Auctions

Michal Jakob
Artificial Intelligence Center,
Dept. of Computer Science and Engineering,
FEE, Czech Technical University

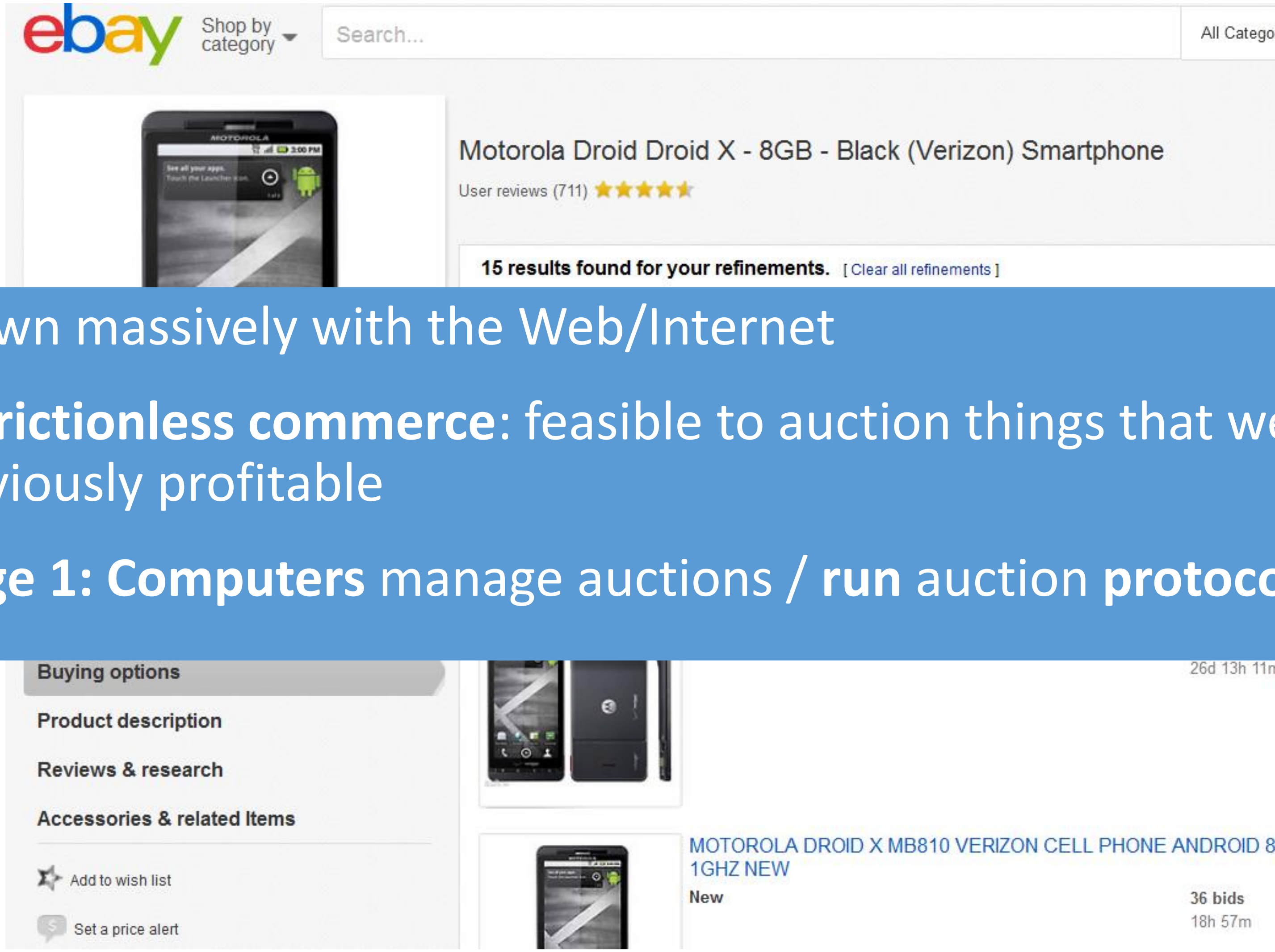
AE4M36MAS Autumn 2018- Lecture 12

Auctions: Traditional

Auctions used in Babylon as early as 500 B.C. but until relatively recently used only for high-value items for which it was difficult to asses the market price

Stage 0: No automation

Auctions: Partial Automation

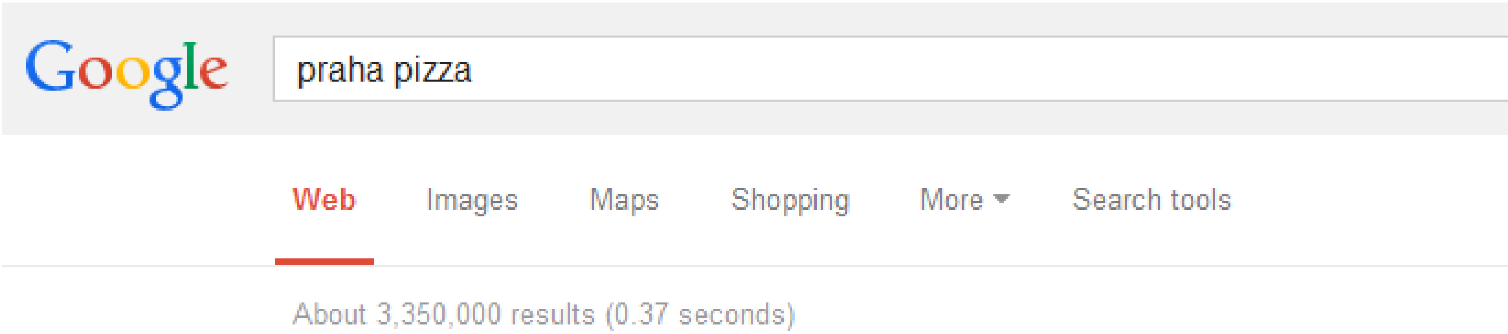


Grown massively with the Web/Internet

→ **Frictionless commerce**: feasible to auction things that weren't previously profitable

