

FSM Testing and Checking Sequences

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1 Finite State Machine

- Definitions

2 Finite state machine testing

- Terminology
- Formal FSM Testing
- Example
- Characterization Set Construction

Outline

- 1 Finite State Machine
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 - Terminology
 - Formal FSM Testing
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Finite Machine in Applications [Bei95, HI98]

- a model for testing of application driven using menu
- a model of communication protocols
- a model used in object-oriented design

Finite State Machine

- an abstract machine which the number of states and input symbols is finite and constant.
- consists of
 - states (nodes) ... future behavior is fully determined by a given state,
 - transitions (edges) ... behavioral rules,
 - input symbols (labels of edges) ... environmental stimuli, and
 - output symbols (labels of edges or nodes) ... external reactions.



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Finite State Machine ^[HI98]

- Let *Input* be a finite alphabet.
- *Finite state machine* over *Input* consists of the following items:
 - 1 A finite set Q of elements called *states*.
 - 2 A subset I of the set Q containing *initial states*.
 - 3 A subset T of the set Q containing *end states*.
 - 4 A finite set of *transitions*, that returns a set of all possible next states for each state and each symbol of the input alphabet.

Transition function

$$F : Q \times Input \rightarrow \mathcal{P}Q$$

- $F(q, input)$ contains all possible states of the automaton, to which it is possible to make a transition if the input symbol *input* is accepted in state q .
- $\mathcal{P}Q$ denotes a set of all subsets of the set Q
(a *power set of the set Q* , CZ *potenční množina množiny*).

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Finite State Machine with Output (Mealy) ^[H198]

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Output function

$$G : Q \times \text{Input} \rightarrow \text{Output}$$

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- F and G might be partial functions.

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Finite State Machines Examples ^[HI98]

A set *Input* of input symbols

- Actions or commands of the user entered through a keyboard,
- Mouse clicks or moves,
- Signals accepted by a sensor.

A set *Q* of states

- Values of certain important variables of the system,
- A behavioral model of the system,
- A formular type visible on the monitor,
- Whether devices are active or not.

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State Diagram ^[Bei95]

- **Nodes:** represent states (a state of the software application).
- **Edges:** represent transitions (a menu item selection).
- **Edge attributes (input symbols):** e.g. mouse actions, Alt+Key, function keys, keyboard keys of cursor movement.
- **Edge attributes (output symbols):** e.g. a menu presentation or a next window open.

Space ship model *Enterprise*

- three modes of the impulse engine:
move forward(d), neutral(n), and move backward(r).
- three possible state of movement:
forward(F), stop(S), and backward(B).
- their combinations creates nine states:
 DF , DS , DB , NF , NS , NB , RF , RS , and RB .
- possible inputs: $d > d$, $r > r$, $n > n$, $d > n$, $n > d$, $n > r$, $r > n$.

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- possible inputs: $d > d, r > r, n > n, d > n, n > d, n > r, r > n$.

Enterprise State Space [Bei95]

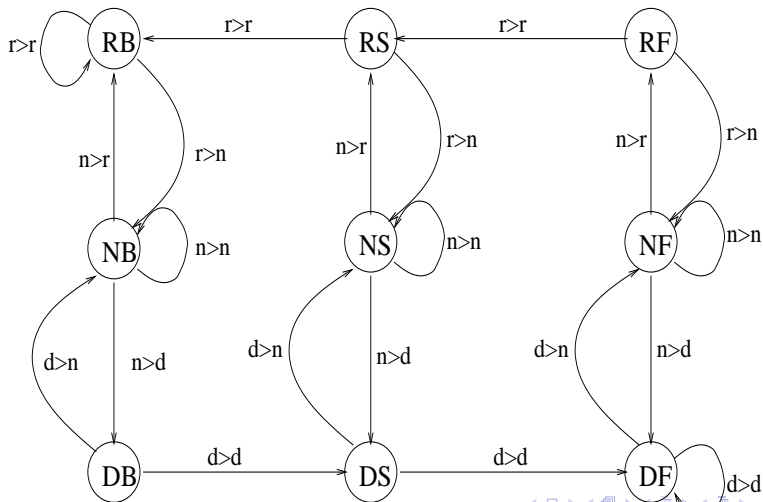
BACKWARD



STOPPED



FORWARD



State Diagram Properties ^[Bei95]

Properties

- A strong connected graph,
- State graphs grow very quickly,
- All possible and impossible inputs are considered in every state
 - the implementation of the system might be incorrect.
- Nice symmetry is a very rare case in real life.

