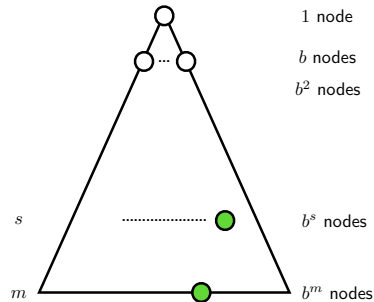
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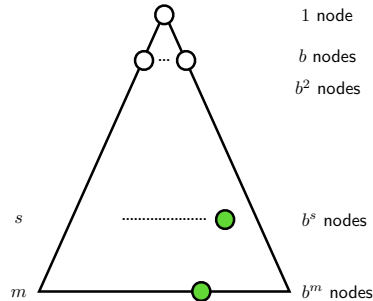
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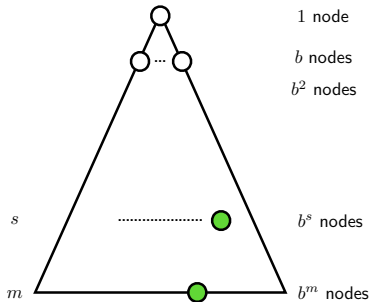
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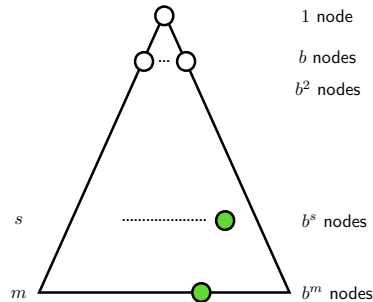
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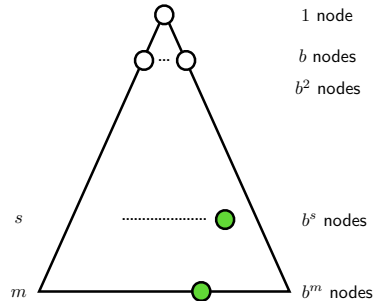
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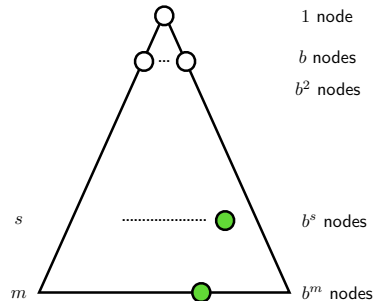
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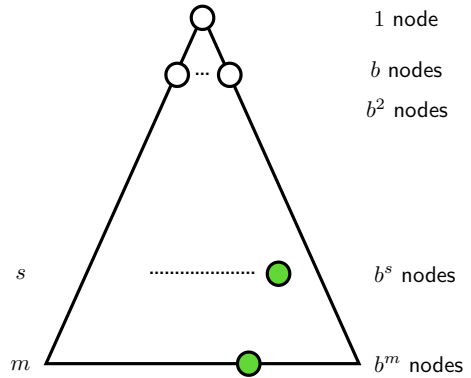
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Strategies

How to traverse/build a search tree?

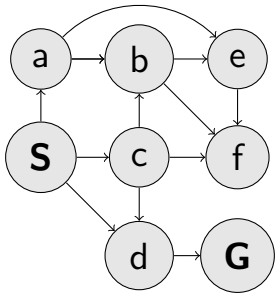
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Notes

It is perhaps worth to remember that the search tree is built as the algorithm goes. Or better said, the tree is a human friendly representation of the machine run.

Depth-First Search (DFS)



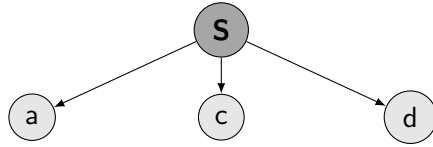
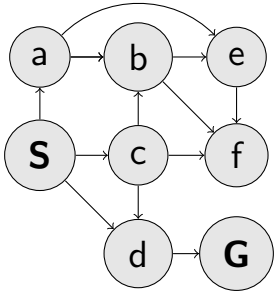
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What are the DFS properties (complete, optimal, time, space)?

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- In animation, we will do the expansion step at once.
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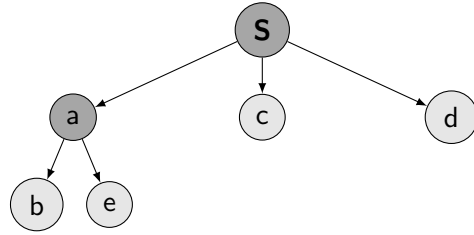
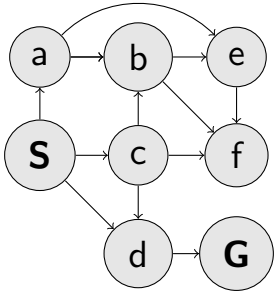


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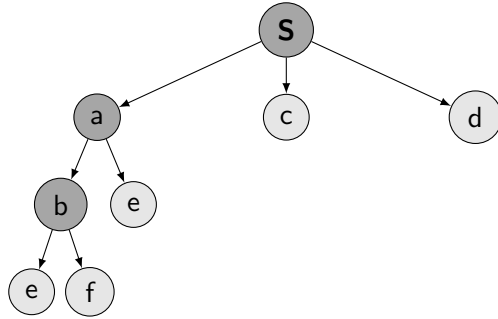
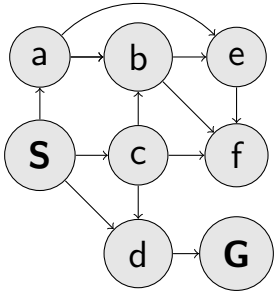


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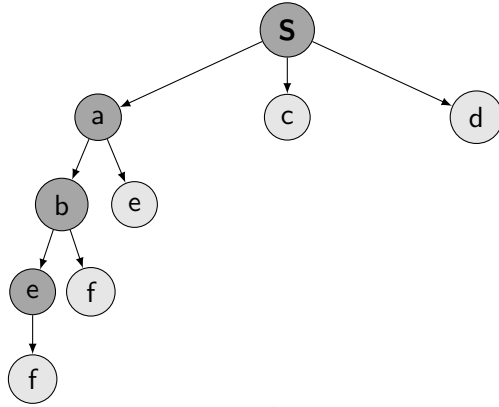
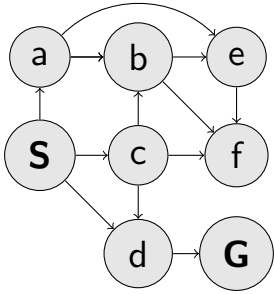


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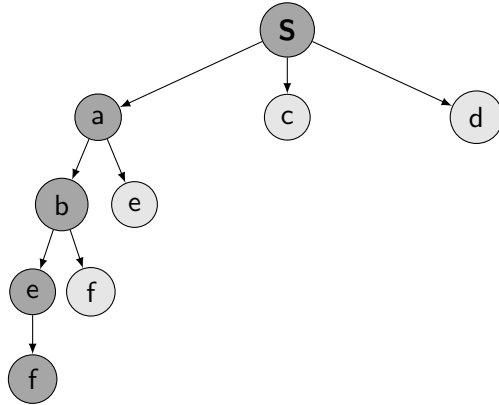
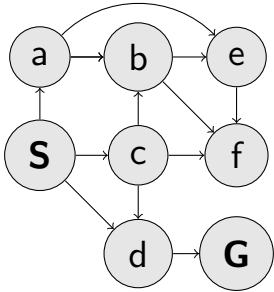


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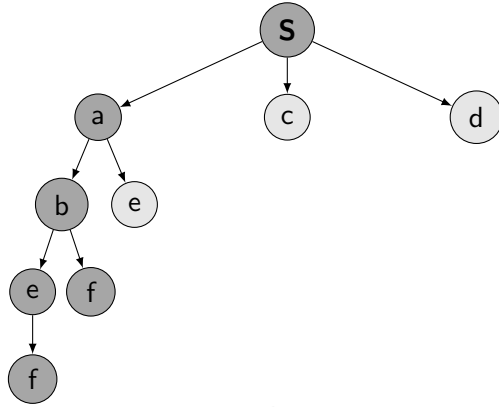
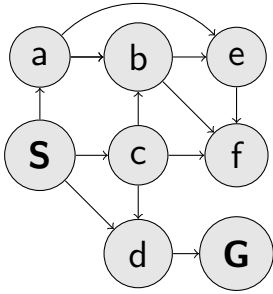


