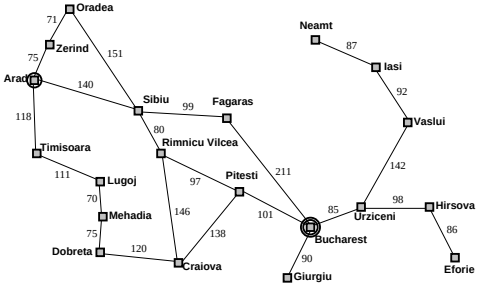
# Problem solving by search
# Finding the optimal sequence of states/decisions/actions

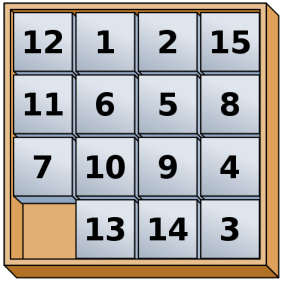
Tomáš Svoboda, Petr Pošík

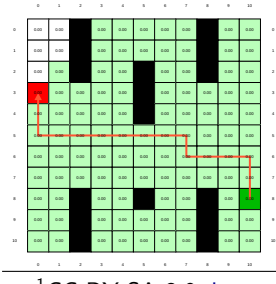
Vision for Robots and Autonomous Systems, Center for Machine Perception
Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University in Prague

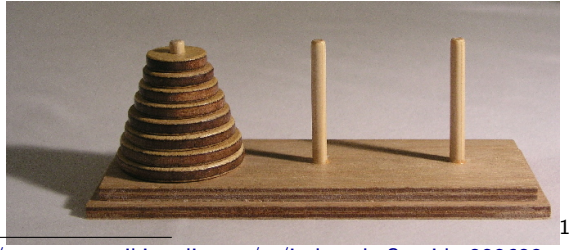
February 28, 2024

# Problems to solve



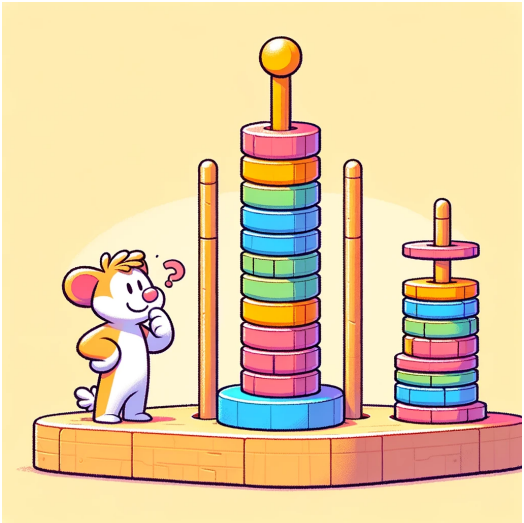





[1]CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=228623

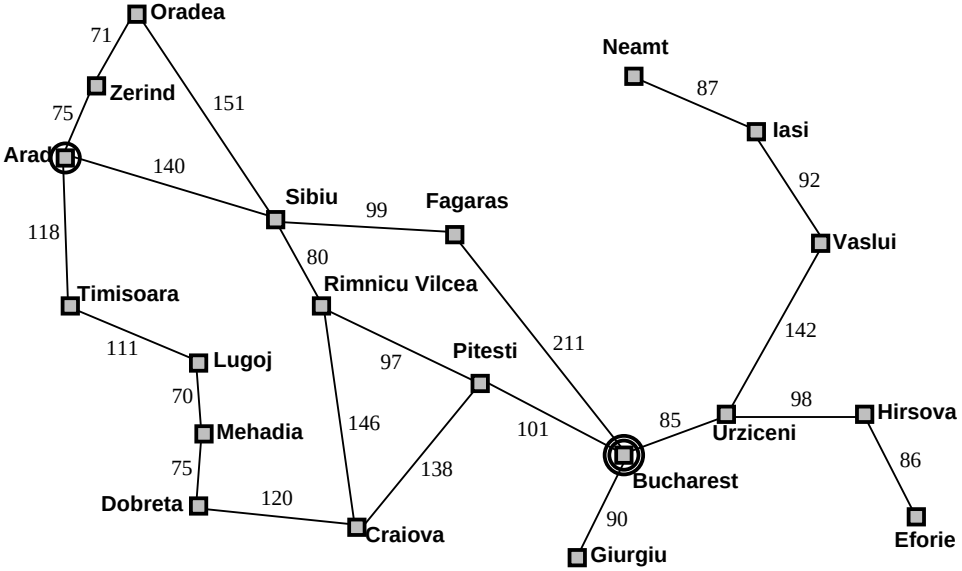
# Understanding the problem is the key, DALL-E.



# Outline

- Search problem. *What do you want to solve?*
- State space graphs. *How do you formalize/represent the problem? Problem abstraction.*
- Search trees. *Visualization of the algorithm run.*
- Strategies: which tree branches to choose?
- Strategy/Algorithm properties. *Memory, time, ...*
- Programming infrastructure.

# Example: Traveling in Romania



# Traveling Example: State and Actions

Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

actions (decisions): select a road

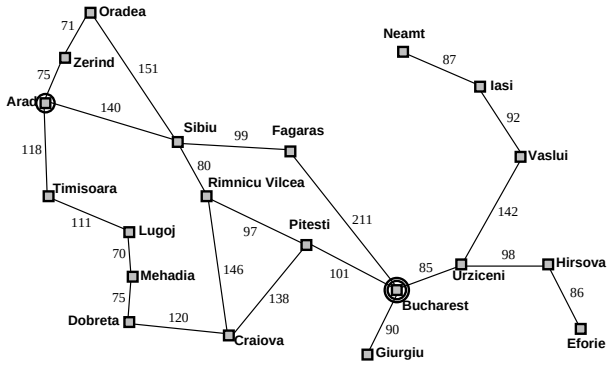
Solution:

Sequence of cities (path)

(sequence of actions/decisions [2])

Optimality – Cost, Loss, Utility, . . . :

Energy, time, tolls, . . .



# Traveling Example: State and Actions

Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

actions (decisions): select a road

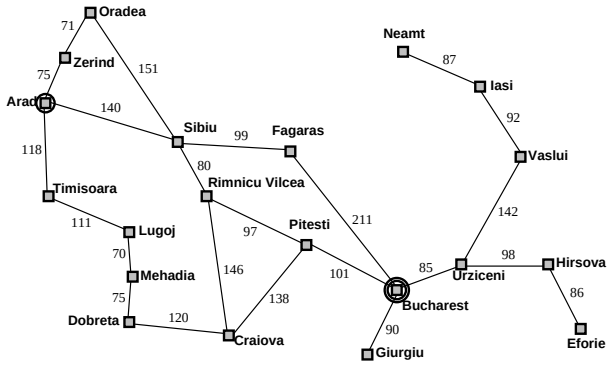
Solution:

Sequence of cities (path)

(sequence of actions/decisions [2])

Optimality – Cost, Loss, Utility, . . . :

Energy, time, tolls, . . .



# Traveling Example: State and Actions

Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

actions (decisions): select a road

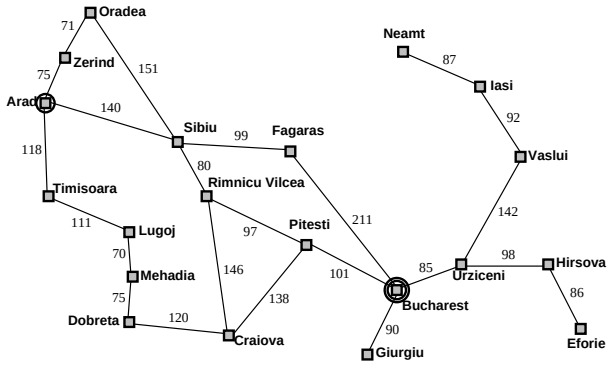
Solution:

Sequence of cities (path)

(sequence of actions/decisions [2])

Optimality – Cost, Loss, Utility, . . . :

Energy, time, tolls, . . .



# Traveling Example: State and Actions

Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

actions (decisions): select a road

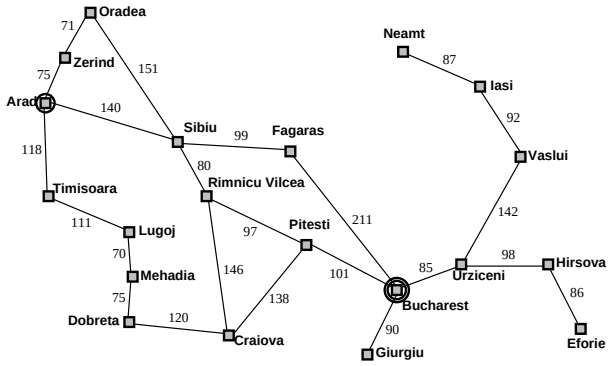
Solution:

Sequence of cities (path)

(sequence of actions/decisions [2])

Optimality – Cost, Loss, Utility, . . . :

Energy, time, tolls, . . .



# Traveling Example: State and Actions

Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

actions (decisions): select a road

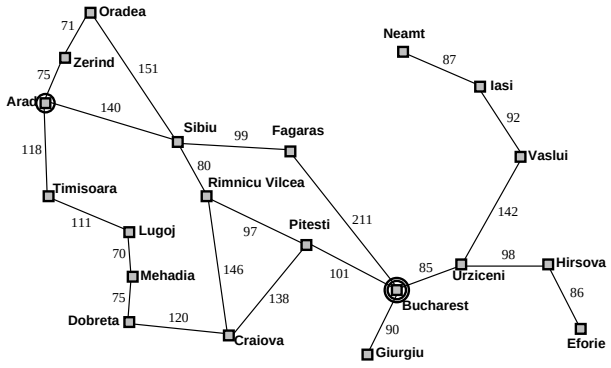
Solution:

Sequence of cities (path)

(sequence of actions/decisions [2])

Optimality – Cost, Loss, Utility, . . . :

Energy, time, tolls, . . .



# Traveling Example: State and Actions

Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

actions (decisions): select a road

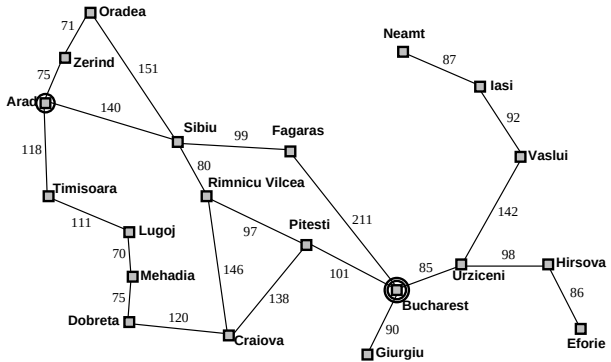
Solution:

Sequence of cities (path)

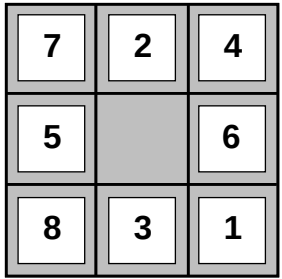
(sequence of actions/decisions [2])

Optimality – Cost, Loss, Utility, . . . :

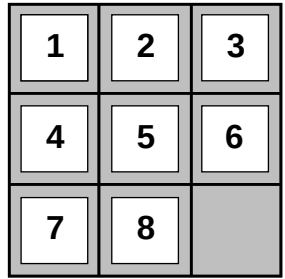
Energy, time, tolls, . . .



# Example: The 8-puzzle




Start State




Goal State

states?
actions?
solution?
cost?

# A Search Problem

- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left ...
- Transition model : Given state and action result state (and cost or reward)
- Goal test : Are we done?

# A Search Problem

- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left ...
- Transition model : Given state and action result state (and cost or reward)
- Goal test : Are we done?

# A Search Problem

- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left ...
- Transition model : Given state and action result state (and cost or reward)
- Goal test : Are we done?

# A Search Problem

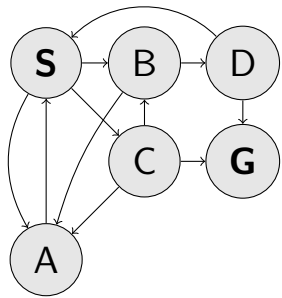
- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left ...
- Transition model : Given state and action result state (and cost or reward)
- Goal test : Are we done?

# Discrete State Space

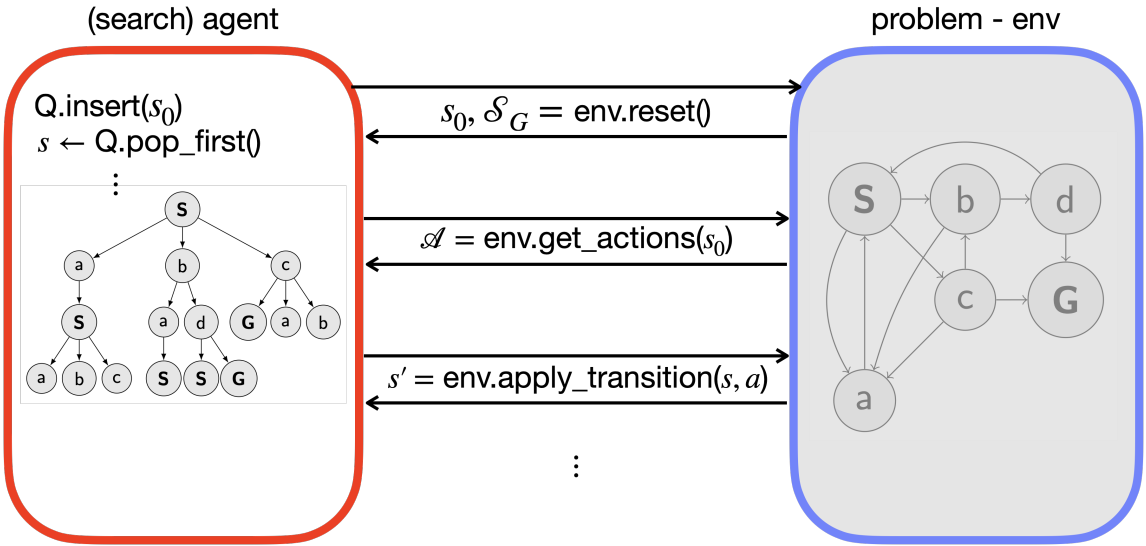
State space graph: a representation of a search problem

- States $s \in \mathcal{S} = \{\mathbf{S}, \mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathbf{G}\}$ (finite set)
- Arcs represent actions $a$, for each state $s$, $a \in \mathcal{A}(s)$ ($\mathcal{A}$ is also finite)
- State transition function $s' = \mathrm{result}(s, a)$
- Start (initial) state $s_0 \in \mathcal{S}$, $s_0 = \mathbf{S}$.
- Goal set $\mathcal{S}_G \subset \mathcal{S}$.

