Problem solving by search
Finding the optimal sequence of states/decisions/actions

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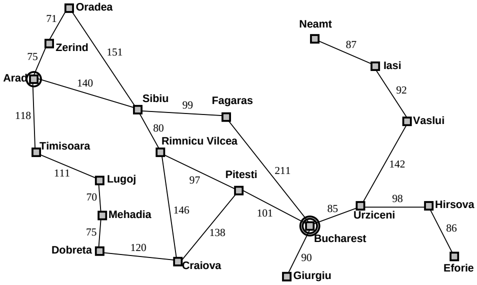
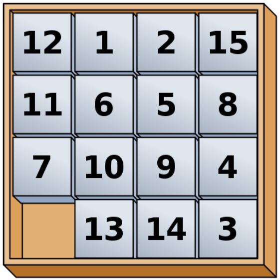
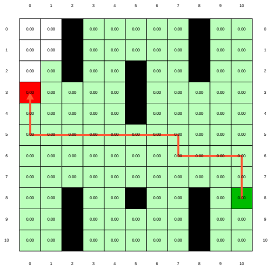
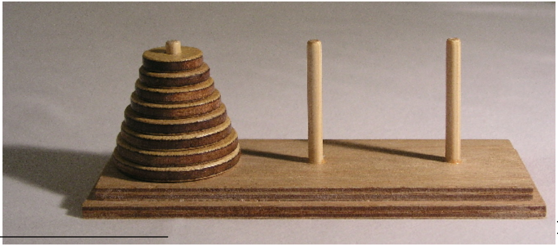
February 21, 2024

Problems to solve





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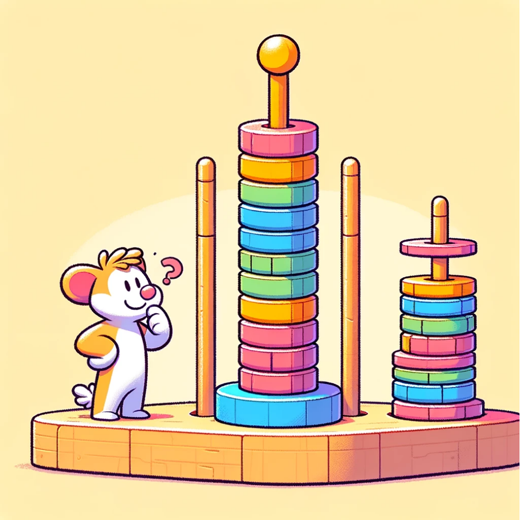
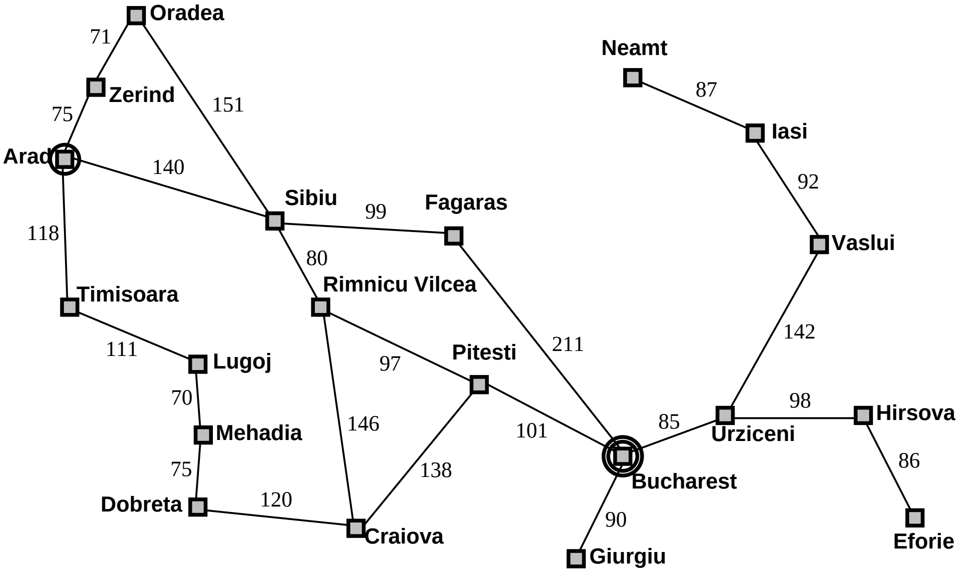
Understanding the problem is the key, DALL-E.



Outline

- Search problem. *What do you want to solve?*
- State space graphs. *How do you formalize/represent the problem? Problem abstraction.*
- Search trees. *Visualization of the algorithm run.*
- Strategies: which tree branches to choose?
- Strategy/Algorithm properties. *Memory, time, ...*
- Programming infrastructure.

Example: Traveling in Romania



Traveling Example: State and Actions

Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

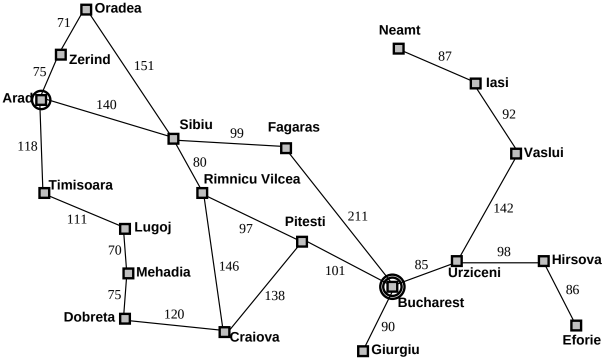
actions (decisions): select a road

Solution:

Sequence of cities (path)
(action/decision sequence [2])

Optimality – Cost:

Energy, time, tolls, ...



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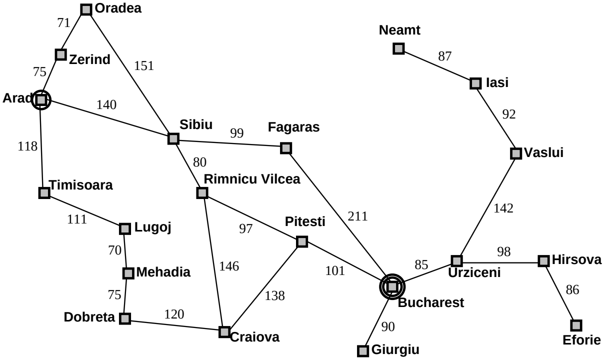

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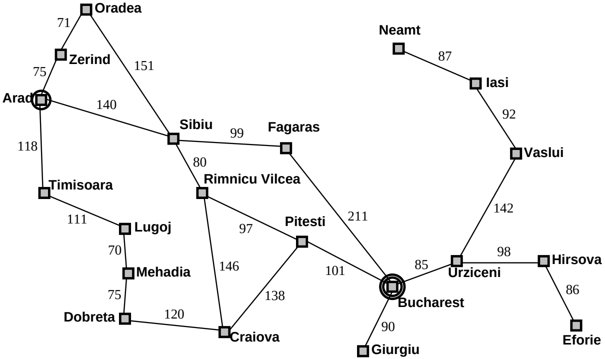

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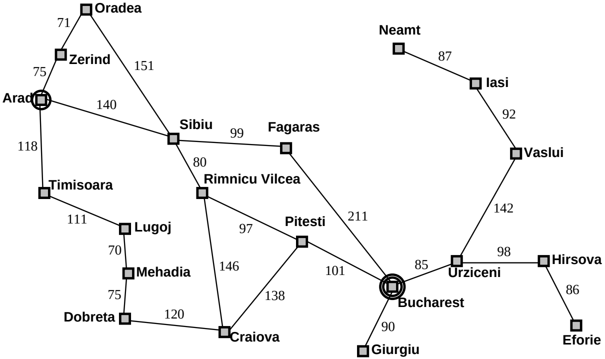

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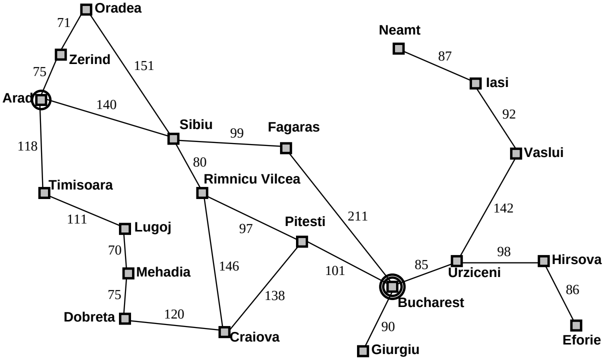

