Auctions

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Auctions: Traditional

Auctions used in Babylon as early as 500 B.C. but until relatively recently used only for high-value items for which it was difficult to assess the market price

Stage 0: No automation
Auctions: Partial Automation

Grown massively with the Web/Internet

→ **Frictionless commerce**: feasible to auction things that weren’t previously profitable

**Stage 1: Computers** manage auctions / run auction **protocols**
Stage 2: Computers also automate the decision making of bidders

Concerns:
1) the most relevant adds are shown (⇒ user’s are reasonably happy)
2) auctioner’s profit is maximized (over longer time)
Lots of Applications

- Industrial procurement
- Transport and logistics
- Energy markets
- Cloud and grid computing
- Internet auctions
- (Electromagnetic spectrum allocation)
- ... and counting!
Introduction to Auctions
English Auction

1. Auctioneer starts the bidding at some reservation price
2. Bidders then shout out ascending prices (with minimum increments)
3. Once bidders stop shouting, the high bidder gets the good at that price
What is an Auction?

An **auction** is a protocol that allows **agents** (=bidders) to indicate their **interests** in one or more **resources** and that uses these indications of interest to determine both an **allocation** of the resources and a set of **payments** by the agents. [Shoham & Leyton-Brown 2009]

Auctions use employ **cardinal preferences** to express interest. Auctions are mechanisms **with money**. Auctions can be viewed as **games** of a specific structure.
Why Auctions?

Market-based price setting: for objects of unknown value, the value is dynamically assessed by the market!

Flexible: any object type can be allocated

Can be automated
- use of simple rules reduces complexity of negotiations
- well-suited for computer implementation

Revenue-maximising and efficient allocations are achievable
Auctions Rules

**Auction mechanism is specified by auction rules** (*→ rules of the game*)

<table>
<thead>
<tr>
<th>Bidding rules</th>
<th>Clearing rules</th>
<th>Information rules</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>How offers are made:</strong></td>
<td>Who gets which goods (<em>allocation</em>) and what money changes hands (<em>payment</em>).</td>
<td>What information about the state of the negotiation is <em>revealed to whom</em> and <em>when</em>.</td>
</tr>
<tr>
<td>• by whom</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• when</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• what their content is</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Payoff

Agent’s payoff from participating in an auction

If winner

\[ \text{payoff} = \text{agent’s valuation of the item} - \text{price paid for the item} \]

If not winner

\[ \text{payoff} = \text{zero} \]

Risk neutrality: the payoff is (as above) a linear function of the difference between the item’s valuation and the price paid

- **risk seeking**: the payoff is a convex function of the difference (aggressively seeking high gains is prioritized)
- **risk aversion**: the payoff is a concave function of the difference (conservatively ensuring at least some gains is prioritized)
## Valuation Models

<table>
<thead>
<tr>
<th>Independent private value (IPV)</th>
<th>Correlated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>An agent A’s valuation of the good is <strong>independent from other agent’s</strong> valuation of the good (e.g. a taxi ride to the airport)</td>
<td>Valuations of the good are <strong>related between agents</strong> (typically the more other agents are prepared to pay, the more agent A prepared to pay – e.g. purchase of items for later resale)</td>
</tr>
</tbody>
</table>
Types of Auctions

- **multi-unit**
  multiple indistinguishable items (A, A, A)

- **single-good**

- **multi-attribute**
  items characterized by multiple attributes (A=$a_1, a_2, a_3$)

- **multi-item**
  multiple different items (A,B,C)

*Combinatorial auctions*
Types of Auctions

Forward **(sell-side)** auction: selling
Reverse **(buy-side)** auction: buying
Single-sided: either selling or buying
Double-sided: both selling and buying (→ exchange)

There are other allocation mechanisms: facility location, allocation of divisible goods (cake cutting), allocation of indivisible goods (CPU, memory), ...
Single-Item Auctions
Basic Auction Mechanisms

English
Japanese
Dutch
First-Price
Second-Price
English Auction

1. Auctioneer starts the bidding at some reservation price

2. Bidders then shout out ascending prices (with minimum increments)

3. Once bidders stop shouting, the high bidder gets the good at that price
Japanese Auctions

Same as an English auction except that the auctioneer calls out the prices

1. All bidders start out **standing**

2. When the price reaches a level that a bidder is not willing to pay, that bidder **sits down**; once a bidder sits down, they **can't get back up**.

