

# Classifiers 2

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Today two examples:

1. Covid-19 testing example
2. Strange loss function for classification

# Covid-19 testing example

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The image shows a screenshot of a news article from the website Seznam Zprávy. At the top left, there is a logo 'SZ' and the text 'Seznam Zprávy'. To the right is a search bar with the placeholder text 'Hledat...'. Below the search bar, the main header features a red and white coronavirus particle icon, the word 'Koronavirus', and a red box containing 'R 0,9' with a question mark. To the right of this, it says 'ČR: Testů 348,849' and 'Nakaže...'. A navigation menu below the header includes links for 'ONLINE', 'ČESKO', 'SVĚT', 'MAPA', 'ZÁCHRANA BYZNYSU', and 'NEWSLETTER'. The breadcrumb trail reads 'Zprávy » Koronavirus » Testy » Přesné testy odhalily, že v Česku už měl koronavirus každý dvacátý'. The main headline is 'Přesné testy odhalily, že v Česku už měl koronavirus každý dvacátý'.

Source: [Seznam zpravy](#)

For details see a post from [Jakub Steiner](#) on Facebook.

## Covid-19 testing example

Let's suppose that 0.5% of a population has already been infected by covid-19. Someone else bought covid-19 tests with specificity=0.9 (specificity =  $\frac{TN}{TN+FP}$ ) and wants to test 2000 people from the population. How many of the tests will be false positive?

- A: 10
- B: 99
- C: 199
- D: 399

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C:  $199 = (0.995 * 2000) * 0.1$

- ▶  $0.995 * 2000 = 1990$  people have not been infected
- ▶ As specificity is 0.9, ten percent of all negative samples (i.e.,  $TN+FP$ ) are determined as false positive.

# Strange loss function for classification

## Strange loss function for classification

Consider a Bayesian decision task in which the set of states is the same as the set of decisions (i.e.,  $S=D$ ) and the loss function is defined as:

$$l(s, d) = K, \quad d = s,$$

$$l(s, d) = 1, \quad d \neq s.$$

What values must be  $K$  to use  $\delta^*(x) = \arg \max_d p(d|x)$  to find the optimal decision?

Let's solve it together. Step by step :)

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What values must be  $K$  to use  $\delta^*(x) = \arg \max_d p(d|x)$  to find the optimal decision?

How to start?

A:  $r(\delta) = \sum_x \sum_s l(s, x)P(x, s)$

B:  $r(\delta) = \sum_x \sum_s l(s, \delta(x))P(x, s)$

C:  $r(\delta) = \sum_x \sum_s l(s, \delta(x))P(\delta(x), s)$

D:  $r(\delta) = \sum_s l(s, \delta(x))P(\delta(x), \delta(s))$



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$$\text{B: } r(\delta) = \sum_x \sum_s l(s, \delta(x)) P(x, s)$$

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$$r(\delta) = \sum_x \sum_s l(s, \delta(x)) P(x, s) = ?$$

A:  $\sum_x P(x) \sum_s l(s, \delta(x)) P(s|x)$

B:  $P(x) \sum_s l(s, \delta(x)) P(s|x)$

C:  $\sum_x P(x) \sum_s l(s, \delta(x)) P(x|s)$

D:  $\sum_x P(x) \sum_s l(s, \delta(x)) P(s, x)$

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$$\text{A: } \sum_x P(x) \sum_s l(s, \delta(x)) P(s|x)$$

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$$r(\delta) = \sum_x \sum_s l(s, \delta(x)) P(x, s) = \sum_x P(x) \sum_s l(s, \delta(x)) P(s|x)$$

Optimal strategy for given  $x$ :

A:  $\delta^*(x) = \arg \min_d \sum_s l(s, d) P(x|s)$

B:  $\delta^*(x) = \arg \max_d \sum_s l(s, d) P(s|x)$

C:  $\delta^*(x) = \arg \min_d \sum_s l(s, d) P(s|x)$

D:  $\delta^*(x) = \arg \max_d \sum_s l(s, d) P(d|x)$

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$$r(\delta) = \sum_x \sum_s l(s, \delta(x)) P(x, s) = \sum_x P(x) \sum_s l(s, \delta(x)) P(s|x)$$