Stage 1: Computers manage auctions / **run** auction **protocols**

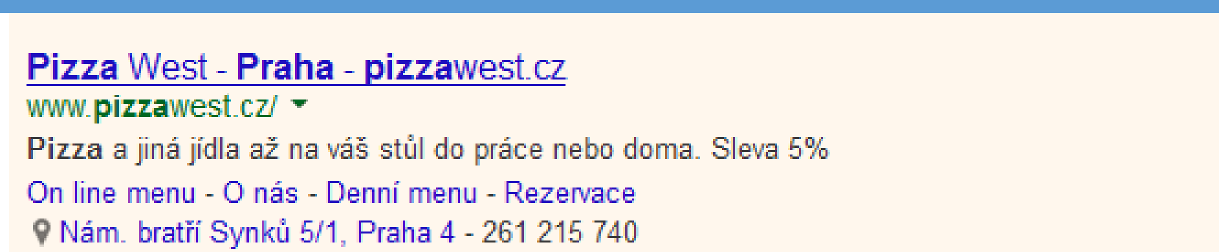
Auctions: (Almost) Full automation



Stage 2: Computers also automate the **decision making of bidders**

Concerns:

1) the most **relevant adds** are shown (→ user's are reasonably happy)
2) auctioner's **profit is maximized** (over longer time)



Lots of Applications

Industrial procurement

Transport and logistics

Energy markets

Cloud and grid computing

Internet auctions

(Electromagnetic spectrum allocation)

... and counting!

Introduction to Auctions

English Auction

1. Auctioneer starts the bidding at some **reservation price**
2. Bidders then shout out **ascending** prices (with minimum increments)
3. Once bidders stop shouting, the *high bidder* gets the good at that price

What is an Auction?

*An **auction** is a protocol that allows **agents** (=bidders) to indicate their **interests** in one or more **resources** and that uses these indications of interest to determine both an **allocation** of the resources and a set of **payments** by the agents.* [Shoham & Leyton-Brown 2009]

Auctions use employ **cardinal preferences** to express interest .

Auctions are mechanisms **with money**.

Auctions can be viewed as **games** of a specific structure.

Why Auctions?

Market-based price setting: for objects of unknown value, the value is dynamically assessed by the market!

Flexible: any object type can be allocated

Can be **automated**

- use of simple rules reduces complexity of negotiations
- well-suited for computer implementation

Revenue-maximising and **efficient allocations** are achievable

Auctions Rules

Auction mechanism is **specified** by **auction rules (→** *rules of the game)*

Bidding rules	Clearing rules	Information rules
How **offers** are made: • by whom • when • what their content is	Who gets which goods (**allocation**) and what money changes hands (**payment**).	**What information** about the state of the negotiation is *revealed* **to whom** and **when**.

Payoff

Agent's payoff
from participating in an auction

If winner

payoff=
agent's **valuation** of the item
− **price paid** for the item

If *not* winner

payoff=**zero**

Risk neutrality: the payoff is (as above) a *linear function* of the difference between the item's valuation and the price paid

- **risk seeking**: the payoff is a convex function of the difference (aggressively seeking high gains is prioritized)
- **risk aversion**: the payoff is a concave function of the difference (conservatively ensuring at least some gains is prioritized)

Valuation Models

Independent private value (IPV)	Correlated value
An agent A's valuation of the good is **independent from other agent's** valuation of the good (e.g. a taxi ride to the airport)	Valuations of the good are **related between agents** (typically the more other agents are prepared to pay, the more agent A prepared to pay – e.g. purchase of items for later resale)

Types of Auctions

multi-unit
multiple indistinguishable items (A, A, A)

single-good

multi-item
multiple different items (A,B,C)

Combinatorial auctions

multi-attribute
items characterized by multiple attributes (A=a_1, a_2, a_3)

Types of Auctions

Forward (sell-side) auction: selling

Reverse (buy-side) auction: buying

Single-sided: either selling or buying

Double-sided: both selling and buying (→ exchange)

There are other allocation mechanisms: facility location, allocation of divisible goods (cake cutting), allocation of indivisible goods (CPU, memory), …

Single-Item Auctions

Basic Auction Mechanisms

English

Japanese

Dutch

First-Price

Second-Price

English Auction

1. Auctioneer starts the bidding at some **reservation price**
2. Bidders then shout out **ascending** prices (with minimum increments)
3. Once bidders stop shouting, the *high bidder* gets the good at that price

Japanese Auctions

Same as an English auction except that the auctioneer calls out the prices

1. All bidders start out **standing**
2. When the price reaches a level that a bidder is not willing to pay, that bidder **sits down;** once a bidder sits down, they **can't get back up.**
3. The **last** person **standing** gets the good

Dutch Auction

1. The auctioneer starts a clock at some high value; it descends
2. At some point, a bidder shouts “mine!" and gets the good at the price shown on the clock

Good when items need to be sold **quickly** (similar to Japanese)

