## Bayesian decision making

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Today two examples:

1. Bayesian decision making basics
2. Prior probabilities in practice

## Bayesian decision making basics

## Bayesian decision making basics

What is correct?
A: $P\left(X=x_{i}\right)=\sum_{j} \frac{P\left(X=x_{i}, Y=y_{j}\right)}{P\left(Y=y_{j}\right)}$
B: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
C: $P\left(X=x_{i}\right)=\sum_{i} P\left(X=x_{i}, Y=y_{j}\right)$
D: $P\left(Y=y_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$

## Bayesian decision making basics

What is correct?

A:

B: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
C:
D:

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$

What is correct?
A: $P\left(X=x_{i} \mid Y=y_{j}\right)=P\left(Y=y_{j}, X=x_{i}\right) P\left(X=x_{i}\right)$
B: $P\left(X=x_{i}, Y=y_{j}\right)=\frac{P\left(Y=y_{j} \mid X=x_{i}\right)}{P\left(X=x_{i}\right)}$
C: $P\left(X=x_{i}, Y=y_{j}\right)=P\left(Y=y_{j} \mid X=x_{i}\right) P\left(Y=y_{i}\right)$
D: $P\left(X=x_{i}, Y=y_{j}\right)=P\left(Y=y_{j} \mid X=x_{i}\right) P\left(X=x_{i}\right)$

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$

What is correct?
A:
B:
C:
D: $P\left(X=x_{i}, Y=y_{j}\right)=P\left(Y=y_{j} \mid X=x_{i}\right) P\left(X=x_{i}\right)$

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
- Product rule of probability: $P\left(X=x_{i}, Y=y_{j}\right)=P\left(Y=y_{j} \mid X=x_{i}\right) P\left(X=x_{i}\right)$

What is correct?
A: $P\left(Y=y_{j} \mid X=x_{i}\right)=\frac{P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)}{P\left(X=x_{i}\right)}$
B: $P\left(Y=y_{i}, X=x_{j}\right)=\frac{P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)}{P\left(X=x_{j}\right)}$
C: $P\left(Y=y_{i} \mid X=x_{j}\right)=P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)$
D: $P\left(Y=y_{i} \mid X=x_{j}\right)=\frac{P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)}{\sum_{i} P\left(X=x_{i}, Y=y_{i}\right)}$

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
- Product rule of probability: $P\left(X=x_{i}, Y=y_{j}\right)=P\left(Y=y_{j} \mid X=x_{i}\right) P\left(X=x_{i}\right)$

What is correct?
A: $P\left(Y=y_{j} \mid X=x_{i}\right)=\frac{P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)}{P\left(X=x_{i}\right)}$
B:
C:
D:

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
- Product rule of probability: $P\left(X=x_{i}, Y=y_{j}\right)=P\left(Y=y_{j} \mid X=x_{i}\right) P\left(X=x_{i}\right)$
- Bayes' theorem: $P\left(Y=y_{j} \mid X=x_{i}\right)=\frac{P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)}{P\left(X=x_{i}\right)}$

What is correct?
A: $\delta^{*}=\arg \max _{\delta} \sum_{x} \sum_{s} l(s, \delta(x)) P(x, s)$
$\mathrm{B}: \delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, \delta(x)) P(x \mid s)$
C: $\delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} l(s, \delta(x)) P(x, s)$
D: $\delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, x) P(x, s)$

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
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What is correct?
A:
B:
C: $\delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, \delta(x)) P(x, s)$
D:

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
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- Bayes optimal strategy: $\delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, \delta(x)) P(x, s)$

What is correct?
$\mathrm{A}: \delta^{*}(x)=\arg \min _{d} \sum_{s} I(s, d) P(s \mid x)$
B: $\delta^{*}(x)=\arg \min _{d} \sum_{s} I(s, d) P(s, x)$
C: $\delta^{*}(x)=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, \delta(x)) P(x, s)$
D: $\delta^{*}(x)=\arg \min _{s} \sum_{d} l(s, d) P(s \mid x)$

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- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
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- Bayes optimal strategy: $\delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, \delta(x)) P(x, s)$

