Social Choice & Voting

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Previously ... on multi-agent systems.

And now ...

Voting Rules

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Positional Scoring Rules Assuming $m = |U|$ alternatives, we define a score vector $s = (s_1, \ldots, s_m) \in \mathbb{R}^m$ such that $s_1 \geq \ldots \geq s_m$ and $s_1 > s_m$. Each time an alternative is ranked i-th by some voter, it gets a particular score s_i.

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- **Plurality rule** the score vector is $s = (1, 0, \ldots, 0)$
- **Anti-plurality rule / Approval voting** the score vector is $s = (1, 1, \ldots, 1, 0)$ (or a subset of alternatives).

Simple Voting Example

Assume there are 7 agents with the following preferences:

- 3 agents: $a > b > c$
- 2 agents: $b > c > a$
- 2 agents: $c > a > b$

Which of the candidates is selected if we use different voting protocols?

- Plurality
- Borda's rule
- Pairwise elimination with ordering: a) (a, b, c), b) (b, c, a), c) (c, a, b)

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Assume that we want to include a fourth candidate d into the profiles. Is there a modification of the current preference profiles such that c can be the winner under Borda voting rule?

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How about if we use Borda voting protocol?

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	unweighted votes, constructive manipulation		weighted votes, constructive				weighted votes, destructive		
# alternatives			2	3	4	≥ 5	2	3	≥ 4
# manipulators	1	≥ 2							
plurality	P	P	P	P	P	P	P	P	P
plurality with runoff	P	P	P	NP-c	NP-c	NP-c	P	NP-c	NP-c
veto	P	P	P	NP-c	NP-c	NP-c	P	P	P
cup	P	P	P	P	P	P	P	P	P
Copeland	P	P	P	P	NP-c	NP-c	P	P	P
Borda	P	NP-c	P	NP-c	NP-c	NP-c	P	P	P
Nanson	NP-c	NP-c	P	P	NP-c	NP-c	P	P	NP-c
Baldwin	NP-c	NP-c	P	NP-c	NP-c	NP-c	P	NP-c	NP-c
Black	P	NP-c	P	NP-c	NP-c	NP-c	P	P	P
STV	NP-c	NP-c	P	NP-c	NP-c	NP-c	P	NP-c	NP-c
maximin	P	NP-c	P	P	NP-c	NP-c	P	P	P
Bucklin	P	P	P	NP-c	NP-c	NP-c	P	P	P
fallback	P	P	P	P	P	P	P	P	P
ranked pairs	NP-c	NP-c	P	P	P	NP-c	P	P	?
Schulze	P	P	P	P	P	P	P	P	P

Table from Conitzer and Walsh, Barriers to Manipulation, Chapter 6 in Handbook of Computational Social Choice

Strategic Manipulation

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Voters may be better off by misrepresenting their preferences.

- 1 voter ranks
  $A \succ B \succ C \succ D$
- 2 voters rank
  $A \succ C \succ B \succ D$
- 2 voters rank
  $B \succ D \succ C \succ A$
- 2 voters rank
  $C \succ B \succ D \succ A$

Plurality winner A ... but B can be the winner if the last two voters vote for B instead of C.

but C wins if the voters in the second row, who prefer C to B move B to the bottom.

Create examples of both constructive and destructive manipulation using the Borda rule setting.