No information is revealed during auction

First-, Second-Price Sealed Bid Auctions

First-price sealed bid auction

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount of **his bid**

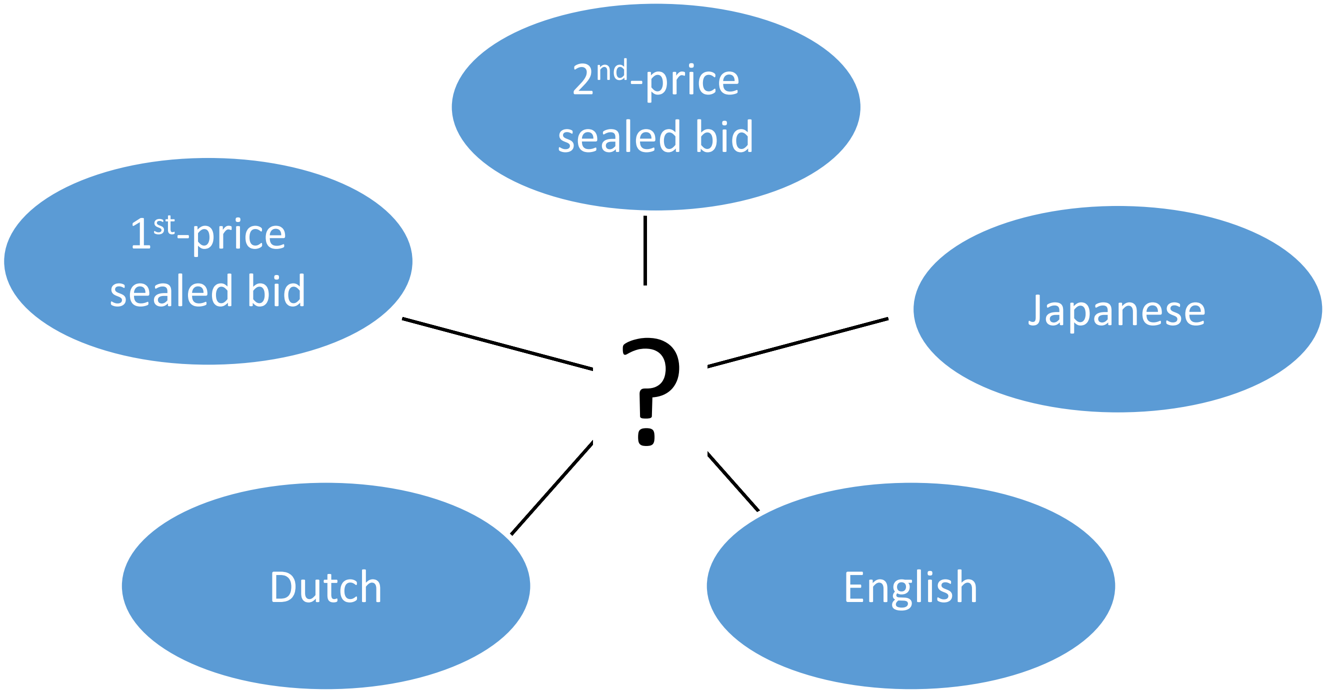
Second-price sealed bid auction (**Vickerey** auction)

- bidders write down bids on pieces of paper
- auctioneer **awards** the good to the bidder with the **highest bid**
- that bidder pays the amount bid by **the second-highest** bidder

Intuitive Comparison

	English	Dutch	Japanese	1^{st}-Price	2^{nd}-Price
Duration	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
Info Revealed	2^{nd}-highest val; bounds on others	winner's bid	all val's but winner's	none	none
Jump bids	yes	n/a	no	n/a	n/a
Price Discovery	yes	no	yes	no	no

Analysing Auctions



Are there fundamental similarities / differences between mechanisms described?

Two Problems

Design of auction mechanisms	**Analysis** of auction mechanisms
• design the auction mechanism (i.e. the game for the bidders) with the desirable properties • methodology: apply mechanism design techniques	• determine the properties of a given auction mechanism • methodology: treat auctions as (extended-form) *Bayesian games* and analyse players' (i.e. bidders') strategies

Bayesian Game

Definition (Bayesian Game)

A Bayesian game is a tuple $\langle N, A, \Theta, p, \boldsymbol{u} \rangle$ where

- N is the set of **players**
- $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, Θ_i is the **type space** of player i
- $A = A_1 \times A_2 \times \cdots \times A_n$ where A_i is the **set of actions** for player i
- $p: \Theta \mapsto [0,1]$ is a **common prior over types**
- $\boldsymbol{u} = (u_1, \dots, u_n)$, where $u_i: \mathrm{A} \times \Theta \mapsto \mathbb{R}$ is the **utility function** of player i

We assume that all of the above is **common knowledge** among the players, and that each **agent knows** his **own type**.

Bayes-Nash equilibrium: rational, risk-neutral players are seeking to maximize their expected payoff, given their beliefs about the other players' types.

Relation to (sealed bid) Auctions

Sealed bid auction under IPV is a Bayesian game in which

- player i's **actions** correspond to his **bids** $\widehat{v_i}$
- player types Θ_i correspond to players' **private valuations** v_i over the auctioned item(s)
- the **payoff** of player i corresponds to i's valuation of the item v_i – price paid (in the case of winning; zero otherwise)

Similar analogies for more complicated auction mechanisms

(Desirable) Properties

Truthfulness: bidders are incentivized to bid their *true* valuations, i.e.

$$v_i = \widehat{v_i} \; \forall i \forall v_i$$

Efficiency: the aggregated value of bidders is maximized, i.e.