3. The **last** person **standing** gets the good
Dutch Auction

1. The auctioneer starts a clock at some high value; it descends
2. At some point, a bidder shouts “mine!” and gets the good at the price shown on the clock

Good when items need to be sold quickly (similar to Japanese)

No information is revealed during auction
First-, Second-Price Sealed Bid Auctions

First-price sealed bid auction
- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- that bidder pays the amount of his bid

Second-price sealed bid auction (Vickerey auction)
- bidders write down bids on pieces of paper
- auctioneer awards the good to the bidder with the highest bid
- that bidder pays the amount bid by the second-highest bidder
## Intuitive Comparison

<table>
<thead>
<tr>
<th></th>
<th>English</th>
<th>Dutch</th>
<th>Japanese</th>
<th>1&lt;sup&gt;st&lt;/sup&gt;-Price</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt;-Price</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Duration</strong></td>
<td>#bidders, increment</td>
<td>starting price, clock speed</td>
<td>#bidders, increment</td>
<td>fixed</td>
<td>fixed</td>
</tr>
<tr>
<td><strong>Info Revealed</strong></td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;-highest val; bounds on others</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;-highest winner’s bid</td>
<td>all val’s but winner’s</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td><strong>Jump bids</strong></td>
<td>yes</td>
<td>n/a</td>
<td>no</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td><strong>Price Discovery</strong></td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>
Analysing Auctions
Are there fundamental similarities / differences between mechanisms described?
Two Problems

**Design of auction mechanisms**

- design the auction mechanism (i.e. the game for the bidders) with the desirable properties
- methodology: apply mechanism design techniques

**Analysis of auction mechanisms**

- determine the properties of a given auction mechanism
- methodology: treat auctions as (extended-form) *Bayesian games* and analyse players’ (i.e. bidders’) strategies
Bayesian Game

A Bayesian game is a tuple \( \langle N, A, \Theta, p, u \rangle \) where

- \( N \) is the set of players
- \( \Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n \), \( \Theta_i \) is the type space of player \( i \)
- \( A = A_1 \times A_2 \times \cdots \times A_n \) where \( A_i \) is the set of actions for player \( i \)
- \( p: \Theta \mapsto [0,1] \) is a common prior over types
- \( u = (u_1, \ldots, u_n) \), where \( u_i: A \times \Theta \mapsto \mathbb{R} \) is the utility function of player \( i \)

We assume that all of the above is common knowledge among the players, and that each agent knows his own type.

Bayes-Nash equilibrium: rational, risk-neutral players are seeking to maximize their expected payoff, given their beliefs about the other players’ types.
Relation to (sealed bid) Auctions

Sealed bid auction under IPV is a Bayesian game in which

- player $i$’s actions correspond to his bids $\hat{v}_i$;
- player types $\Theta_i$ correspond to players’ private valuations $v_i$ over the auctioned item(s);
- the payoff of player $i$ corresponds to $i$’s valuation of the item $v_i$ – price paid (in the case of winning; zero otherwise).

Similar analogies for more complicated auction mechanisms.
(Desirable) Properties

**Truthfulness**: bidders are incentivized to bid their *true* valuations, i.e.

$$v_i = \hat{v}_i \ \forall i \forall v_i$$

**Efficiency**: the aggregated value of bidders is maximized, i.e.

$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x')$$

**Optimality**: maximization of seller’s revenue

**Strategy**: existence of dominant strategy

**Manipulation vulnerability**: lying auctioner, shills, bidder collusion

Other consideration: communication complexity, private information revelation, ...
Are there fundamental similarities / differences between mechanisms described?
Second-Price Sealed Bid

**Theorem**

*Truth-telling is a dominant strategy* in a second-price sealed bid auction (assuming independent private values – IPV).

**Proof:** Assume that the other bidders bid in some arbitrary way. We must show that $i$'s best response is always to bid truthfully. We'll break the proof into two cases:

- Bidding honestly, $i$ would win the auction
- Bidding honestly, $i$ would lose the auction
Second-Price Sealed Bid Proof

Bidding honestly, $i$ is the winner

If $i$ bids higher, he will still win and still pay the same amount.

If $i$ bids lower, he will either still win and still pay the same amount. . .

... or lose and get the payoff of zero.

