

# Extensive-Form Games

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Previously ... on multi-agent systems (tutorials and lectures).

- 1 Extensive-Form Games (game trees)

# Normal-form LP

Reminder from lectures:

$$\max_{s,U} U \quad (1)$$

$$\text{s.t.} \quad \sum_{a_1 \in \mathcal{A}_1} s(a_1) u_1(a_1, a_2) \geq U \quad \forall a_2 \in \mathcal{A}_2 \quad (2)$$

$$\sum_{a_1 \in \mathcal{A}_1} s(a_1) = 1 \quad (3)$$

$$s(a_1) \geq 0 \quad \forall a_1 \in \mathcal{A}_1 \quad (4)$$

# Sequence-form LP

Reminder from lectures:

$$\max_{r_1, v} v(\text{root}) \quad (5)$$

$$\text{s.t.} \quad r_1(\emptyset) = 1 \quad (6)$$

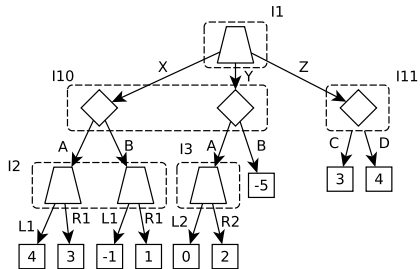
$$0 \leq r_1(\sigma_1) \leq 1 \quad \forall \sigma_1 \in \Sigma_1 \quad (7)$$

$$\sum_{a \in \mathcal{A}(I_1)} r_1(\sigma_1 a) = r_1(\sigma_1) \quad \forall \sigma_1 \in \Sigma_1, \forall I_1 \in \text{inf}_1(\sigma_1) \quad (8)$$

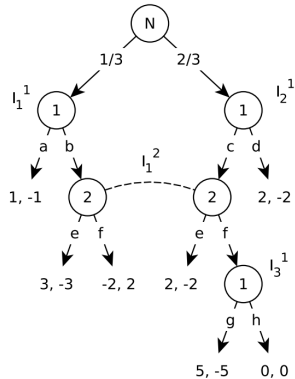
$$\sum_{I' \in \mathcal{I}_2: \sigma_2 a = \text{seq}_2(I')} v(I') + \sum_{\sigma_1 \in \Sigma_1} g(\sigma_1, \sigma_2 a) r_1(\sigma_1) \geq v(I) \quad \forall I \in \mathcal{I}_2, \sigma_2 = \text{seq}_2(I), \forall a \in \mathcal{A}(I) \quad (9)$$

- $\text{seq}_i(I)$  is a sequence of player  $i$  to information set,
- $I \in \mathcal{I}_i$ ,  $v_I$  is an expected utility in an information set,
- $\text{inf}_i(\sigma_i)$  is an information set, where the last action of  $\sigma_i$  has been executed,
- $\sigma_i a$  denotes an extension of a sequence  $\sigma_i$  with action  $a$

Task 1: Consider the following games. Write down a sequence-form linear program for both players:



Task 2: Consider the following games. Write down a sequence-form linear program for both players:



Task 3: Write down a sequence-form linear program for both players for the following game of a small “poker”:

- there is an ante of 1\$
- there is a limited deck of cards  $\{J, J, Q, Q\}$
- each player receives a card
- player 1 either folds or bets 2\$
- player 2 either calls or folds
- player with the higher card wins