Each state occurs only *once* in a state (search) space.



# agent – problem dialog (a programmer's viewpoint)



# BFS, Q is FIFO data structure (queue)

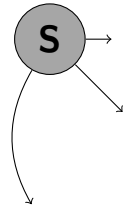
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```



Q: (_,**S**)
visited: **S**

# BFS, Q is FIFO data structure (queue)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

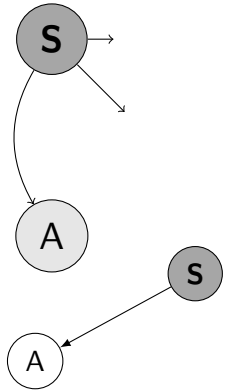




Q:
visited: **S**

# BFS, Q is FIFO data structure (queue)

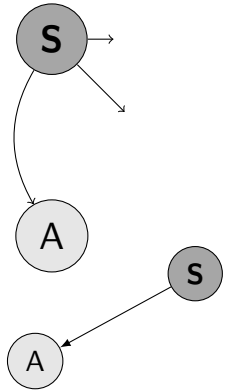
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q:
visited: **S**

# BFS, Q is FIFO data structure (queue)

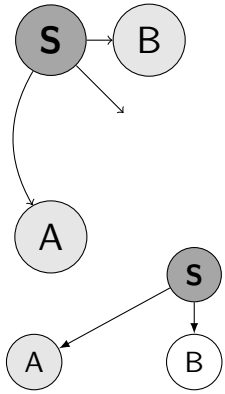
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A)
visited: **S** A

# BFS, Q is FIFO data structure (queue)

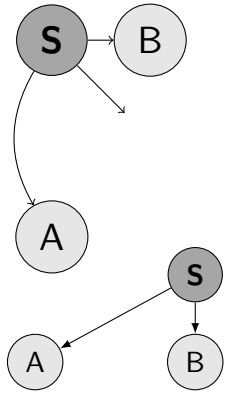
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A)
visited: **S** A

# BFS, Q is FIFO data structure (queue)

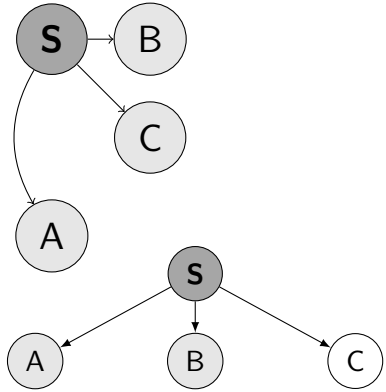
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A) (S,B)
visited: **S** A B

# BFS, Q is FIFO data structure (queue)

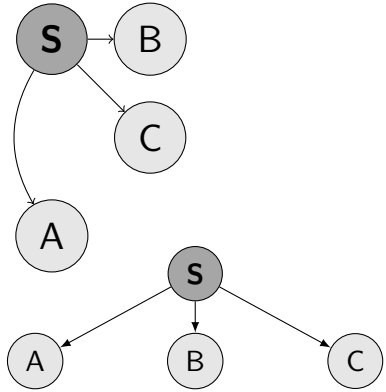
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A) (S,B)
visited: **S** A B

# BFS, Q is FIFO data structure (queue)

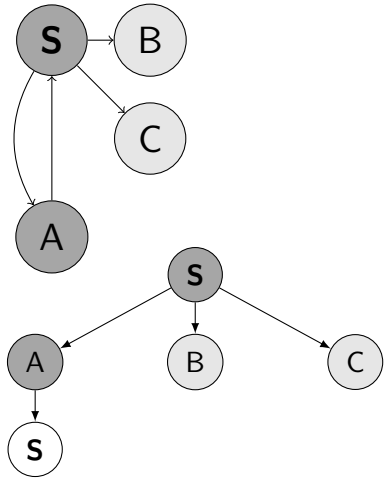
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A) (S,B) (S,C)
visited: **S** A B C

# BFS, Q is FIFO data structure (queue)

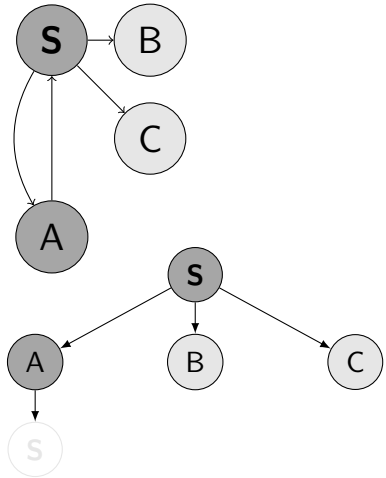
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,B) (S,C)
visited: **S** A B C

# BFS, Q is FIFO data structure (queue)

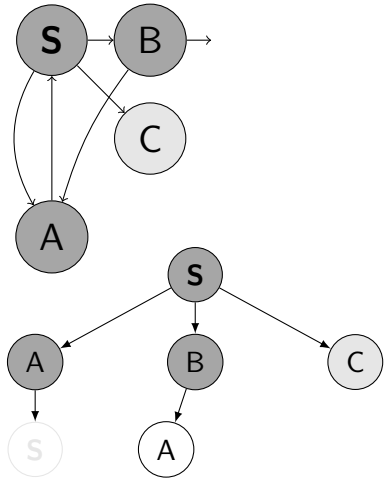
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,B) (S,C)
visited: **S** A B C

# BFS, Q is FIFO data structure (queue)

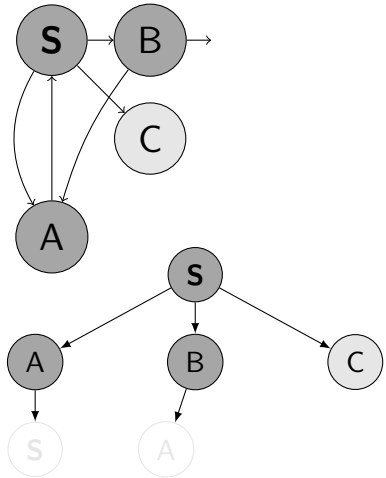
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,C)
visited: **S** A B C

# BFS, Q is FIFO data structure (queue)

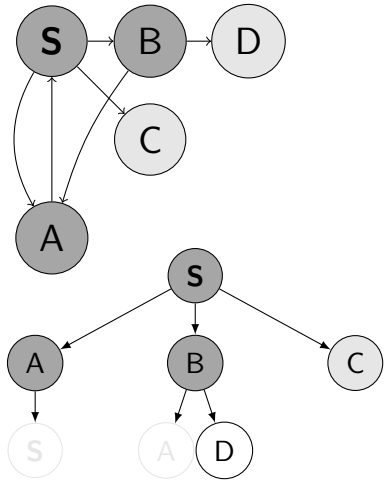
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,C)
visited: **S** A B C

# BFS, Q is FIFO data structure (queue)

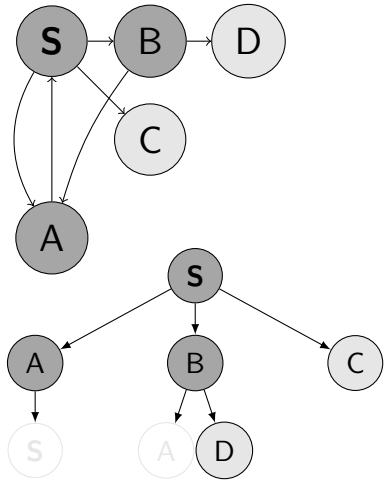
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
    return Failure
```




Q: (S,C)
visited: **S** A B C

# BFS, Q is FIFO data structure (queue)

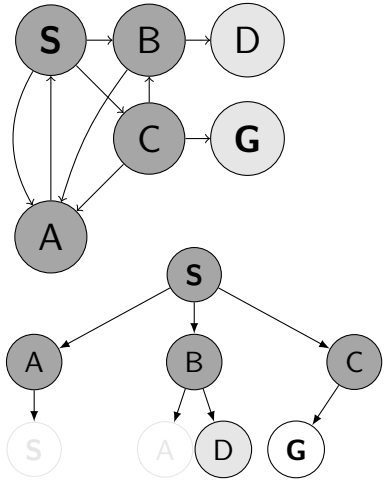
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
    return Failure
```




Q: (S,C) (B,D)
visited: **S** A B C D

# BFS, Q is FIFO data structure (queue)

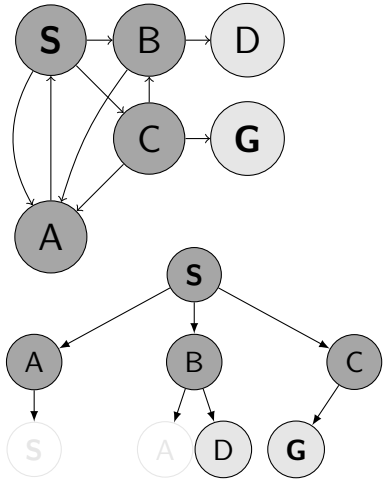
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (B,D)
visited: **S** A B C D

# BFS, Q is FIFO data structure (queue)

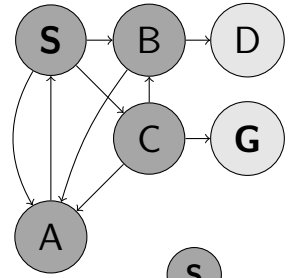
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (B,D) (C,**G**)
visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

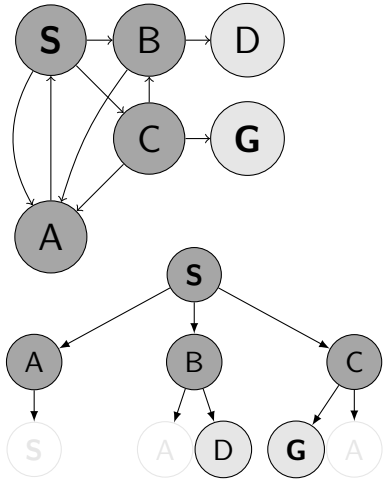
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (B,D) (C,**G**)
visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

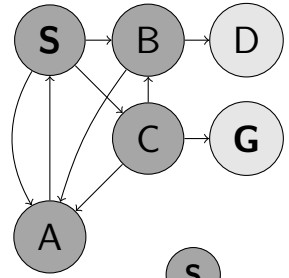
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (B,D) (C,**G**)
visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

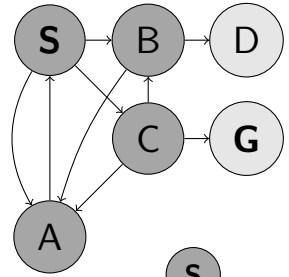
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (B,D) (C,**G**)
visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

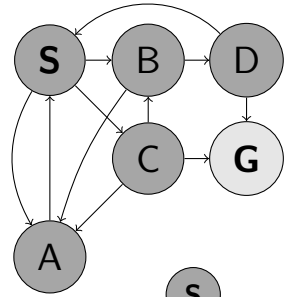
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

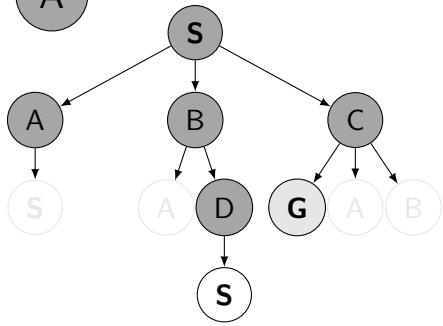



Q: (B,D) (C,**G**)
visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```



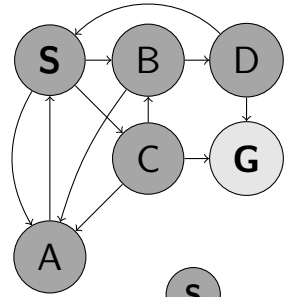



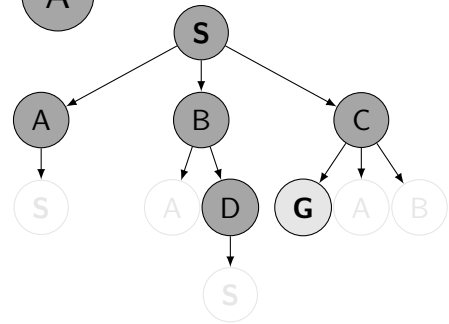

Q: (C,**G**)

visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

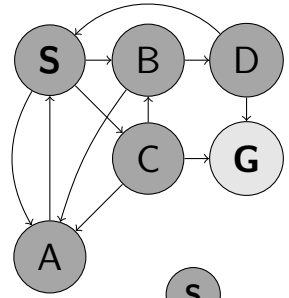


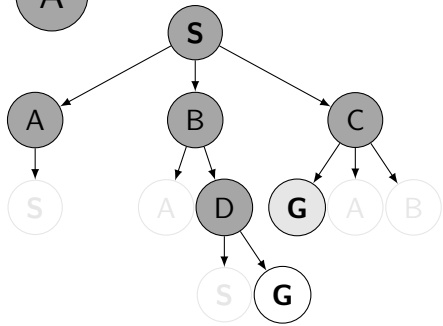




Q: (C,**G**)
visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```



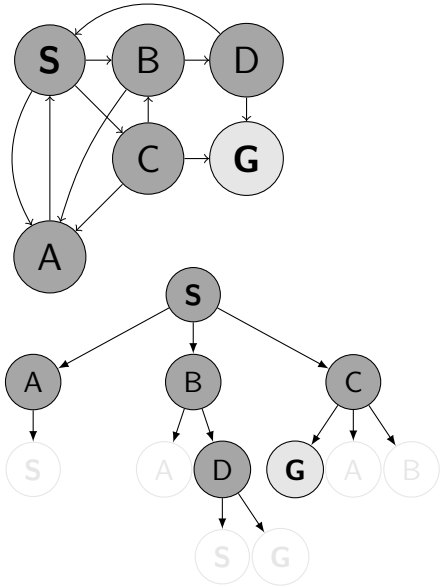




Q: (C,**G**)

visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

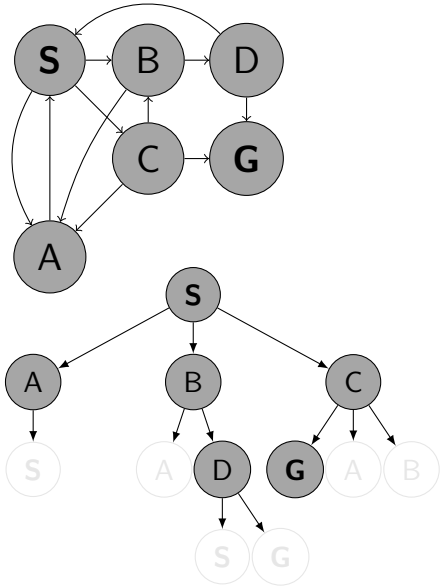
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (C,**G**)
visited: **S** A B C D **G**

