LP-based Heuristics for Cost-optimal Classical Planning

1. Overview and Background

Florian Pommerening   Gabriele Röger   Malte Helmert

Based on: ICAPS 2015 Tutorial

March 2018

Background: Linear Programs

Linear Programs and Integer Programs

Linear Program

A linear program (LP) consists of:

- a finite set of real-valued variables V
- a finite set of linear inequalities (constraints) over V
- an objective function, which is a linear combination of V
- which should be minimized or maximized.

Integer program (IP): ditto, but with integer-valued variables

Linear Program: Example

Example:

$$\text{maximize} \quad 2x - 3y + z \quad \text{subject to}$$

$$\begin{aligned} x + 2y + z &\leq 10 \\ x \phantom{{}+2y} - z &\leq 0 \end{aligned}$$

$$x \geq 0, \quad y \geq 0, \quad z \geq 0$$

$\rightsquigarrow$ unique optimal solution:
$x = 5$, $y = 0$, $z = 5$ (objective value 15)

Solving Linear Programs and Integer Programs

Complexity:

- LP solving is a polynomial-time problem.
- Finding solutions for IPs is NP-complete.

Common idea:

- Approximate IP solution with corresponding LP (LP relaxation).

Tutorial Structure

1. ~~Introduction and Overview~~
2. Cost Partitioning
3. Operator Counting
4. Potential Heuristics

Cost Partitioning

Cost Partitioning

Idea 1: Cost Partitioning

- create copies $\Pi_1, \ldots, \Pi_n$ of planning task Π
- each has its own operator cost function $cost_i : \mathcal{O} \to \mathbb{R}_0^+$ (otherwise identical to Π)
- for all o: require $cost_1(o) + \cdots + cost_n(o) \leq cost(o)$

⇝ sum of solution costs in copies is admissible heuristic: $h^*_{\Pi_1} + \cdots + h^*_{\Pi_n} \leq h^*_{\Pi}$

Cost Partitioning

- for admissible heuristics $h_1, \ldots, h_n$,
  $h(s) = h_{1,\Pi_1}(s) + \cdots + h_{n,\Pi_n}(s)$
  is an admissible estimate
- $h(s)$ can be better or worse than any $h_{i,\Pi}(s)$
  $\rightarrow$ depending on cost partitioning
- strategies for defining cost-functions
  - uniform: $cost_i(o) = cost(o)/n$
  - zero-one: full operator cost in one copy, zero in all others
  - ...

Can we find an optimal cost partitioning?

Optimal Cost Partitioning

Optimal Cost Partitioning

Optimal Cost Partitioning with LPs

- Use variables for cost of each operator in each task copy
- Express heuristic values with linear constraints
- Maximize sum of heuristic values subject to these constraints

LPs known for

- abstraction heuristics
- landmark heuristic

Optimal Cost Partitioning for Abstractions

Abstractions

- Simplified versions of the planning task, e.g. projections
- Cost of optimal abstract plan is admissible estimate

How to express the heuristic value as linear constraints?

Optimal Cost Partitioning for Abstractions

Abstractions

- Simplified versions of the planning task, e.g. projections
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How to express the heuristic value as linear constraints?
$\rightsquigarrow$ Shortest path problem in abstract transition system

LP for Shortest Path in State Space

Variables

Distance_s for each state s,
GoalDist

Objective

Maximize GoalDist

Subject to

$$\text{Distance}_{s_I} = 0 \qquad \text{for the initial state } s_I$$

$$\text{Distance}_{s'} \leq \text{Distance}_s + cost(o) \qquad \text{for all transition } s \xrightarrow{o} s'$$

$$\text{GoalDist} \leq \text{Distance}_{s_\star} \qquad \text{for all goal states } s_\star$$

Optimal Cost Partitioning for Abstractions I

Variables

For each abstraction α:
Distance_s^α for each abstract state s,
$\text{cost}^\alpha(o)$ for each operator o,
GoalDist^α

Objective

Maximize $\sum_\alpha \text{GoalDist}^\alpha$

...