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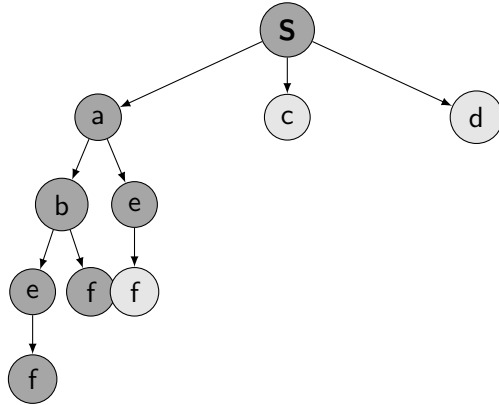
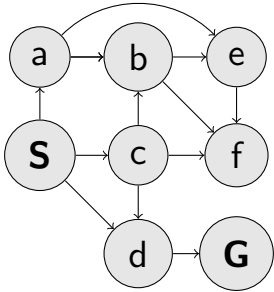


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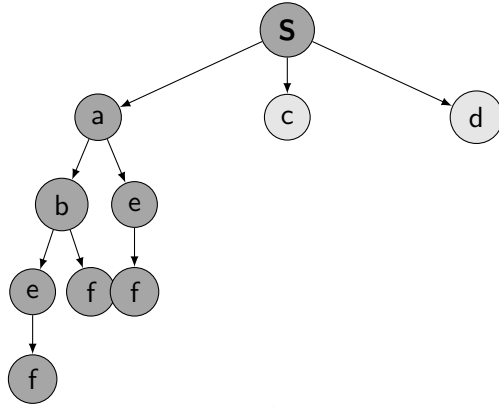
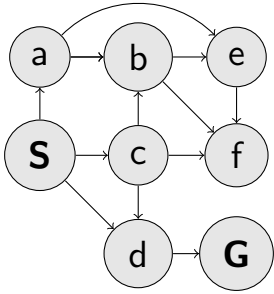


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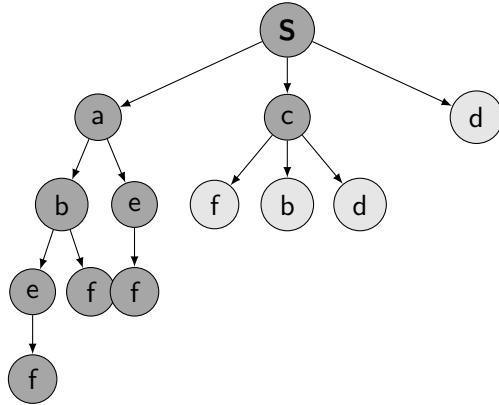
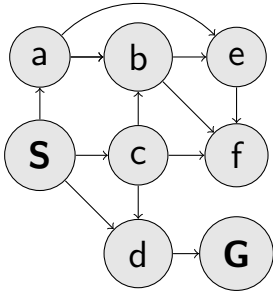
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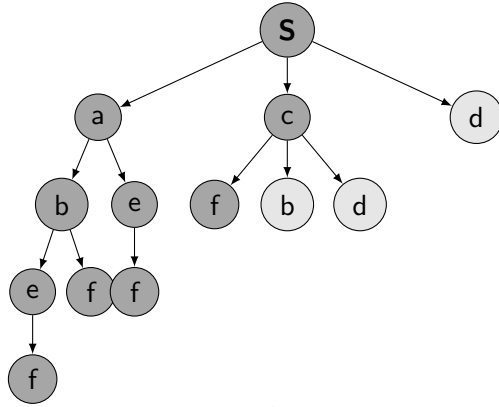
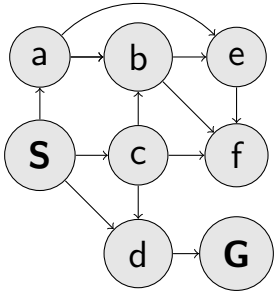


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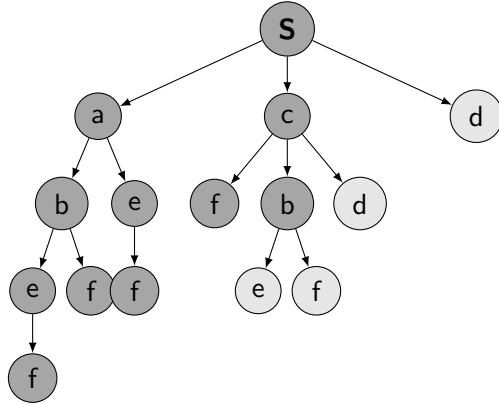
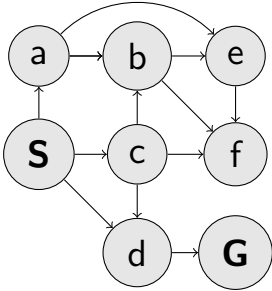


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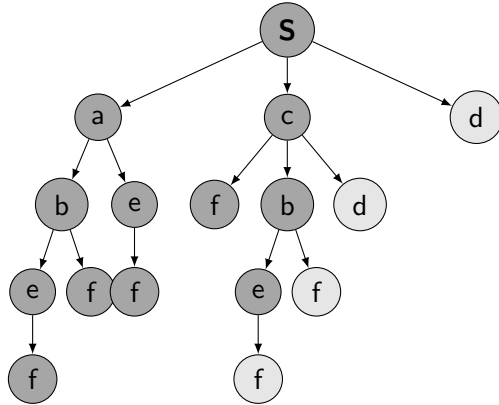
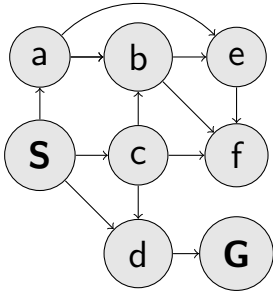


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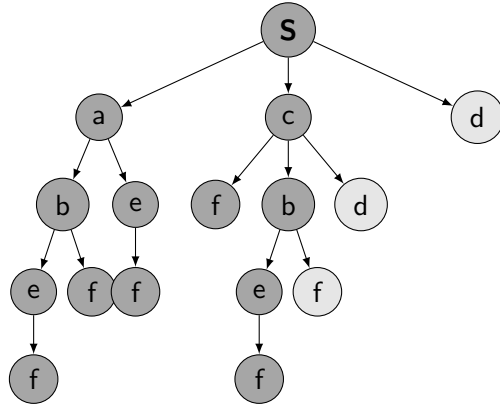
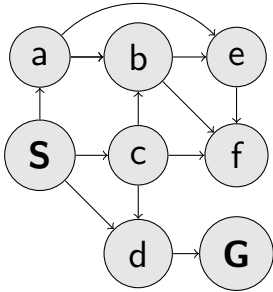
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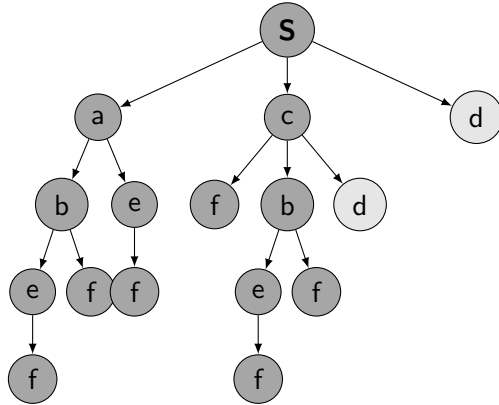
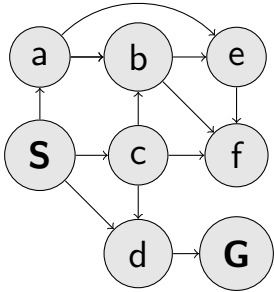


