Auctions

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Simple Auction Example

Consider a second-price, sealed-bid auction with two bidders who have independent, private values v_i which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 0.5 and they both play equilibrium strategies.

- What is the seller‘s expected revenue?
- Now let‘s suppose that there are three bidders who have independent, private values v_i which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 0.5. What is the sellers expected revenue in this case?
- How the situation changes in both cases if we have first-price sealed bid auction?

Simple Auction Example

A seller runs a second-price, sealed-bid auction for an object. There are two bidders, a and b, who have independent, private values v_i which are either 0 or 1. For both bidders the probabilities of $v_i = 0$ and $v_i = 1$ are 0.5 each. Both bidders understand the auction, but bidder b sometimes makes a mistake about his value for the object. Half of the time his value is 1 and he is aware that it is 1 the other half of the time his value is 0 but occasionally he mistakenly believes that his value is 1. Lets suppose that when b's value is 0 he acts as if it is 1 with probability 0.5 and as if it is 0 with probability 0.5. So in effect bidder b sees value 0 with probability 0.25 and value 1 with 0.75 probability. Bidder a never makes mistakes about his value for the object, but he is aware of the mistakes that bidder b makes. Assume that if there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.

- Assume bidder b is not aware of his mistake and bids optimally given the perceptions of the value of the object. Is bidding his true value still a dominant strategy for bidder a?

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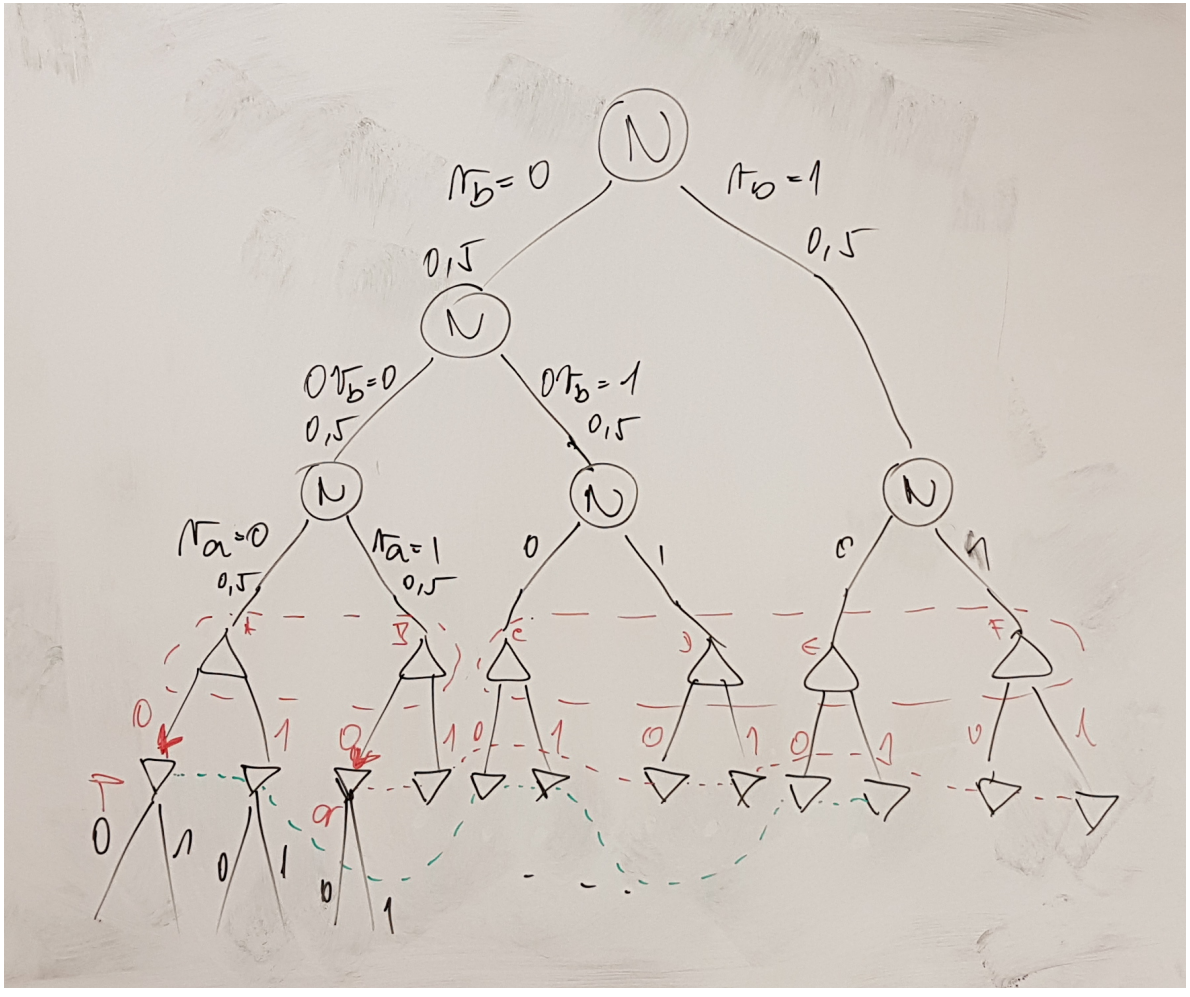
- Assume that b is aware of his mistake and bids must be integers. What are the optimal strategies?

Simple Auction Example

Solution:



When player b observes 1, the expected value for bidding 0 is 0.125; the expected value for bidding 1 is 0.1825. Bidding truthfully (w.r.t. to observed values) is still the optimal strategy.

Simple Auction Example

Consider a first-price, sealed-bid auction with two bidders who have independent, private values v_i which are independent and uniformly distributed over the set $\{0, 1, 2\}$. The bids in the auction must be nonnegative integers. Assume that ties are broken randomly.

- What is an equilibrium strategy?
- Find all equilibria.