Optimal Cost Partitioning for Abstractions II

Subject to

for all operators o

$$\sum_{\alpha} \text{Cost}_o^\alpha \leq cost(o)$$

$$\text{Cost}_o^\alpha \geq 0 \qquad \text{for all abstractions } \alpha$$

and for all abstractions α

$$\text{Distance}_{s_I}^\alpha = 0 \qquad \text{for the abstract initial state } s_I$$

$$\text{Distance}_{s'}^\alpha \leq \text{Distance}_s^\alpha + \text{Cost}_o^\alpha \qquad \text{for all transition } s \xrightarrow{o} s'$$

$$\text{GoalDist}^\alpha \leq \text{Distance}_{s_\star}^\alpha \qquad \text{for all abstract goal states } s_\star$$

Optimal Cost Partitioning for Landmarks

Disjunctive action landmark

- Set of operators
- Every plan uses at least one of them
- Landmark cost = cost of cheapest operator

Optimal Cost Partitioning for Landmarks

Variables

Cost_L for each landmark L

Objective

Maximize $\sum_L \text{Cost}_L$

Subject to

$$\sum_{L:o \in L} \text{Cost}_L \leq cost(o) \quad \text{for all operators } o$$

Caution

A word of warning

- optimization for every state gives best-possible cost partitioning
- but takes time

Better heuristic guidance often does not outweigh the overhead.

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Operator-counting Framework

Operator Counting

Idea 2: Operator Counting Constraints

- linear constraints whose variables denote number of occurrences of a given operator
- must be satisfied by every plan that solves the task

Examples:

- $Y_{o_1} + Y_{o_2} \geq 1$ "must use o_1 or o_2 at least once"
- $Y_{o_1} - Y_{o_3} \leq 0$ "cannot use o_1 more often than o_3"

Motivation:

- declarative way to represent knowledge about solutions
- allows reasoning about solutions to derive heuristic estimates

Operator Counting Heuristics

Operator occurrences in potential plans

(2,1,0)

(1,1,2) (0,0,0)

(0,0,1)

(1,2,1)

(3,0,2)

(1,3,1)

(3,2,2)

(2,2,0)

(2,2,1)

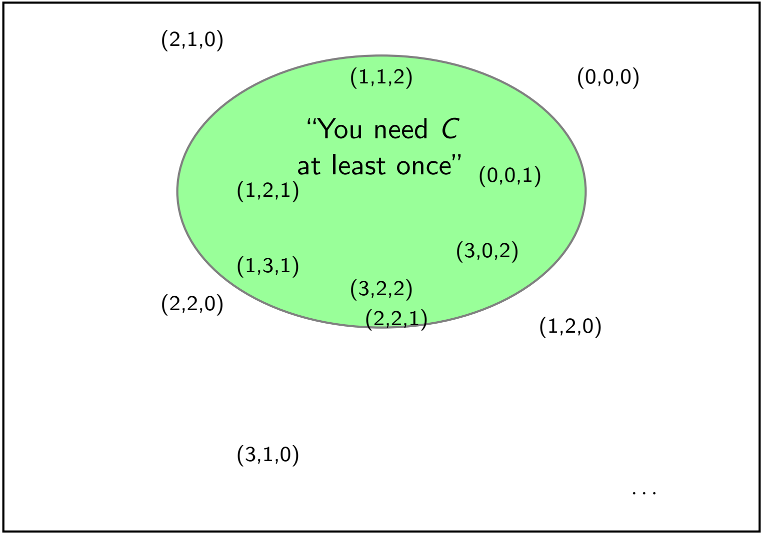
(1,2,0)

(3,1,0)

. . .