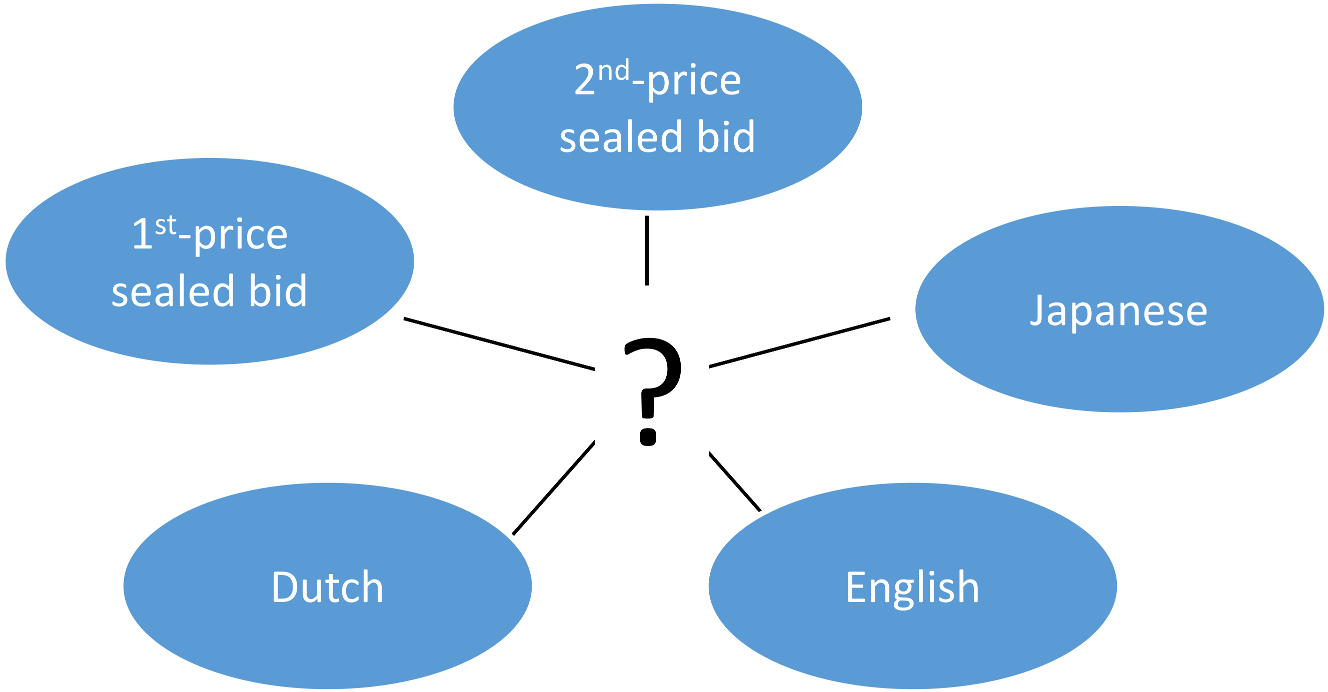
$$\forall v \forall x', \sum_i v_i(x) \geq \sum_i v_i(x')$$

Optimality: maximization of seller's revenue

Strategy: existence of dominant strategy

Manipulation vulnerability: lying auctioner, shills, bidder collusion

Other consideration: communication complexity, private information revelation, ...



Are there fundamental similarities / differences between mechanisms described?

Second-Price Sealed Bid

Theorem

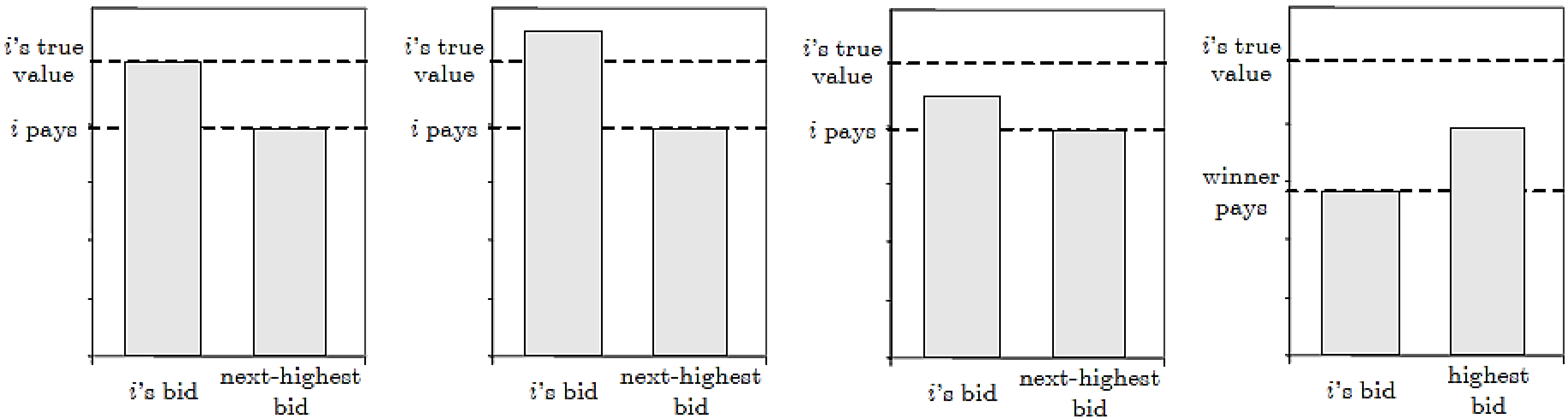
Truth-telling is a **dominant strategy** in a second-price sealed bid auction (assuming independent private values – IPV).