$\Rightarrow$ There is a disadvantage bidding lower and no advantage bidding higher.
Second-Price Sealed Bid Proof

Bidding honestly, \( i \) is not the winner

If \( i \) bids lower, he will still lose and still pay nothing

If \( i \) bids higher, he will either still lose and still pay nothing...

... or win and pay more than his valuation (⇒ negative payoff).

\( \Rightarrow \) There is a disadvantage bidding higher and no advantage bidding lower
Second-Price Sealed Bid

Advantages:
- **Truthful** bidding is dominant strategy
- No incentive for counter-speculation
- Computational efficiency

Disadvantages:
- Lying auctioneer
- Bidder collusion self-enforcing
- **Not revenue maximizing**
Dutch and First-price Sealed Bid

Strategically equivalent: an agent bids without knowing about the other agents’ bids
- a bidder must decide on the amount he's willing to pay, conditional on having placed the highest bid

Differences
- First-price auctions can be held asynchronously
- Dutch auctions are fast, and require minimal communication
Bidding in Dutch / First Price Sealed Bid

Bidders don't have a dominant strategy any more:

\[ \Leftarrow \text{there's a trade-off between probability of winning vs. amount paid upon winning} \]

- individually optimal strategy depends on the assumptions about others’ valuations

Assume a first-price auction with two risk-neutral bidders whose valuations are drawn independently and uniformly at random from the interval \([0, 1]\) - what is the equilibrium strategy?

\[ \Rightarrow \left( \frac{1}{2} v_1, \frac{1}{2} v_2 \right) \text{ is the Bayes-Nash equilibrium strategy profile} \]

\[ \Rightarrow \text{Dutch / FPSB auctions not incentive compatible, i.e., there are incentives to counter-speculate.} \]
In a first-price sealed bid auction with \( n \) risk-neutral agents whose valuations \( v_1, v_2, \ldots, v_n \) are independently drawn from a uniform distribution on the same bounded interval of the real numbers, the unique symmetric equilibrium is given by the strategy profile \( \left( \frac{n-1}{n} v_1, \ldots, \frac{n-1}{n} v_n \right) \).

For non-uniform valuation distributions: Each bidder should bid the expectation of the second-highest valuation, conditioned on the assumption that his own valuation is the highest.
A much more complicated strategy space
- extensive-form game
- bidders are able to condition their bids on information revealed by others
- in the case of English auctions, the ability to place jump bids

Intuitively, though, the revealed information does not make any difference in the independent-private value (IPV) setting.
In correlated-value auctions, it can be worthwhile to counter-speculate.
Revenue Equivalence

Which auction should an auctioneer choose?
To some extent, it doesn't matter...

Theorem (Revenue Equivalence)

Assume that each of \( n \) risk-neutral agents has an independent private valuation for a single good at auction, drawn from a common cumulative distribution \( F(v) \) that is strictly increasing and atomless on \([\underline{v}, \overline{v}]\). Then any auction mechanism in which

1. the good will be allocated to the agent with the highest valuation; and
2. any agent with valuation \( \underline{v} \) has an expected utility of zero yields the same expected revenue, and hence results in any bidder with valuation \( v \) making the same expected payment.
Revenue Equivalence

Assuming bidders are risk neutral and have independent private valuations, all the auctions we have spoken about so far—English, Japanese, Dutch, and all sealedbid auction protocols—are revenue equivalent.
What about Efficiency?

**Efficiency** in single-item auctions: the item allocated to the agent who values it the most.

With independent private values (IPV):

<table>
<thead>
<tr>
<th>Auction</th>
<th>Efficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>English (without reserve price)</td>
<td>yes</td>
</tr>
<tr>
<td>Japanese</td>
<td>yes</td>
</tr>
<tr>
<td>Dutch</td>
<td>no</td>
</tr>
<tr>
<td>Sealed bid second price</td>
<td>yes</td>
</tr>
<tr>
<td>Sealed bid first price</td>
<td>no</td>
</tr>
</tbody>
</table>

Efficiency (often) lost in the correlated value setting.
Optimal Auctions
The seller's problem is to design an auction mechanism which has a Nash equilibrium giving him/her the highest possible expected utility.

- assuming individual rationality

Second-prize sealed bid auction does not maximize expected revenue → not the best choice if profit maximization is important (in the short term).
Can we get better revenue?

