Auctions and Resource Allocations

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Simple Auction Mechanisms

single-item one-sided auctions:

- **English** - The auctioneer sets a starting price for the good, and agents then have the option to announce successive bids, each of which must be higher than the previous bid (usually by some minimum increment set by the auctioneer).
- **Japanese** - The auctioneer sets a starting price for the good that is (continuously) increasing and the agents must confirm that they still want to buy the good for that price.
- **Dutch** - The auctioneer begins by announcing a high price and then successively lower the price. The auction ends when the first agent signals the auctioneer that she buys the good for the current price.

Auctions and Mechanism Design

single-item one-sided auctions (continued):

- **1st price sealed-bid** - Each agent submits to the auctioneer a secret, "sealed" bid for the good that is not accessible to any of the other agents. The agent with the highest bid must purchase the good. In a first-price sealed-bid auction (or simply first-price auction) the winning agent pays an amount equal to his own bid.
- **2nd price sealed-bid** - In a second-price auction the winning agent pays an amount equal to the next highest bid (i.e., the highest rejected bid).

Dutch and First-price Sealed-bid Auctions

Equilibria in two-player FPSB Auctions

Assume a first-price auction with two risk-neutral bidders whose valuations are drawn independently and uniformly at random from the interval $[0, 1]$ – what is the equilibrium strategy?

$$\left(\frac{v_1}{2}, \frac{v_2}{2}\right)$$

Can be generalized to the n-player case.

Theorem

In a first-price sealed bid auction with n risk-neutral agents whose valuations $v_1, v_2, \ldots, v_n$ are independently drawn from a uniform distribution on the same bounded interval of the real numbers, the unique symmetric equilibrium is given by the strategy profile $\left(\frac{n-1}{n}v_1, \ldots, \frac{n-1}{n}v_n\right)$

Simple Auction Example

Consider a second-price, sealed-bid auction with two bidders who have independent, private values v_i which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 0.5 and they both play equilibrium strategies.

- What is the seller's expected revenue?
- Now let's suppose that there are three bidders who have independent, private values v_i which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 0.5. What is the seller's expected revenue in this case?
- How does the situation change in both cases if we have first-price sealed bid auction?

Simple Auction Example

A seller runs a second-price, sealed-bid auction for an object. There are two bidders, a and b, who have independent, private values v_i which are either 0 or 1. For both bidders the probabilities of $v_i = 0$ and $v_i = 1$ are 0.5 each. Both bidders understand the auction, but bidder b sometimes makes a mistake about his value for the object. Half of the time his value is 1 and he is aware that it is 1 the other half of the time his value is 0 but occasionally he mistakenly believes that his value is 1. Let's suppose that when b's value is 0 he acts as if it is 1 with probability 0.5 and as if it is 0 with probability 0.5. So in effect bidder b sees value 0 with probability 0.25 and value 1 with 0.75 probability. Bidder a never makes mistakes about his value for the object, but he is aware of the mistakes that bidder b makes. Assume that if there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.

- Draw the game tree of the auction.

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- Assume bidder b is not aware of his mistake and bids optimally given the perceptions of the value of the object. Is bidding his true value still a dominant strategy for bidder a?

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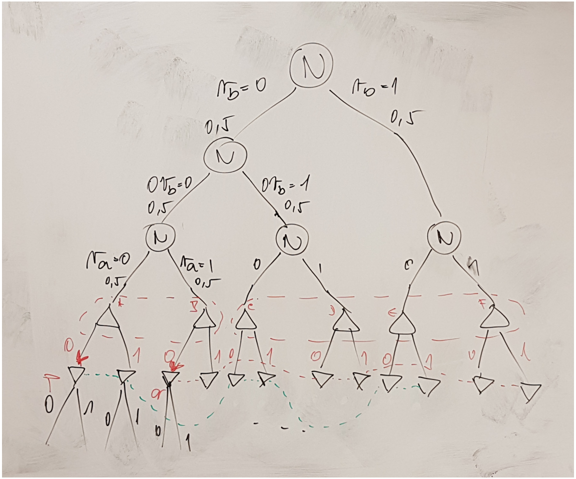
- Assume that b is aware of his mistake and bids must be integers. What are the optimal strategies?

Simple Auction Example

Solution:



When player b observes 1, the expected value for bidding 0 is 0.125; the expected value for bidding 1 is 0.1825. Bidding truthfully (w.r.t. to observed values) is still the optimal strategy.

Simple Auction Example

Consider a first-price, sealed-bid auction with two bidders who have independent, private values v_i which are independent and uniformly distributed over the set $\{0, 1, 2\}$. The bids in the auction must be nonnegative integers. Assume that ties are broken randomly.

- What is an equilibrium strategy?
- Find all equilibria.