1. Calculate the sum of the given sequence

\[ \frac{n}{1} + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} + \frac{n}{16} + \frac{n}{32} + \ldots \]

A. \( n \)
B. \( 2n \)
C. \( n \cdot \log(n) \)
D. \( n^2 \)
E. \( n! \)
2. A skip list contains N keys. The number of pointers (separate references to some of the next items) in the skip list is roughly:

A. $N/2$
B. $N$
C. $2N$
D. $N \cdot \log(N)$
E. $N^2$
3A. A skip list is originally empty and then the keys are inserted into the list one by one in the given order. The numbers in the parentheses denote the level of the corresponding node given by the number of coin tosses. Each time, the coin is being tossed until it comes up tails. Draw the resulting list.

\[16(3) \ 23(2) \ 18(2) \ 5(2) \ 15(1) \ 19(1) \ 33(1) \ 11(2) \ 21(2) \ 4(1) \ 22(2) \ 6(2) \ 17(4) \ 10(1) \ 9(1) \ 28(4)\]

Next, operation Search(22) is performed on the list. What is the number of comparisons of the value 22 to the values stored in the list during the search process?

A. 4 or less
B. 5
C. 6
D. 7
E. 8 or more
3B. A skip list is originally empty and then the keys are inserted into the list one by one in the given order. The numbers in the parentheses denote the level of the corresponding node given by the number of coin tosses. Each time, the coin is being tossed until it comes up tails. Draw the resulting list.

16(3)  23(2)  18(2)  5(2)  15(1)  19(1)  33(1)  11(2)  21(2)  4(1)  22(2)  6(2)  17(4)  10(1)  9(1)  28(4)

Next, operation Insert(14) is performed on the list. What is the number of comparisons of the value 14 to the values stored in the list during the search process?

A.  5 or less
B.  6
C.  7
D.  8
E.  9 or more
4. There are two skip lists of length $N$. Describe an effective algorithm which merges these two skip lists into one skip list of length $2N$. What is the asymptotic complexity of your solution?

A. $O(1)$
B. $\Theta(N)$
C. $\Theta(N \cdot \log(N))$
D. $\Theta(N \cdot \log^2(N))$
E. $\Theta(N^2)$
5. There is a skip list of length N. Split the list into two separate skip lists, first of which will contain only the odd keys of the original list and the second one will contain only the even keys of the original skip list. Can this task be accomplished in time $\Theta(N)$?

A. Yes  
B. No  
C. Yes, but only in case when the parity of all keys in the list is the same  
D. No, the case when the parity of all keys in the list is the same takes longer time
6. Is it possible to reverse the order of the keys in a skip list in time which is asymptotically less than $\Theta(N \cdot \log(N))$?

A. Yes
B. No
7. Describe an effective operation IncreaseKey and DecreaseKey in a skip list. The amount by which the key value is increased/decreased will be the parameter of the operation.

The complexity of the operation will be (N is the skip list size):

A. O(1)
B. O(log(N))
C. O(N)
D. O(N \cdot \log(N))
E. O(N^2)
8. Describe how to use a skip list as a priority queue. Compare the complexity of operations Insert and ExtractMin to the complexity of the same operations in a binary heap. Which of these two data structures performs asymptotically better?

The asymptotic complexity and the amortized complexity of ExtractMin operation would be (N is the skip list size):

A. O(1) and O(1)
B. O(log(N)) and O(1)
C. O(log(N)) and O(log(N))
D. O(N) and O(log(N))
E. O(N) and O(N)
9. There is a discussion about the sum of level values of all nodes in a skip list of length N. Professor Highman says that the sum is on average proportional to \(N \cdot \log(N)\). Professor Lowman says that the sum is on average proportional to just N. Professor Middleman says that both cases might happen and that it depends on the data. Decide whether any of the three professors is right.