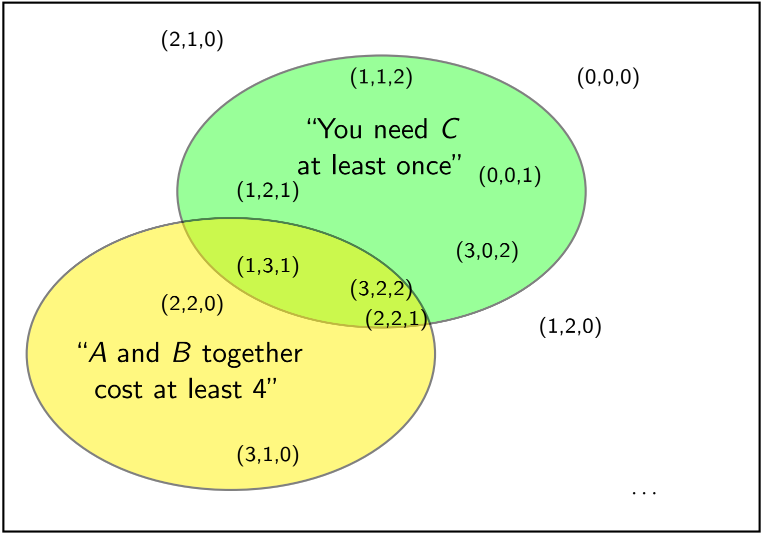
Operator Counting Heuristics

Operator occurrences in potential plans



Operator Counting Heuristics

Operator occurrences in potential plans



Operator Counting Heuristics

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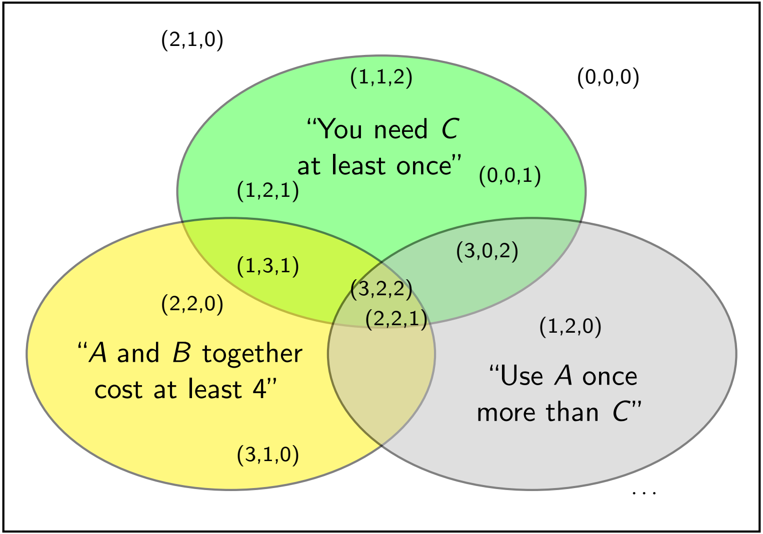