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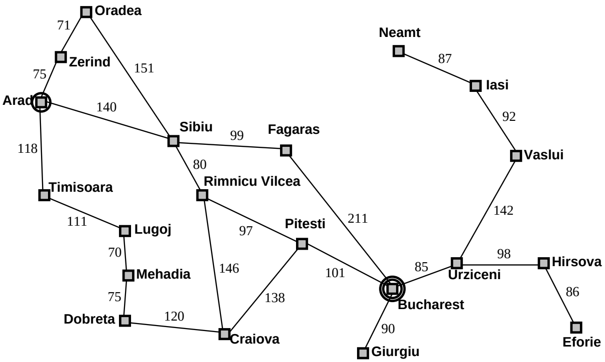
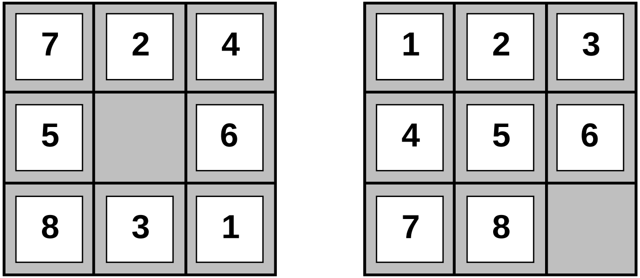


Example: The 8-puzzle



Start State **Goal State**

states?
actions?
solution?
cost?

A Search Problem

- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left . . .
- Transition model : Given state and action return state (and cost)
- Goal test : Are we done?

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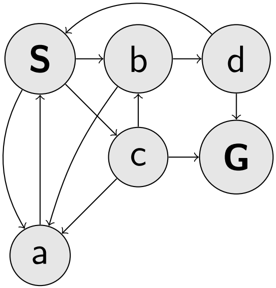
Discrete State Space

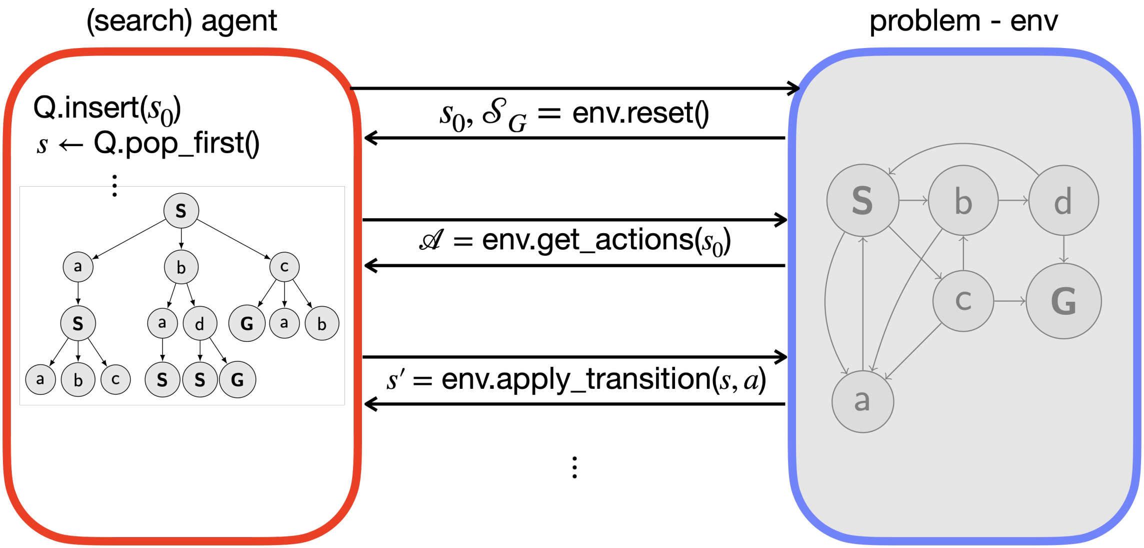
State space graph: a representation of a search problem

- States $s \in \mathcal{S} = \{\mathbf{S}, \mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathbf{G}\}$ (finite set)
- Arcs represent actions a, for each state s, $a \in \mathcal{A}(s)$ ($\mathcal{A}$ is also finite)
- State transition function $s' = \mathrm{result}(s, a)$
- Start (initial) state $s_0 \in \mathcal{S}$, $s_0 = \mathbf{S}$.
- Goal set $\mathcal{S}_G \subset \mathcal{S}$.

Each state occurs only *once* in a state (search) space.



agent – problem dialog (a programmer's viewpoint)



BFS, Q is FIFO data structure (queue)

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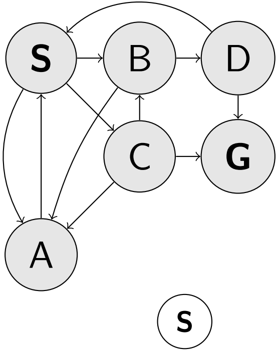

Q: (_,**S**)
visited: **S**

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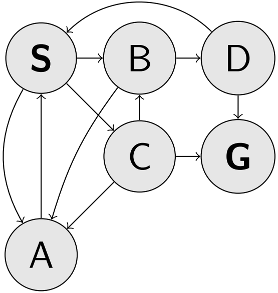

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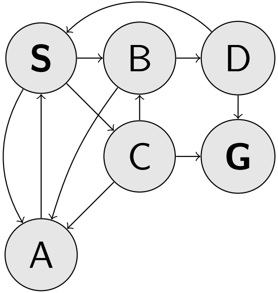

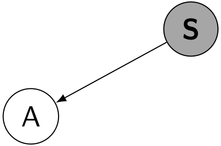

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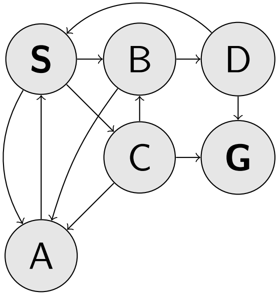

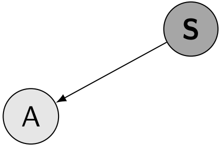

Q: (S,A)
visited: **S** A

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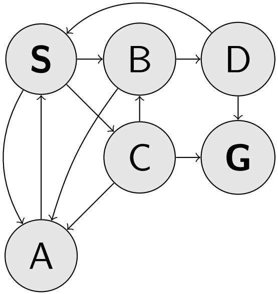

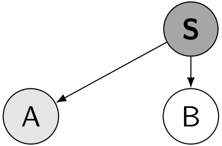

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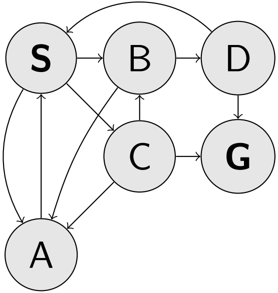

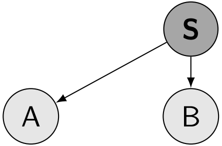

Q: (S,A) (S,B)
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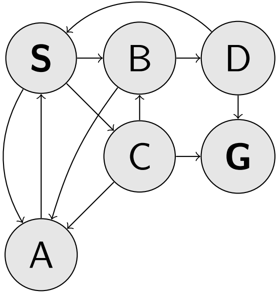

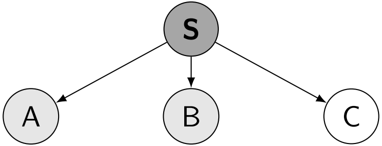

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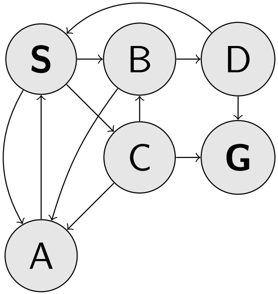

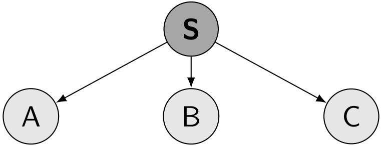

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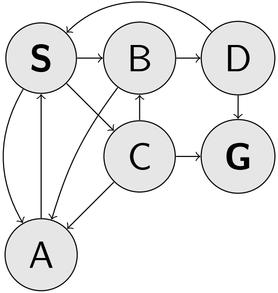