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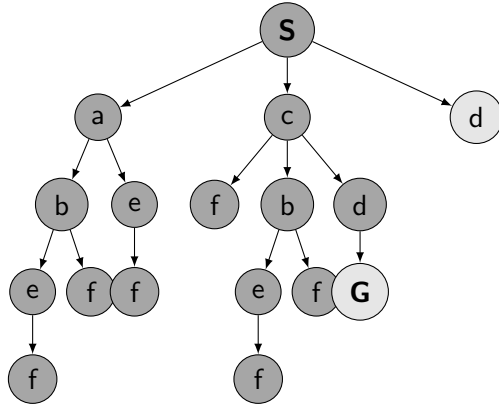
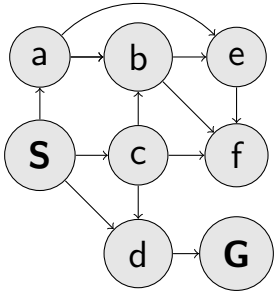


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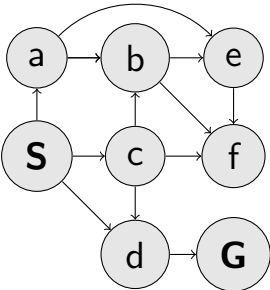


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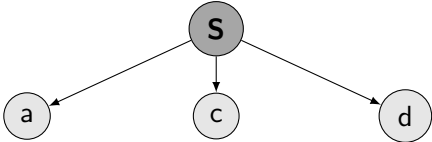
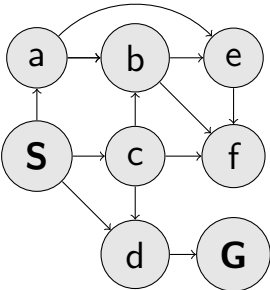
Breadth-First Search (BFS)



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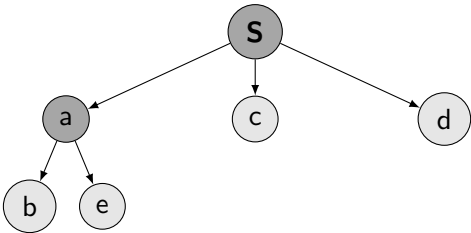
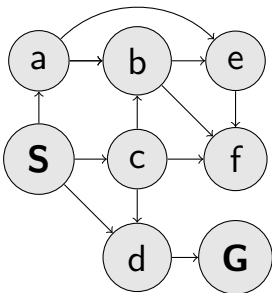
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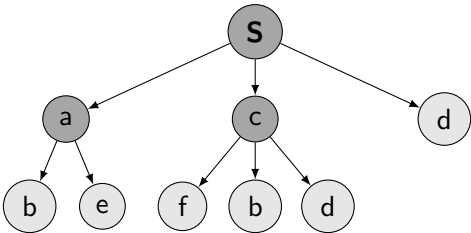
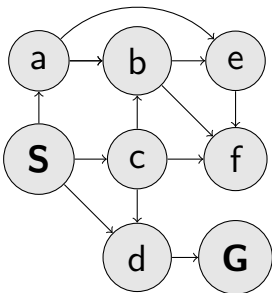
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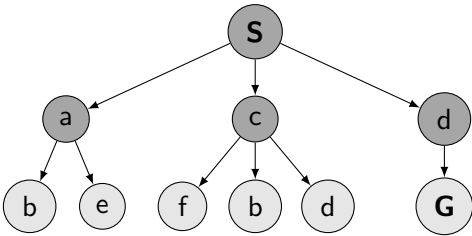
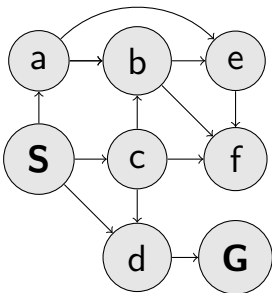
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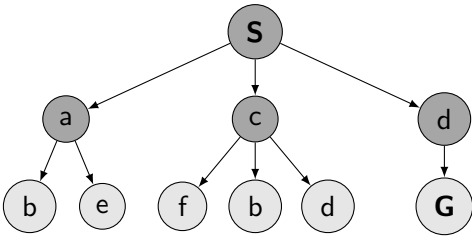
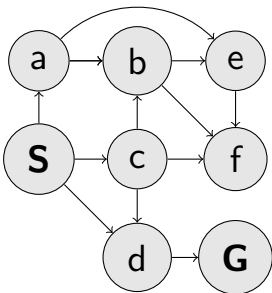
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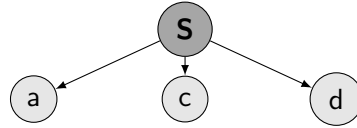
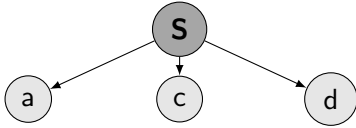
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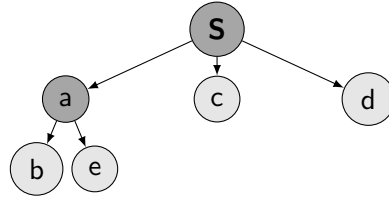
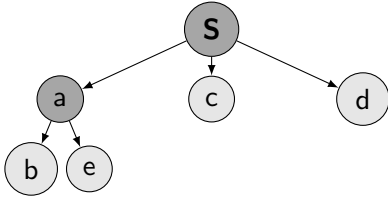
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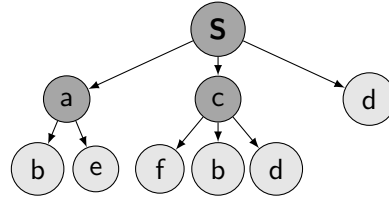
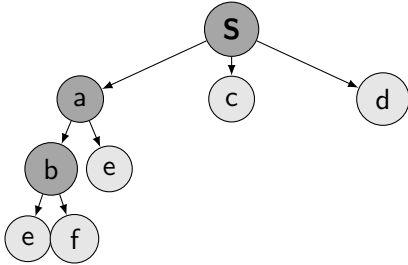
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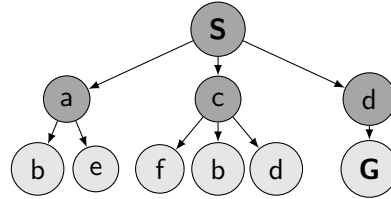
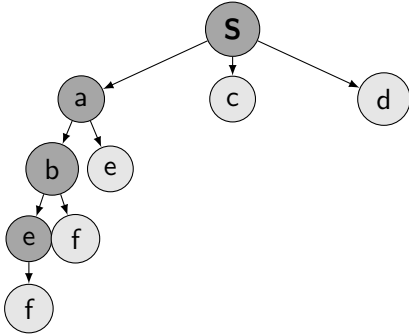
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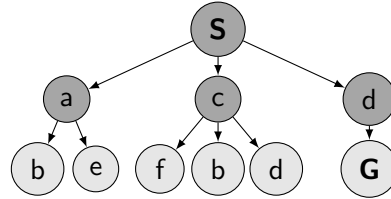
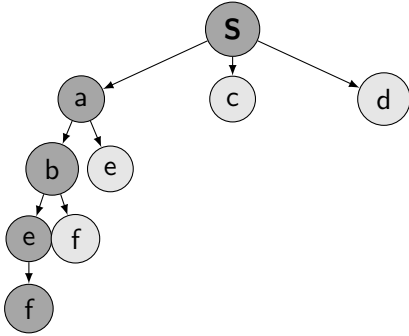
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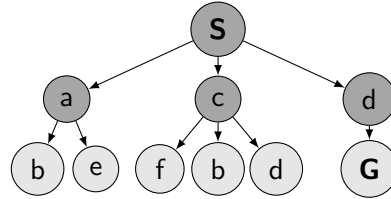
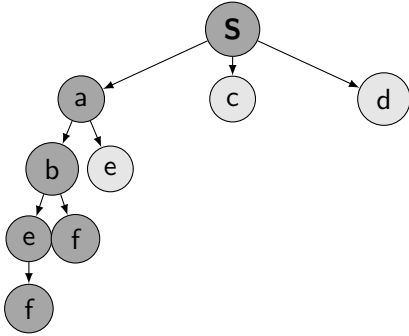
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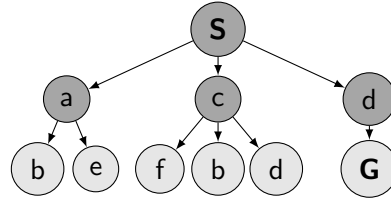
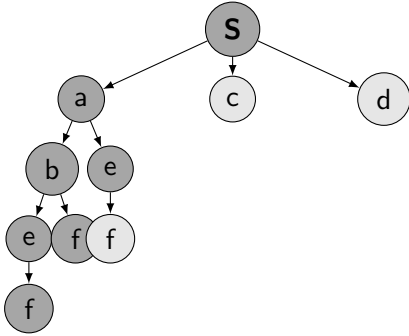
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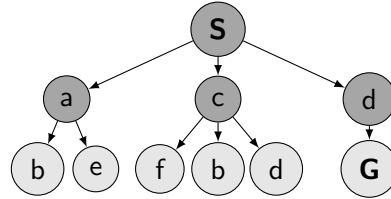
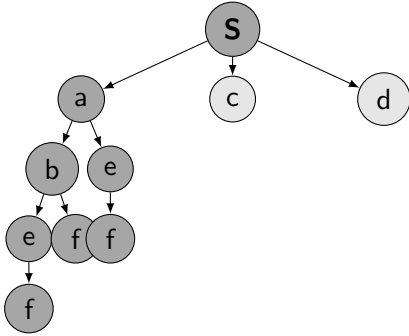
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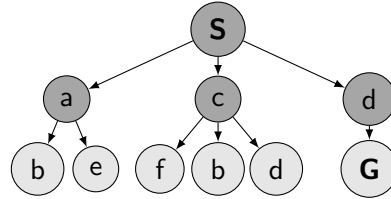
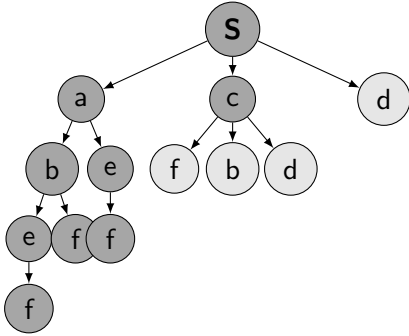
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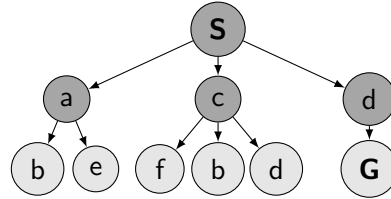
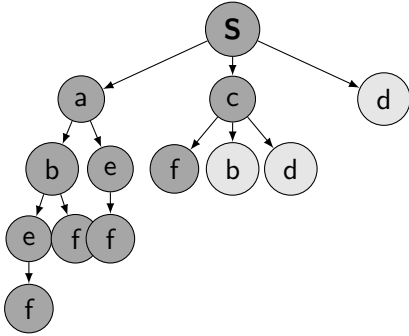
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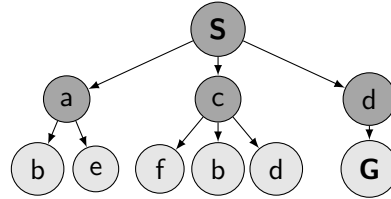
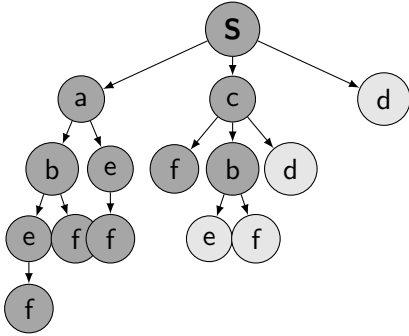
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DFS vs BFS