Transition Table ^[Bei95]

A transition table

- has a row for each state
- has a column for each input.
- In fact, there are two tables with the same shape:
 - a transition table,
 - an output table.
- A value in the transition table represents the next state.
- A value in the output table is the output code for a given transition.
- **Hierachical (nested) automata** are the only way how huge tables can be avoided (e.g. statechart, starchart, etc.)



Enterprise Transition Table ^[Bei95]

Enterprise

STATE	$r > r$	$r > n$	$n > n$	$n > r$	$n > d$	$d > d$	$d > n$	$r > d$	$d > r$
RB	RB	NB							
RS	RB	NS							
RF	RS	NF							
NB			NB	RB	DB				
NS			NS	RS	DS				
NF			NF	RF	DF				
DB						DS	NB		
DS						DF	NS		
DF						DF	NF		



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State Reachability ^[Bei95]

- **Reachable state:** a state B is reachable from a state A , if there is a input sequence such that the system is transferred from the state A to the state B .
- **Unreachable state:** a state is unreachable if it is not reachable, especially from the initial state. Unreachable states implies typically a mistake.
- **Strong connectivity:** all state of the finite automaton are reachable from the initial state. Most practical models are strongly connected if they do not contain mistakes.
- **Isolated states:** a set of states that are not reachable from the initial state. If they exist, then they are very suspicious, mistaken states.
- **Reset:** a special input symbol/action causing the transition from any state to the initial state.



State Categories ^[Bei95]

- **The set of the initial state:** If a transition leading from this set is performed, then there is no way back to this set (e.g. a boot of the system).
- **Working states:** When the set of the initial state is left, then the system works in a strongly connected set of states in which a majority of testing is performed.
- **The initial state of the working set:** a state of the working set which can be considered as the “initial state”.
- **The set of ending state:** If the system reaches this set, then there is not way back to the working set, e.g. a finalizing sequence, a shutdown.
- The system is **fully specified** if transitions and output symbols are defined for all combinations of input symbols and states.
- **A round trip of the state A :** a sequence of transitions going from the state A to a state B and back to the state A .



Test Design ^[Bei95]

- Each state begins in the initial state.
- The system is transferred
 - from the initial state using the shortest path to the selected state,
 - the given transition is performed,
 - and the system is transferred using the shortest path back into the initial state,
 - i.e. we create a round trip.
- Each test is build upon the preceding simpler tests.
- The input symbol is determined for each transition of the round trip.
- The output symbol is determined for all associated transitions of the round trip.
- **We verify**
 - input codes,
 - output codes,
 - states,
 - each transition.
- **Are all end states reachable?**

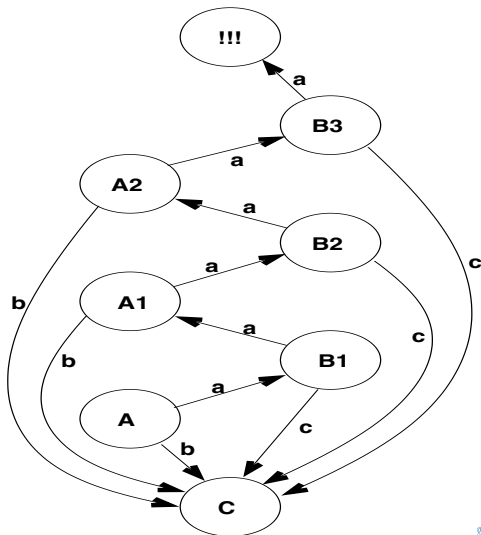
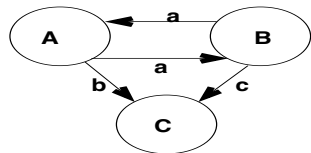


Hidden States

- **Is the system in the initial state?**
 - A test cannot be started if there is no confirmation that the system is in the initial state.
 - Applications store their settings in a persistent way.
 - If a previous test fails, in what state is the application?
- **Hidden state:** an unknown state that is different from a given state but it has all transitions with the same input and output codes, i.e. it cannot be distinguish from the given state.
- **Has the implementation hidden states?**
 - During the software testing we might assume conditions that are not valid generally.
 - e.g. we know in which state the state is.
 - Often, we do not dealt with one or two hidden states, but the state space doubles and is multiplied in other way.