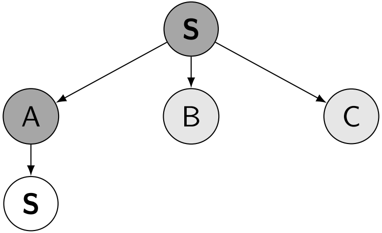

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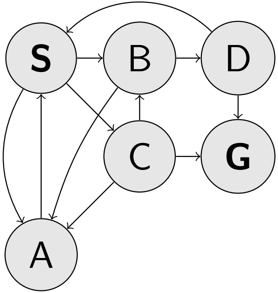

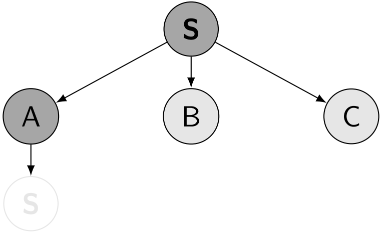

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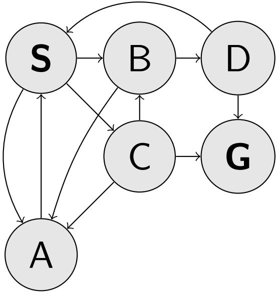

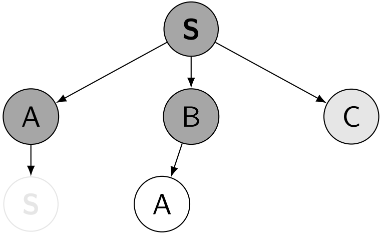

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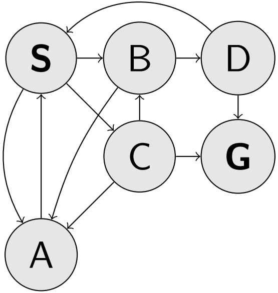

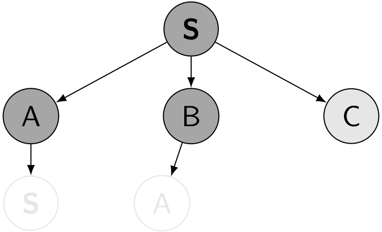

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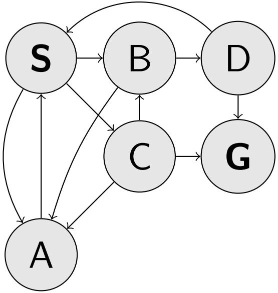

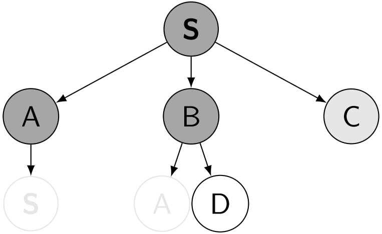

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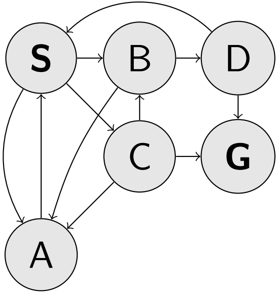

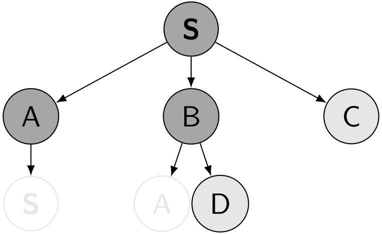

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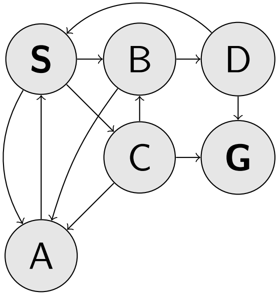

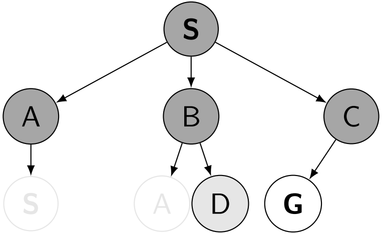

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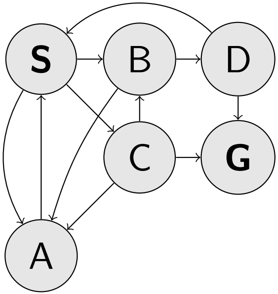

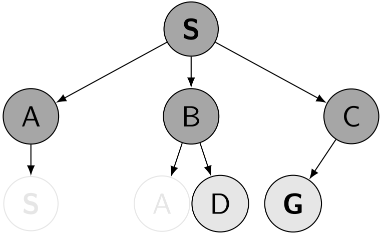

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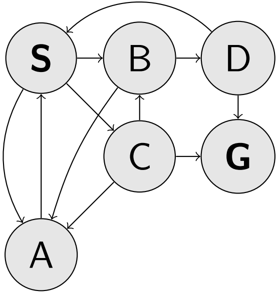

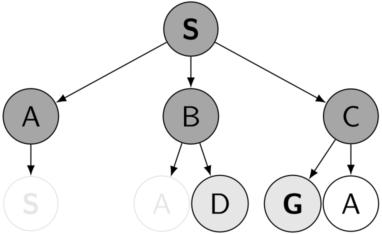

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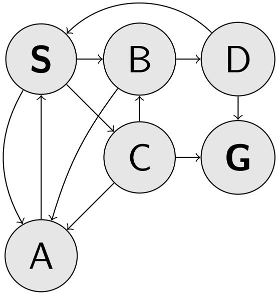

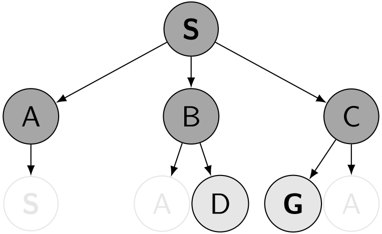

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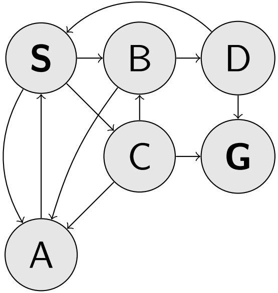

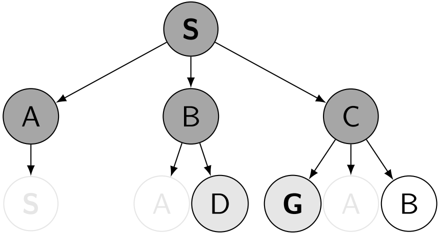

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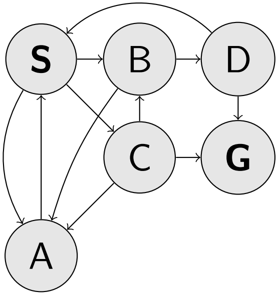

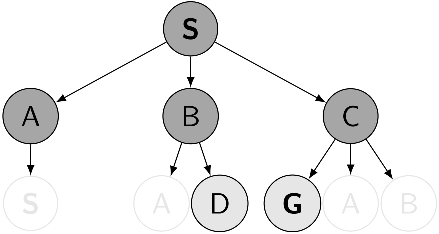

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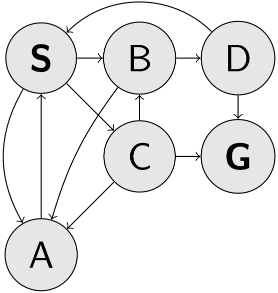

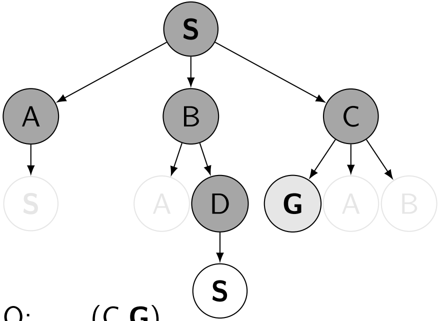

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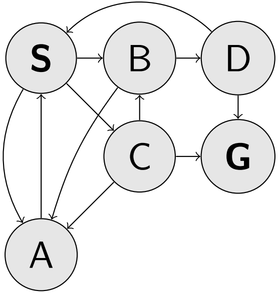