What are (dis)advantages of the individual strategies?



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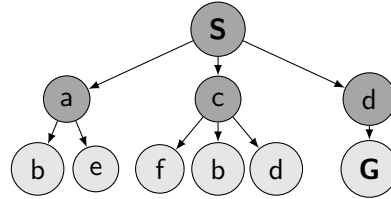
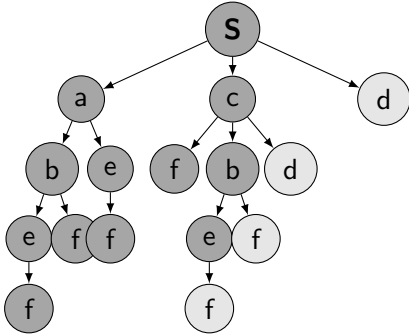
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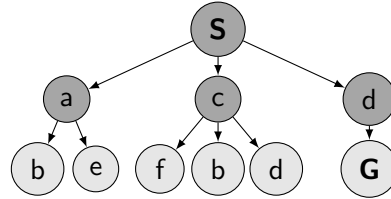
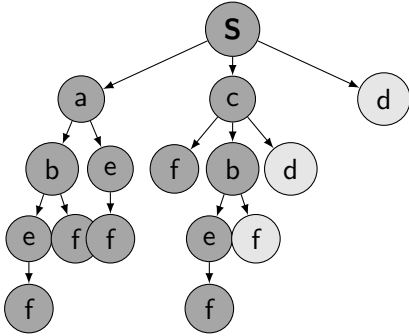
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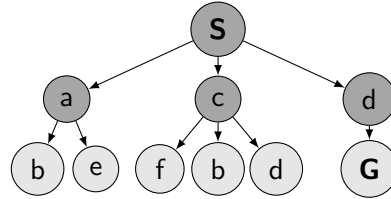
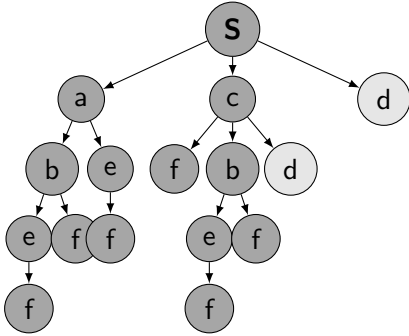
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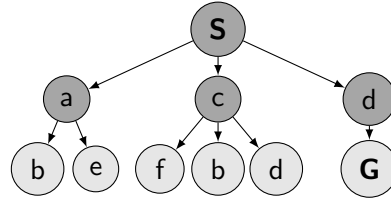
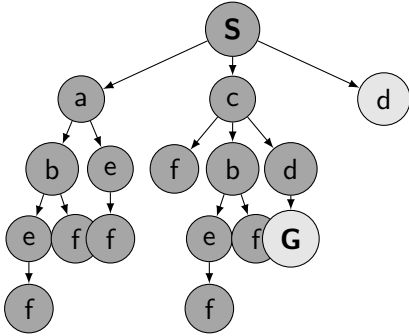
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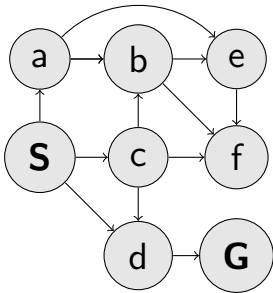
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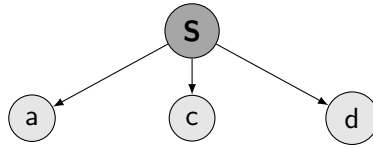
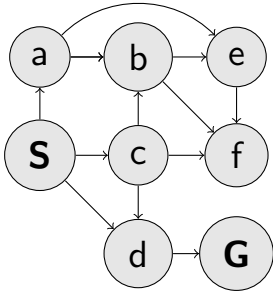


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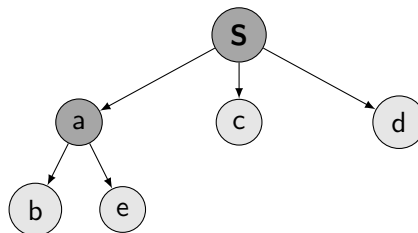
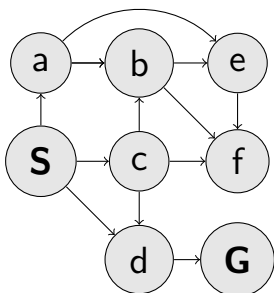