What is correct?
$\mathrm{A}: \delta^{*}(x)=\arg \min _{d} \sum_{s} I(s, d) P(s \mid x)$
B:
C:
D:
$\arg \min _{s} \sum_{d} I(s, d) P(s \mid x)$

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
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- Bayes' theorem: $P\left(Y=y_{j} \mid X=x_{i}\right)=\frac{P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)}{P\left(X=x_{i}\right)}$
- Bayes optimal strategy: $\delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, \delta(x)) P(x, s)$
- BOS solution: $\delta^{*}(x)=\arg \min _{d} \sum_{s} I(s, d) P(s \mid x)$

Assume $I(s, d)=1$, if $d \neq s, I(s, d)=0$ otherwise. What is correct?
A: $\delta^{*}(x)=\arg \min _{d} P(d \mid x)$
B: $\delta^{*}(x)=\arg \max _{d} P(d \mid x)$
C: $\delta^{*}(x)=\arg \max _{d} P(d \mid x) P(x)$
$D: \delta^{*}(x)=\arg \max _{d} P(d \mid x) P(s)$

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
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- Bayes' theorem: $P\left(Y=y_{j} \mid X=x_{i}\right)=\frac{P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)}{P\left(X=x_{i}\right)}$
- Bayes optimal strategy: $\delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, \delta(x)) P(x, s)$
- BOS solution: $\delta^{*}(x)=\arg \min _{d} \sum_{s} I(s, d) P(s \mid x)$

Assume $I(s, d)=1$, if $d \neq s, I(s, d)=0$ otherwise. What is correct?
A:
B: $\delta^{*}(x)=\arg \max _{d} P(d \mid x)$
C:
D:
$\arg \max _{d} P(d \mid x) P(s)$

## Bayesian decision making basics

- Sum rule of probability: $P\left(X=x_{i}\right)=\sum_{j} P\left(X=x_{i}, Y=y_{j}\right)$
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- Bayes' theorem: $P\left(Y=y_{j} \mid X=x_{i}\right)=\frac{P\left(X=x_{i} \mid Y=y_{j}\right) P\left(Y=y_{j}\right)}{P\left(X=x_{i}\right)}$
- Bayes optimal strategy: $\delta^{*}=\arg \min _{\delta} \sum_{x} \sum_{s} I(s, \delta(x)) P(x, s)$
- BOS solution: $\delta^{*}(x)=\arg \min _{d} \sum_{s} I(s, d) P(s \mid x)$
- $L_{0,1}$ classification: $\delta^{*}(x)=\arg \max _{d} P(d \mid x)$

Prior probabilities in practice

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| ${ }_{\text {cm }}^{\text {cm }}$ |  | $\underset{(100-125)}{S}$ | $\underset{(125-150)}{M}$ | $(150-175)$ | $\begin{gathered} \begin{array}{c} \mathrm{XL} \\ (175-200) \end{array} \\ \hline \end{gathered}$ | $\begin{gathered} \text { XXL } \\ (200-\infty) \end{gathered}$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ (x\|male) | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P$ (x\|female) | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female.

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| ${ }_{\text {cm }}^{\text {cm }}$ |  | $\underset{(100-125)}{S}$ | $\underset{(125-150)}{M}$ | $(150-175)$ | $\begin{gathered} \begin{array}{c} \mathrm{XL} \\ (175-200) \end{array} \\ \hline \end{gathered}$ | $\begin{gathered} \text { XXL } \\ (200-\infty) \end{gathered}$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ (x\|male) | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P$ (x\|female) | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female.

A: Male
B: Female

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $\stackrel{\times}{\text { cm }}$ | $\xrightarrow[(0-100)]{\text { xS }}$ | ${ }_{(100-125)}^{\text {S }}$ | $\underset{(125-150)}{\text { M }}$ | ${ }_{(150-175)}^{\text {L }}$ | ${ }_{(175-200)}^{\text {XL }}$ | $\xrightarrow[\substack{\text { XXL } \\(200-\infty)}]{ }$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P$ (x\|male) | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P$ (x\|female) | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female.

A: Male
B: Female (if we assume that there are same the number of men and women.)