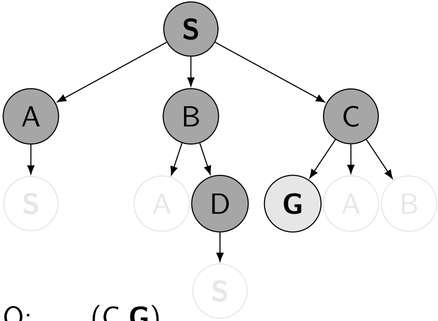

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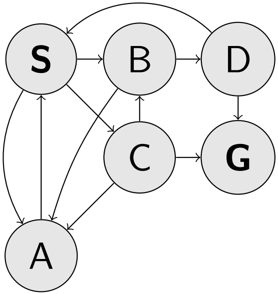

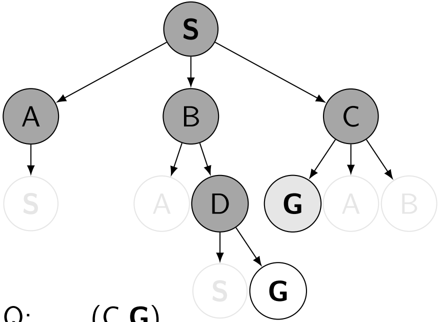

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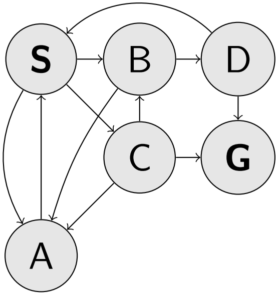

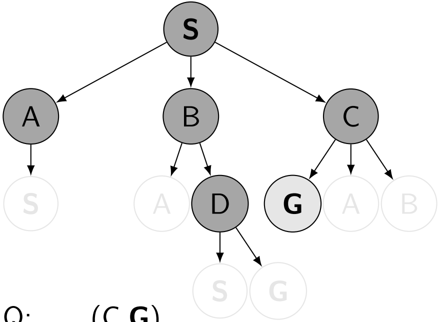

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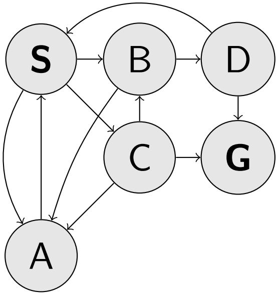

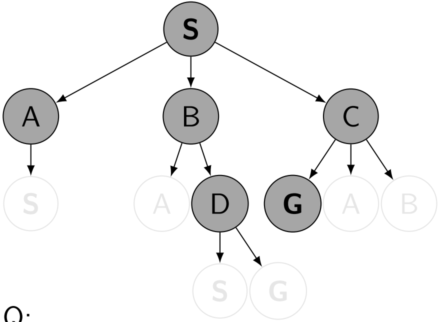

Q:
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Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
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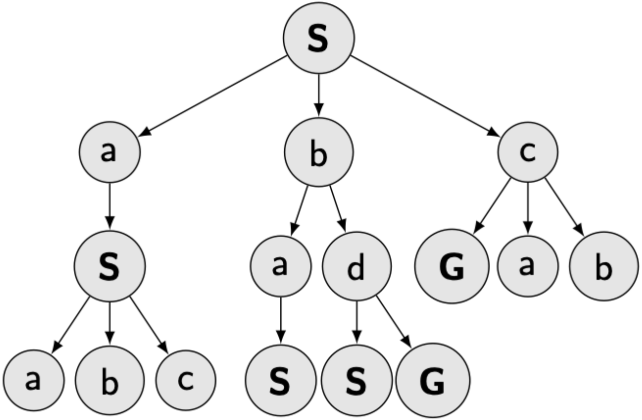
How to traverse/build a search tree?

- Depth of the tree d. A node is in depth d.
- Max-Depth of the tree m. Can be ∞.
- (Average) Branching factor b.
- s denotes the depth of the shallowest Goal .
- How many nodes in the whole tree?



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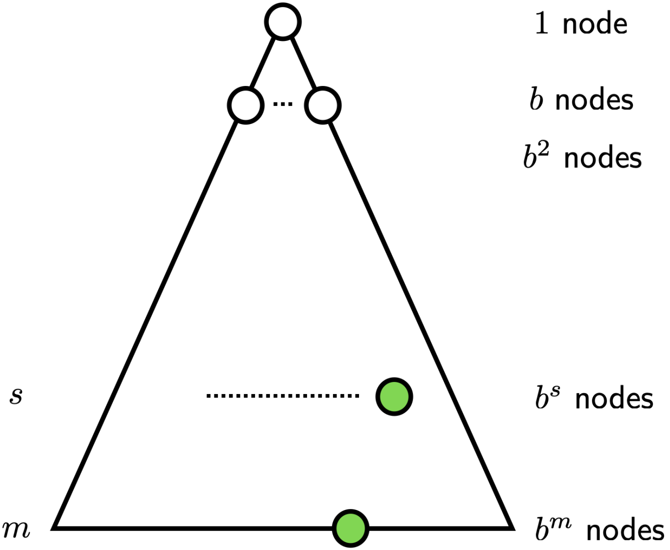

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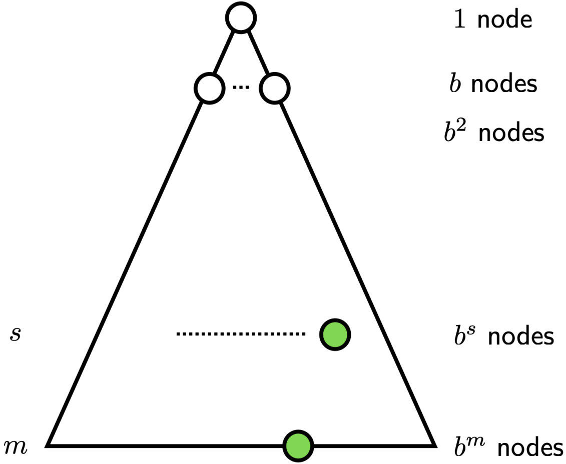

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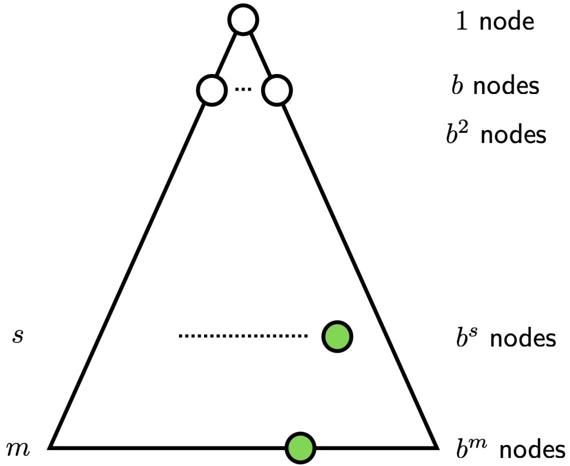

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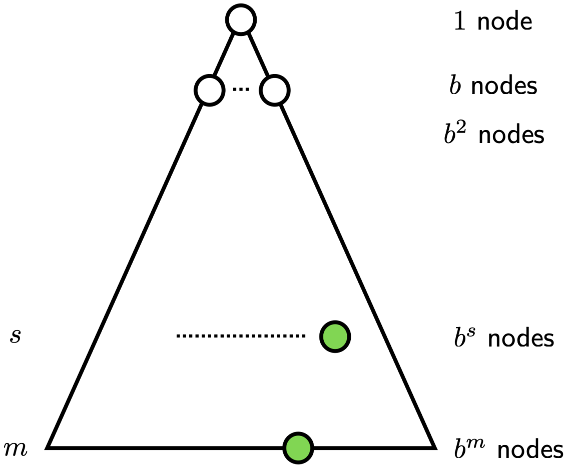

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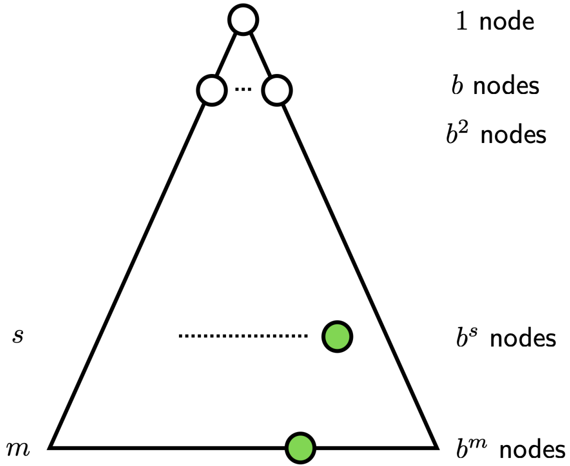