Operator Counting Heuristics

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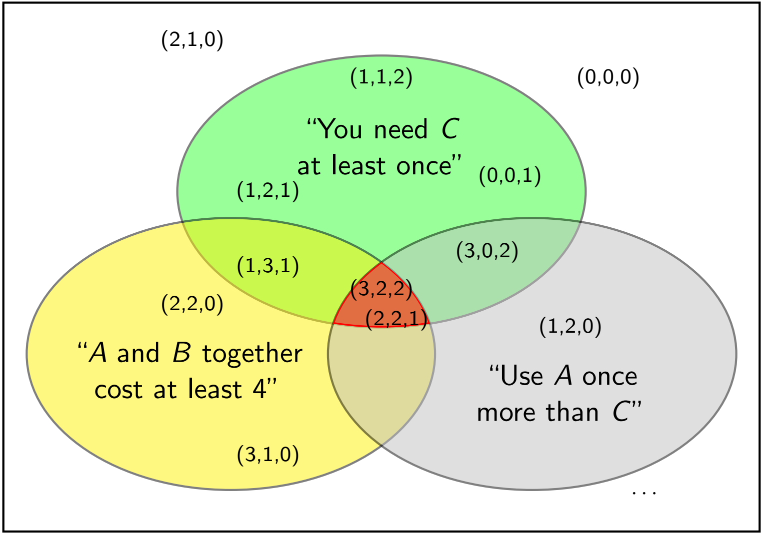
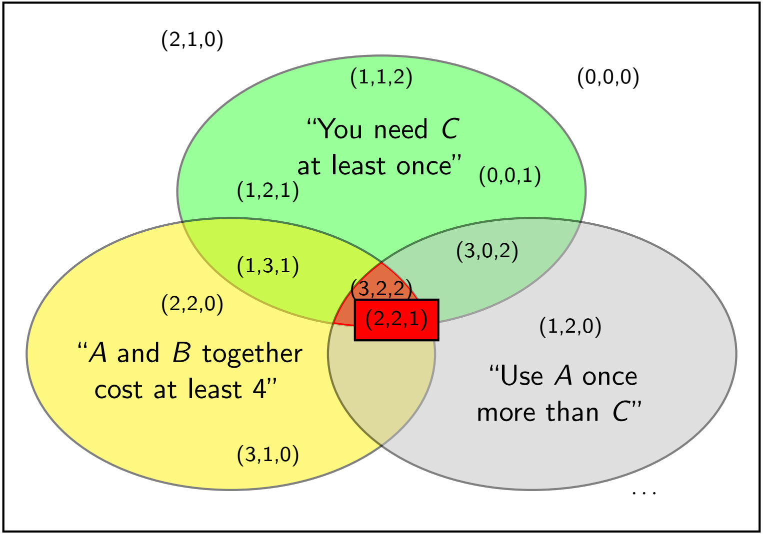


Operator Counting Heuristics

Operator occurrences in potential plans



Operator-counting Heuristics

Operator-counting IP/LP Heuristic

$$\text{Minimize } \sum_{o} Y_o \cdot cost(o) \text{ subject to}$$

$Y_o \geq 0$ and some operator-counting constraints

Operator-counting constraint

- Set of linear inequalities
- For every plan π there is an LP-solution where Y_o is the number of occurrences of o in π .

Properties of Operator-counting Heuristics

Admissibility

Operator-counting (IP and LP) heuristics are admissible.

Computation time

Operator-counting LP heuristics are solvable in polynomial time.

Adding constraints

Adding constraints can only make the heuristic more informed.

State-equation Heuristic

State-equation Heuristic (SEQ)

Also known as

- Order-relaxation heuristic (van den Briel et al. 2007)
- State-equation heuristic (Bonet 2013)
- Flow-based heuristic (van den Briel and Bonet 2014)

Main idea:

- Facts can be produced (made true) or consumed (made false) by an operator
- Number of producing and consuming operators must balance out for each fact

State-equation Heuristic

Net-change constraint for fact f

$$G(f) - S(f) = \sum_{f \in \mathit{eff}(o)} Y_o - \sum_{f \in \mathit{pre}(o)} Y_o$$

Remark:

- Assumes transition normal form (not a limitation)
  - Operator mentions same variables in precondition and effect
  - General form of constraints more complicated

$\rightsquigarrow$

State-equation Heuristic (Constraints)

Net-change constraint for fact f

$$0 = \sum_{o \text{ produces } f} Y_o - \sum_{o \text{ consumes } f} Y_o$$

State-equation Heuristic (Constraints)

Net-change constraint for fact f

$$G(f) - S(f) = \sum_{o \text{ produces } f} Y_o - \sum_{o \text{ consumes } f} Y_o$$

- Special cases for goal and initial state
  - Add/Subtract one from net change

Connection to Cost Partitioning

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Overview

Potential Heuristics

Idea 3: Potential Heuristics

Heuristic design as an optimization problem:

- Define simple numerical state features $f_1, \dots, f_n$.
- Consider heuristics that are linear combinations of features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

  with weights (potentials) $w_i \in \mathbb{R}$
- Find potentials for which h is admissible and well-informed.

Motivation:

- declarative approach to heuristic design
- heuristic very fast to compute if features are

Comparison to Previous Parts (1)

What is the same as in operator-counting constraints:

- We again use LPs to compute (admissible) heuristic values (spoiler alert!)