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $\times$ <br> cm | xS <br> $(0-100)$ | ${ }_{(100-125)}^{\mathrm{S}}$ | $(125-150)$ | ${ }_{(150-175)}^{\mathrm{L}}$ | $\underset{(175-200)}{\mathrm{XL}}$ | XXL <br> $(200-\infty)$ | $\sum$ |
| :---: | :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | $\mathbf{1}$ |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | $\mathbf{1}$ |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female. Female Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one.

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $x$ <br> cm | xS <br> $(0-100)$ | S <br> $(100-125)$ | $(125-150)$ | ${ }_{(150-175)}^{\mathrm{L}}$ | $\underset{(175-200)}{\mathrm{XL}}$ | XXL <br> $(200-\infty)$ | $\sum$ |
| :---: | :---: | :---: | :--- | :--- | :--- | :--- | :--- |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | $\mathbf{1}$ |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | $\mathbf{1}$ |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female. Female Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one.

Right step?
A: $P(X=$ male, $Y=L)=P(X=$ female, $Y=L)$
B: $P(X=$ male $\mid Y=L)=P(X=$ female $\mid Y=L)$
C: $P(X=$ male $\mid Y>L)=P(X=$ female $\mid Y<L)$
D: $P(X=$ male $\mid Y>L)>P(X=$ female $\mid Y<L)$

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $\times$ <br> cm | xS <br> $(0-100)$ | ${ }_{(100-125)}^{\mathrm{S}}$ | $(125-150)$ | ${ }_{(150-175)}^{\mathrm{L}}$ | $\underset{(175-200)}{\mathrm{XL}}$ | XXL <br> $(200-\infty)$ | $\sum$ |
| :---: | :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | $\mathbf{1}$ |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | $\mathbf{1}$ |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female. Female Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one.

Right step?
A:
B: $P(X=$ male $\mid Y=L)=P(X=$ female $\mid Y=L)$
C:
D:
$P(X$
male $Y$
$Y$
L)
$P(X=$
fermale $Y<L$

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $x$ <br> cm | XS <br> $(0-100)$ | S <br> $(100-125)$ | $(125-150)$ | ${ }_{(150-175)}^{\mathrm{L}}$ | $\underset{(175-200)}{\mathrm{XL}}$ | XXL <br> $(200-\infty)$ | $\sum$ |
| :---: | :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | $\mathbf{1}$ |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | $\mathbf{1}$ |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female. Female Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one. $P(X=$ male $\mid Y=L)=P(X=$ female $\mid Y=L)$

From the equation get value of?
A: $P(X=$ male $)$
B: $P(X=$ male $\mid Y=L)$
C: $P(X=$ female $\mid Y<L)$
D: $P(X=$ male $\mid Y>L)$

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $x$ <br> cm | XS <br> $(0-100)$ | S <br> $(100-125)$ | $(125-150)$ | ${ }_{(150-175)}^{\mathrm{L}}$ | $\underset{(175-200)}{\mathrm{XL}}$ | XXL <br> $(200-\infty)$ | $\sum$ |
| :---: | :---: | :---: | :---: | :--- | :--- | :--- | :--- |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | $\mathbf{1}$ |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | $\mathbf{1}$ |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female. Female
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one. $P(X=$ male $\mid Y=L)=P(X=$ female $\mid Y=L)$

From the equation get value of?
A: $P(X=$ male $)$
B:
C: $P(X=$ female $Y<L)$
D:
male $\mid Y>L$ )

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $\stackrel{\text { cm }}{\text { cm }}$ | $\begin{gathered} \text { XS } \\ (0-100) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{S} \\ (100-125) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{M} \\ (125-150) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{L} \\ (150-175) \\ \hline \end{gathered}$ | $\begin{gathered} X L \\ (175-200) \\ \hline \end{gathered}$ | $\begin{gathered} \text { XXL } \\ (200-\infty) \\ \hline \end{gathered}$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female. Female
Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one. $P(X=$ male $\mid Y=L)=P(X=$ female $\mid Y=L)$

From the equation get value of? $P(X=$ male $)$ Calculate $P(X=$ male $)$ :
A: $\frac{5}{11}$
B: $\frac{6}{11}$
C: $\frac{6}{10}$
D: $\frac{7}{12}$