# BFS, Q is FIFO data structure (queue)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

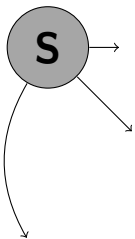



Q:
visited: **S** A B C D **G**

# Search algorithm partitions state space into 3 disjoint sets

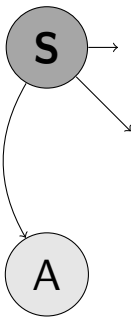

Find good names

# Search algorithm partitions state space into 3 disjoint sets



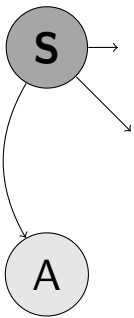

Find good names

# Search algorithm partitions state space into 3 disjoint sets



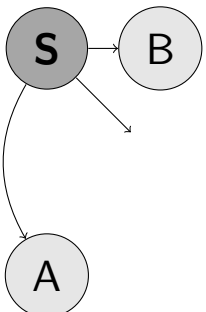

Find good names

# Search algorithm partitions state space into 3 disjoint sets



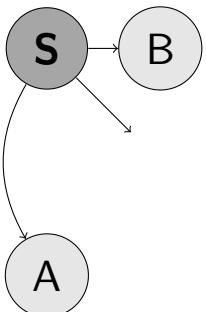

Find good names

# Search algorithm partitions state space into 3 disjoint sets



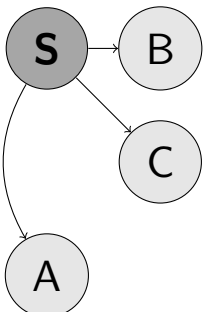

Find good names

# Search algorithm partitions state space into 3 disjoint sets



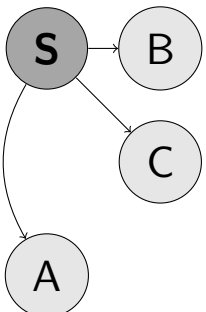

Find good names

# Search algorithm partitions state space into 3 disjoint sets



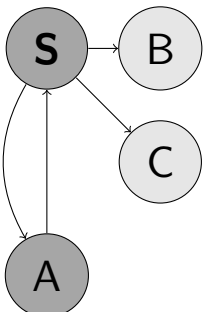

Find good names

# Search algorithm partitions state space into 3 disjoint sets



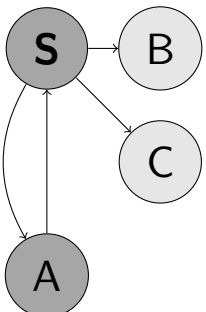

Find good names

# Search algorithm partitions state space into 3 disjoint sets



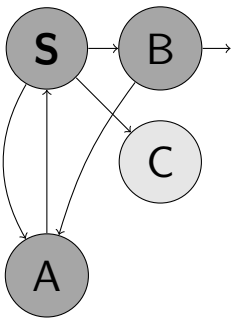

Find good names

# Search algorithm partitions state space into 3 disjoint sets



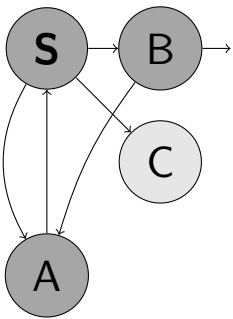

Find good names

# Search algorithm partitions state space into 3 disjoint sets



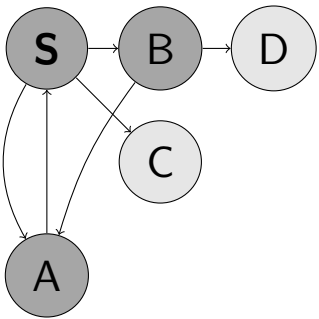

Find good names

# Search algorithm partitions state space into 3 disjoint sets



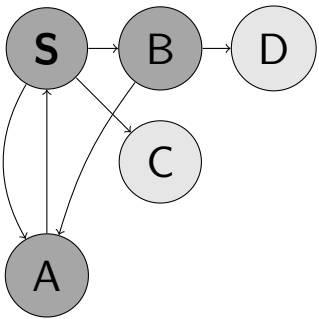

Find good names

# Search algorithm partitions state space into 3 disjoint sets



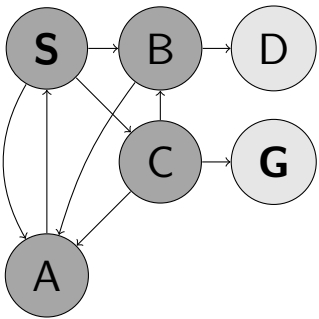

Find good names

# Search algorithm partitions state space into 3 disjoint sets



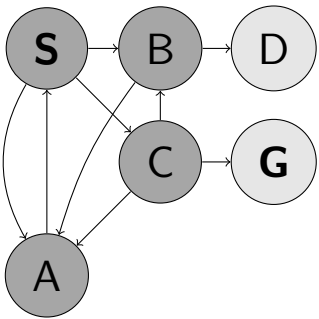

Find good names

# Search algorithm partitions state space into 3 disjoint sets



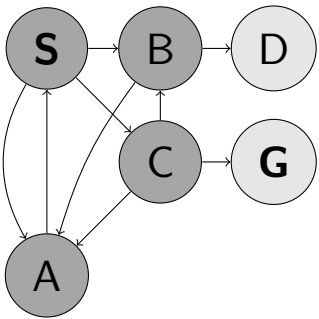

Find good names

# Search algorithm partitions state space into 3 disjoint sets



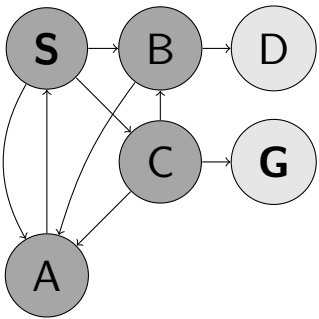

Find good names

# Search algorithm partitions state space into 3 disjoint sets



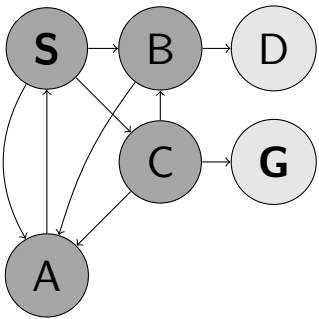

Find good names

# Search algorithm partitions state space into 3 disjoint sets



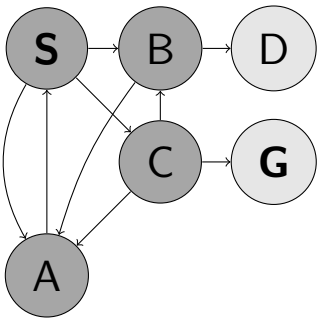

Find good names

# Search algorithm partitions state space into 3 disjoint sets



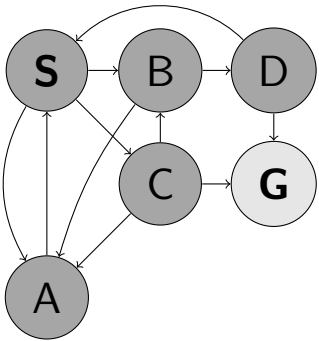

Find good names

# Search algorithm partitions state space into 3 disjoint sets



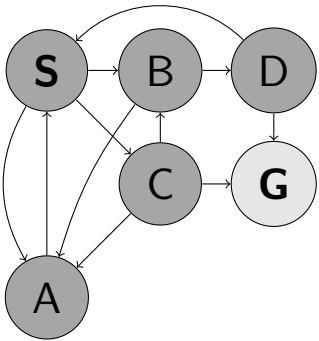

Find good names

# Search algorithm partitions state space into 3 disjoint sets



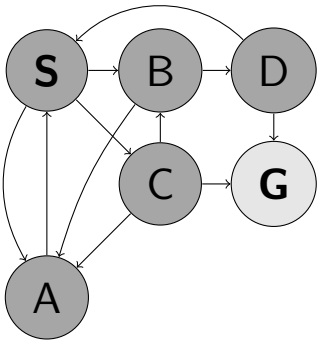

Find good names

# Search algorithm partitions state space into 3 disjoint sets



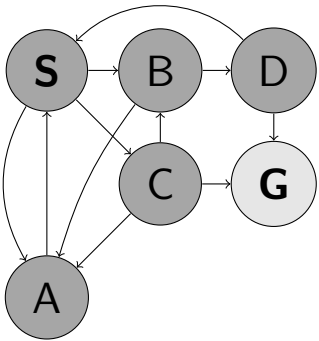

Find good names

# Search algorithm partitions state space into 3 disjoint sets



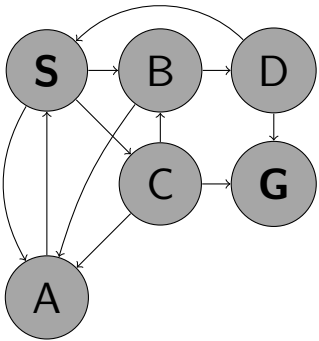

Find good names

# Search algorithm partitions state space into 3 disjoint sets




Find good names

# Search algorithm partitions state space into 3 disjoint sets




Find good names

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Search (algorithm) properties

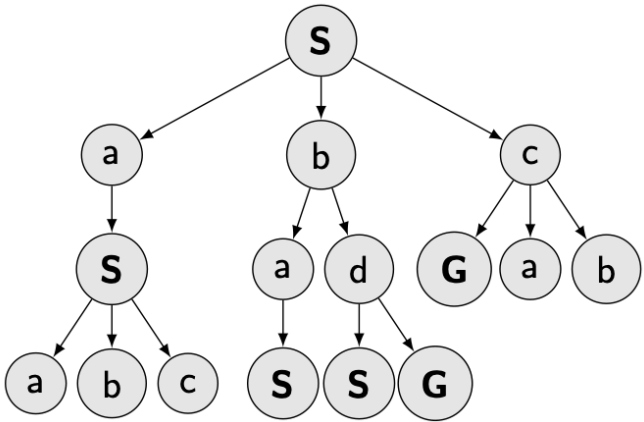
- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

# Strategies

How to traverse/build a search tree?

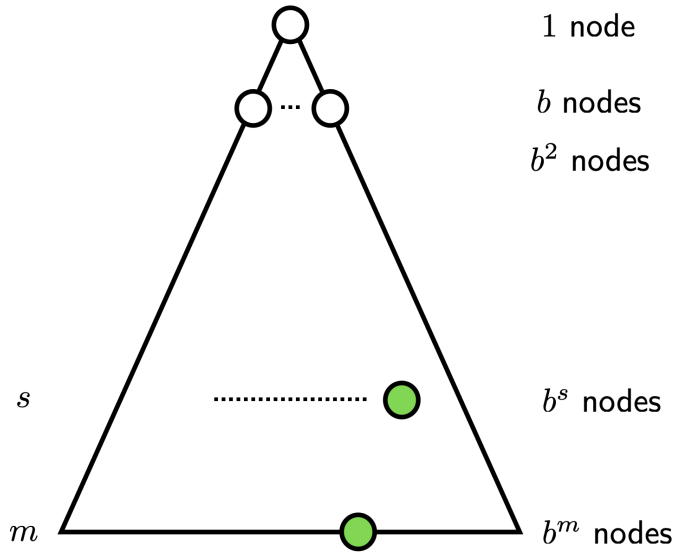
- Depth $d$ of a node in the tree.
- Max-Depth of the tree $m$. Can be $\infty$.
- (Average) Branching factor $b$.
- $s$ denotes the depth of the shallowest Goal .
- How many nodes in the whole tree?



# Strategies

How to traverse/build a search tree?

- Depth $d$ of a node in the tree.
- Max-Depth of the tree $m$. Can be $\infty$.
- (Average) Branching factor $b$.
- $s$ denotes the depth of the shallowest Goal .
- How many nodes in the whole tree?



# BFS properties

Complete?

Optimal?

Time complexity?