Let’s have another look at 2nd price auctions:

Bidder 1 wins and pays $x$ (his lowest winning bid)

lost revenue

1 wins and pays $x$ (his lowest winning bid)
Can we get better revenue?

Some reserve price improves revenue.
Can we get better revenue?

We will be here with probability $R^2$

Loss is always at most $R$

Gain is at least: $\frac{2R(1-R)R}{2} = R^2 - R^3$

Loss is at most: $R^2 R = R^3$

When $R^2 - 2R^3 > 0$, reserve price of $R$ is beneficial.

(for example, $R = 1/4$)
Optimal Single Item Auction

Definition (Virtual valuations)
Consider an **IPV setting** where bidders are **risk neutral** and each bidder $i$’s valuation is drawn from some **strictly increasing** cumulative density function $F_i(v)$, having probability density function $f_i(v)$. We then define:

where

- Bidder $i$’s **virtual valuation** is $\psi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$
- Bidder $i$’s **bidder-specific reserve price** $r_i^*$ is the value for which $\psi_i(r_i^*) = 0$

Example: uniform distribution over $[0,1]$: $\psi(v) = 2v - 1$
Theorem (Optimal Single-item Auction)

The optimal (single-good) auction is a **sealed-bid auction** in which every agent is asked to declare his valuation. The good is sold to the agent \( i = \arg\max_i \psi_i(\widehat{v}_i) \), as long as \( \widehat{v}_i > r_i^* \).

If the good is sold, the winning agent \( i \) is charged the smallest valuation that he could have declared while still remaining the winner:

\[
\inf\{v_i^* : \psi_i(v_i^*) \geq 0 \land \forall j \neq i, \psi_i(v_i^*) \geq \psi_j(\widehat{v}_j)\}
\]

Can be understood as a second-price auction with a reserve price, held in **virtual valuation space** rather than in the space of actual valuations.

Remains **dominant-strategy truthful**.
Symmetric case: second-price auction with reserve price $r^*$ satisfying:

$$\psi(r^*) = r^* - \frac{1 - F(r^*)}{f(r^*)} = 0$$

- Truthful mechanism when $\psi(v)$ is non-decreasing.
- Uniform distribution over $[0, p]$: optimum reserve price $p/2$.

Second-price sealed bid auction with Reserve Price is not efficient!
Optimal Auctions: Remarks

Always: $\text{revenue} \leq \text{efficiency}$
- due to individual rationality
- more efficiency makes the pie larger!

However, for optimal revenue one needs to sacrifice some efficiency.

Optimal auctions are not detail-free:
- they require the seller to incorporate information about the bidders’ valuation distributions into the mechanism.

Theorem (Bulow and Klemperer): revenue of an efficiency-maximizing auction with $k+1$ bidder is at least as high as that of the revenue-maximizing one with $k$ bidders.

$\Rightarrow$ better to spend energy on attracting more bidders
Auctions are mechanisms for **allocating scarce resource** among **self-interested agent**

Mechanism-design and game-theoretic perspective


**Desirable** properties: truthfulness, efficiency, optimality, ...

Rapidly expanding list of **applications** worth billions of dollars

Reading:
- [Shoham] – Chapter 11
Final Notes

Rapidly evolving field with the exploding number of applications
→ http://aic.fel.cvut.cz/ for (Ph.D.) opportunities

Exams: 15/1, 23/1 and 29/1 9:00-12:00

Survey/Anketa: be as specific possible: we do care
How to get around impossibility results

Mechanisms with money

Measure not just that a preferred to b, but also “by how much”...

Each individual $j$ (or player $j$) has a “valuation” for each alternative $a$ in $A$. Denoted as $v_j(a)$

Also, each player values money the same.

So, if we choose alternative $a$, and give $m$ to $j$, then $j$’s “utility” is $v_j(a) + m$
Auction Protocols

Auctions are centralised mechanisms for the allocation of goods amongst several agents. Agents report their preferences (bidding) and the auctioneer decides on the final allocation (and on prices).

- Distinguish *direct* and *reverse* auctions (auctioneer buying).
- Bidding may be *open-cry* (English) or by *sealed bids*.
- Open-cry: *ascending* (English) or *descending* bids (Dutch).
- Pricing rule: *first-price* or *second-price* (Vickrey).
- *Combinatorial auctions*: several goods, sold/bought in bundles.