Hidden States - Example



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Finite State Machine Testing ^[HI98]

- Based on the isomorphism of finite state machines,

- $\mathcal{A} = (\text{Input}, Q, \mathbf{F}, q_0)$

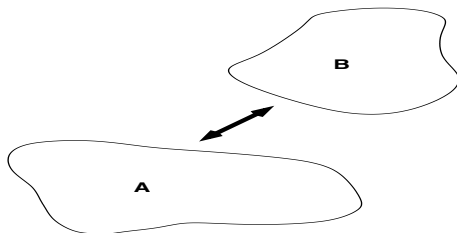
- $\mathcal{A}' = (\text{Input}, Q', \mathbf{F}', q_0')$

- $g : \mathcal{A} \rightarrow \mathcal{A}'$

- $g : Q \rightarrow Q'$

- 1 $g(q_0) = q_0'$

- 2 $\forall q \in Q, \text{input} \in \text{Input},$
 $g(\mathbf{F}(q, \text{input})) = \mathbf{F}'(g(q), \text{input})$



Test Set Construction [HI98, Cho78]

Chow's W method

- Let L be a set of input sequences and q, q' be two states.
- L *distinguishes* (CZ rozliší) the state q from q' if there is a sequence $k \in L$ such that the output sequence obtained by the application of k to the machine in the state q is different from the output sequence obtained by the application of k to the state q' .
- The machine is *minimal* if it does not contain redundant states.
- A set of input sequences W is called a *characterization set* if it can distinguish any two state of the machine.
- **A state cover** is a set L of input sequences such that it is possible to find an element of L using which we can reach the given state from the initial state q_0 .
- **A transition cover** of the minimal machine is a set T of input sequences such that it is a state cover closed under the right composition with the input set $Input$.
 - $sequence \in T = L \bullet (Input^1 \cup \{\langle \rangle\})$



Test Set Generation [HI98, Cho78]

- How many times are there more states than in the specification? (k)
- $Z = Input^k \bullet W \cup Input^{k-1} \bullet W \cup \dots \cup Input^1 \bullet W \cup W$
 - If A and B are two sets of sequences over the same alphabet, then $A \bullet B$ denotes a set of sequences composed from the sequences of the set A followed by a sequence from B .
 - k steps into an “unknown” space are performed followed by the verification of the state.

- Finite **test set**:

$$T \bullet Z$$

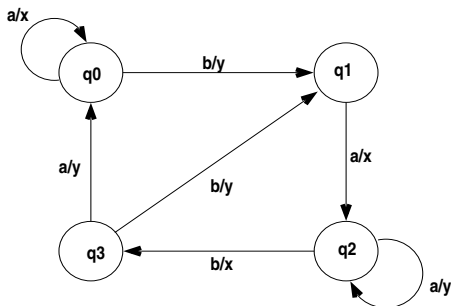
- Transition cover ensures
 - that all state and transition of the specification are implemented.
 - The set Z ensures that the implementation is in the same state as specified.
 - The parameter k ensures that all hidden states into the level k are tested.



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A Simple Example ^[HI98]



- $Input = \{a, b\}$
- $L = \{\langle \rangle, b, b::a, b::a::b\}$, $\langle \rangle \dots$ null input sequence
- $T = \{\langle \rangle, a, b, b::a, b::b, b::a::a, b::a::b, b::a::b::a, b::a::b::b\}$
- $W = \{a, b\}$ ^[Chy84], pp. 31–34
 - $Z = Input \bullet W \cup W$
 - $= \{a, b\} \bullet \{a, b\} \cup \{a, b\}$
 - $= \{a, b, a::a, a::b, b::a, b::b\}$