A $\mathcal{O}(bm)$

B $\mathcal{O}(b^m)$

C $\mathcal{O}(m^b)$

D $\mathcal{O}(b^s)$

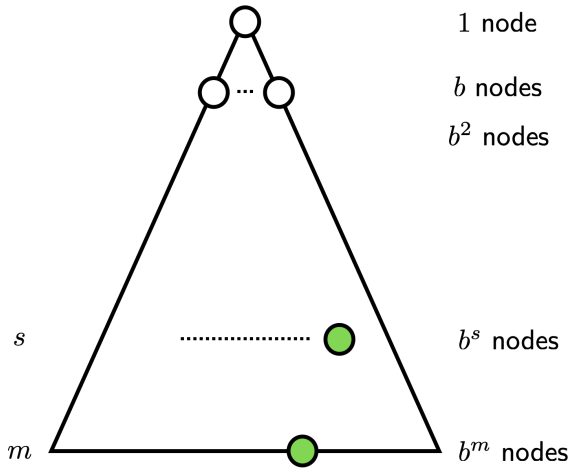
Space complexity?

A $\mathcal{O}(bm)$

B $\mathcal{O}(b^m)$

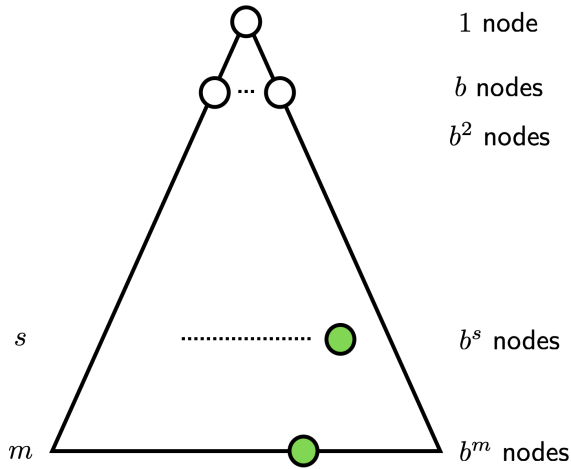
C $\mathcal{O}(m^b)$

D $\mathcal{O}(b^s)$



# BFS properties

Complete?
Optimal?
Time complexity?
A $\mathcal{O}(bm)$
B $\mathcal{O}(b^m)$
C $\mathcal{O}(m^b)$
D $\mathcal{O}(b^s)$
Space complexity?
A $\mathcal{O}(bm)$
B $\mathcal{O}(b^m)$
C $\mathcal{O}(m^b)$
D $\mathcal{O}(b^s)$



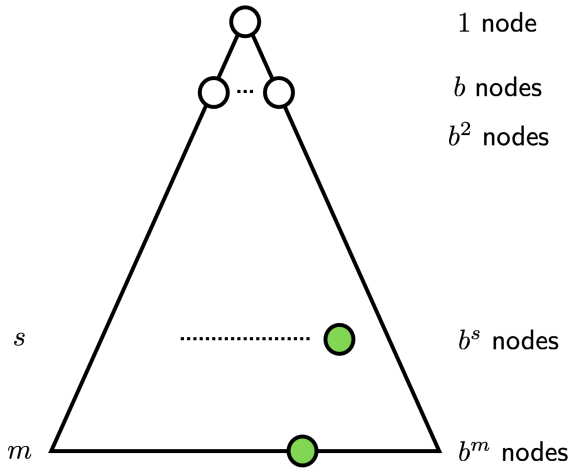
# BFS properties

Complete?
Optimal?
Time complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$

Space complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$



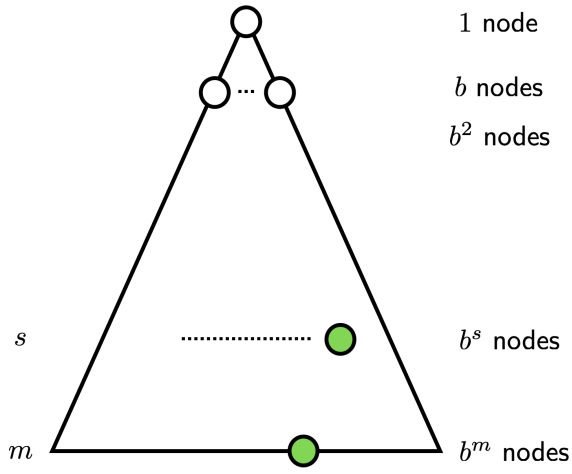
# BFS properties

Complete?
Optimal?
Time complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$

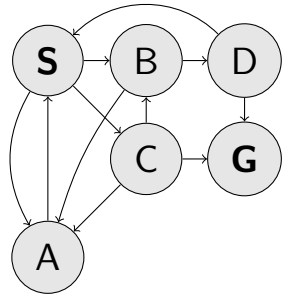
Space complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$



# DFS, Q is LIFO data structure (stack)

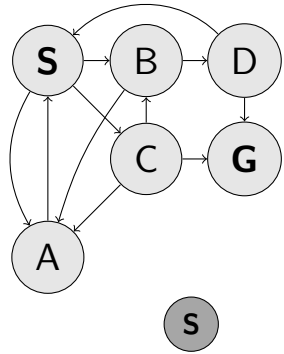
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (_,**S**)
visited: **S**

# DFS, Q is LIFO data structure (stack)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
    return Failure
```

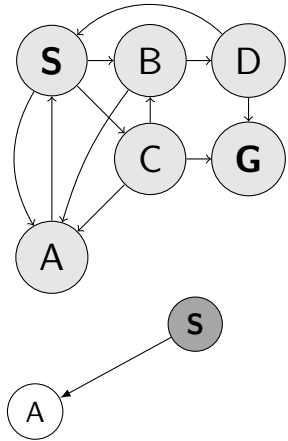



Q:

visited: **S**

# DFS, Q is LIFO data structure (stack)

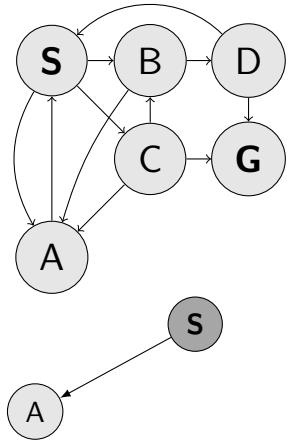
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
    return Failure
```




Q:
visited: **S**

# DFS, Q is LIFO data structure (stack)

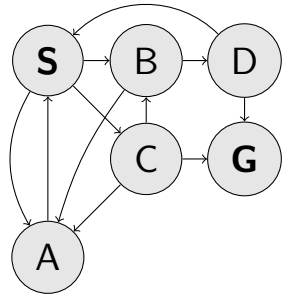
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A)
visited: **S** A

# DFS, Q is LIFO data structure (stack)

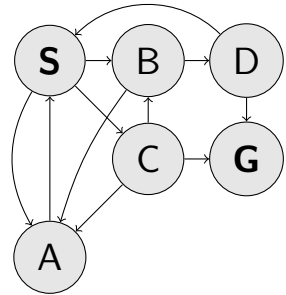
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A)
visited: **S** A

# DFS, Q is LIFO data structure (stack)

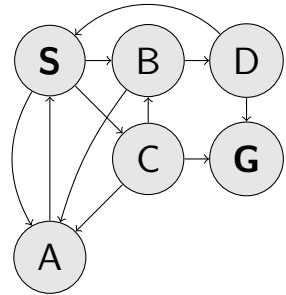
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A) (S,B)
visited: **S** A B

# DFS, Q is LIFO data structure (stack)

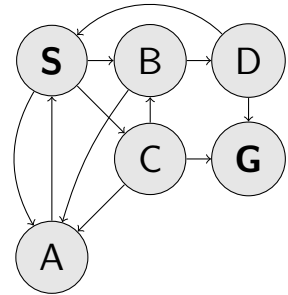
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
    return Failure
```




Q: (S,A) (S,B)
visited: **S** A B

# DFS, Q is LIFO data structure (stack)

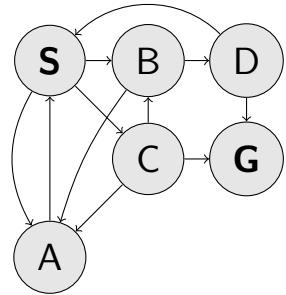
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

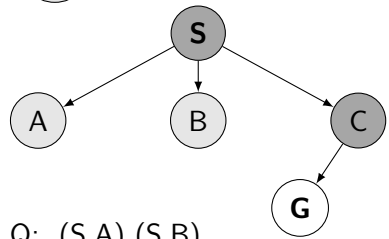



Q: (S,A) (S,B) (S,C)
visited: **S** A B C

# DFS, Q is LIFO data structure (stack)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

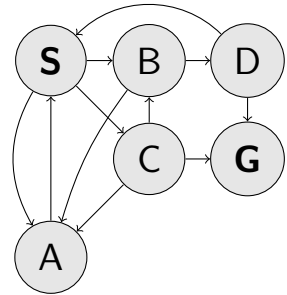


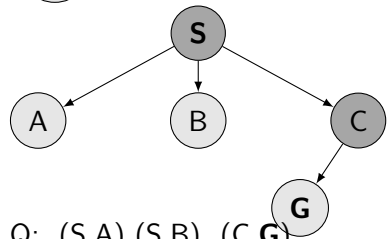




Q: (S,A) (S,B)
visited: **S** A B C

# DFS, Q is LIFO data structure (stack)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

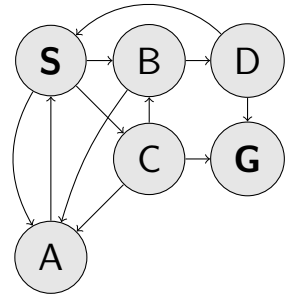


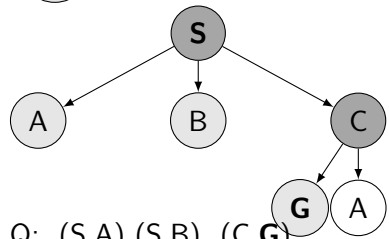




Q: (S,A) (S,B) (C,**G**)
visited: **S** A B C **G**

# DFS, Q is LIFO data structure (stack)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

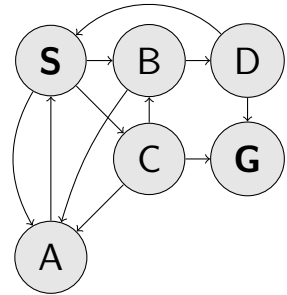


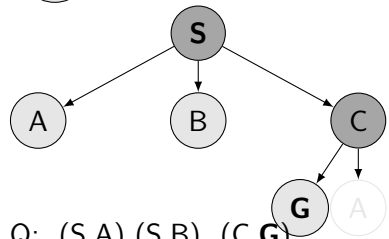




Q: (S,A) (S,B) (C,**G**)
visited: **S** A B C **G**

# DFS, Q is LIFO data structure (stack)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
    return Failure
```