Optimal strategy for given  $x$ :

$$C: \delta^*(x) = \arg \min_d \sum_s l(s, d) P(s|x)$$

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$$\delta^*(x) = \arg \min_d \sum_s l(s, d) P(s|x) = ?$$

A:  $\arg \min_d (P(s = d|x)K + \sum_{s \neq d} P(s|x))$

B:  $\arg \max_d (P(s = d|x) + \sum_{s \neq d} P(s|x)K)$

C:  $\arg \min_d (P(s = d|x) + \sum_{s \neq d} P(s|x)K)$

D:  $\arg \min_d (P(s \neq d|x)K + \sum_{s \neq d} P(s|x))$

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$$\delta^*(x) = \arg \min_d \sum_s l(s, d)P(s|x) = ?$$

$$\text{A: } \arg \min_d (P(s = d|x)K + \sum_{s \neq d} P(s|x))$$

See:

$$\begin{aligned} \sum_s l(s, d)P(s|x) &= l(s_1, d)P(s_1|x) + l(s_2, d)P(s_2|x) + \dots \\ &+ l(s_k = d, d)P(s_k = d|x) + \dots + l(s_n, d)P(s_n|x) = 1P(s_1|x) + 1P(s_2|x) + \dots + KP(s_k = d|x) + \\ &\dots + 1P(s_n|x) = P(s = d|x)K + \sum_{s \neq d} P(s|x) \end{aligned}$$

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A:  $\arg \min_d (P(s = d|x)K + (P(s = d|x) - 1))$

B:  $\arg \max_d (P(s = d|x) + \sum_{s \neq d} P(s|x)K)$

C:  $\arg \max_d (P(s = d|x)K + (1 - P(s = d|x)))$

D:  $\arg \min_d (P(s = d|x)K + (1 - P(s = d|x)))$



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$$\delta^*(x) = \arg \min_d \left( \sum_s l(s, d) P(s|x) \right) = \arg \min_d \left( P(s = d|x)K + \sum_{s \neq d} P(s|x) \right) = ?$$

$$D: \arg \min_d \left( P(s = d|x)K + (1 - P(s = d|x)) \right)$$

Notice that:

$$\sum_{s \neq d} P(s|x) + P(s = d|x) = 1$$

$$\sum_{s \neq d} P(s|x) = 1 - P(s = d|x)$$

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$$\begin{aligned} \delta^*(x) &= \arg \min_d \sum_s l(s, d)P(s|x) = \arg \min_d P(s = d|x)K + \sum_{s \neq d} P(s|x) = \\ &= \arg \min_d (P(s = d|x)K + (1 - P(s = d|x))) =? \end{aligned}$$

A:  $\arg \max_d (P(s = d|x)K + (1 - P(s = d|x)))$

B:  $\arg \min_d (P(s = d|x)(K - 1))$

C:  $\arg \max_d (P(s = d|x)K)$

D:  $\arg \min_d (P(s = d|x)(K + 1))$

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$$\text{B: } \arg \min_d (P(s = d|x)(K - 1))$$

See:

$$\begin{aligned} \arg \min_d P(s = d|x)K + (1 - P(s = d|x)) &= \arg \min_d P(s = d|x)(K - 1) + 1 = \\ &= \arg \min_d P(s = d|x)(K - 1) \end{aligned}$$

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$$\dots = \arg \min_d (P(s = d|x)K + (1 - P(s = d|x))) = \arg \min_d (P(s = d|x)(K - 1))$$

What values must be  $K$  to  $\arg \min_d (P(s = d|x)(K - 1)) = \arg \max_d P(s = d|x)$ ? Select the most general option.

A:  $K \leq 0$

B:  $K < 1$

C:  $K < 2$

D:  $K \leq 2$

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What values must be  $K$  to  $\arg \min_d (P(s = d|x)(K - 1)) = \arg \max_d P(s = d|x)$ ? Select the most general option.

**B:**  $K < 1$

For  $K < 1$  value of  $(K - 1)$  is negative. Therefore, minimization  $\min_d (P(s = d|x)(K - 1))$  is equivalent to maximization  $\max_d P(s = d|x)$ .

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$$= \arg \min_d (P(s = d|x)K + (1 - P(s = d|x))) = \arg \min_d (P(s = d|x)(K - 1)) =$$

For  $K < 1$ :

$$= \arg \max_d P(s = d|x) = \arg \max_d P(d|x)$$

:)