Magnetic resonance imaging Part 1

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Introduction

MRI physics

Nuclear spin Spectroscopy Excitation Relaxation

Bloch equation

First human MRI

MR přístroje

První obraz člověka (1977)





MRI scanner

selenoid, closed-bore magnet



Permanentní magnety - architektura "OPEN"



11.7T MRI



MRI – Example

Brain slice:



MRI – Example

Brain slice:



MRI – Example

Spine:



MRI – Example



MRI principles

- 1. Insert object (subject) into a strong magnetic field.
- 2. (Repeatedly) send a radio-frequency impuls.
- 3. Spins are excited and then relax to equilibrium.
- 4. Receive and record emitted radio-frequency waves.
- 5. Reconstruct image from data.
- 6. Remove the object (subject) from the magnetic field.

Brief history of MRI

- 1946 Felix Bloch, Edward Purcell, independent discovery
- 1950–1970 NMR, spectroscopic analysis
- 1971 Raymond Damadian, tissue relaxation times differ
- 1973 Hounsfield, CT (showed demand for medical imaging)
- 1973 Paul Lauterbur, tomographic MRI (backprojection)
- 1975 Richard Ernst, Fourier MRI
- 1977 Peter Mansfield, echo-planar imaging (EPI), later 30 ms/slice

Brief history of MRI (2)

- 1980 Edelstein, whole-body MRI (3D), 5 min/slice
- 1986 whole-body MRI, 5 s/slice
- 1986 MRI microscopy, resolution 10 μm
- 1987 beating heart imaging
- 1987 MRA angiography without contrast agents, blood flow
- 1992 functional MRI, brain mapping

Nobel prizes

- 1952 Felix Bloch, Edward Purcell, physics, discovery
- 1991 Richard Ernst, chemistry, Fourier MRI
- 2003 Paul Lauterbur, Peter Mansfield, medicine, MRI in medicine

Numbers related to MRI

- About 40000 MRI scanners worldwide
- About 20 examination per day per scanner
- 110 scanners in Czech Republic in 2018
- One scanner costs $10 \sim 100$ mil. Kč (millions of EUR)
- One examination 5 \sim 20 tis. Kč (hundreds of EUR)

Introduction MRI physics Nuclear spin Spectroscopy Excitation Relaxation Bloch equat

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Spin

- Human body: fat and water. 63 % hydrogen (10 % by mass)
- Hydrogen nucleus = proton.
- Proton has a property called a **spin** (besides a mass and charge), related to angular momentum.
- Non-zero spin particles behave like small magnets \rightarrow MRI signal

Nuclear spin

- $\bullet\,$ Free electrons, protons, and neutrons have a spin of 1/2
- Spins may pair and compensate
- Nuclear spin *I* is a multiple of 1/2
- $I \neq 0 \Rightarrow (\text{small}) \text{ magnet},$

magnetic moment
$$\boldsymbol{\mu}$$
 $\|\boldsymbol{\mu}\| = \gamma \hbar \sqrt{I(I+1)} \quad \begin{bmatrix} N \cdot m \\ T \end{bmatrix}$
torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \quad [N \cdot m]$

Isotopes useful for MRI

- Only unpaired spins $(I \neq 0)$ are useful for MRI
- Total nuclear spin no easy rule
- Even atomic number Z (number of protons) and even mass number A (total number of protons and neutrons) $\Rightarrow I = 0$ $({}^{12}C, {}^{16}O)$
- Most abundant isotopes for even Z have I = 0
- Isotopes with $I \neq 0$ are often
 - rare isotopes (1.11% for ¹³C)
 - biologically rare elements
 - give small signal

Biological abundance of elements (by count)

Element	Abundance [%]	
Н	63	
0	26	main isotope ¹⁶ O with zero spin
С	9.4	main isotope ¹² C with zero spin
Ν	1.5	
Р	0.24	
Ca	0.22	
Na	0.041	

Abundance of MRI active isotopes

by count

lsotope	Abundance [%]
^{1}H	99.985
² H	0.015
¹³ C	1.11
^{14}N	99.63
¹⁵ N	0.37
²³ Na	100
³¹ P	100
³⁹ K	93.1
⁴³ Ca	0.145

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Practical MRI exists for

- ¹H most often used, strongest signal, best quality
- ¹⁹F, ²³Na, ³¹P... mostly research

Spins in magnetic field





magnetic induction $B_0 = 0$, random orientation

Spins in magnetic field



 $B_0 \neq$ 0 (around 0.1 \sim 10 T needed)

- Spin packet
- Spins will partially align (on the average)
- Macroscopically observable magnetisation ${f M}=\sum{m \mu}$

- For B₀ along z axis
- M rotates around z axis
- $\boldsymbol{\tau} = \boldsymbol{\mu} imes \mathbf{B}$



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RF signal

Precession in the xy plane gives measurable RF signal.



RF signal

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Larmor frequency

$$f = \gamma B$$

- B [Tesla] magnetic field intensity
- γ gyromagnetic constant
- Pro $^1\mathrm{H}$, $\gamma =$ 42.58 MHz/T
- f is a frequency of:
 - the precession
 - the received signal
 - the excitation signal



- low (parallel) and high (antiparallel) orientations
- For *H*, typically $f = 15 \sim 80$ MHz.
- Energy difference \sim signal amplitude

Boltzmann statistics

For a closed (non-quantum) system in thermal equilibrium:

- Number of low-energy spins N⁻
- Number of high-energy spins N^+

$$\frac{N^+}{N^-} = \mathrm{e}^{-\frac{\Delta E}{kT}}$$

Boltzmann constant $k = 1.3805 \cdot 10^{-23}$ Temperature T [K]

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- MRI signal $\propto \|\mathbf{M}\| \propto N^- N^+$
- low T, high $B \rightarrow$ high signal
- high T, low $B \rightarrow$ low signal

Boltzmann statistics example

Je-li stav β obsazen 10° spinů, stav α obsahuje10°+přebytek.