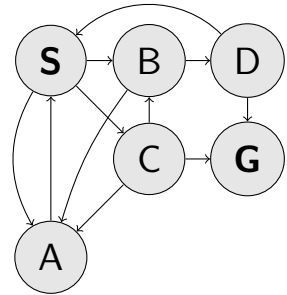


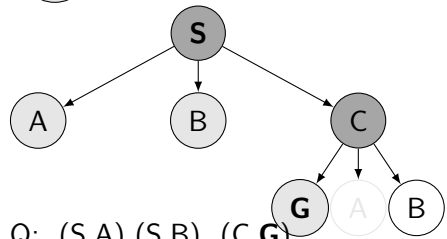




Q: (S,A) (S,B) (C,**G**)
visited: **S** A B C **G**

# DFS, Q is LIFO data structure (stack)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

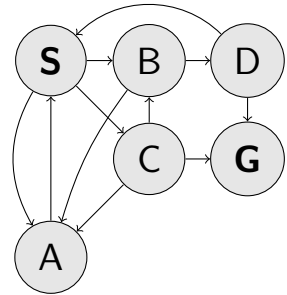


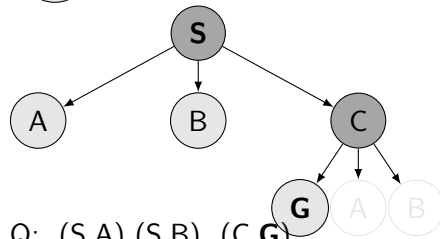




Q: (S,A) (S,B) (C,**G**)
visited: **S** A B C **G**

# DFS, Q is LIFO data structure (stack)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```

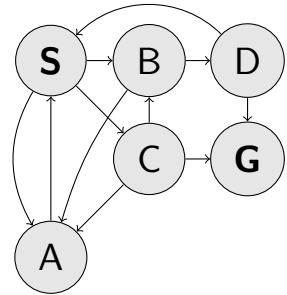


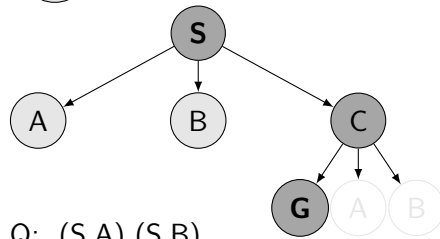




Q: (S,A) (S,B) (C,**G**)
visited: **S** A B C **G**

# DFS, Q is LIFO data structure (stack)

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s' ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s')
12:            else
13:                Resolve duplicate s'
   return Failure
```




Q: (S,A) (S,B)
visited: **S** A B C **G**

# DFS properties

Complete?

Optimal?

Time complexity?

A $\mathcal{O}(bm)$

B $\mathcal{O}(b^m)$

C $\mathcal{O}(m^b)$

D $\mathcal{O}(b^s)$

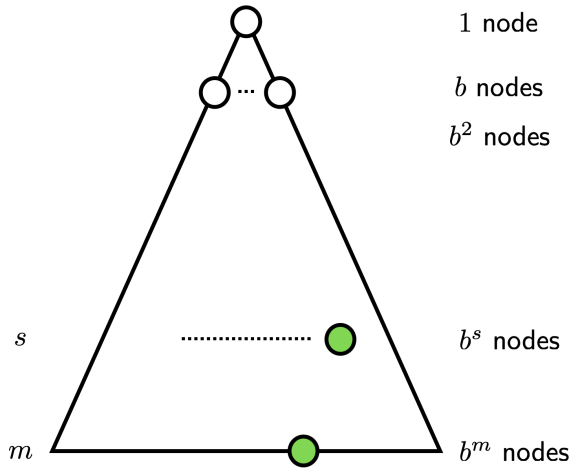
Space complexity?

A $\mathcal{O}(bm)$

B $\mathcal{O}(b^m)$

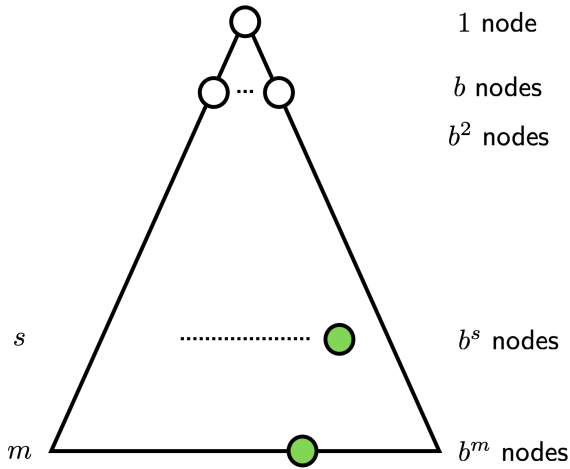
C $\mathcal{O}(m^b)$

D $\mathcal{O}(b^s)$



# DFS properties

Complete?
Optimal?
Time complexity?
A $\mathcal{O}(bm)$
B $\mathcal{O}(b^m)$
C $\mathcal{O}(m^b)$
D $\mathcal{O}(b^s)$
Space complexity?
A $\mathcal{O}(bm)$
B $\mathcal{O}(b^m)$
C $\mathcal{O}(m^b)$
D $\mathcal{O}(b^s)$



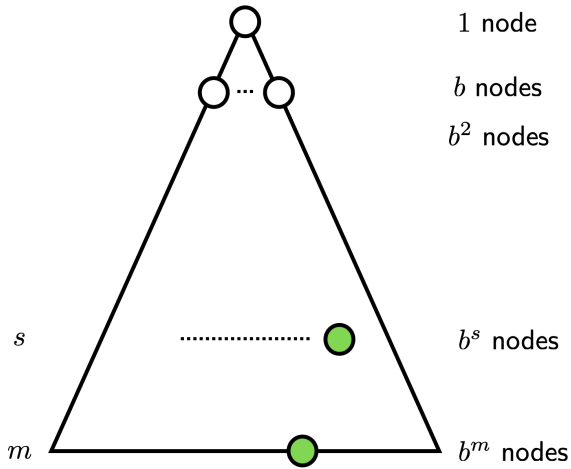
# DFS properties

Complete?
Optimal?
Time complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$

Space complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$



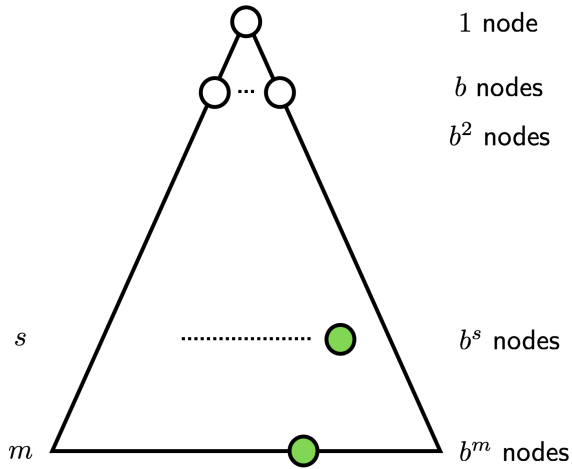
# DFS properties

Complete?
Optimal?
Time complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$

Space complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$



# Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

# Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

# Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

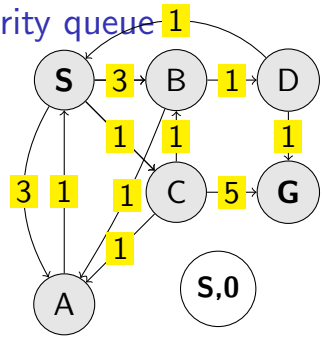
# Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

# Uniform Cost Search (Dijkstra), Q is priority queue

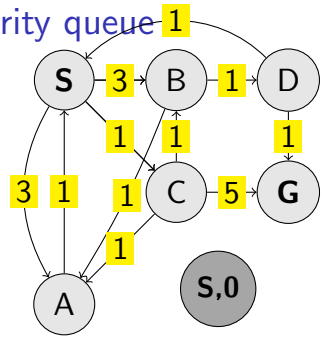
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (_,**S**,0)
visited: **S**

# Uniform Cost Search (Dijkstra), Q is priority queue

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```

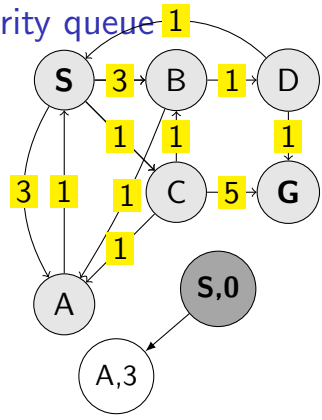



Q:

visited: **S**

# Uniform Cost Search (Dijkstra), Q is priority queue