A. Highman is right
B. Middleman is right
C. Lowman is right
D. All professors are wrong, there are more cases to consider
10A. Insert keys 7, 5 (in this order) into the given B-tree. What will be the contents of the root after the insertion?

A. 3
B. 5
C. 7
D. 8
E. 25
10B. Insert keys 14 and 10 (in this order) into the given B-tree. What will be the contents of the root after the insertion?

A. 8
B. 10
C. 11
D. 14
E. 15
11A. Suppose that a B-tree of order 1 is originally empty. Insert, in the given order, the given keys into the tree 25, 13, 37, 32, 40, 20, 22. Draw the tree after each insertion.

What is the contents of the root after the insertion?

A. 13
B. 20
C. 22
D. 25
E. 32
11B. Next, delete the keys from the B-tree built in the previous problem, in the order:
13, 25
When a key K in an internal node is deleted substitute it with the smallest key bigger than K.
Draw the tree after each deletion.

What is the contents of the root after these two deletions?

A. 22
B. 32
C. 22, 32
D. 22, 37
E. 32, 37
11C. Next, delete the key 40 from the B-tree obtained in the previous problem:

What is the contents of the root after this deletion?

A. 22  
B. 32  
C. 37  
D. 22, 32  
E. 22, 37
12A. Suppose that a B+ tree of order 1 is originally empty. Insert, in the given order, the given keys into the tree 32, 18, 31, 59, 20

What will be the contents of the root after the insertions?

A. 20
B. 20, 31
C. 31
D. 31, 32
E. 32
12B. Insert the given keys into the B+ tree built in the previous problem.
23, 24, 36, 60

What will be the contents of the root after the insertions?

A. 31
B. 31, 32
C. 31, 36
D. 32
E. 32, 36
12C. Next, delete the keys 23, 31 from the resulting B+ tree in the order:

What will be the contents of the root after the deletions?

A. 23, 32,
B. 23, 36
C. 32
D. 32, 36
E. 36
13A. We are given a B-tree. The order of the tree is 10 (maximum number of children = 11, in any node) and it contains exactly 100,000 keys. What is the minimum and the maximum possible height of the tree?

A. 2, 4
B. 2, 5
C. 3, 4
D. 3, 5
E. 4, 6
13B. We are given a B+ tree. The order of the tree is 10 (maximum number of children = 11, in any node) and it contains exactly 100 000 keys. What is the minimum and the maximum possible height of the tree?

A. 2, 4  
B. 2, 5  
C. 3, 4  
D. 3, 5  
E. 4, 6
14. The rule governing the choice of the level value of a node in a skip list were experimentally modified. The new rule says, that the choice of level value of each node is randomly uniformly distributed on an interval $<1, \lceil \log_2 N \rceil >$.

$N$ is the actual size of the list. There is a hypothesis that Insert and Delete will perform asymptotically slower in this list than they do in a standard skip list. Confirm or disapprove the hypothesis.
15A. We are given a B-tree. The order of the tree is 10 and it contains exactly 100,000 keys. What is the maximum and minimum possible number of keys in the tree?
15B. We are given a B+ tree. The order of the tree is 10 and it contains exactly 100,000 keys. What is the maximum and minimum possible number of keys in the tree?
16. Two empty B-trees of order 1 (max 2 keys in a node) are isomorphic. Let T1 and T2 be two unempty B-trees with the respective roots R1 and R2. T1 and T2 are isomorphic iff both 1. and 2. holds:
   1. The root of T1 contains the same number of keys as the root of T2.
   2. The left subtree of R1 is isomorphic to the left subtree of R2, the right subtree of R1 is isomorphic to the right subtree of R2 and the middle subtree (if it exists) of R1 is isomorphic to the middle subtree of R2.

What is the number of non-isomorphic B-trees with A) 0, B) 1, C) 3, D) 4, E) 7 nodes?
17. Extend your solution to the previous problem and find a general recursive formula which specifies the number of non-isomorphic B-trees of order 1 for any given number of nodes in the tree.