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $x$ <br> cm | XS <br> $(0-100)$ | S <br> $(100-125)$ | M <br> $(125-150)$ | L <br> $(150-175)$ | XL <br> $(175-200)$ | XXL <br> $(200-\infty)$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 |

Task 1: Estimate whether a 168 cm tall (i.e. L) person is male or female. Female Task 2: What would be the minimum ratio of men to change the previous answer into the opposite one. $P(X=$ male $\mid Y=L)=P(X=$ female $\mid Y=L)$

From the equation get value of? $P(X=$ male $)$
Calculate $P(X=$ male $)$ :
B: $\frac{6}{11}$
$P(X=$ male $\mid Y=L)=P(X=$ female $\mid Y=L)$
$\frac{P(L \mid \text { male }) \cdot P(\text { male })}{P(L)}=\frac{P(L \mid \text { female }) \cdot P(\text { female })}{P(L)}, P($ female $)=1-P($ male $)$
$P(L \mid$ male $) \cdot P($ male $)=P(L \mid$ female $) \cdot(1-P($ male $))$
$0.25 \cdot P($ male $)=0.3-0.3 \cdot P($ male $) \Rightarrow P($ male $)=\frac{6}{11}$

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $\stackrel{\times}{\text { cm }}$ | ¢ ${ }_{\text {x }}$ | $\underset{(100-125)}{\text { S }}$ | ${ }_{(125-150)}^{\text {M }}$ | (150-175) | ${ }_{(175-200)}$ | $\underset{\substack{\text { XXL } \\(200-\infty)}}{ }$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 3: Assuming there are $70 \%$ men and $30 \%$ women, consider the loss function / (s - state, d decision $): I(s=$ female,$d=$ male $)=2, I(s=$ male,$d=$ female $)=1$,
$I(s=$ male,$d=$ male $)=I(s=$ female, $d=$ female $)=0$.
How do you classify a person under consideration of L?
How?
A: $\delta^{*}(X=L)=\operatorname{argmin}_{s} \sum_{s} I(s, d) \cdot P(s \mid X=L)$
B: $\delta^{*}(X=L)=\operatorname{argmin}_{d} I(s, d) \cdot P(s \mid X=L)$
C: $\delta^{*}(X=L)=\operatorname{argmin}_{s} \sum_{d} I(s, d) \cdot P(s \mid X=L)$
D: $\delta^{*}(X=L)=\operatorname{argmin}_{d} \sum_{s} I(s, d) \cdot P(s \mid X=L)$

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $\begin{gathered} x \\ \mathrm{~cm} \\ \hline \end{gathered}$ | $\begin{gathered} \text { XS } \\ (0-100) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{S} \\ (100-125) \\ \hline \end{gathered}$ | $\begin{array}{r} \mathrm{M} \\ (125-150) \\ \hline \end{array}$ | $\begin{gathered} \mathrm{L} \\ (150-175) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{XL} \\ (175-200) \\ \hline \hline \end{gathered}$ | $\begin{gathered} \text { XXL } \\ (200-\infty) \end{gathered}$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 3: Assuming there are $70 \%$ men and $30 \%$ women, consider the loss function $I(s=s t a t e, d=$ decision $): I(s=$ female,$d=$ male $)=2, I(s=$ male,$d=$ female $)=1$,
$I(s=$ male,$d=$ male $)=I(s=$ female,$d=$ female $)=0$.
How do you classify a person under consideration of $L$ ?
How?
D: $\delta^{*}(X=L)=\operatorname{argmin}_{d} \sum_{s} I(s, d) \cdot P(s \mid X=L)$

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $\stackrel{\text { cm }}{\text { cm }}$ | $\begin{gathered} \text { XS } \\ (0-100) \end{gathered}$ | $\begin{gathered} \mathrm{S} \\ (100-125) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{M} \\ (125-150) \end{gathered}$ | $\begin{gathered} \mathrm{L} \\ (150-175) \\ \hline \end{gathered}$ | $\begin{gathered} X L \\ (175-200) \\ \hline \end{gathered}$ | $\begin{gathered} \text { XXL } \\ (200-\infty) \end{gathered}$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 3: Assuming there are $70 \%$ men and $30 \%$ women, consider the loss function $/(\mathrm{s}=$ state, $\mathrm{d}=$ decision $): I(s=$ female,$d=$ male $)=2, I(s=$ male,$d=$ female $)=1$,
$I(s=$ male,$d=$ male $)=I(s=$ female,$d=$ female $)=0$.
How do you classify a person under consideration of L ?
How? $\delta^{*}(X=L)=\operatorname{argmin}_{d} \sum_{s} I(s, d) \cdot P(s \mid X=L)$
Result?
A: female
B: male