```
1:  function FORWARD_SEARCH
2:      Q.insert(_, s_0, 0) and mark s_0 as visited
3:      while Q not empty do
4:          p, s, _ ← Q.pop_first()
5:          parent[s] ← p
6:          if s ∈ S_G then return Success
7:          for all a ∈ A(s) do
8:              s', c ← result(s, a)                ▷ c cost
9:              if s' not visited then
10:                 Mark s' as visited
11:                 Q.insert(s, s', cost_from_start)
12:             else
13:                 Resolve duplicate s'
        return Failure
```

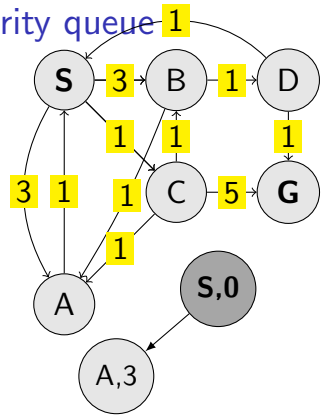



Q:

visited: **S**

# Uniform Cost Search (Dijkstra), Q is priority queue

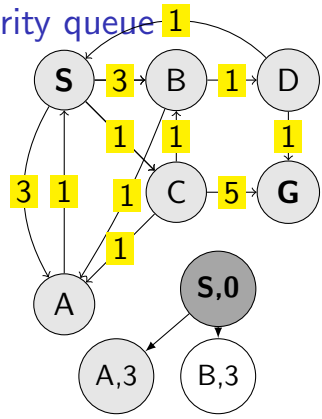
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,A,3)
visited: **S** A

# Uniform Cost Search (Dijkstra), Q is priority queue

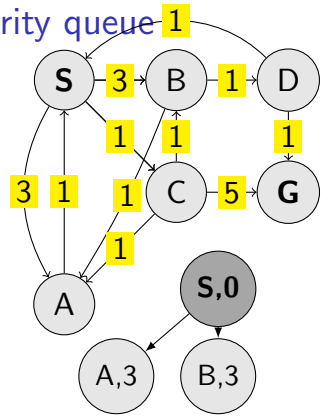
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,A,3)
visited: **S** A

# Uniform Cost Search (Dijkstra), Q is priority queue

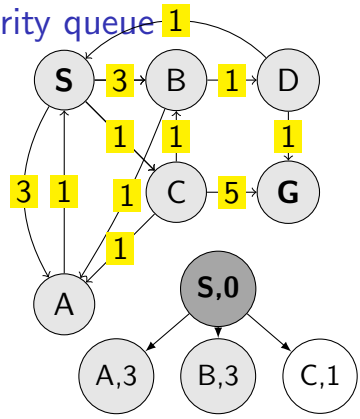
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,A,3) (S,B,3)
visited: **S** A B

# Uniform Cost Search (Dijkstra), Q is priority queue

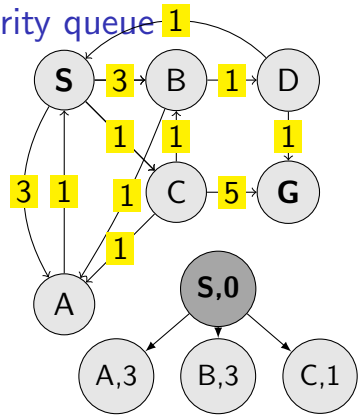
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,A,3) (S,B,3)
visited: **S** A B

# Uniform Cost Search (Dijkstra), Q is priority queue

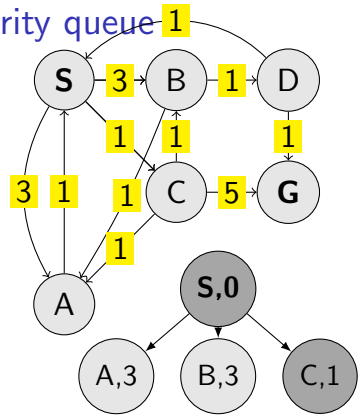
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,C,1) (S,A,3) (S,B,3)
visited: **S** A B C

# Uniform Cost Search (Dijkstra), Q is priority queue

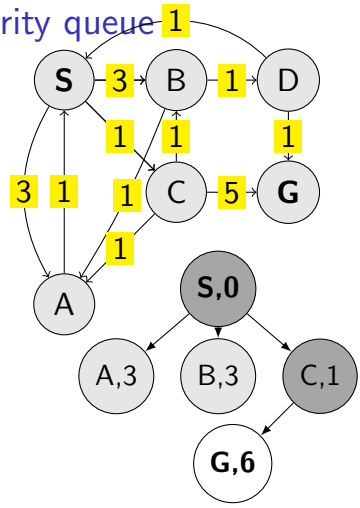
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,A,3) (S,B,3)
visited: **S** A B C

# Uniform Cost Search (Dijkstra), Q is priority queue

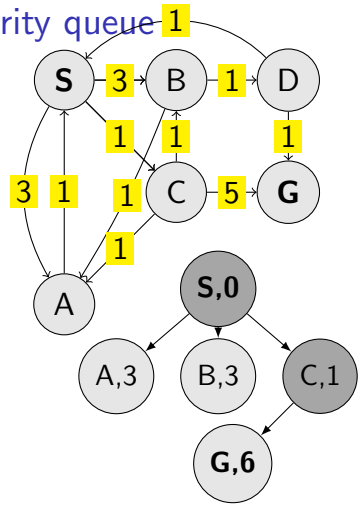
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,A,3) (S,B,3)
visited: **S** A B C

# Uniform Cost Search (Dijkstra), Q is priority queue

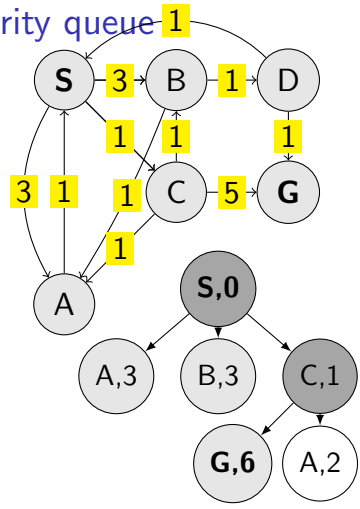
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,A,3) (S,B,3) (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

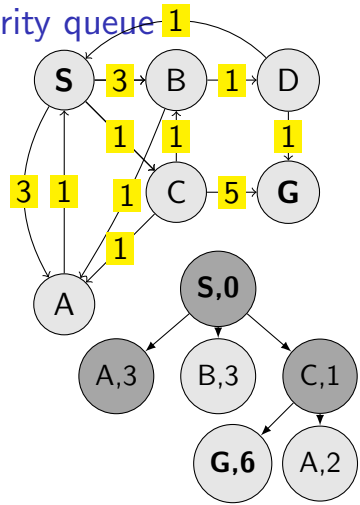
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (S,A,3) (S,B,3) (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

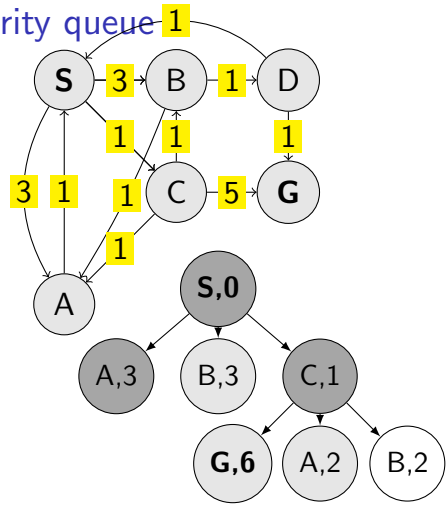
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,A,2) (S,B,3) (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

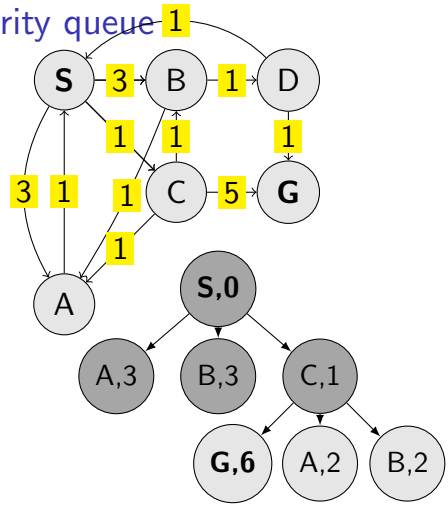
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,A,2) (S,B,3) (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

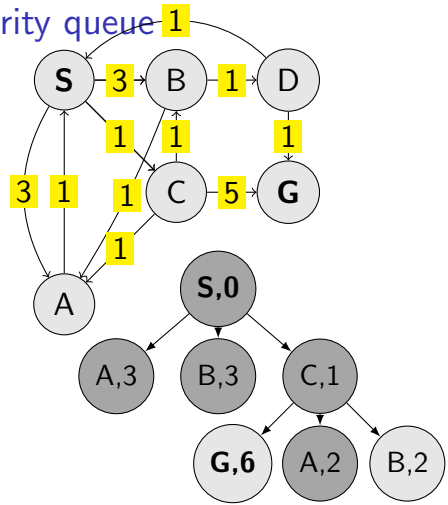
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,A,2) (C,B,2) (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

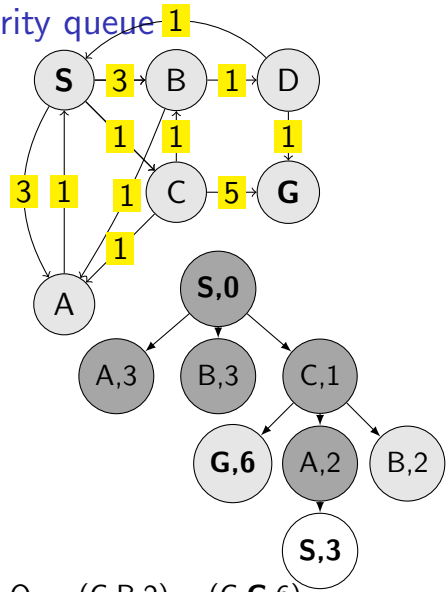
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,B,2) (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

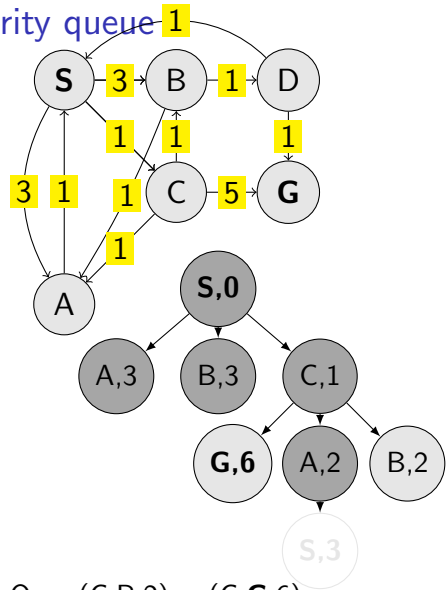
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,B,2) (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

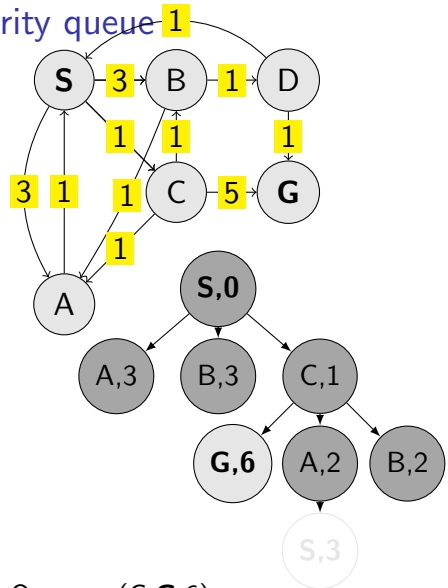
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,B,2) (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

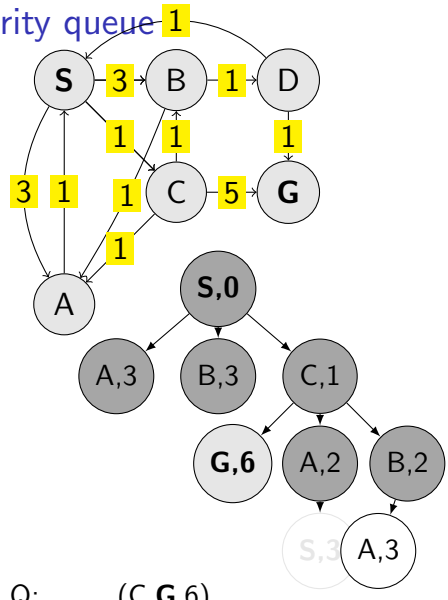
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

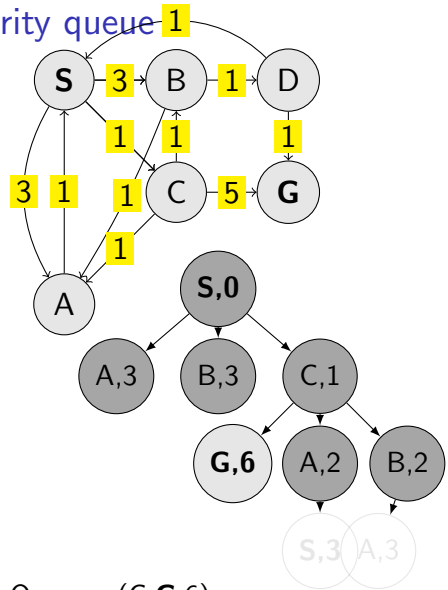
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

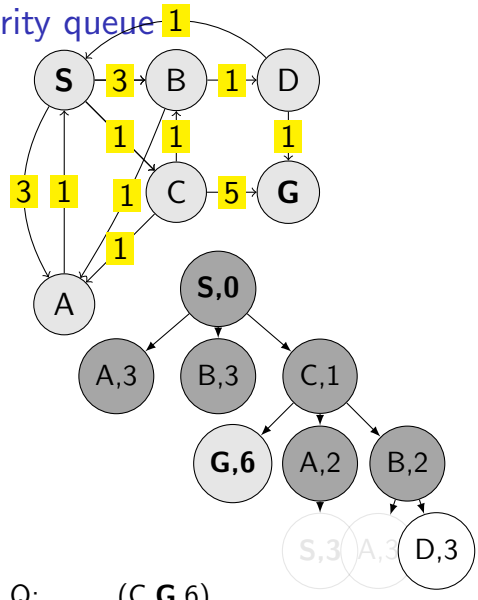
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

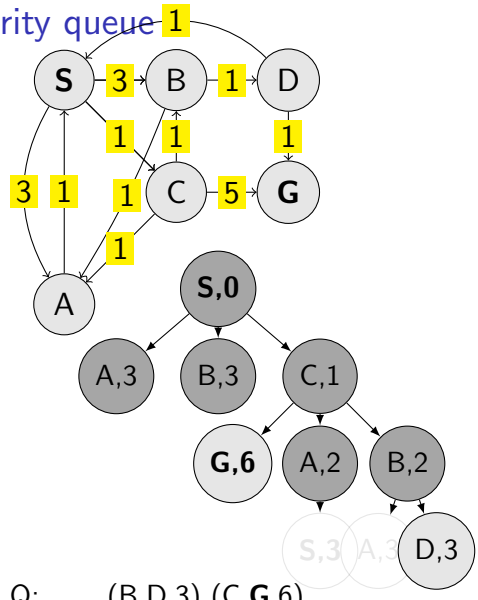
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,**G**,6)
visited: **S** A B C **G**

# Uniform Cost Search (Dijkstra), Q is priority queue

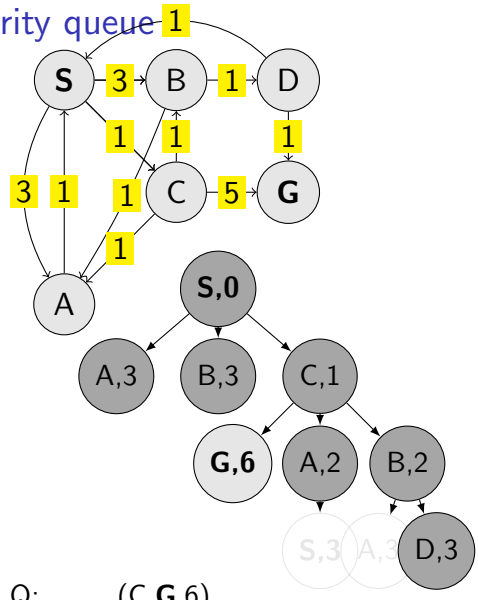
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (B,D,3) (C,**G**,6)
visited: **S** A B C **G** D

# Uniform Cost Search (Dijkstra), Q is priority queue

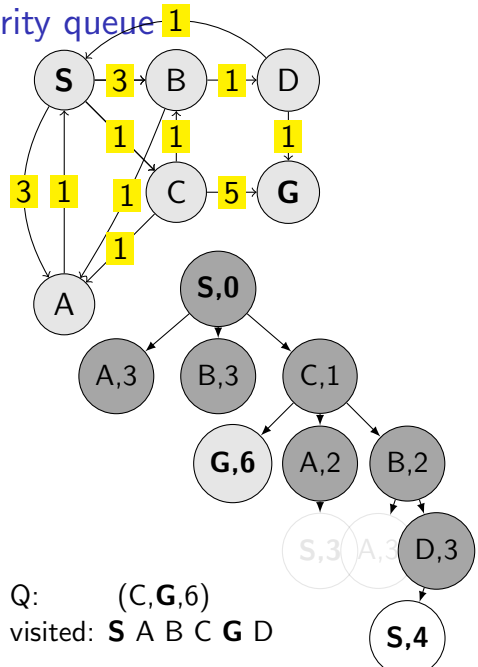
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,**G**,6)
visited: **S** A B C **G** D

# Uniform Cost Search (Dijkstra), Q is priority queue

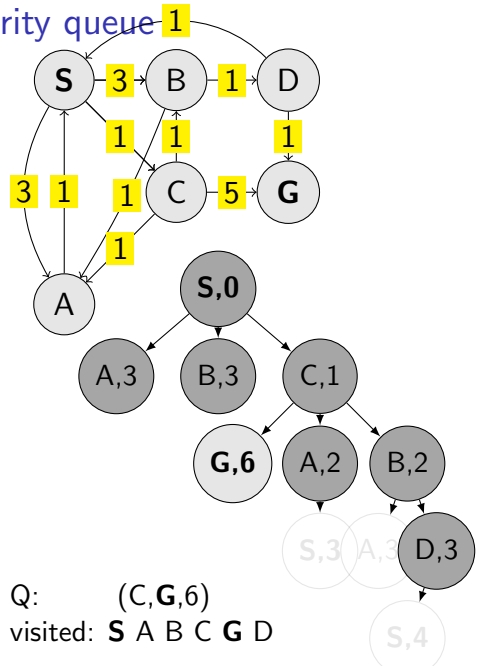
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,**G**,6)
visited: **S** A B C **G** D

# Uniform Cost Search (Dijkstra), Q is priority queue

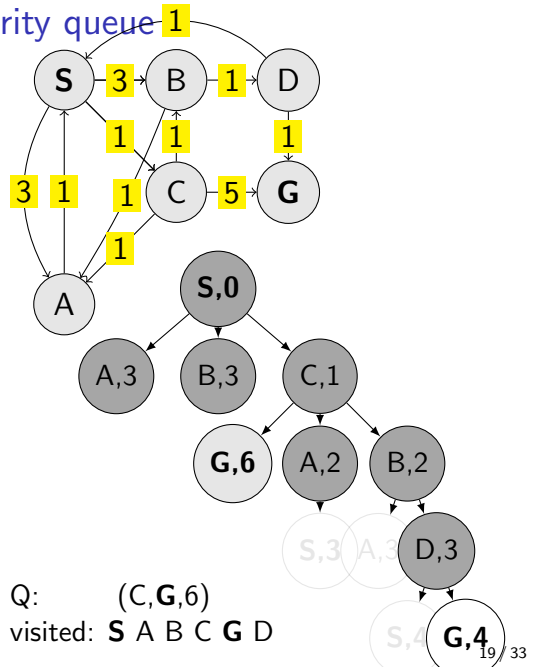
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,**G**,6)
visited: **S** A B C **G** D

# Uniform Cost Search (Dijkstra), Q is priority queue

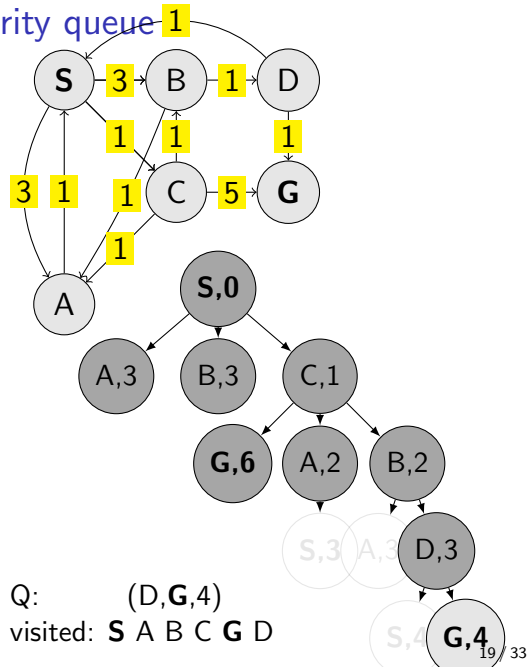
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (C,**G**,6)
visited: **S** A B C **G** D

# Uniform Cost Search (Dijkstra), Q is priority queue

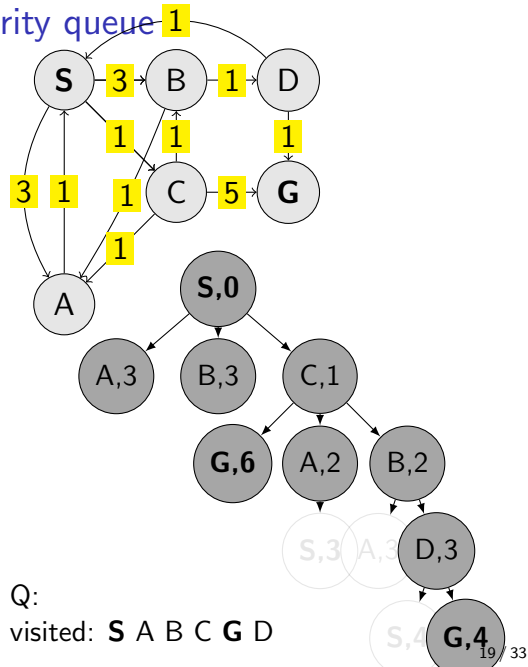
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q: (D,**G**,4)
visited: **S** A B C **G** D

# Uniform Cost Search (Dijkstra), Q is priority queue

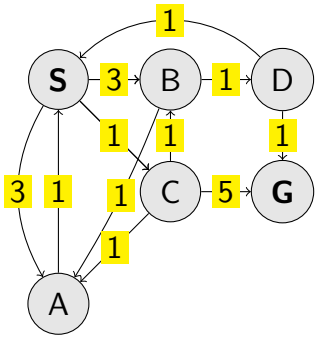
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 as visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop_first()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)                ▷ c cost
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start)
12:            else
13:                Resolve duplicate s'
       return Failure
```




Q:

visited: **S** A B C **G** D

# UCS properties

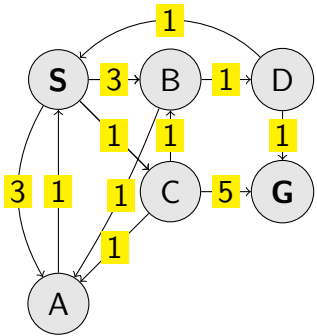



Complete?