DFS, Q is LIFO data structure (stack)

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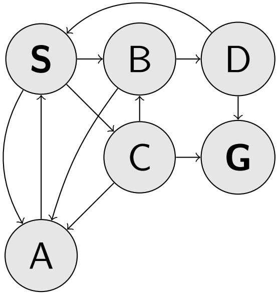

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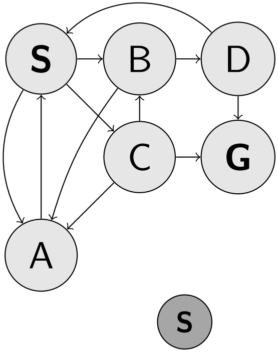

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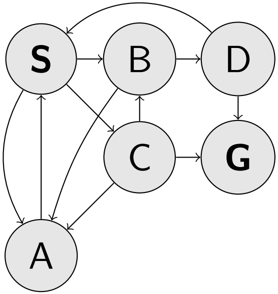

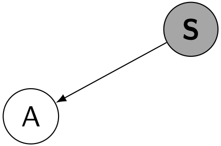

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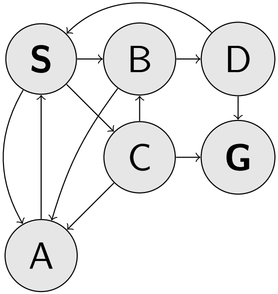

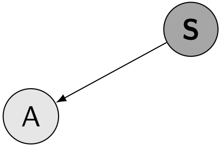

Q: (S,A)
visited: **S** A

DFS, Q is LIFO data structure (stack)

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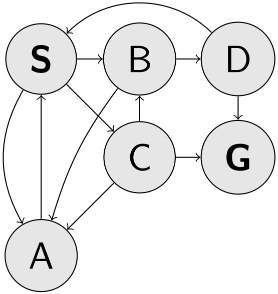

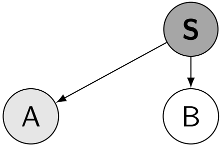

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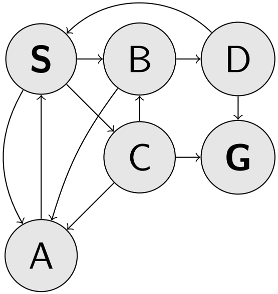

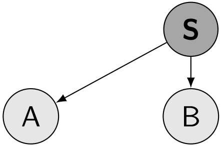

Q: (S,A) (S,B)
visited: **S** A B

DFS, Q is LIFO data structure (stack)

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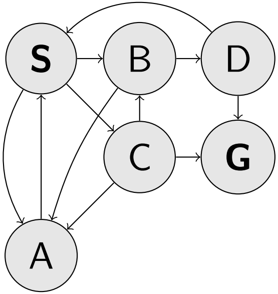

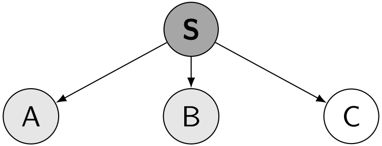

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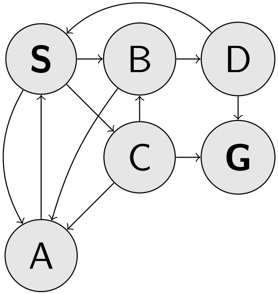