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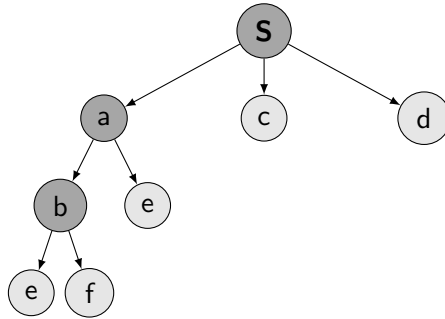
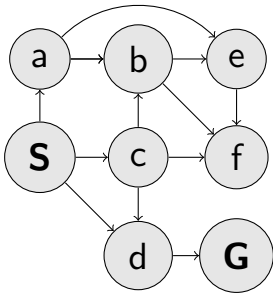


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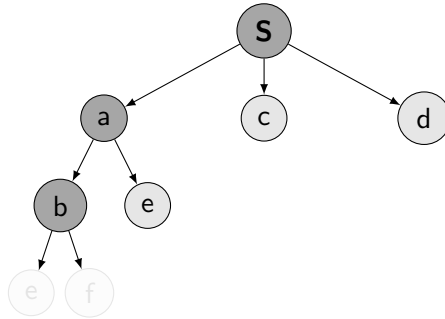
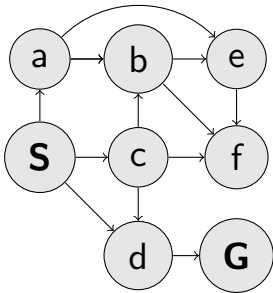


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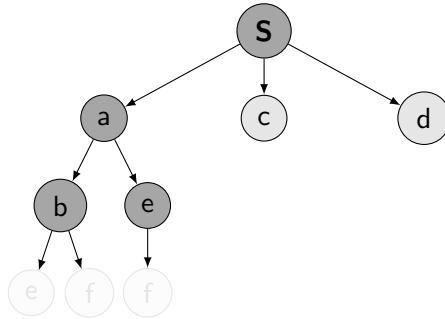
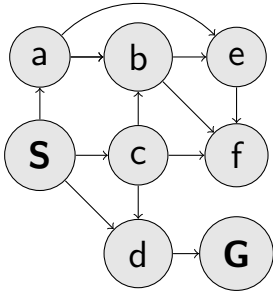


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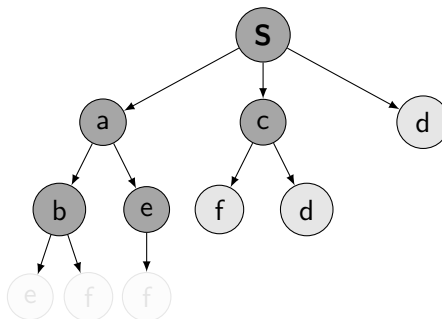
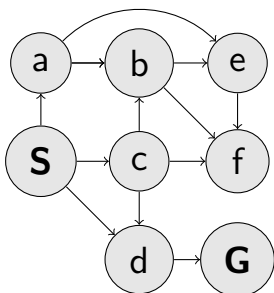


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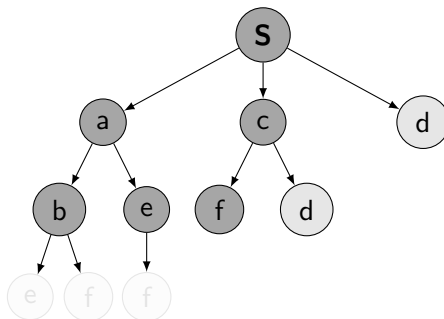
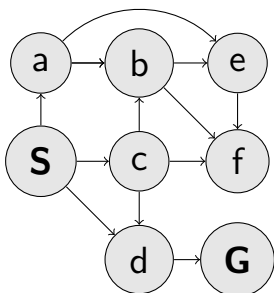


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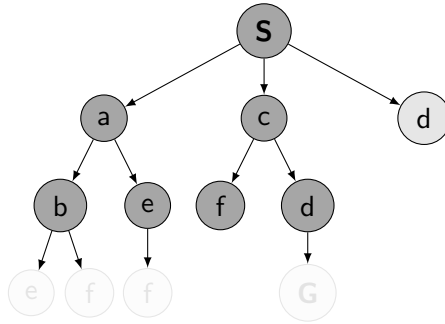
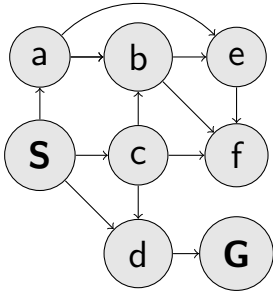


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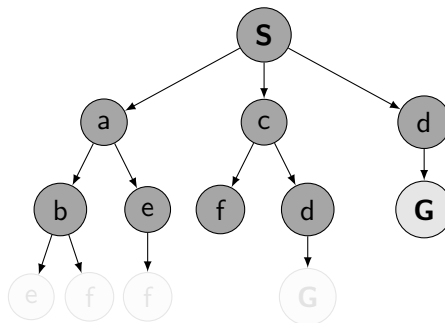
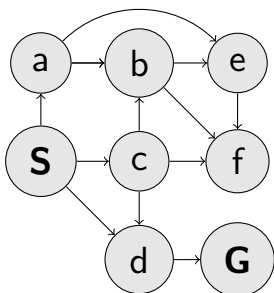


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Iterative deepening DFS (ID-DFS)

- ▶ Start with `maxdepth = 1`
 - ▶ Perform DFS with limited depth. Report success or failure.
 - ▶ If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

Notes

Really, how much do we repeat/waste? The “upper levels”, close to the root, are repeated many times. However, in a tree, most nodes are the bottom levels and nr. nodes traversed is what counts. More specifically, for a solution at depth s , the nodes on the bottom level are generated only once, those on the next-to-bottom level $2x$... children of the root are generated $s \times$. Compare the number of nodes generated ID-DFS vs. BFS:

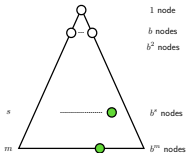
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Try some calculations for various s and b . For $b = 10$ and $d = 5$:

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(Example from [2].)

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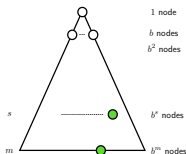
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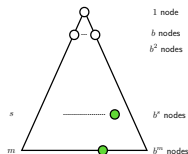
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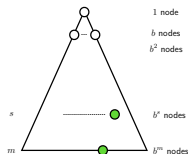
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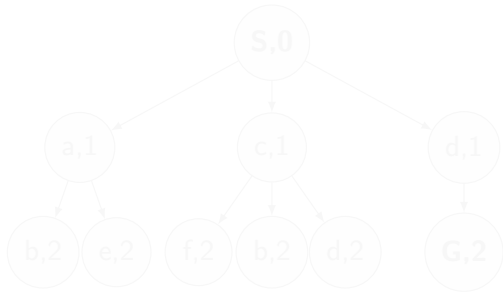
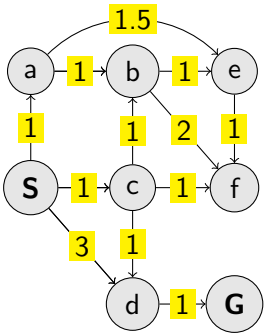
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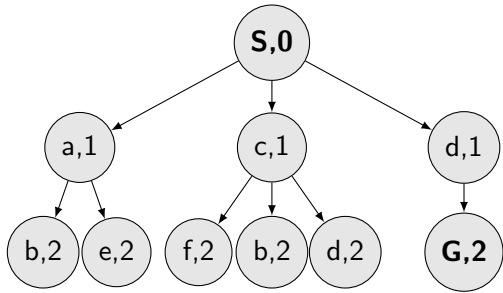
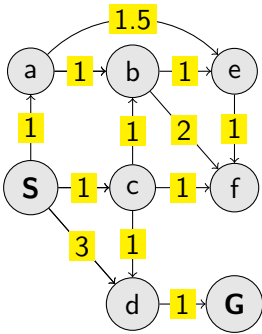
Cost sensitive search