Test Set of the Example ^[HI98]

$$\begin{aligned}
 T \bullet Z &= \\
 &= \{ \langle \rangle, a, b, b::a, b::b, b::a::a, b::a::b, b::a::b::a, b::a::b::b \} \\
 &\quad \bullet \{ a, b, a::a, a::b, b::a, b::b \} \\
 &= \{ a, b, a::a, a::b, b::a, b::b, \\
 &\quad a::a, a::b, a::a::a, a::a::b, a::b::a, a::b::b, \\
 &\quad b::a, b::b, b::a::a, b::a::b, b::b::a, b::b::b, \\
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 &\quad b::a::b::b::a, b::a::b::b::b, b::a::b::b::a::a, \\
 &\quad b::a::b::b::a::b, b::a::b::b::b::a, b::a::b::b::b::b \} \\
 &= \dots \text{ simplifications}
 \end{aligned}$$



Test Set of the Example ^[HI98]

$$T \bullet Z =$$

$$= \{ \langle \rangle, a, b, b::a, b::b, b::a::a, b::a::b, b::a::b::a, b::a::b::b \}$$

$$\bullet \{ a, b, a::a, a::b, b::a, b::b \}$$

$$= \{ a, b, a::a, a::b, b::a, b::b,$$

$$a::a, a::b, a::a::a, a::a::b, a::b::a, a::b::b,$$

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$$= \dots \text{simplifications}$$


Applications ^[Bei95]

- Menu driven software,
- Object-oriented software,
- Protocols,
- Device drivers,
- Legacy hardware,
- Microcomputers of industrial and home devices,
- Software instalation,
- Archive and backup software,
- Safety software models,
- WEB applications.

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Mealy Machine

[Mea55, Mat13]

Definition 1 (Mealy machine with a finite number of states is)

- 6-tuple $M(X, Y, Q, q_0, \delta, \lambda)$:
 - X is a finite set of input symbols (the input alphabet),
 - Y is a finite set of output symbols (the output alphabet)
 - Q is a finite set of state,
 - $q_0 \in Q$ is the initial state,
 - $D \subseteq Q \times X$ is a specification domain,
 - $\delta : D \rightarrow Q$ is a state transition function,
 - $\lambda : D \rightarrow Y$ is an output function.
- If $D = Q \times X$, then M is a **complete** Mealy machine ^[SP10].
 - A sequence $\alpha = x_1 \dots x_k, \alpha \in I^*$ is a **defined input sequence** for a state $q \in Q$ if there are $q_1, \dots, q_{k+1} \in Q$, where $q_1 = q$ such that $(q_i, x_i) \in D$ and $\delta(q_i, x_i) = q_{i+1}$ for all $1 \leq i \leq k$.



machine Minimality [SP10, Mat13]

Let $M(X, Y, Q, q_0, \delta, \lambda)$ be a Mealy machine with a finite number of states.

- Extended transition and state functions applied to an input symbol x of a defined input sequence α including the empty sequence ϵ :
 - for $q \in Q$, $\delta(q, \epsilon) = q$ and $\lambda(q, \epsilon) = \epsilon$
 - $\delta(q, \alpha x) = \delta(\delta(q, \alpha), x)$
 - $\lambda(q, \alpha x) = \lambda(\delta(q, \alpha), x)$
- $\Omega(q)$ is the set of all defined input sequences for state $q \in Q$.
- Two states $q, q' \in Q$ are **distinguishable**, if there is $\gamma \in \Omega(q) \cap \Omega(q')$ such that $\lambda(q, \gamma) \neq \lambda(q', \gamma)$. Then, we say that γ **distinguishes** the states q and q' .
- Two states $q_1, q_2 \in Q$; $q_1 \neq q_2$ are **state equivalent**, if they lead to the same of equivalent states after an application of any input sequence.
- M is **minimal** if no its two states are equivalent [Ner58, Gil60].



C -equivalence of States [SP10, Mat13]

Let $M(X, Y, Q, q_0, \delta, \lambda)$ be a Mealy machine with a finite number of states.

- Let $C \subseteq \Omega(q) \cap \Omega(q')$ be a set.
- The states $q_1, q_2 \in Q$ are **C -equivalent**, if $\lambda(q, \gamma) \neq \lambda(q', \gamma)$ for all $\gamma \in C$.

