Basics of Description Logic $\mathcal{ALC}$

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1 Understanding $\mathcal{ALC}$

Consider the following $\mathcal{ALC}$ theory $\mathcal{K} = (\mathcal{T}, \{\})$, where $\mathcal{T}$ contains the following axioms:

$$\begin{aligned} Man &\sqsubseteq Person \\ Woman &\sqsubseteq Person \sqcap \neg Man \\ Father &\equiv Man \sqcap \exists hasChild \cdot Person \\ GrandFather &\equiv \exists hasChild \cdot \exists hasChild \cdot \top \\ Sister &\equiv Person \sqcap \neg Man \sqcap \exists hasSibling \cdot Person \end{aligned}$$

Ex. 1 — What is the meaning of these particular axioms ? Do they reflect your understanding of reality ? Formulate them in natural language.

Ex. 2 — Rewrite the last axiom into the semantically equivalent FOPL formula.

Ex. 3 — Consider the following interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \bullet^{\mathcal{I}})$:

$$\begin{aligned} \Delta^{\mathcal{I}} &= Person^{\mathcal{I}} = \{B, A\} \\ Man^{\mathcal{I}} &= \{B\} \\ Woman^{\mathcal{I}} &= \{A\} \\ Father^{\mathcal{I}} &= GrandFather^{\mathcal{I}} = \{B\} \\ hasChild^{\mathcal{I}} &= \{(B, B)\} \\ hasSibling^{\mathcal{I}} &= \{\} \\ Sister^{\mathcal{I}} &= \{B\} \end{aligned} \tag{1}$$

1. Is $\mathcal{I}$ a model $\mathcal{K}$? If yes, decide, whether $\mathcal{I}$ reflects reality.
2. We know that $\mathcal{ALC}$ has the *tree model property* and *finite model property*. In case $\mathcal{I}$ is a model, is $\mathcal{I}$ tree-shaped ? If not, find a model that is tree-shaped.

Ex. 4 — How does the situation change when we consider the same $\mathcal{I}$, except that $Sister^{\mathcal{I}} = \{\}$?

Ex. 5 — Using the vocabulary from $\mathcal{K}$, define the concept "A father having just sons."

Ex. 6 — Using the vocabulary from $\mathcal{K}$, define the concept "A man who has no brother, but at least one sister with more than one child."

Ex. 7 — During knowledge modeling, it is often necessary to specify:

global domain and range of given role, i.e. statement of the type "By $hasChild$ we always connect a $Person$ (domain) with another $Person$ (range)".

local range of given role, e.g. "Every father having only sons (domain) can be connected by $hasChild$ (domain) just with a Man (range)".

Show, in which way it is possible to model global domain and range of these roles in $\mathcal{ALC}$.

2 Using Protégé

1. Go through the Protégé Crash Course on the tutorial web pages.

2. Create a new ontology in Protégé 4 and insert there all the definitions from Section 1. Verify correctness of your solution of the previous task (e.g. in the DL query tab).