Proof: Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

- Bidding honestly, i would win the auction
- Bidding honestly, i would lose the auction

Second-Price Sealed Bid Proof

Bidding honestly, i is the winner



If i bids higher, he will still win and still pay the same amount

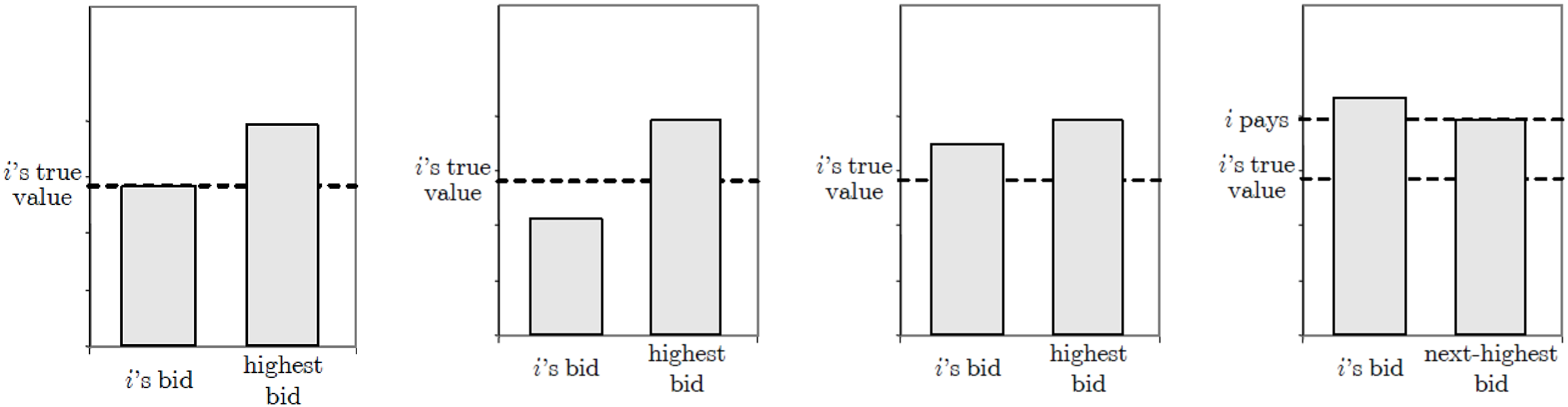
If i bids lower, he will either still win and still pay the same amount. . .

... or lose and get the payoff of zero.

➔ There is a disadvantage bidding lower and no advantage bidding higher

Second-Price Sealed Bid Proof

Bidding honestly, i is not the winner



If i bids lower, he will still lose and still pay nothing

If i bids higher, he will either still lose and still pay nothing...

... or win and pay more than his valuation ($\Rightarrow$ negative payoff).

➔ There is a disadvantage bidding higher and no advantage bidding lower

Second-Price Sealed Bid

Advantages:

- **Truthful** bidding is dominant strategy
- No incentive for counter-speculation
- Computational efficiency

Disadvantages:

- Lying auctioneer
- Bidder collusion self-enforcing
- **Not revenue maximizing**

Dutch and First-price Sealed Bid

Strategically equivalent: an agent bids without knowing about the other agents' bids

- a bidder must decide on the amount he's willing to pay, conditional on having placed the highest bid

Differences

- First-price auctions can be held **asynchronously**
- Dutch auctions are **fast**, and require **minimal communication**

Bidding in Dutch / First Price Sealed Bid

Bidders don't have a **dominant strategy** any more:

⇐ there's a **trade-off** between **probability of winning** vs. **amount paid** upon winning

- **individually optimal** strategy depends on the **assumptions** about **others' valuations**

Assume a **first-price auction** with **two risk-neutral bidders** whose valuations are drawn independently and **uniformly** at random from the interval [0, 1] - what is the equilibrium strategy?

→ $\left(\frac{1}{2}v_1, \frac{1}{2}v_2\right)$ is the Bayes-Nash equilibrium strategy profile

⇒ Dutch / FPSB auctions **not incentive compatible,** i.e., there are incentives to counter-speculate.

Bidding in Dutch / First Price Sealed Bid

Theorem

In a first-price sealed bid auction with n **risk-neutral** agents whose valuations $v_1, v_2, \ldots, v_n$ are **independently** drawn from a **uniform distribution** on the **same** bounded **interval** of the real numbers, the **unique** symmetric **equilibrium** is given by the **strategy profile** $(\frac{n-1}{n}v_1,\ldots,\frac{n-1}{n}v_n)$.

For non-uniform valuation distributions: Each bidder should bid **the expectation of the second-highest valuation**, conditioned on the assumption that his own valuation is the highest.

English and Japanese Auctions Analysis

A much more complicated **strategy space**

- extensive-form game
- bidders are able to condition their bids on information revealed by others
- in the case of English auctions, the ability to place jump bids

Intuitively, though, the **revealed information** does not make any **difference** in the **independent-private value** (IPV) setting.

English and Japanese Auctions Analysis

Theorem

Under the IPV model, it is a **dominant strategy** for bidders to bid **up to** (and not beyond) their valuations in both Japanese and English auctions.

In correlated-value auctions, it can be worthwhile to counter-speculate

Revenue Equivalence

Which auction should an auctioneer choose?

To some extent, it doesn't matter...

Theorem (Revenue Equivalence)

Assume that each of n **risk-neutral** agents has an **independent private valuation** for a single good at auction, drawn from a **common cumulative distribution** $F(v)$ that is **strictly increasing** and **atomless** on $[\underline{v}, \overline{v}]$. Then any auction mechanism in which

1. the good will be allocated to the agent with the highest valuation; and
2. any agent with valuation $\underline{v}$ has an expected utility of zero

yields the **same expected revenue**, and hence results in any bidder with valuation v making the same expected payment.

Revenue Equivalence

Assuming bidders are risk neutral and have independent private valuations, **all the auctions** we have spoken about so far—English, Japanese,Dutch, and all sealedbid auction protocols—are **revenue equivalent**.