Optimal?

Complexities?

# UCS properties

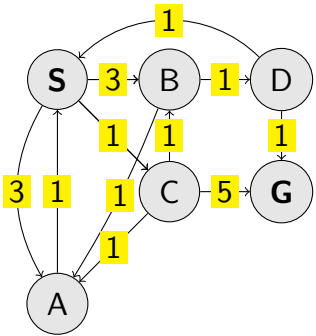


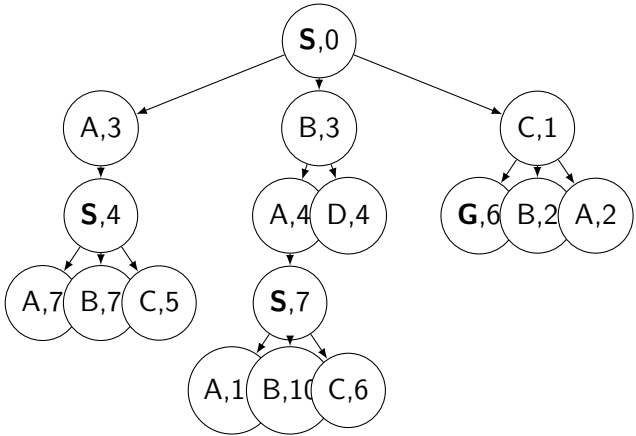

Complete?

Optimal?

Complexities?

# UCS properties




Complete?
Optimal?
Complexities?

# Node selection, take argmin $f(n)$. Search Node: $n = (p, s, \text{cost\_value})$

Selecting next node to explore (pop operation):

$$\texttt{node} \leftarrow \operatorname*{argmin}_{n \in \texttt{Q}} f(n)$$

What is $f(n)$ for DFS, BFS, and UCS?

- DFS:
- BFS:
- UCS:

- $f(n) = n.\text{cost\_from\_start}$
- $f(n) = n.\text{depth}$
- $f(n) = -n.\text{depth}$

The good: (one) frontier as a priority queue
(I.e., priority queue will work universally. Still, stack (LIFO) and queue (FIFO) are (conceptually) the perfect data structures for DFS and BFS, respectively.)
The bad: All the $f(n)$ correspond to the accumulated cost from start to $n$, cost_from_start .

# Node selection, take argmin $f(n)$. Search Node: $n = (p, s, \text{cost\_value})$

Selecting next node to explore (pop operation):

$$\texttt{node} \leftarrow \operatorname*{argmin}_{n \in \texttt{Q}} f(n)$$

What is $f(n)$ for DFS, BFS, and UCS?

- DFS:
  - $f(n) = n.\text{cost\_from\_start}$
- BFS:
  - $f(n) = n.\text{depth}$
- UCS:
  - $f(n) = -n.\text{depth}$

The good: (one) frontier as a priority queue
(I.e., priority queue will work universally. Still, stack (LIFO) and queue (FIFO) are (conceptually) the perfect data structures for DFS and BFS, respectively.)
The bad: All the $f(n)$ correspond to the accumulated cost from start to $n$, cost_from_start .

# Node selection, take argmin $f(n)$. Search Node: $n = (p, s, \text{cost\_value})$

Selecting next node to explore (pop operation):

$$\texttt{node} \leftarrow \operatorname*{argmin}_{n \in \texttt{Q}} f(n)$$

What is $f(n)$ for DFS, BFS, and UCS?

- DFS:
  - $f(n) = n.\text{cost\_from\_start}$
- BFS:
  - $f(n) = n.\text{depth}$
- UCS:
  - $f(n) = -n.\text{depth}$

The good: (one) frontier as a priority queue
(I.e., priority queue will work universally. Still, stack (LIFO) and queue (FIFO) are (conceptually) the perfect data structures for DFS and BFS, respectively.)

The bad: All the $f(n)$ correspond to the accumulated cost from start to $n$, cost_from_start .

# Node selection, take argmin $f(n)$. Search Node: $n = (p, s, \text{cost\_value})$

Selecting next node to explore (pop operation):

$$\texttt{node} \leftarrow \operatorname*{argmin}_{n \in \texttt{Q}} f(n)$$

What is $f(n)$ for DFS, BFS, and UCS?

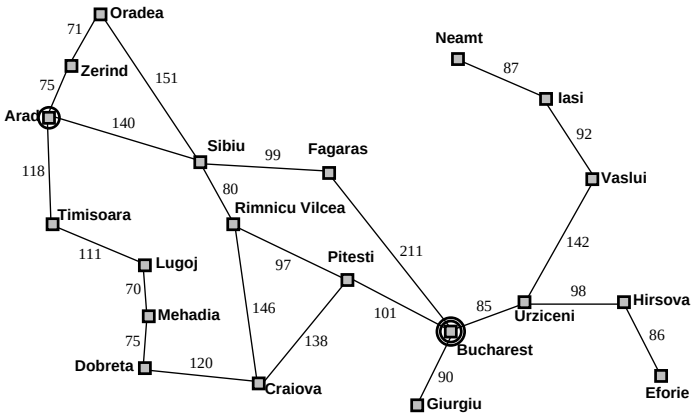
- DFS:
  - $f(n) = n.\text{cost\_from\_start}$
- BFS:
  - $f(n) = n.\text{depth}$
- UCS:
  - $f(n) = -n.\text{depth}$

The good: (one) frontier as a priority queue
(I.e., priority queue will work universally. Still, stack (LIFO) and queue (FIFO) are (conceptually) the perfect data structures for DFS and BFS, respectively.)
The bad: All the $f(n)$ correspond to the accumulated cost from start to $n$, cost_from_start .

# How far are we from the goal cost-to-go ? – Heuristics

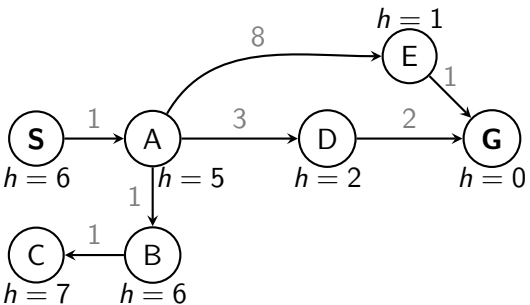
- A function that estimates how close a *state* is to the goal.
- Designed for a particular problem.
- $h(s)$ – it is function of the state (attribute of the search node)
- It is often shortened as $h(n)$ – heuristic value of node $n$.

# Example of heuristics



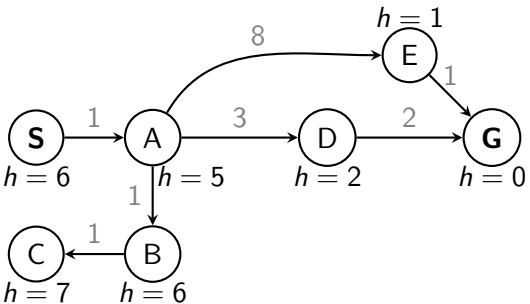

| | | | |
|---|---|---|---|
| **Arad** | 366 | **Mehadia** | 241 |
| **Bucharest** | 0 | **Neamt** | 234 |
| **Craiova** | 160 | **Oradea** | 380 |
| **Drobeta** | 242 | **Pitesti** | 100 |
| **Eforie** | 161 | **Rimnicu Vilcea** | 193 |
| **Fagaras** | 176 | **Sibiu** | 253 |
| **Giurgiu** | 77 | **Timisoara** | 329 |
| **Hirsova** | 151 | **Urziceni** | 80 |
| **Iasi** | 226 | **Vaslui** | 199 |
| **Lugoj** | 244 | **Zerind** | 374 |

# Greedy, take the $n^* = \operatorname{argmin}_{n \in \mathtt{Q}} h(n)$



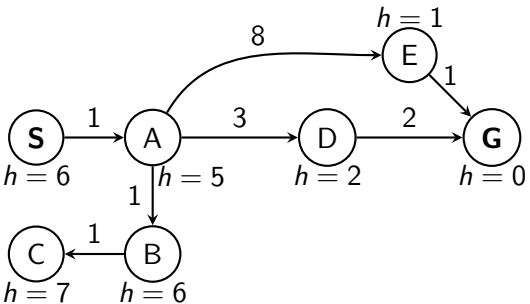

What is wrong (and nice) with the Greedy?

# Greedy, take the $n^* = \operatorname{argmin}_{n \in \mathtt{Q}} h(n)$




What is wrong (and nice) with the Greedy?

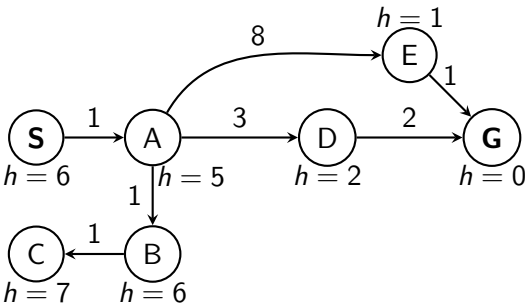
# A* combines UCS and Greedy




UCS orders (path) cost_from_start $g(n)$
Greedy uses heuristics (goal proximity) $h(n)$

A* orders nodes by: $f(n) = g(n) + h(n)$

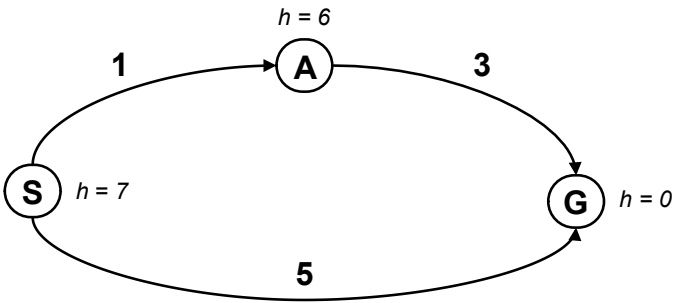
# A* combines UCS and Greedy




UCS orders (path) cost_from_start $g(n)$
Greedy uses heuristics (goal proximity) $h(n)$

A* orders nodes by: $f(n) = g(n) + h(n)$

# Is A* optimal?

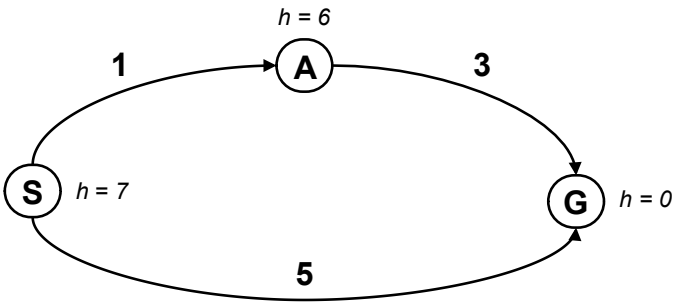



[2]

What is the problem?

[2] Graph example: Dan Klein and Pieter Abbeel

# Is A* optimal?




[2]

What is the problem?

[2] Graph example: Dan Klein and Pieter Abbeel

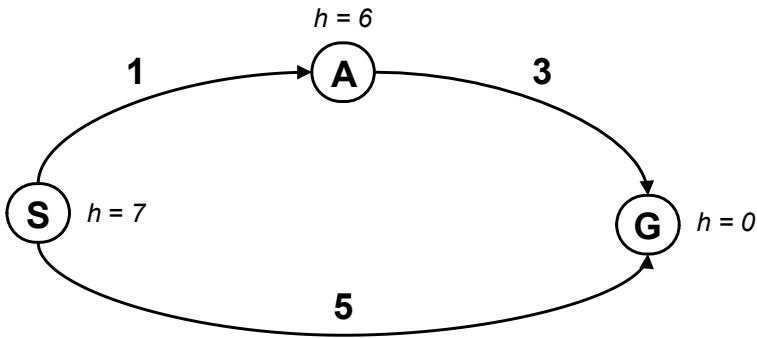
# What is the right $h(A)$?