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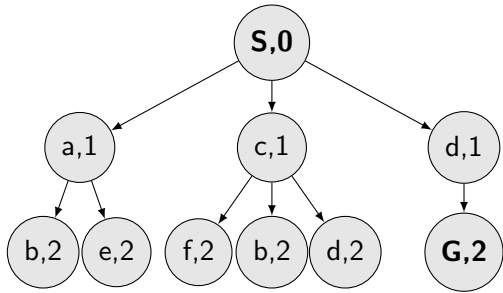
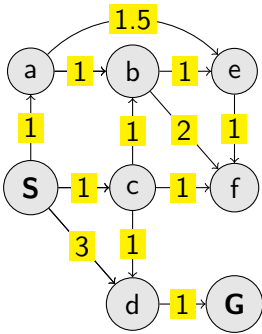
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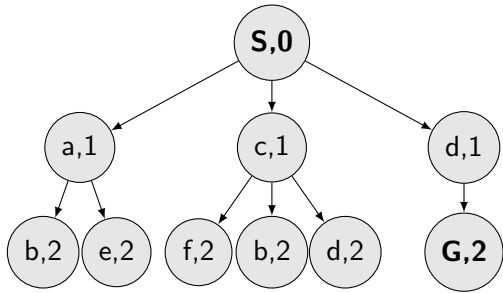
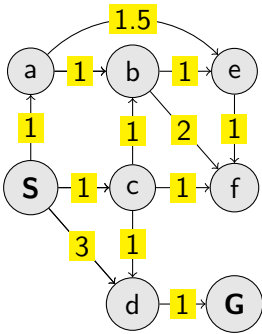
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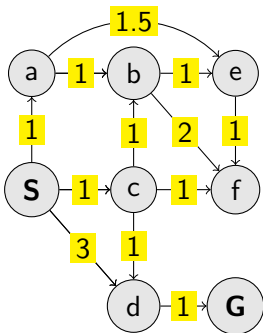
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Uniform Cost Search (UCS)



When to check the goal (and stop) the search? When visiting or expanding the node?

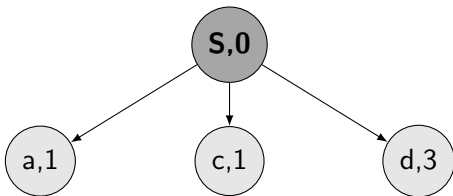
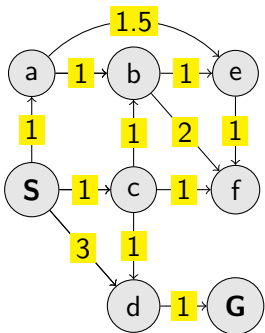
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Simple extension of BFS. Instead of expanding shallowest node, the node with smallest path cost so far is expanded.

Two differences:

- Goal test applied to a node when *selected for expansion* – not when first generated. (First goal generated may be on a suboptimal path.)
- Test is added in case a better path is found to a node currently on the frontier.

Uniform Cost Search (UCS)



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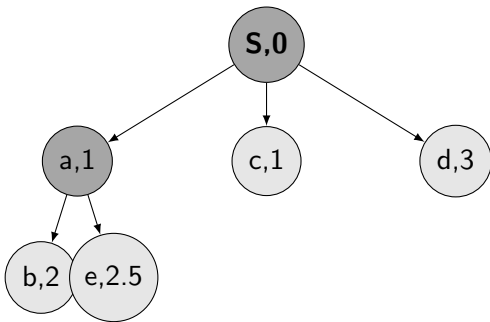
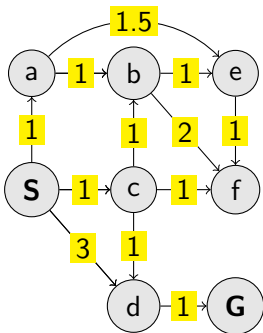
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Uniform Cost Search (UCS)



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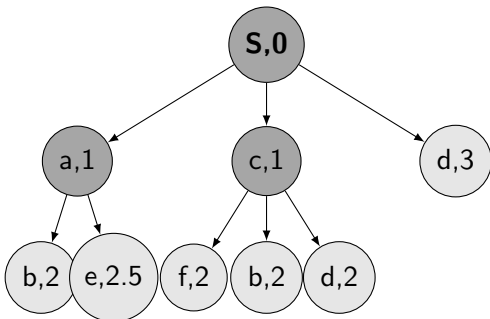
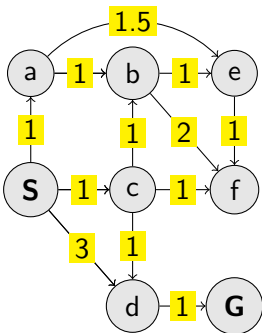
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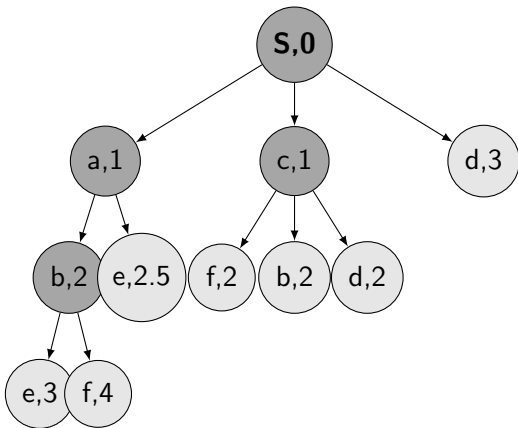
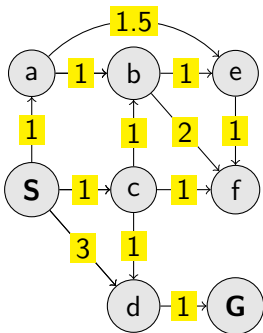
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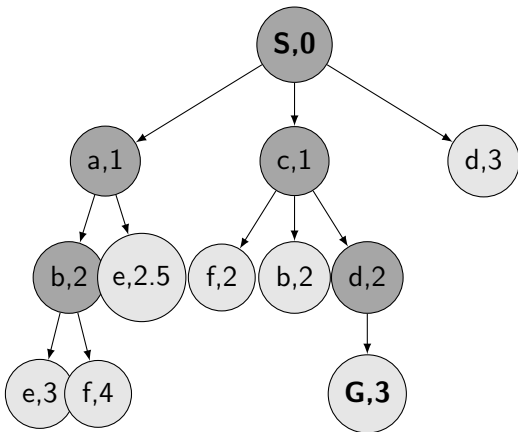
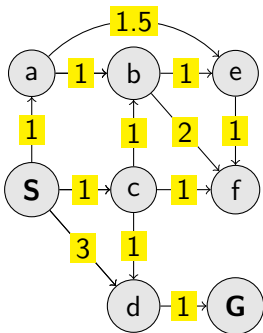
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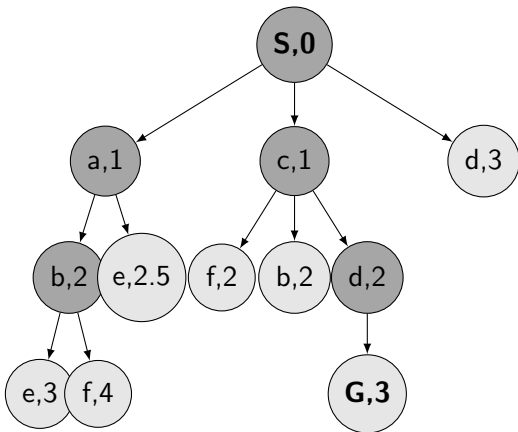
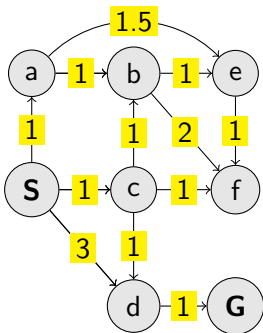
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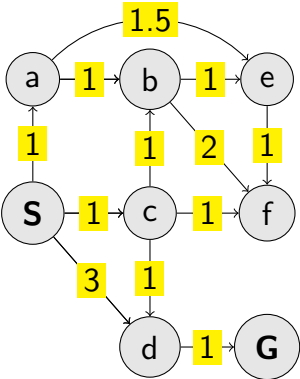
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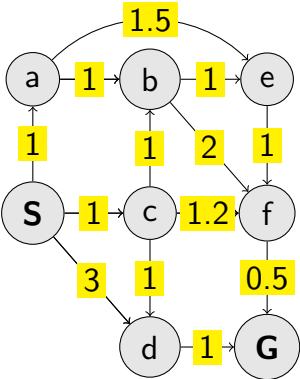
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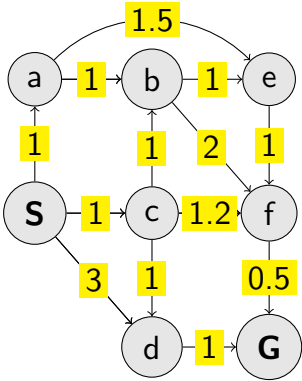