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The probability distribution of height of men and women is known (see table).

| $\stackrel{\text { cm }}{\text { cm }}$ | $\begin{gathered} \text { XS } \\ (0-100) \end{gathered}$ | $\begin{gathered} \mathrm{S} \\ (100-125) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{M} \\ (125-150) \end{gathered}$ | $\begin{gathered} \mathrm{L} \\ (150-175) \\ \hline \end{gathered}$ | $\begin{gathered} X L \\ (175-200) \\ \hline \end{gathered}$ | $\begin{gathered} \text { XXL } \\ (200-\infty) \end{gathered}$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P(x \mid$ female $)$ | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 3: Assuming there are $70 \%$ men and $30 \%$ women, consider the loss function $/(\mathrm{s}=$ state, $\mathrm{d}=$ decision $): I(s=$ female,$d=$ male $)=2, I(s=$ male,$d=$ female $)=1$,
$I(s=$ male,$d=$ male $)=I(s=$ female,$d=$ female $)=0$.
How do you classify a person under consideration of L ?
How? $\delta^{*}(X=L)=\operatorname{argmin}_{d} \sum_{s} I(s, d) \cdot P(s \mid X=L)$
Result?
A: female
B: male

## Prior probabilities in practice

The probability distribution of height of men and women is known (see table).

| $\begin{gathered} \times \\ \mathrm{cm} \\ \hline \end{gathered}$ | $\begin{gathered} \text { XS } \\ (0-100) \\ \hline \end{gathered}$ | $\underset{(100-125)}{S}$ | $\begin{gathered} \mathrm{M} \\ (125-150) \end{gathered}$ | $\begin{gathered} \mathrm{L} \\ (150-175) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{XL} \\ (175-200) \\ \hline \end{gathered}$ | $\begin{array}{r} \text { XXL } \\ (200-\infty) \\ \hline \end{array}$ | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $P(x \mid$ male $)$ | 0.05 | 0.15 | 0.2 | 0.25 | 0.3 | 0.05 | 1 |
| $P$ (x\|female) | 0.05 | 0.1 | 0.3 | 0.3 | 0.25 | 0.0 | 1 |

Task 3: Assuming there are $70 \%$ men and $30 \%$ women, consider the loss function / ( $s=$ state, $\mathrm{d}=$ decision $): I(s=$ female, $d=$ male $)=2, I(s=$ male,$d=$ female $)=1$,
$I(s=$ male,$d=$ male $)=I(s=$ female,$d=$ female $)=0$.

## How do you classify a person under consideration of $\mathbf{L}$ ?

$$
\begin{aligned}
& P(\text { male } \mid L)=\frac{P(L \mid \text { male }) \cdot P(\text { male })}{P(L)}=\frac{P(L \mid \text { male }) \cdot P(\text { male })}{P(L \mid \text { male }) \cdot P(\text { male })+P(L \mid \text { female }) \cdot P(\text { female })}=\frac{0.25 \cdot 0.7}{0.25 \cdot 0.7+0.3 \cdot 0.3}=0.66 \\
& P(\text { female } \mid L)=1-0.66=0.34 \\
& \delta^{*}(X)=\operatorname{argmin}_{d}(I(\text { female }, d) \cdot P(\text { female } \mid L)+I(\text { male }, d) \cdot P(\text { male } \mid L))
\end{aligned}
$$

$$
\delta^{*}(X)=\operatorname{argmin}_{d}\left\{\begin{array}{c}
d=\text { female }: 0 \cdot 0.34+1 \cdot 0.66=0.66 \\
d=\text { male }: 2 \cdot 0.34+0 \cdot 0.66=0.68
\end{array}\right\} \Rightarrow d=\text { female }
$$