A: $0 \leq h(A) \leq 4$

B: $h(A) \leq 3$

C: $0 \leq h(A) \leq 3$

D: $0 \leq h(A)$



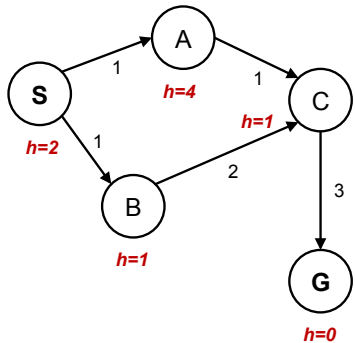
# Admissible heuristics

A heuristic function $h$ is admissible if:

$$\begin{aligned} h(n) &\leq \text{cost}(n.state, \text{Goal}_{\text{nearest}}) \\ h(\text{Goal}) &= 0 \end{aligned}$$

# Consistent heuristic

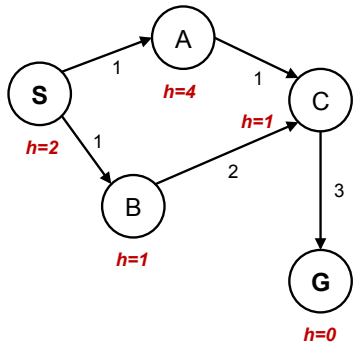
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```




S,2

# Consistent heuristic

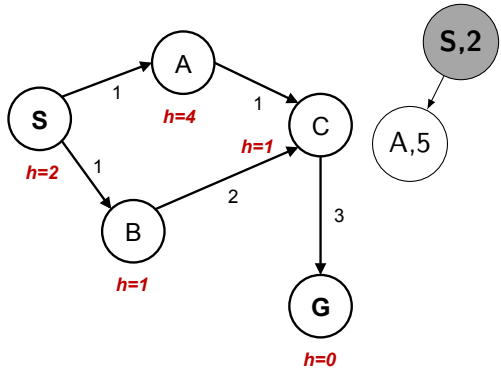
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```




S,2

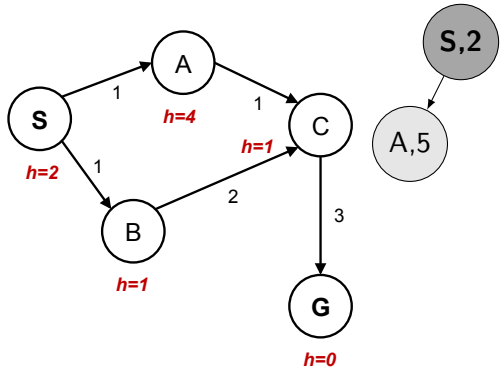
# Consistent heuristic

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```



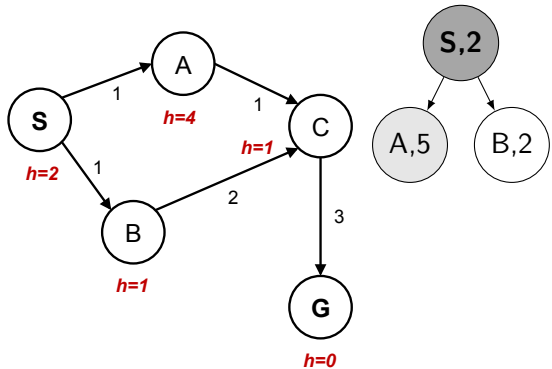
# Consistent heuristic

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```



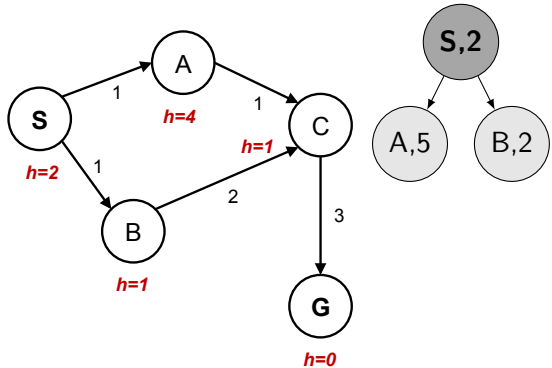
# Consistent heuristic

```
1:  function FORWARD_SEARCH
2:      Q.insert(_, s_0, 0) and mark s_0 visited
3:      while Q not empty do
4:          p, s, _ ← Q.pop()
5:          parent[s] ← p
6:          if s ∈ S_G then return Success
7:          for all a ∈ A(s) do
8:              s', c ← result(s, a)
9:              if s' not visited then
10:                 Mark s' as visited
11:                 Q.insert(s, s', cost_from_start + h(s'))
12:             else
13:                 Resolve duplicate s'
        return Failure
```



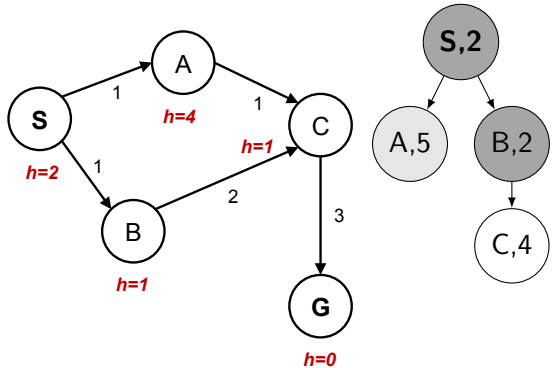
# Consistent heuristic

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```



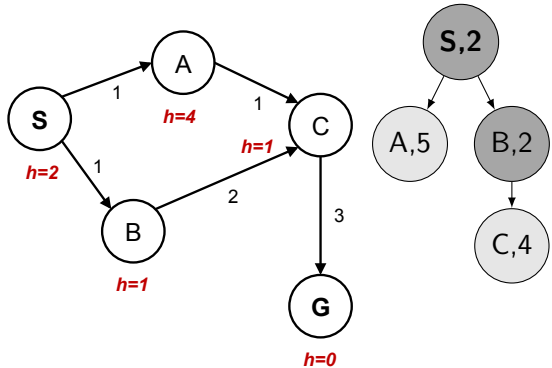
# Consistent heuristic

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```



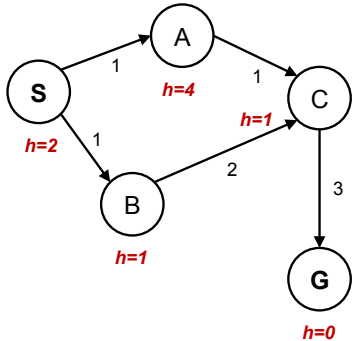
# Consistent heuristic

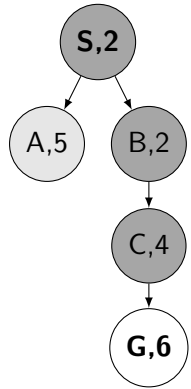
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```



# Consistent heuristic

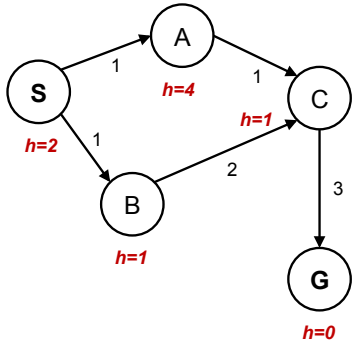
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```

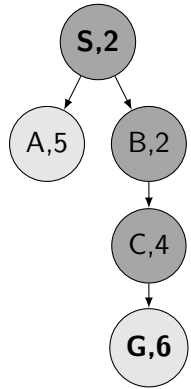


# Consistent heuristic

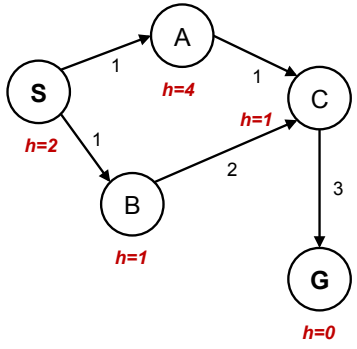
```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```



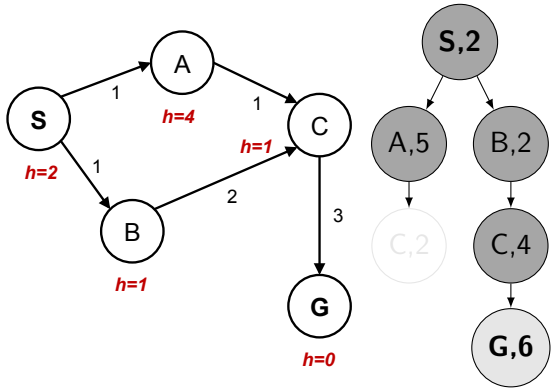
# Consistent heuristic

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```



# Consistent heuristic

```
1: function FORWARD_SEARCH
2:     Q.insert(_, s_0, 0) and mark s_0 visited
3:     while Q not empty do
4:         p, s, _ ← Q.pop()
5:         parent[s] ← p
6:         if s ∈ S_G then return Success
7:         for all a ∈ A(s) do
8:             s', c ← result(s, a)
9:             if s' not visited then
10:                Mark s' as visited
11:                Q.insert(s, s', cost_from_start + h(s'))
12:            else
13:                Resolve duplicate s'
       return Failure
```



# What would be the proper $h(A)$?

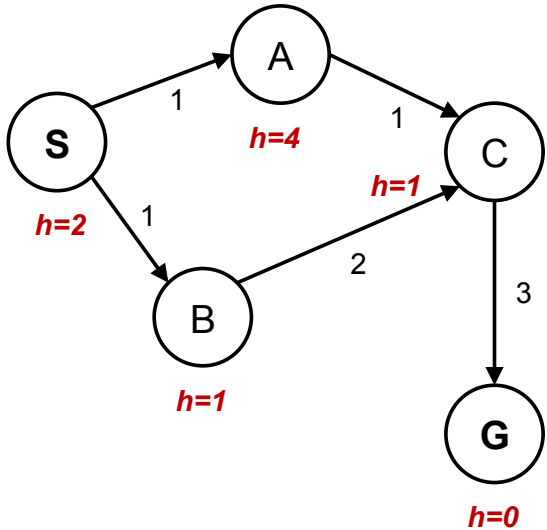
Consider other $h(s)$ fixed.

A: $h(A) = 1$

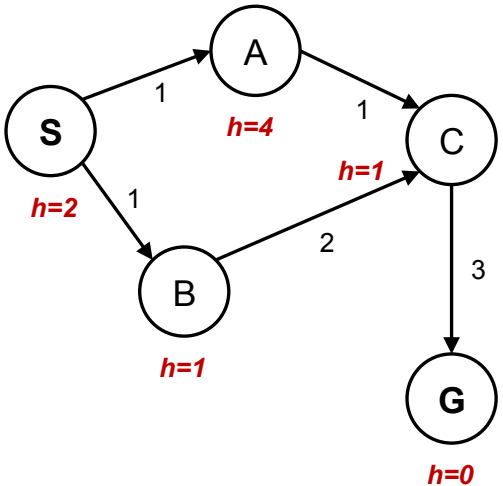
B: $h(A) = 2$

C: $1 \leq h(A) \leq 2$

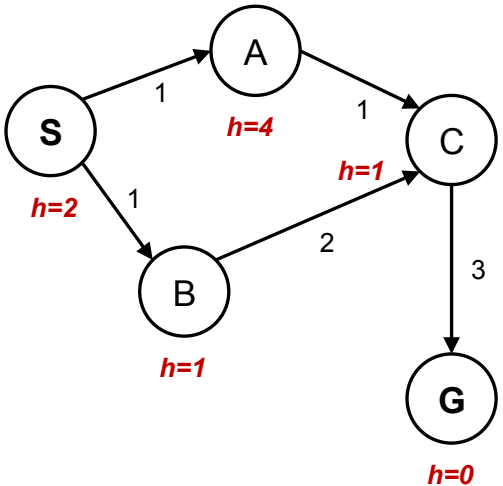
D: $0 \leq h(A) \leq 1$



# Consistent heuristics



# Consistent heuristics

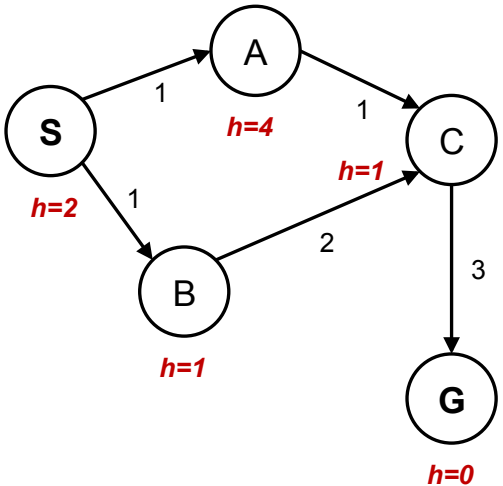



Admissible $h$:
$h(A) \leq$ true cost $A \to G$

Consistent $h$:
$h(A) - h(C) \leq$ true cost $A \to C$
in general:
$h(p) - h(s) \leq$ true cost $p \to s$ for any pair:
parent $p$ and its successor $s$

$f(n) = g(n) + h(n)$ along a path never decreases!

# Consistent heuristics

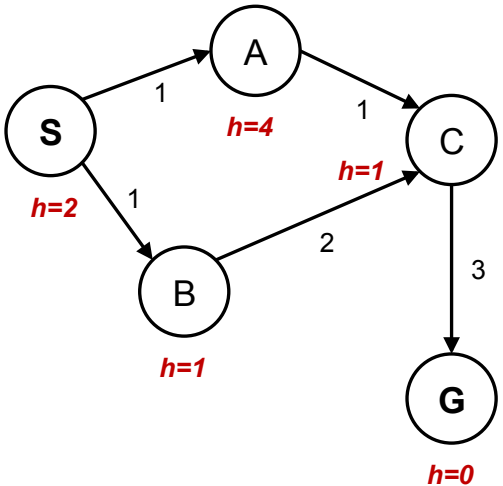



Admissible $h$:
$h(A) \leq$ true cost $A \to G$

Consistent $h$:
$h(A) - h(C) \leq$ true cost $A \to C$
in general:
$h(p) - h(s) \leq$ true cost $p \to s$ for any pair:
parent $p$ and its successor $s$

$f(n) = g(n) + h(n)$ along a path never decreases!

# Consistent heuristics




Admissible $h$:
$h(A) \leq$ true cost $A \to G$

Consistent $h$:
$h(A) - h(C) \leq$ true cost $A \to C$
in general:
$h(p) - h(s) \leq$ true cost $p \to s$ for any pair:
parent $p$ and its successor $s$

$f(n) = g(n) + h(n)$ along a path never decreases!

# Summary

- State space graph vs. Search Tree
- Search strategies - properties, complexities
- Evaluating states - cost_from_start and cost_to_go
- Effectivness – adding heuristic estimates of cost_to_go
- Not all heuristics are equally good (admissibility, consistence, informativeness)

# References, further reading

Some figures from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms.


[1] Steven M. LaValle.
*Planning Algorithms*.
Cambridge, 1st edition, 2006.
Online version available at: http://planning.cs.uiuc.edu.

[2] Stuart Russell and Peter Norvig.
*Artificial Intelligence: A Modern Approach*.
Prentice Hall, 4th edition, 2021.
http://aima.cs.berkeley.edu/.