Auctions

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Simple Auction Example

Consider a second-price, sealed-bid auction with two bidders who have independent, private values $v_i$ which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 0.5 and they both play equilibrium strategies.

- What is the seller’s expected revenue?
- Now let’s suppose that there are three bidders who have independent, private values $v_i$ which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 0.5. What is the sellers expected revenue in this case?
- How the situation changes in both cases if we have first-price sealed bid auction?
A seller runs a second-price, sealed-bid auction for an object. There are
two bidders, $a$ and $b$, who have independent, private values $v_i$ which are
either 0 or 1. For both bidders the probabilities of $v_i = 0$ and $v_i = 1$ are
0.5 each. Both bidders understand the auction, but bidder $b$ sometimes
makes a mistake about his value for the object. Half of the time his value
is 1 and he is aware that it is 1 the other half of the time his value is 0
but occasionally he mistakenly believes that his value is 1. Lets suppose
that when $b$’s value is 0 he acts as if it is 1 with probability 0.5 and as if
it is 0 with probability 0.5. So in effect bidder $b$ sees value 0 with
probability 0.25 and value 1 with 0.75 probability. Bidder $a$ never makes
mistakes about his value for the object, but he is aware of the mistakes
that bidder $b$ makes. Assume that if there is a tie at a bid of $x$ for the
highest bid the winner is selected at random from among the highest
bidders and the price is $x$.

Assume bidder $b$ is not aware of his mistake and bids optimally
given the perceptions of the value of the object. Is bidding his true
value still a dominant strategy for bidder $a$?
A seller runs a second-price, sealed-bid auction for an object. There are two bidders, \(a\) and \(b\), who have independent, private values \(v_i\) which are either 0 or 1. For both bidders the probabilities of \(v_i = 0\) and \(v_i = 1\) are 0.5 each. Both bidders understand the auction, but bidder \(b\) sometimes makes a mistake about his value for the object. Half of the time his value is 1 and he is aware that it is 1 the other half of the time his value is 0 but occasionally he mistakenly believes that his value is 1. Let’s suppose that when \(b\)’s value is 0 he acts as if it is 1 with probability 0.5 and as if it is 0 with probability 0.5. So in effect bidder \(b\) sees value 0 with probability 0.25 and value 1 with 0.75 probability. Bidder \(a\) never makes mistakes about his value for the object, but he is aware of the mistakes that bidder \(b\) makes. Assume that if there is a tie at a bid of \(x\) for the highest bid the winner is selected at random from among the highest bidders and the price is \(x\).

- Assume that \(b\) is aware of his mistake and bids must be integers.
  What are the optimal strategies?
Solution:

When player $b$ observes 1, the expected value for bidding 0 is 0.125; the expected value for bidding 1 is 0.1825. Bidding truthfully (w.r.t. to observed values) is still the optimal strategy.
Consider a first-price, sealed-bid auction with two bidders who have independent, private values $v_i$ which are independent and uniformly distributed over the set \{0, 1, 2\}. The bids in the auction must be nonnegative integers. Assume that ties are broken randomly.

- What is an equilibrium strategy?
- Find all equilibria.