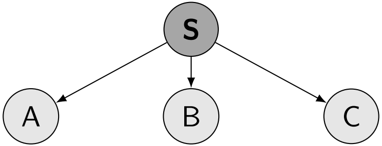

Q: (S,A) (S,B) (S,C)
visited: **S** A B C

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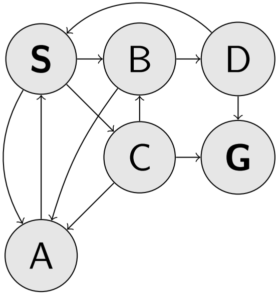
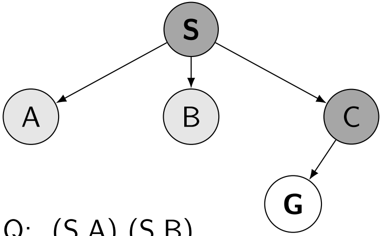
```





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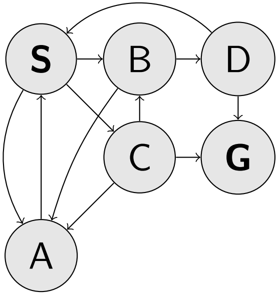

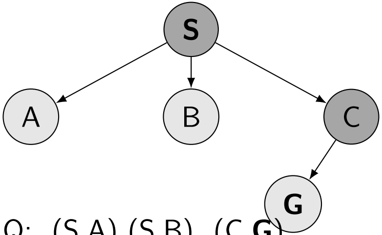

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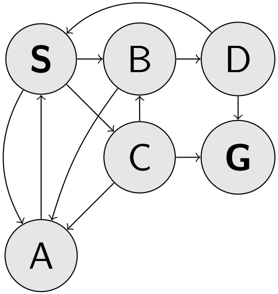

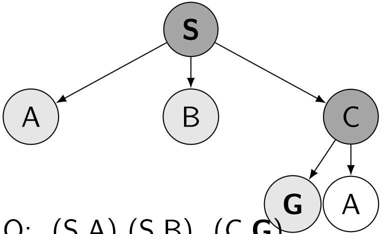

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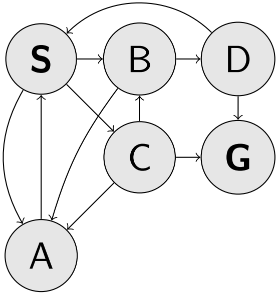

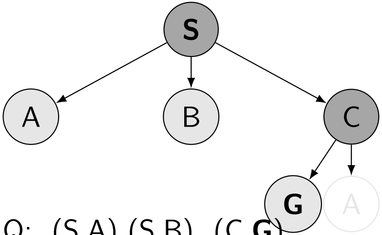

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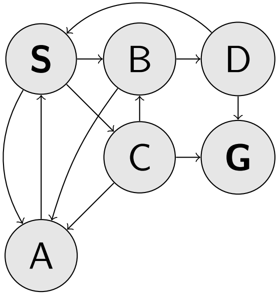

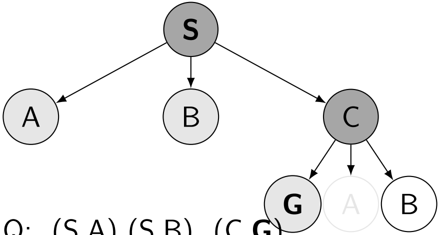

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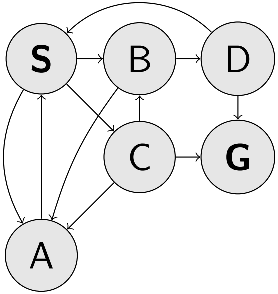

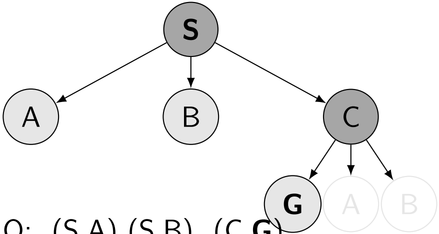

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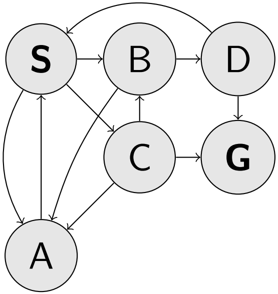

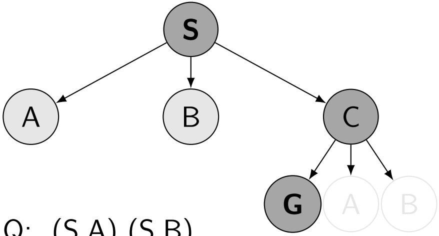

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DFS properties

Complete?

Optimal?

Space complexity?

- **A** $\mathcal{O}(bm)$
- **B** $\mathcal{O}(b^m)$
- **C** $\mathcal{O}(m^b)$
- **D** $\mathcal{O}(b^s)$

Time complexity?

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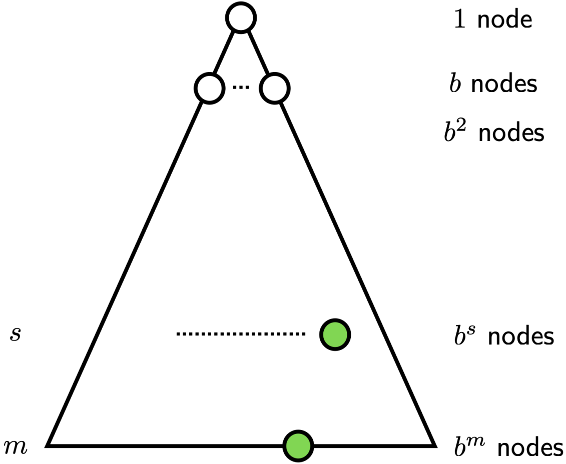

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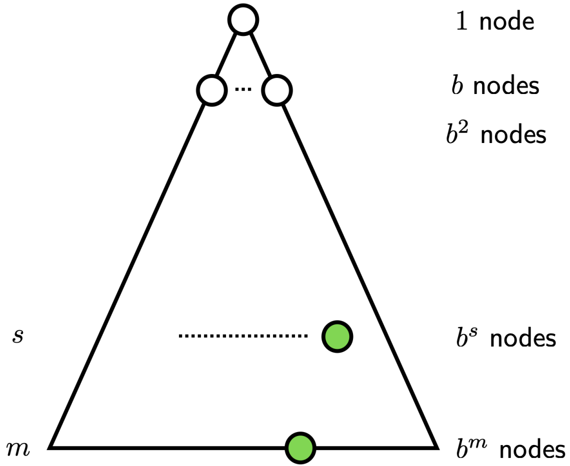

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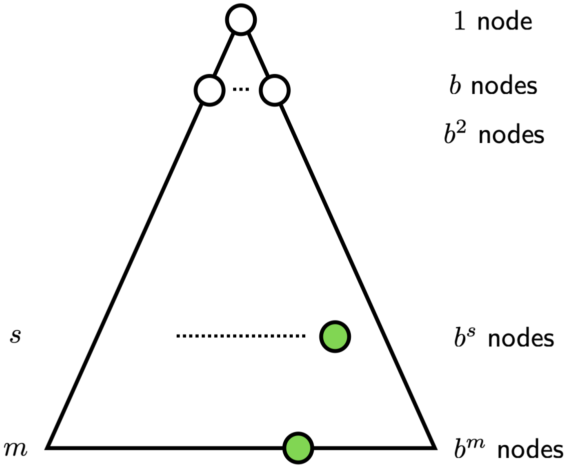

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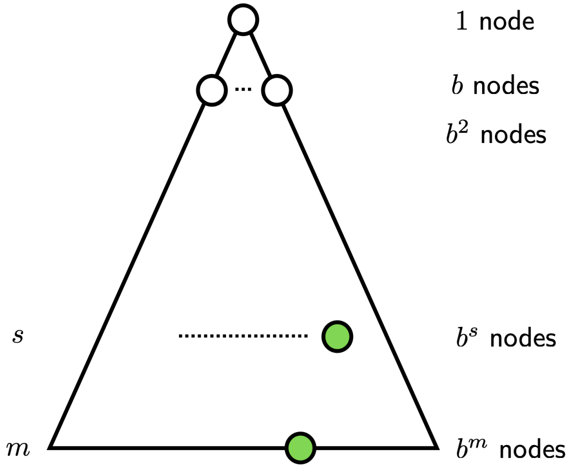

Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

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Uniform Cost Search (Dijkstra), Q is priority queue

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    while Q not empty do
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        for all a ∈ A(s) do
            s', c ← result(s, a)                    ▷ c cost
            if s' not visited then
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                Q.insert(s, s', cost_from_start)
            else
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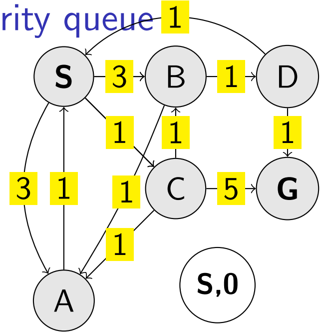
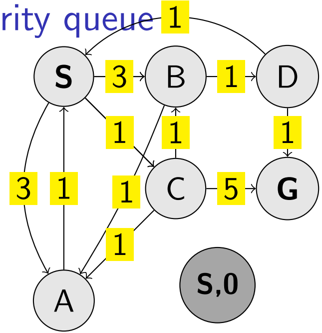
    return Failure
```



Q: (_,**S**,0)
visited: **S**

Uniform Cost Search (Dijkstra), Q is priority queue

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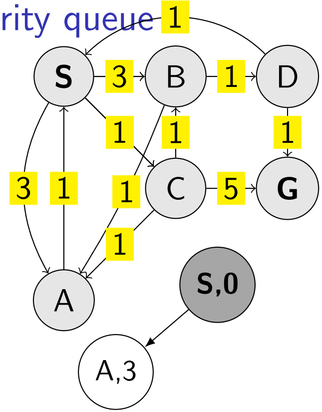

Q:

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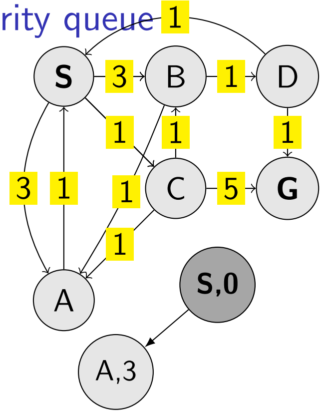

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      else
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  return Failure

Q: (S,A,3)
visited: **S** A

Uniform Cost Search (Dijkstra), Q is priority queue

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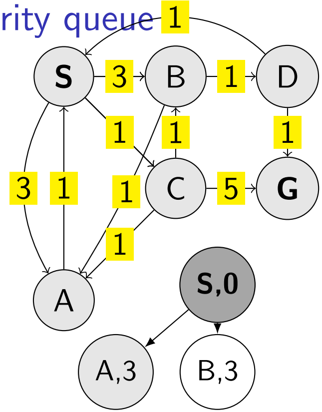

Q: (S,A,3)

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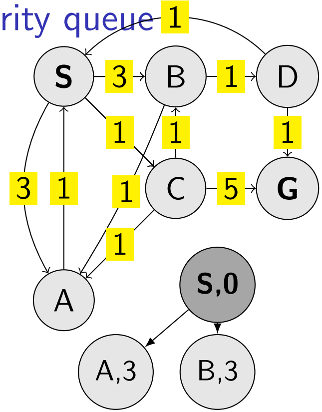

Q: (S,A,3) (S,B,3)
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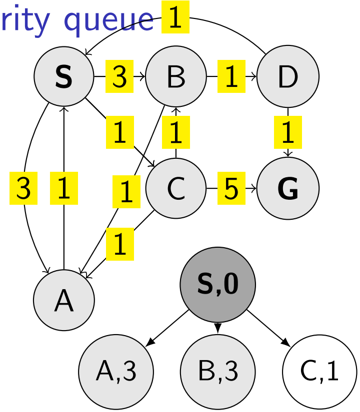

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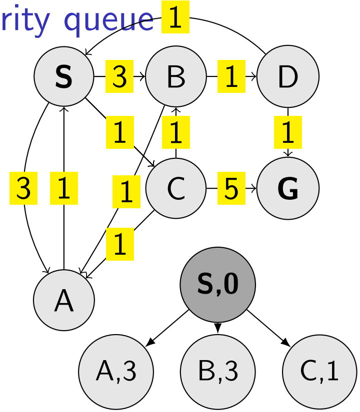
        p, s, _ ← Q.pop_first()
        parent[s] ← s
        if s ∈ S_G then return Success
        for all a ∈ A(s) do
            s′, c ← result(s, a)                    ▷ c cost
            if s′ not visited then
                Mark s′ as visited
                Q.insert(s, s′, cost_from_start)
            else
                Resolve duplicate s′
    return Failure
```



Q: (S,C,1) (S,A,3) (S,B,3)

visited: **S** A B C

Uniform Cost Search (Dijkstra), Q is priority queue