Comparison to Previous Parts (2)

What is different from operator-counting constraints (computationally):

- With potential heuristics, solving one LP defines the entire heuristic function, not just the estimate for a single state.
- Hence we only need one LP solver call, making LP solving much less time-critical.

Comparison to Previous Parts (3)

What is different from operator-counting constraints (conceptually):

- axiomatic approach for defining heuristics:
  - What should a heuristic look like mathematically?
  - Which properties should it have?
- define a space of interesting heuristics
- use optimization to pick a good representative

Potential Heuristics

Features

Definition (feature)

A (state) feature for a planning task is a numerical function defined on the states of the task: $f : S \to \mathbb{R}$.

Potential Heuristics

Definition (potential heuristic)

A potential heuristic for a set of features $\mathcal{F} = \{f_1, \dots, f_n\}$ is a heuristic function h defined as a linear combination of the features:

$$h(s) = w_1 f_1(s) + \dots + w_n f_n(s)$$

with weights (potentials) $w_i \in \mathbb{R}$.

$\rightsquigarrow$ cf. evaluation functions for board games like chess

Atomic Potential Heuristics

Atomic features test if some proposition is true in a state:

Definition (atomic feature)

Let $X = x$ be an atomic proposition of a planning task.

The atomic feature $f_{X=x}$ is defined as:

$$f_{X=x}(s) = \begin{cases} 1 & \text{if variable } X \text{ has value } x \text{ in state } s \\ 0 & \text{otherwise} \end{cases}$$

- We only consider atomic potential heuristics, which are based on the set of all atomic features.
- Example for a task with state variables X and Y:

$$h(s) = 3f_{X=a} + \frac{1}{2}f_{X=b} - 2f_{X=c} + \frac{5}{2}f_{Y=d}$$

Finding Good Potential Heuristics

How to Set the Weights?

We want to find good atomic potential heuristics:

- admissible
- consistent
- well-informed

How to achieve this? Linear programming to the rescue!

Admissible and Consistent Potential Heuristics

Constraints on potentials characterize (= are necessary and sufficient for) admissible and consistent atomic potential heuristics:

Goal-awareness (i.e., $h(s) = 0$ for goal states)

$$\sum_{\text{goal facts } f} w_f = 0$$

Consistency

$$\sum_{\substack{f \text{ consumed} \\ \text{by } o}} w_f - \sum_{\substack{f \text{ produced} \\ \text{by } o}} w_f \leq cost(o) \quad \text{for all operators } o$$

Remarks:

- assumes transition normal form (not a limitation)
- goal-aware and consistent = admissible and consistent

Well-Informed Potential Heuristics

How to find a well-informed potential heuristic?

⇝ encode quality metric in the objective function
and use LP solver to find a heuristic maximizing it

Examples:

- maximize heuristic value of a given state (e.g., initial state)
- maximize average heuristic value of all states
(including unreachable ones)
- maximize average heuristic value of some sample states
- minimize estimated search effort

⇝

Connections

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So what does this have to do with what we talked about before?

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Theorem (Pommerening et al., AAAI 2015)

For state s, let $h^{\text{maxpot}}(s)$ denote the maximal heuristic value of all admissible and consistent atomic potential heuristics in s.

Then $h^{\text{maxpot}}(s) = h^{\text{SEQ}}(s) = h^{\text{gOCP}}(s)$.

- h^{SEQ}: state equation heuristic a.k.a. flow heuristic
- h^{gOCP}: optimal general cost partitioning of atomic projections

proof idea: compare dual of $h^{\text{SEQ}}(s)$ LP
to potential heuristic constraints optimized for state s

What Do We Take From This?

- general cost partitioning, operator-counting constraints and potential heuristics: facets of the same phenomenon
- study of each reinforces understanding of the others
- potential heuristics: fast admissible approximations of h^{SEQ}
- clear path towards generalization beyond h^{SEQ}: use non-atomic features

The End

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Thank you for your attention!