What about Efficiency?

Efficiency in single-item auctions: the item allocated to the agent who values it the most.

With independent private values (IPV):

Auction	Efficient
English (without reserve price)	yes
Japanese	yes
Dutch	no
Sealed bid second price	yes
Sealed bid first price	no

Efficiency (often) lost in the correlated value setting.

Optimal Auctions

Optimal Auction Design

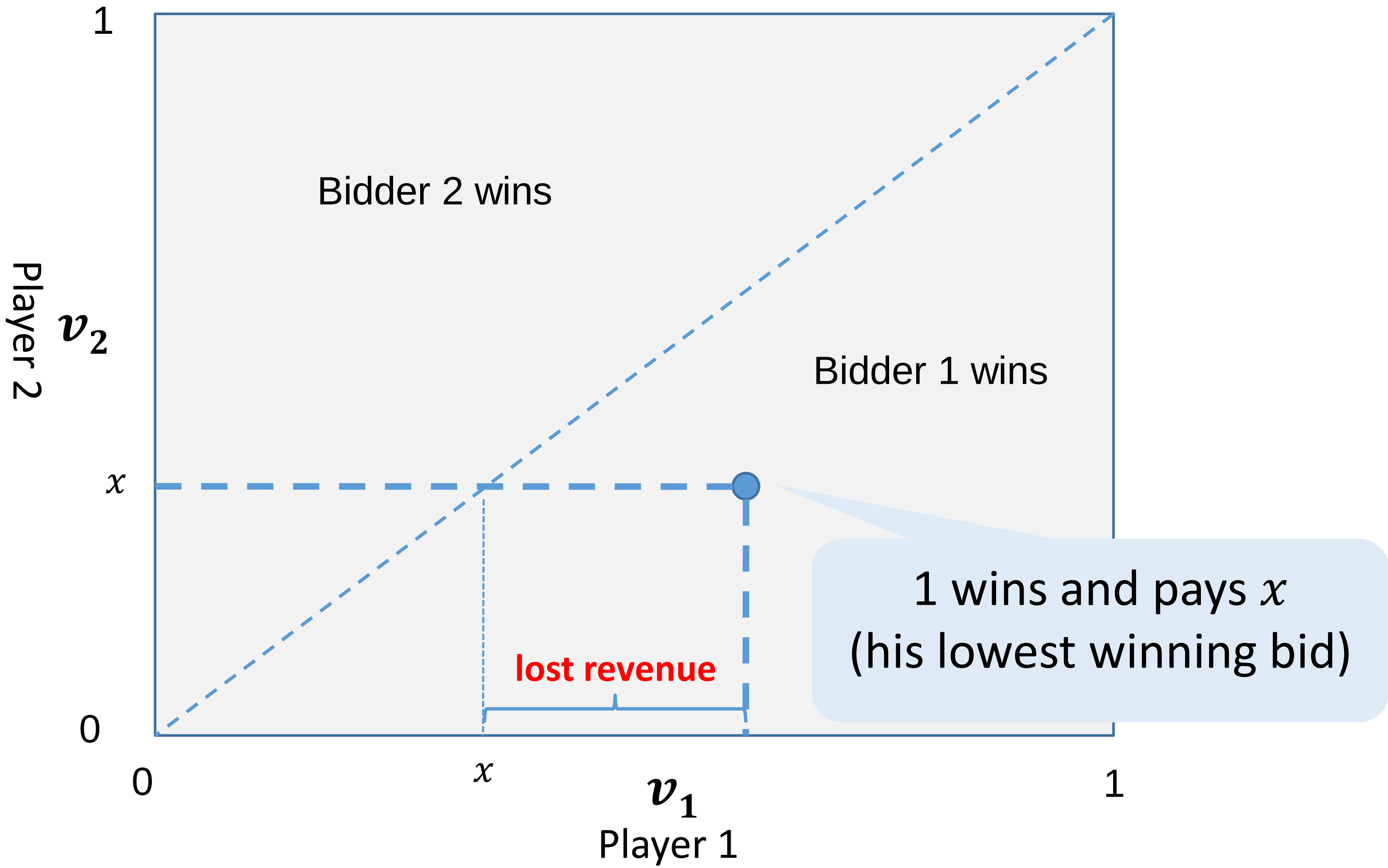
The seller's problem is to **design an auction mechanism** which has a Nash equilibrium giving him/her the **highest possible expected utility**.

- assuming individual rationality

Second-prize sealed bid auction **does not maximize** expected revenue → not the best choice if profit maximization is important (in the short term).