```
function FORWARD_SEARCH
    Q.insert(s_0) and mark s_0 as visited
    while Q not empty do
        p, s, _ ← Q.pop_first()
        parent[s] ← s
        if s ∈ S_G then return Success
        for all a ∈ A(s) do
            s', c ← result(s, a)                    ▷ c cost
            if s' not visited then
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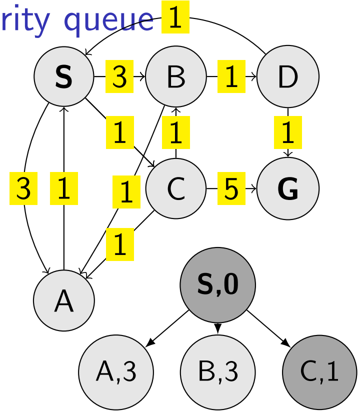
                Q.insert(s, s', cost_from_start)
            else
                Resolve duplicate s'
    return Failure
```



Q: (S,A,3) (S,B,3)

visited: **S** A B C

Uniform Cost Search (Dijkstra), Q is priority queue

function FORWARD_SEARCH
  Q.insert(s_0) and mark s_0 as visited
  while Q not empty **do**
    $p, s, \_ \leftarrow$ Q.pop_first()
    parent[s] $\leftarrow s$
    if $s \in \mathcal{S}_G$ **then return** Success
    for all $a \in \mathcal{A}(s)$ **do**
      $s', c \leftarrow$ result(s, a)   ▷ c cost
      if s' not visited **then**
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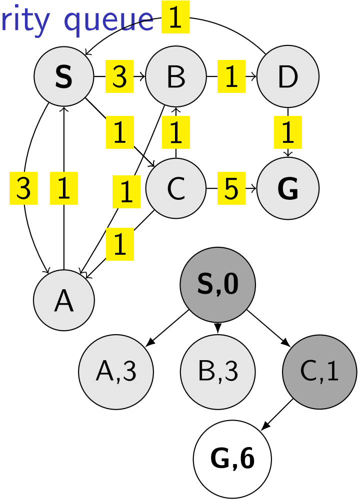
        Q.insert(s, s', cost_from_start)
      else
        Resolve duplicate s'
  return Failure



Q: (S,A,3) (S,B,3)
visited: **S** A B C

Uniform Cost Search (Dijkstra), Q is priority queue

function FORWARD_SEARCH
  Q.insert(s_0) and mark s_0 as visited
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        Q.insert(s, s', cost_from_start)
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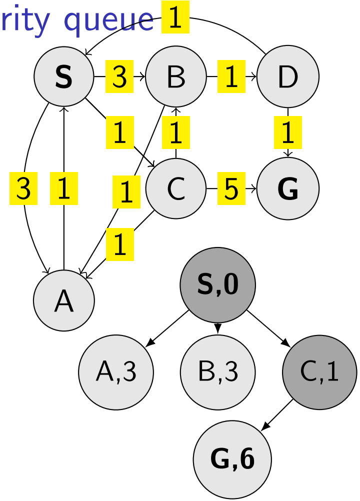
  return Failure



Q: (S,A,3) (S,B,3) (C,**G**,6)
visited: **S** A B C **G**

Uniform Cost Search (Dijkstra), Q is priority queue

function FORWARD_SEARCH
  Q.insert(s_0) and mark s_0 as visited
  while Q not empty **do**
    $p, s, \_ \leftarrow$ Q.pop_first()
    parent[s] $\leftarrow s$
    if $s \in \mathcal{S}_G$ **then return** Success
    for all $a \in \mathcal{A}(s)$ **do**
      $s', c \leftarrow$ result(s, a) ▷ c cost
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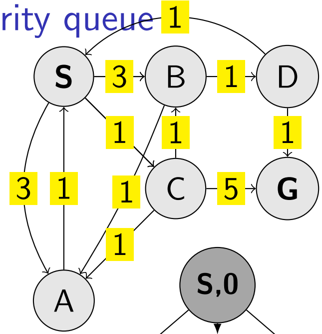
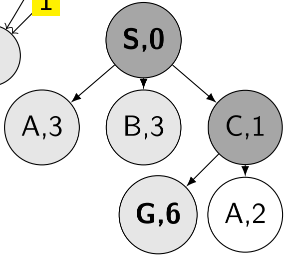
        Q.insert(s, s', cost_from_start)
      else
        Resolve duplicate s'
  return Failure





Q: (S,A,3) (S,B,3) (C,**G**,6)
visited: **S** A B C **G**

Uniform Cost Search (Dijkstra), Q is priority queue

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function FORWARD_SEARCH
    Q.insert(s_0) and mark s_0 as visited
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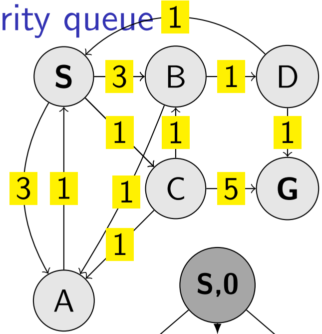