Two machines $M_1(X, Y, Q_1, q_0^1, \delta_1, \lambda_1)$ and $M_2(X, Y, Q_2, q_0^2, \delta_2, \lambda_2)$ are **equivalent**, if

- 1 for each state $q \in M_1$ there is $q' \in M_2$ such that q and q' are equivalent and
- 2 for each state $q \in M_2$ there is $q' \in M_1$ such that q and q' are equivalent.

k -equivalence

- Let $M_1(X, Y, Q_1, q_0^1, \delta_1, \lambda_1)$ and $M_2(X, Y, Q_2, q_0^2, \delta_2, \lambda_2)$ be two machines.
- The states $q_i \in Q_1$ and $q_j \in Q_2$ are considered to be **k -equivalent**, if they produce identical output sequences after excited with any input sequence of the length k .



Characterization set W [SP10, Mat13]

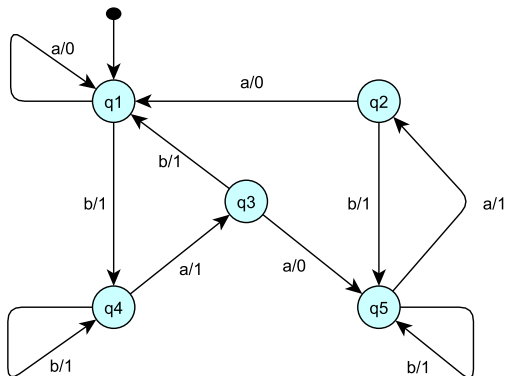
Let $M(X, Y, Q, q_0, \delta, O)$ be a minimal and complete Mealy machine with a finite number of states.

- W is a finite set of input sequences that distinguishes any pair of different states $q_i, q_j \in Q$.
- Each input sequence $\gamma \in W$ has a finite length.
- For each pair of different states $q_i, q_j \in Q$ the set W contains at least one input sequence γ such that

$$\lambda(q_i, \gamma) \neq \lambda(q_j, \gamma)$$



Characterization Set Example ^[HI98]



- $Input = \{a, b\}$
- $W = \{baaa, aa, aaa\}$
- $\lambda(q_1, baaa) = 1 \dots (1101)$
- $\lambda(q_2, baaa) = 0 \dots (1100)$
- $\lambda(q_1, baaa) \neq \lambda(q_2, baaa) \implies baaa$ distinguishes the states q_1 a q_2

k -equivalence Partition of States Q ^[Mat13]

- k -equivalence partition of states Q , denoted as P_k , is a collection of n finite sets $\Sigma_{k,1}, \Sigma_{k,2}, \dots, \Sigma_{k,n}$ such that

$$\bigcup_{i=1}^n \Sigma_{k,i} = Q$$

- The states in $\Sigma_{k,i}$ are k -equivalent.
- If states $ql_1 \in \Sigma_{k,j}$ and $ql_2 \in \Sigma_{k,j}$ for $i \neq j$, then ql_1 and ql_2 are k -distinguishable.



W Set Construction ^[Mat13]

The Algorithm

- 1 Create a sequence of k -equivalence partitions of states Q denoted as $P_1, P_2, \dots, P_m, m > 0$
 - 2 Backward search k -equivalence partitions while constructing distinguishing sequences for each pair of the states.
- Algorithm convergence is guaranteed.
 - When the algorithm stops each class $\Sigma_{K,j}$ of the finite partition P_K defines a class of equivalent states (1 for minimal machines).

Informally:

- First, find what can be distinguished in one step,
- then in two steps,
- etc.



W Set Construction ^[Mat13]

Tabular representation M .

0-equivalence partition $P_0 = \{\Sigma_1 = \{q_1, q_2, q_3, q_4, q_5\}\}$

Current state	Output		Next state	
	a	b	a	b
q_1	0	1	q_1	q_4
q_2	0	1	q_1	q_5
q_3	0	1	q_5	q_1
q_4	1	1	q_3	q_4
q_5	1	1	q_2	q_5



1-equivalence Partition P_1 Construction ^[Mat13]

1-equivalence partition $P_1 = \{\Sigma_1 = \{q_1, q_2, q_3\}, \Sigma_2 = \{q_4, q_5\}\}$.