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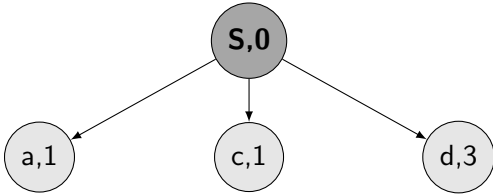
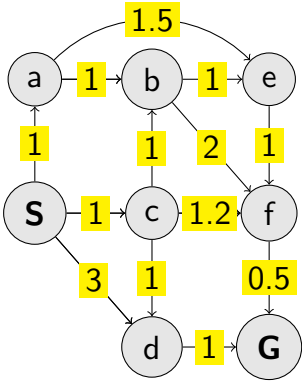
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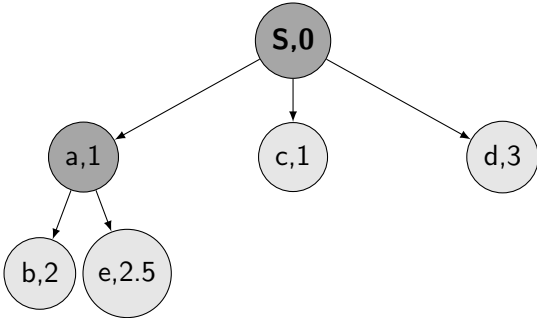
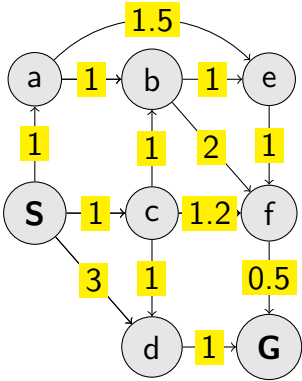
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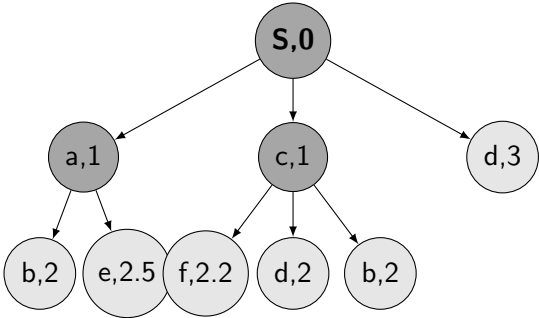
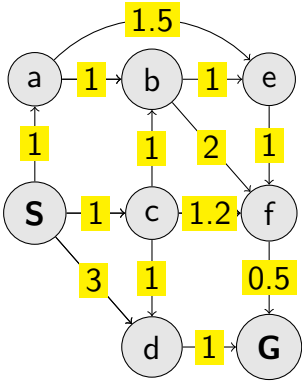
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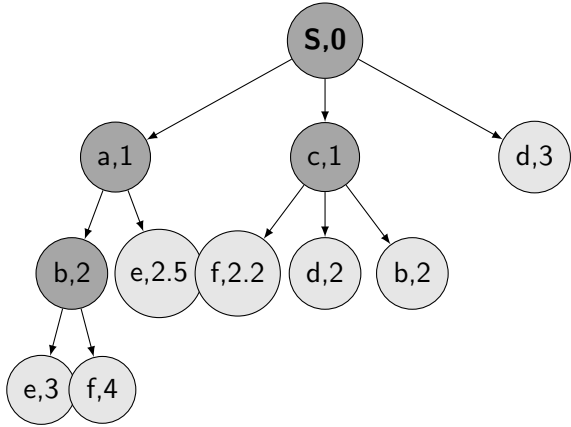
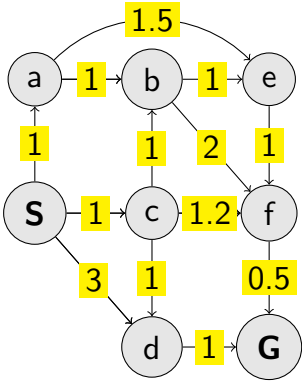
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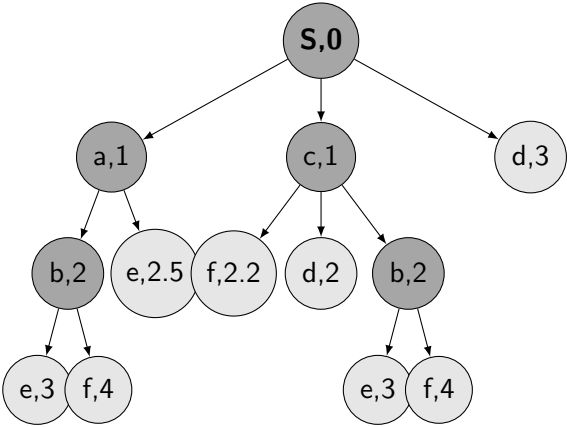
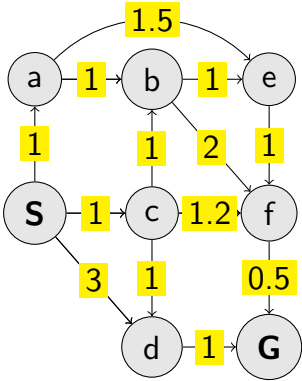
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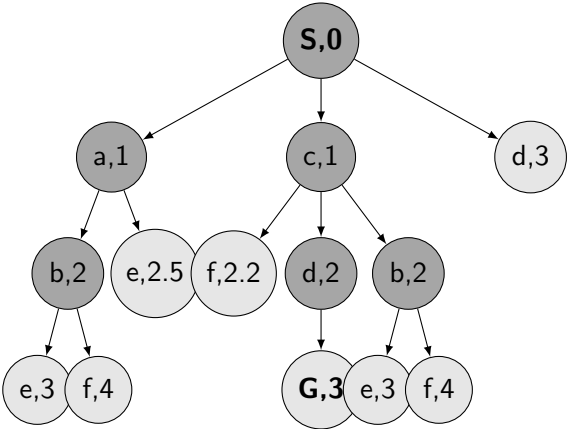
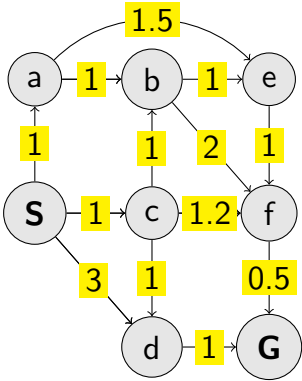
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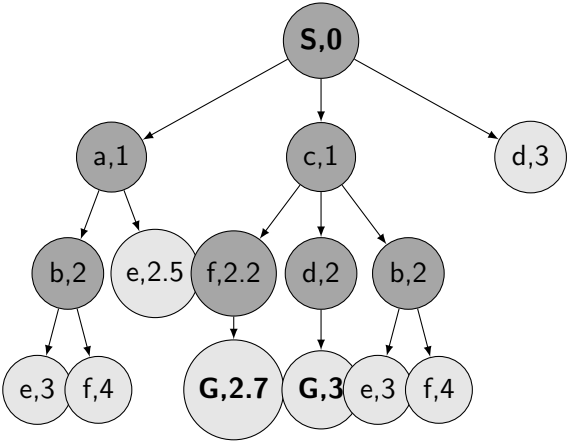
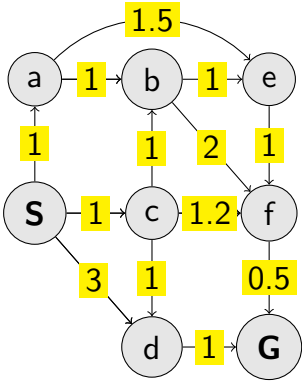
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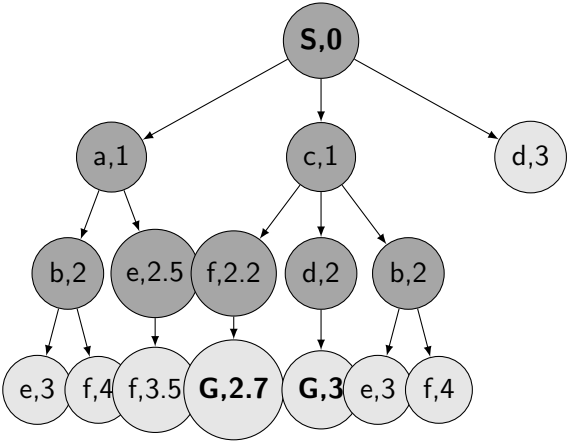
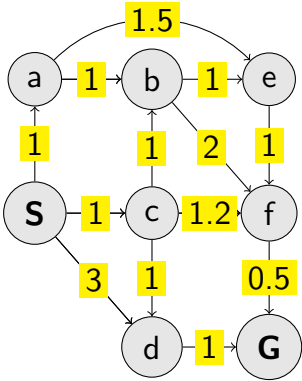
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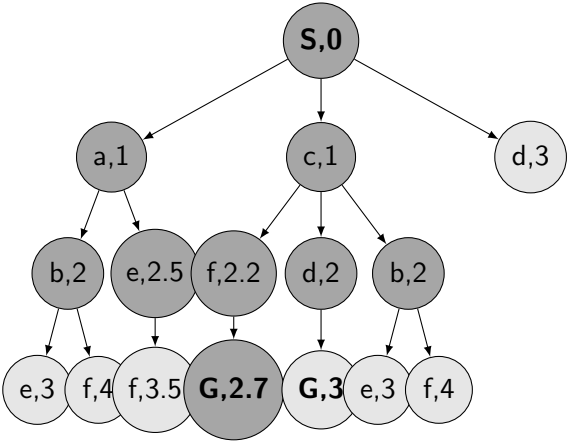
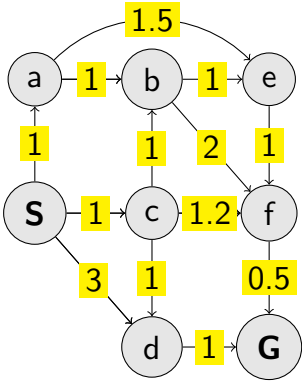
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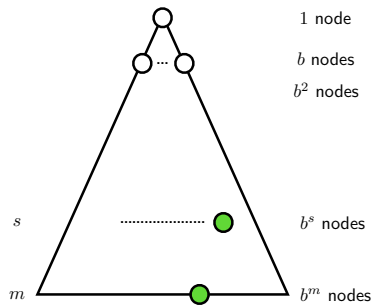
UCS properties