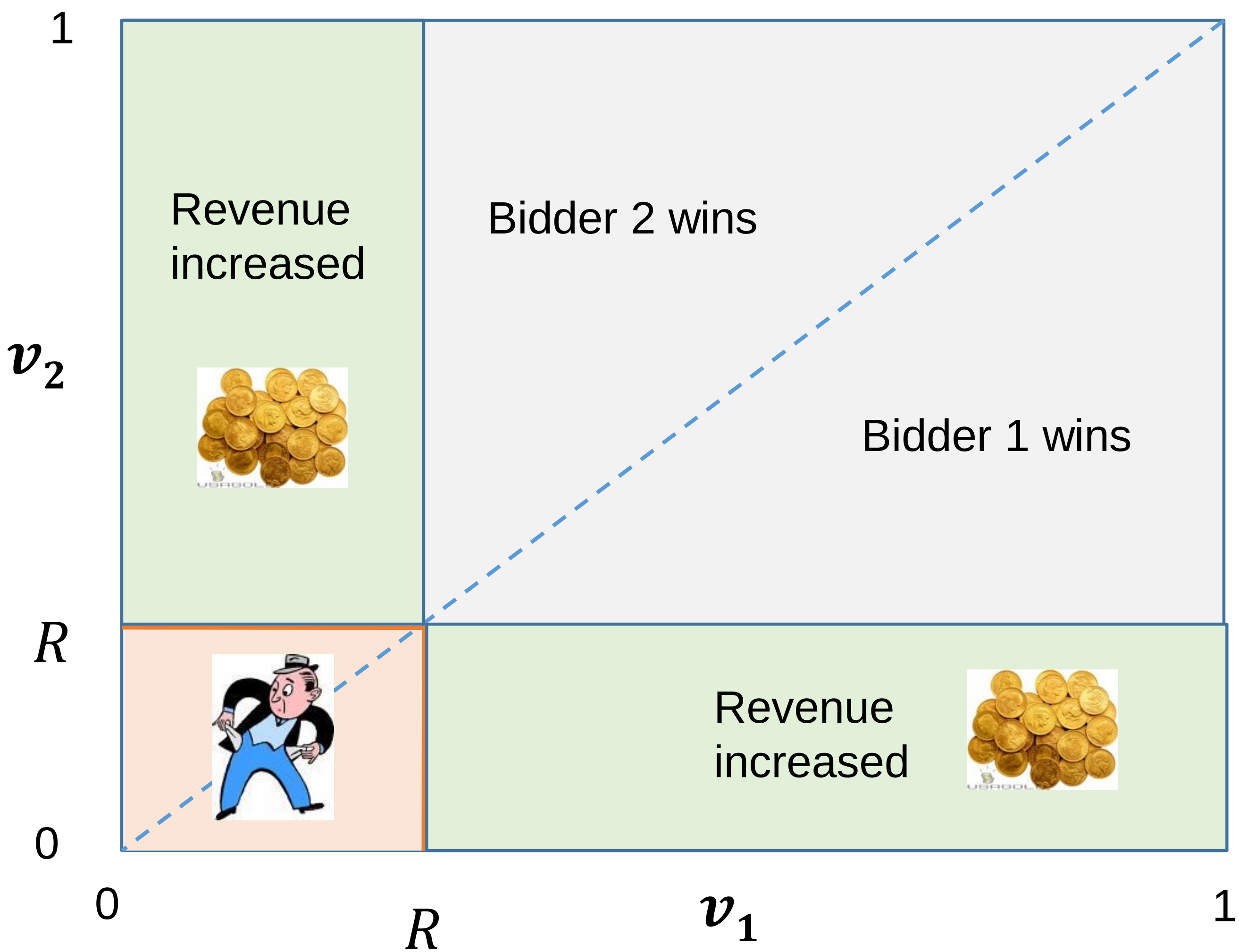
Can we get better revenue?

Let's have another look at 2nd price auctions:

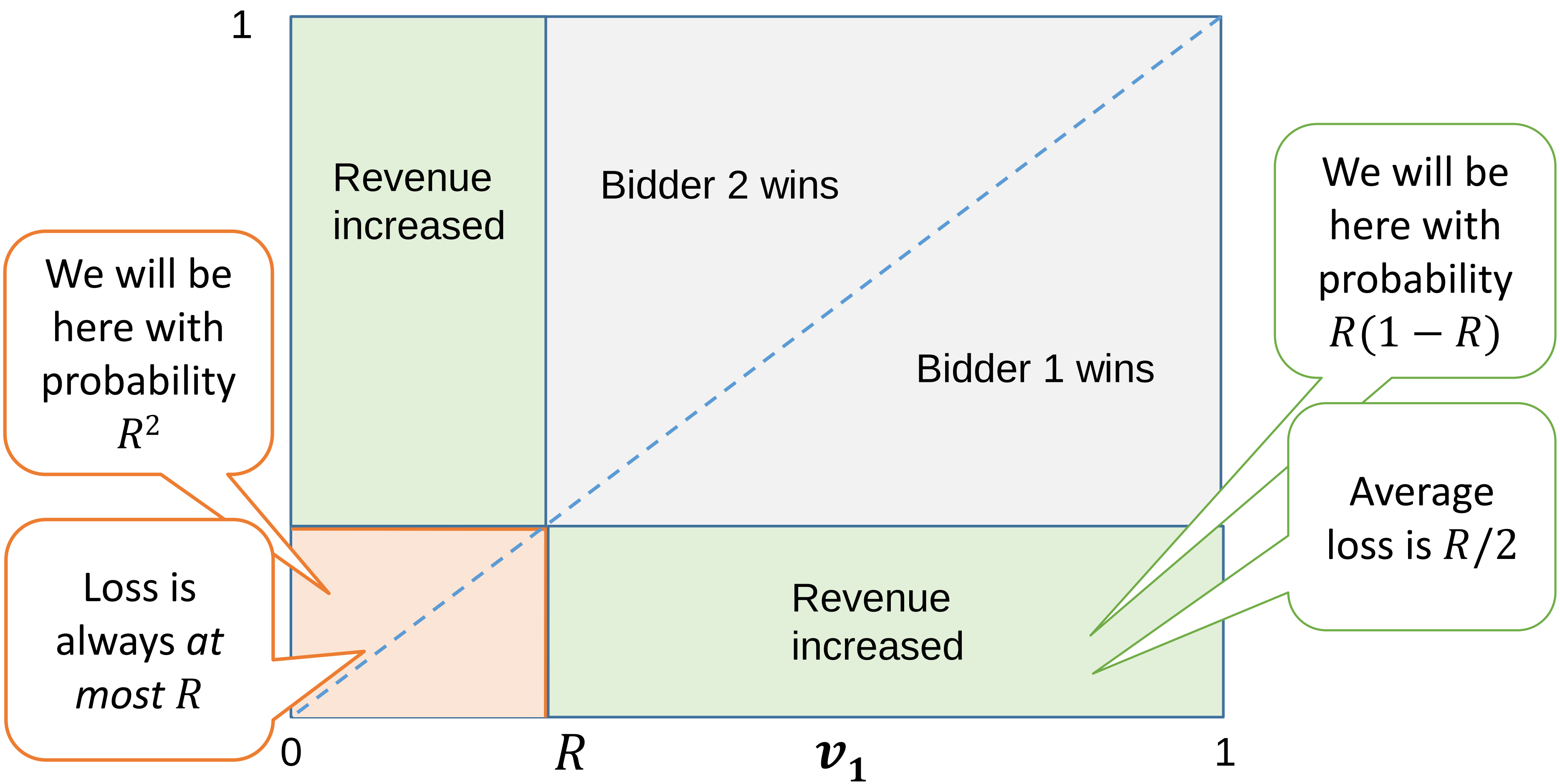


Can we get better revenue?

Some reserve price **improves revenue**.



Can we get better revenue?



Gain is at least: $\frac{2R(1-R)R}{2} = R^2 - R^3$

Loss is at most: $R^2\,R = R^3$

When $R^2 - 2R^3 > 0$,

reserve price of $\boldsymbol{R}$ is beneficial.

(for example, $R = 1/4$)

Optimal Single Item Auction

Definition (Virtual valuations)

Consider an **IPV setting** where bidders are **risk neutral** and each bidder i's valuation is drawn from some **strictly increasing** cumulative density function $F_i(v)$, having probability density function $f_i(v)$. We then define:
where

- Bidder i's **virtual valuation** is $\psi_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$
- Bidder i's **bidder-specific reserve price** r_i^* is the value for which $\psi_i(r_i^*) = 0$

Example: uniform distribution over $[0,1]$: $\psi(v) = 2v - 1$

Optimal Single Item Auction

Theorem (Optimal Single-item Auction)

The optimal (single-good) auction is a **sealed-bid auction** in which every agent is asked to declare his valuation. The good is sold to the agent $i = \mathrm{argmax_i} \psi_i(\hat{v}_i)$, as long as $\hat{v}_i > r_i^*$.
If the good is sold, the winning agent i is charged the smallest valuation that he could have declared while still remaining the winner:

$$\inf\{v_i^* : \psi_i(v_i^*) \geq 0 \wedge \forall j \neq i, \psi_i(v_i^*) \geq \psi_j(\hat{v}_j)\}$$

Can be understood as a second-price auction with a reserve price, held **in virtual valuation space** rather than in the space of actual valuations.

Remains **dominant-strategy truthful.**

Second-Price Auction with Reservation Price

Symmetric case: second-price auction with reserve price r^* satisfying: $\psi(r^*) = r^* - \frac{1-F(r^*)}{f(r^*)} = 0$

- Truthful mechanism when $\psi(v)$ is non-decreasing.
- Uniform distribution over $[0, p]$: optimum reserve price $p/2$.

Second-price sealed bid auction with Reserve Price is **not efficient**!

Optimal Auctions: Remarks

Always: **revenue ≤ efficiency**

- due to **individual rationality**
- more efficiency makes the pie larger!

However, for **optimal revenue** one needs to **sacrifice** some **efficiency**.

Optimal auctions are not **detail-free:**

- they require the seller to incorporate information about the bidders' valuation distributions into the mechanism.

Theorem (Bulow and Klemperer): *revenue* of an efficiency-maximizing auction with *k+1* bidder is at least as high as that of the revenue-maximizing one with *k* bidders.

➔ better to spend energy on attracting more bidders

Auctions Summary

Auctions are mechanisms for **allocating scarce resource** among **self-interested agent**

Mechanism-design and game-theoretic perspective

Many auction mechanisms: English, Dutch, Japanese, First-price sealed bid, Second-price sealed bid

Desirable properties: truthfulness, efficiency, optimality, ...

Rapidly expanding list of **applications** worth billions of dollars

Reading:

- [Shoham] – Chapter 11

Final Notes

Rapidly evolving field with the exploding number of applications
→ http://aic.fel.cvut.cz/ for (Ph.D.) opportunities

Exams: 15/1, 23/1 and 29/1 9:00-12:00

Survey/Anketa: be as specific possible: we *do* care

How to get around impossibility results

Mechanisms with money

Measure not just that a preferred to b, but also "by how much"…

Each individual j (or player j) has a "valuation" for each alternative a in A. Denoted as $v_j(a)$

Also, each player values money the same.

So, if we choose alternative a, and give \$m to j, then j's "utility" is $v_j(a) + m$

Auction Protocols

Auctions are centralised mechanisms for the allocation of goods amongst several agents. Agents report their preferences (bidding) and the auctioneer decides on the final allocation (and on prices).

- Distinguish *direct* and *reverse* auctions (auctioneer buying).
- Bidding may be *open-cry* (English) or by *sealed bids*.
- Open-cry: *ascending* (English) or *descending* bids (Dutch).
- Pricing rule: *first-price* or *second-price* (Vickrey).
- *Combinatorial auctions:* several goods, sold/bought in bundles.

R.P. McAfee and J. McMillan. Auctions and Bidding. *Journal of Economic Literature*, 25:699–738, 1987.

P. Cramton, Y. Shoham, and R. Steinberg (eds.). *Combinatorial Auctions*. MIT Press, 2006.