Σ	Current state	Output		Next state	
		a	b	a	b
1	q_1	0	1	q_1	q_4
	q_2	0	1	q_1	q_5
	q_3	0	1	q_5	q_1
2	q_4	1	1	q_3	q_4
	q_5	1	1	q_2	q_5



2-equivalence Partition Construction: P_1 Rewrite ^[Mat13]

Rewrite P_1 , state q_i is replaced by $q_{i,j}$ if $q_i \in \Sigma_j$.

Σ	Current state	Next state	
		a	b
1	q_1	$q_{1,1}$	$q_{4,2}$
	q_2	$q_{1,1}$	$q_{5,2}$
	q_3	$q_{5,2}$	$q_{1,1}$
2	q_4	$q_{3,1}$	$q_{4,2}$
	q_5	$q_{2,1}$	$q_{5,2}$

2-equivalence Partition Construction: P_2 Construction [Mat13]

Construct P_2 . Divide $\Sigma_{1,j}$ with regard to the groups of next states.

Σ	Current state	Next state	
		a	b
1	q_1	$q_{1,1}$	$q_{4,3}$
	q_2	$q_{1,1}$	$q_{5,3}$
2	q_3	$q_{5,3}$	$q_{1,1}$
3	q_4	$q_{3,2}$	$q_{4,3}$
	q_5	$q_{2,1}$	$q_{5,3}$

3-equivalence Partition Construction: P_3 Construction [Mat13]

Construct P_3 . Divide $\Sigma_{2,j}$ with regard to the groups of next states.

Σ	Current state	Next state	
		a	b
1	q_1	$q_{1,1}$	$q_{4,3}$
	q_2	$q_{1,1}$	$q_{5,4}$
2	q_3	$q_{5,4}$	$q_{1,1}$
3	q_4	$q_{3,2}$	$q_{4,3}$
4	q_5	$q_{2,1}$	$q_{5,4}$



4-equivalence Partition Construction: P_4 Construction [Mat13]

Construct P_4 . Divide $\Sigma_{3,j}$ with regard to the groups of next states.

Σ	Current state	Next state	
		a	b
1	q_1	$q_{1,1}$	$q_{4,4}$
2	q_2	$q_{1,1}$	$q_{5,5}$
3	q_3	$q_{5,5}$	$q_{1,1}$
4	q_4	$q_{3,3}$	$q_{4,4}$
5	q_5	$q_{2,2}$	$q_{5,5}$



Distinguishing Sequence Construction: Example ^[Mat13]

- 1 Find a distinguishing sequence of the states q_1 a q_2 .
- 2 Init the distinguishing sequence: $z = \epsilon$.
- 3 Find tables P_i and P_{i+1} such that (q_1, q_2) are in the same group in P_i and in different groups in P_{i+1} :
 - P_3 and P_4 are obtained.
- 4 Find the input symbol distinguishing q_1 and q_2 in table P_3
 - b is the distinguishing symbol.
 - Update the distinguishing sequence: $z := z.b = \epsilon.b = b$.
- 5 Find the next states if the symbol b is applied to q_1 and q_2 ,
 - q_4 and q_5 are obtained.
- 6 Find tables P_i and P_{i+1} such that (q_4, q_5) are in the same group in P_i and in different groups in P_{i+1} :
 - P_2 and P_3 are obtained.
- 7 $(q_4, q_5) \rightarrow P_2, P_3 \rightarrow a \rightarrow z = ba$
- 8 $(q_3, q_2) \rightarrow P_1, P_2 \rightarrow a \rightarrow z = baa$
- 9 $(q_1, q_5) \rightarrow P_0, P_1 \rightarrow a \rightarrow z = baaa$
- 10 Repeat for each pair (q_i, q_j) : $W = \{a, aa, aaa, baaa\}$



Summary

- Finite state machines
- How to test finite state machines
- Test set construction using Chow's W method
- Characterization set construction

Competencies

- Define finite state machine.
- Describe the concept of hidden states.
- Describe Chow's W method of test set construction.
- Define characterization set and describe its construction algorithm.

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