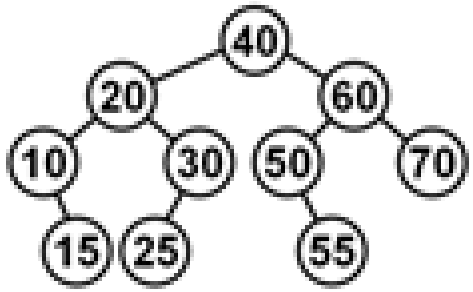
PAL cv. 12

December 7, 2022

There is an AVL tree at the figure. We further delete nodes with 50, 30, and 25 in this order. Decide if some rotation is going to take a place and if so, which kind of rotation it is (e.g. right-right).



Draw and AVL tree with 8 keys such that after inserting a key with 19 a rotation will have to take a place

a) a left-right rotation in the root

b) a left-right rotation in a non-root node

We have an unempty AVL tree. We firstly remove key x, and then we insert x again into the tree. Do the original and final trees have the same shape? Is it dependent on the value x?

Start with an empty splay tree and insert 2, 7, 1, 4, 3, 9, 5, 6. Draw a tree after each insertion.

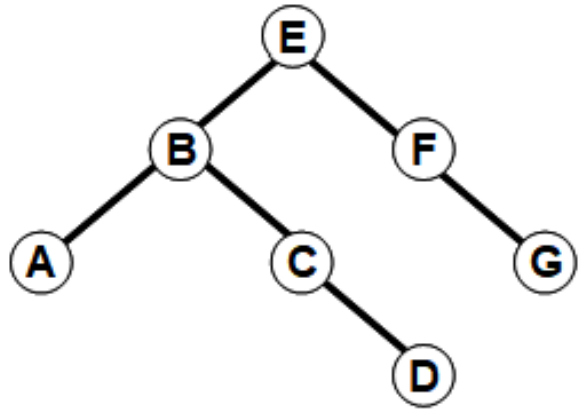
There is a splay tree which was initially empty and we insert 1, 2, 3, ..., 10 into the tree. Thereafter, we delete keys 1, 2, and 3. Draw the final tree. Assume that the deletion operation uses *findMaxKey* applied on the left subtree.

Splay tree contains N keys, there are two of them x and y (which are distinct). There is this sequence of operation (Find(x), Find(y), Find(x), Find(y), ..., Find(x), Find(y)) applied on the tree, length of the sequence is $2N$. What is the average number of visited nodes for one Find operation? Derive the most precise upper and lower bound.

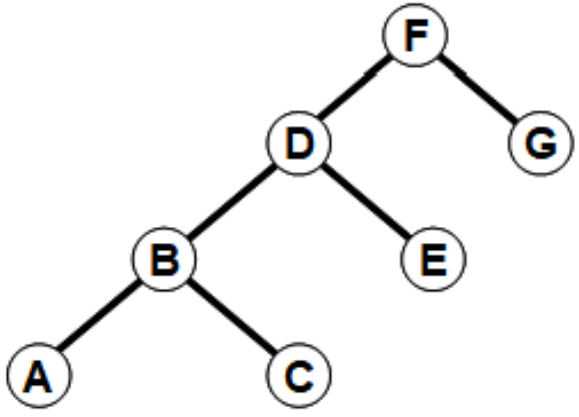
Splay tree contains $2^n - 1$ key with values $1, 2, 3, ..., 2^n - 1$ and is ideally balanced, i.e. the depth is $n - 1$. After finding the node with key 1, that node becomes a root. What will be the depth of the final tree? Solve for even and odd n separately.

Design a red-black coloring of these trees, so you obtain a correctly colored red-black tree. The empty (nil) leaves are not shown.

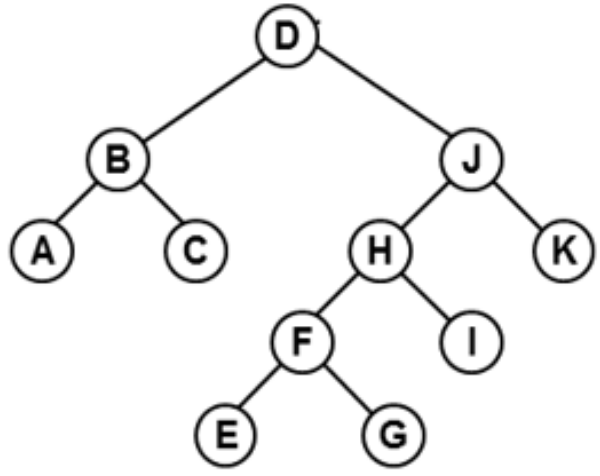
A.



B.



C.



Red-black tree has black depth of 11. Black depth is the number of black nodes on the path from root to a leaf (containing a key) decreased by 1. Decide what is the maximal possible number of

A) black nodes

B) red nodes

C) all nodes

Decide if there exists a regular binary tree (each inner node has two children), which cannot be colored according to red-black trees.