 \rightarrow very low SNR in MRI.

Introduction MRI physics Nuclear spin Spectroscopy Excitation Relaxation Bloch equat

Introduction

MRI physics

Nuclear spin Spectroscopy

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Continous wave NMR (1)

- Constant frequency
- Variable magnetic field
- Measuring absorbed energy



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NMR spektroskopie





Fyzikální základy - pokračování

FID (pokračování)

 V reálném vzorku je mnoho spinových systémů, jejichž frekvence jsou odlišné od frekvence B, (carrier frequency) Protože jsme efektivné excitovali všechny tyto spiny, dostaneme kombinaci signálů a řůzné frekvenci Free Induction Decay (FD);



· Po zpracování Fourierovou transformací dostaneme:



Introduction MRI physics Nuclear spin Spectroscopy Excitation Relaxation Bloch equat

Introduction

MRI physics

Nuclear spin Spectroscopy Excitation Relaxation Bloch equation









 $\pmb{\mu}$ rotating with frequency f appears stationary











 μ rotating slower than f appears rotating in the opposite direction



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Electromagnetic excitation

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- **B**⁻₁ has frequency 2*f* there, far from the resonance, can be neglected.
- \to alternating field B_1 will appear stationary along x' in the rotating frame of reference.

- RF impuls at Larmor frequency f (at resonance), with amplitude B_1 and duration τ
- \rightarrow magnetization **M** will turn around B_1 (= x') by angle

 $\alpha = 2\pi\gamma\tau B_1$ flip angle



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- 90° impuls turns **M** from z to y'
- 180° impuls turns M from z to -z'





















- Magnetization is turned by angle α from any initial position
- e.g. 180° impuls for $\mathbf{M} || y'$



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Introduction MRI physics Nuclear spin Spectroscopy Excitation Relaxation Bloch equat

Introduction

MRI physics

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• In equilibrium, $\mathbf{M} = M_0 \mathbf{e}_z$, $M_z = M_0$



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T_1 relaxation (2)

After the pulse M_z returns to equilibrium.

$$M_z = M_0 \left(1 - \mathrm{e}^{-rac{t}{T_1}}
ight)$$



 T_1 — spin-lattice relaxation time (mřížková relaxační časová konstanta) the energy is dissipated into the lattice as heat














T_1 relaxation (3)

A stronger/longer pulse may cause e.g. $M_z = -M_0$.

$$M_z = M_0 \left(1 - 2\mathrm{e}^{-\frac{t}{T_1}} \right)$$



• Once **M** turns away from the z axis...



• Once **M** turns away from the z axis...



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- Once **M** turns away from the z axis...
- ... it starts to rotate around z with $f = \gamma B$



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T_2 relaxation (2)

Transversal magnetization M_{xy} decreases



 T_2 — spin-spin relaxation time (spinová relaxační časová konstanta), $T_2 < T_1$

- Transversal magnetization M_{xy} decreases
- At the same time (but more slowly) M_z returns to M_0 .



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Reasons for the T_2 relaxation

- Molecular interation (*T*₂)
- Inhomogeneity of the magnetic field (T_2^{inhom})

Combined time constant T_2^* :

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2^{\mathsf{inhom}}}$$

Other factors influencing relaxation

- Molecular movement (because of the magnetic field inhomogeneity)
- Temperature
- Viscosity

Typical relaxation times				
	1.5 T		3 T	
tissue	T_1 [ms]	<i>T</i> ₂ [ms]	T_1 [ms]	T ₂ [ms]
fat	260	80	420	100
muscle	870	45	1300	40
brain (gray matter)	900	100	1600	100
brain (white matter)	780	90	900	70
liver	500	40	800	34
cerebrospinal fluid	2400	160	4100	500

Typical relayation times

Reported values differ significantly.

Introduction MRI physics Nuclear spin Spectroscopy Excitation Relaxation Bloch equat

Introduction

MRI physics

Nuclear spin Spectroscopy Excitation Relaxation

Bloch equation

Bloch equation

$$rac{\mathrm{d} {f M}}{\mathrm{d} t} = \gamma {f M} imes {f B}$$
 where {f B} is the total magnetic field $({f B}_0 + {f B}_1).$

Bloch equation (2)

$$\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = \gamma \mathbf{M} \times \mathbf{B}$$

substituting for ${\bf B},$ add losses and use the rotating frame of reference

$$\begin{aligned} \frac{\mathrm{d}M_{x'}}{\mathrm{d}t} &= (\omega_0 - \omega)M_{y'} - \frac{M_{x'}}{T_2}\\ \frac{\mathrm{d}M_{y'}}{\mathrm{d}t} &= -(\omega_0 - \omega)M_{x'} + 2\pi\gamma B_1 M_z - \frac{M_{y'}}{T_2}\\ \frac{\mathrm{d}M_z}{\mathrm{d}t} &= -2\pi\gamma B_1 M_{y'} - \frac{M_z - M_{z0}}{T_1} \end{aligned}$$

where $\omega_0=2\pi f_0=2\pi\gamma B_0$, ω is the spin rotation frequency.