- ▶ Time complexity?
- ▶ Space complexity?
- ▶ Complete?
- ▶ Optimal?

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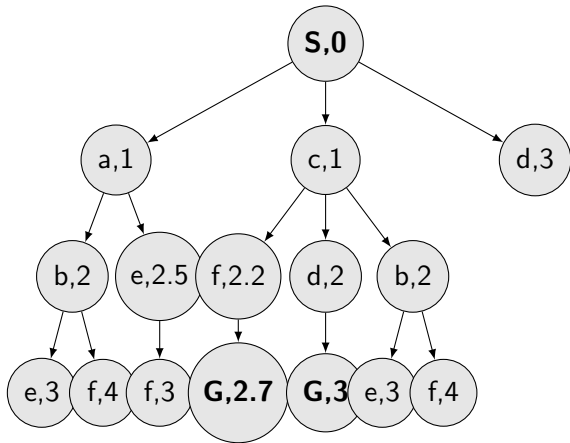
Solution cost C^* , transition cost at least ϵ . Effective depth, roughly C^*/ϵ .

- Time: $b^{C^*/\epsilon}$
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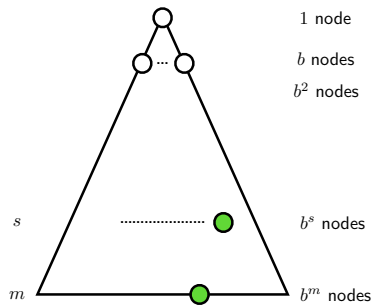
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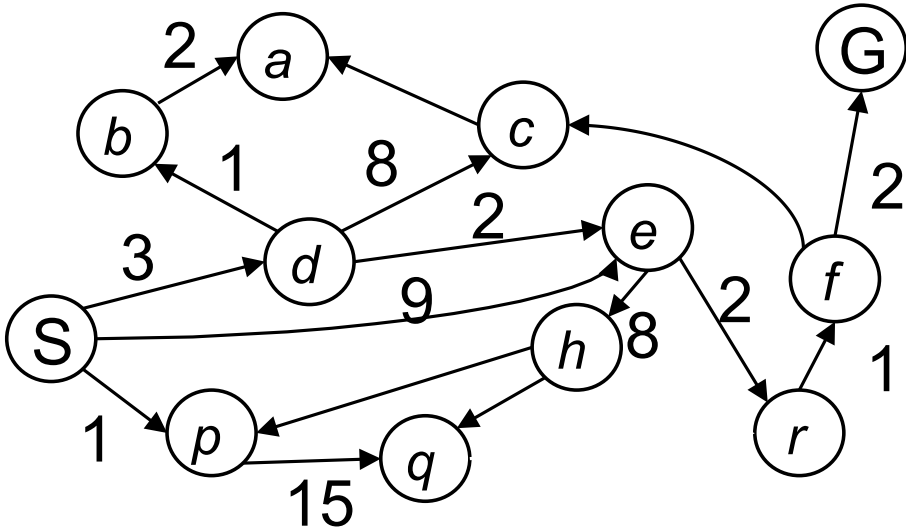
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Example: Graph with costs



Notes

Try it on paper, mark which nodes are in frontier, mark lines of equal cost.

Infrastructure for (tree) search algorithms

What should a **tree node** `n` know?

- ▶ `n.state`
- ▶ `n.parent`
- ▶ `n.pathcost`

Perhaps we may add something later, if needed ...

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How to organize nodes?

The Python examples are just suggestions, ...

- ▶ A dynamically linked structure (`list()`).
- ▶ Add a node (`list.insert(node)`).
- ▶ Take a node and remove from the structure (`node=list.pop()`).
- ▶ Check the Python modules `heapq`¹ and `queue`² for inspiration.

¹<https://docs.python.org/3.5/library/heapq.html>

²<https://docs.python.org/3.5/library/queue.html>

Notes

Very likely, you discussed `heapq` and `queue` in some programming, algorithms or data structures related courses.

What is the solution?

- ▶ We stop when **Goal** is reached.
- ▶ How do we construct the **path**?

References, further reading

Some figures if from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms.

[1] Steven M. LaValle.

Planning Algorithms.

Cambridge, 1st edition, 2006.

Online version available at: <http://planning.cs.uiuc.edu>.

[2] Stuart Russell and Peter Norvig.

Artificial Intelligence: A Modern Approach.

Prentice Hall, 3rd edition, 2010.

<http://aima.cs.berkeley.edu/>.