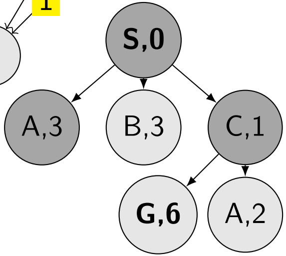

Q: (C,A,2) (S,B,3) (C,**G**,6)

visited: **S** A B C **G**

Uniform Cost Search (Dijkstra), Q is priority queue

function FORWARD_SEARCH
  Q.insert(s_0) and mark s_0 as visited
  while Q not empty **do**
    $p, s, \_ \leftarrow$ Q.pop_first()
    parent[s] $\leftarrow s$
    if $s \in \mathcal{S}_G$ **then return** Success
    for all $a \in \mathcal{A}(s)$ **do**
      $s', c \leftarrow \text{result}(s, a)$ ▷ c cost
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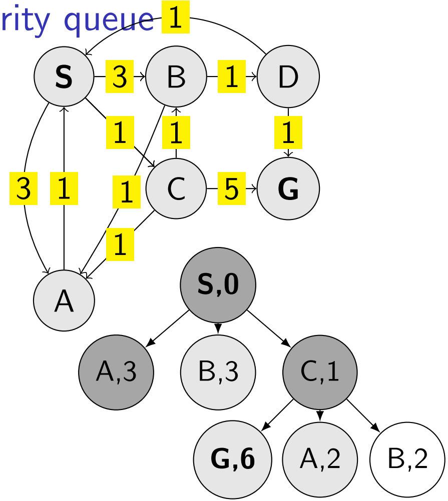
        Resolve duplicate s'
  return Failure



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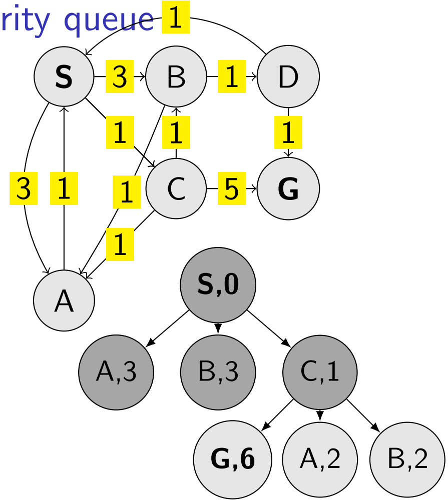

Q: (C,A,2) (C,B,2) (C,**G**,6)
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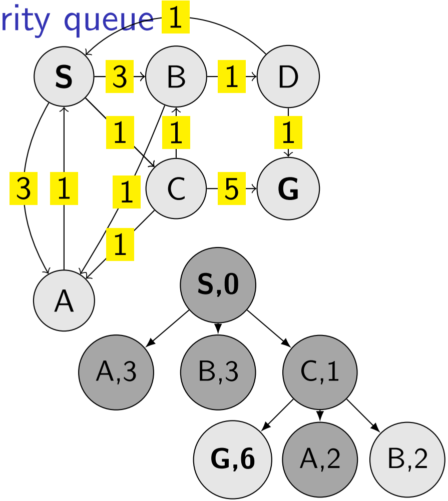

Q: (C,B,2) (C,**G**,6)

visited: **S** A B C **G**

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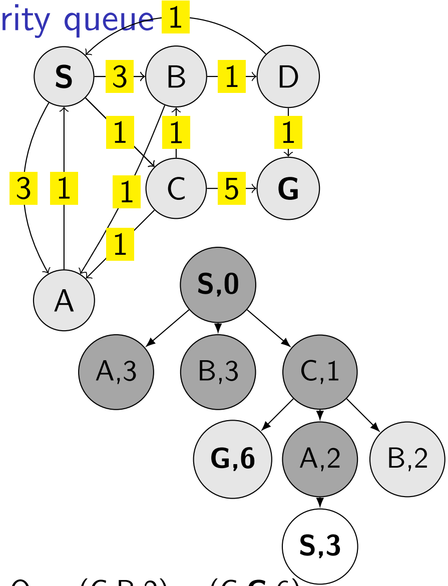

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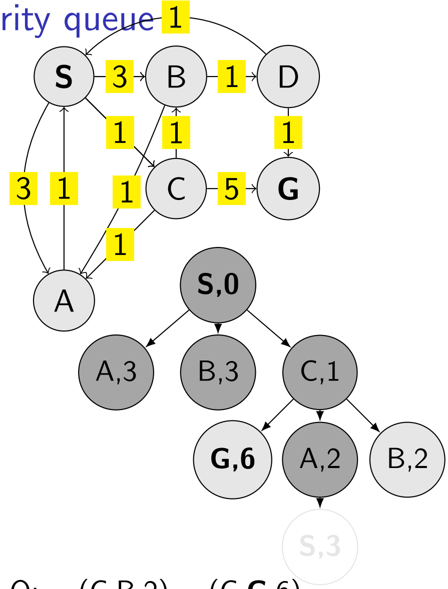

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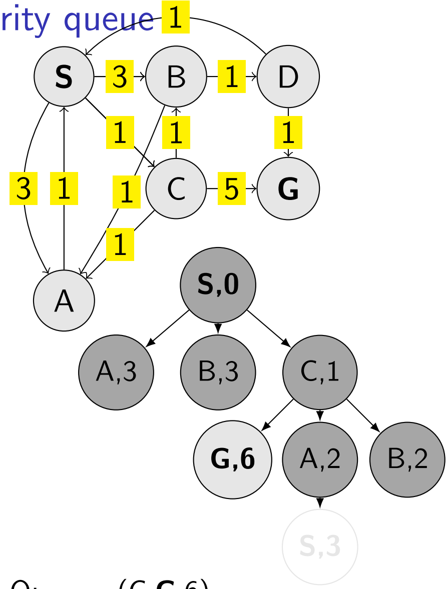

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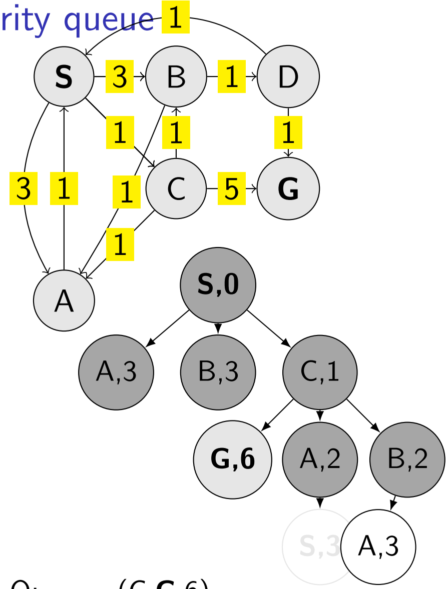

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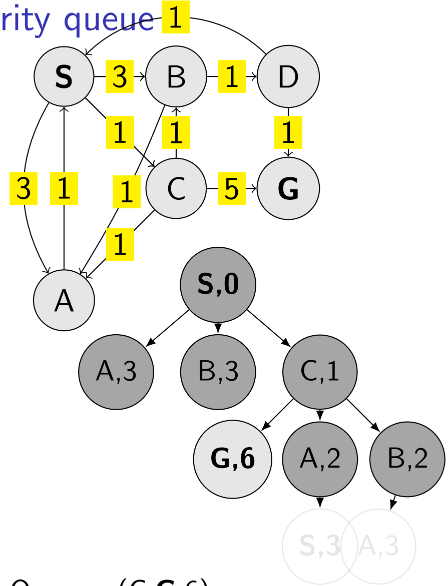

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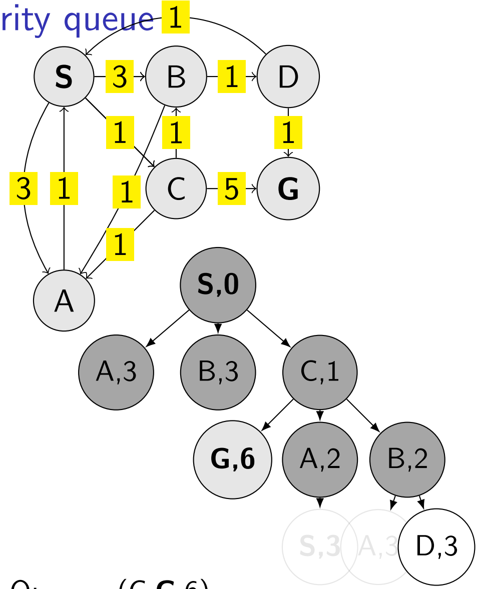

Q: (C,**G**,6)
visited: **S** A B C **G**

Uniform Cost Search (Dijkstra), Q is priority queue

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    Q.insert(s_0) and mark s_0 as visited
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            else
                Resolve duplicate s'
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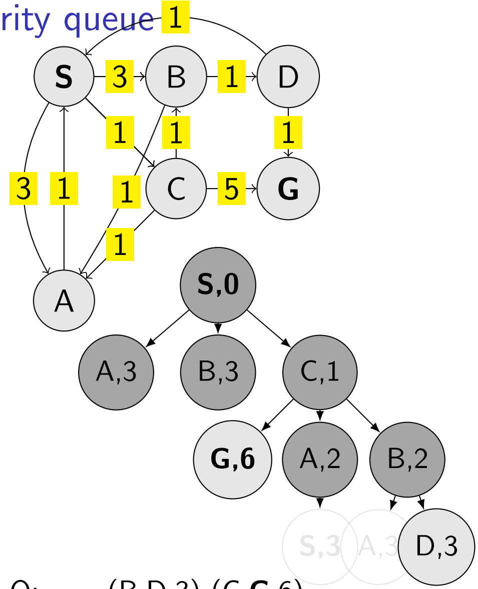

Q: (B,D,3) (C,**G**,6)
visited: **S** A B C **G** D

Uniform Cost Search (Dijkstra), Q is priority queue

function FORWARD_SEARCH
  Q.insert(s_0) and mark s_0 as visited
  while Q not empty **do**
    $p, s, \_ \leftarrow$ Q.pop_first()
    parent[s] $\leftarrow s$
    if $s \in \mathcal{S}_G$ **then return** Success
    for all $a \in \mathcal{A}(s)$ **do**
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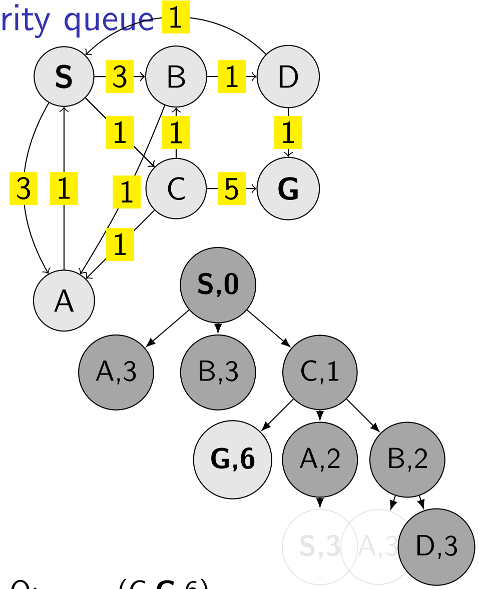

Q: (C,**G**,6)
visited: **S** A B C **G** D

Uniform Cost Search (Dijkstra), Q is priority queue

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  Q.insert(s_0) and mark s_0 as visited
  while Q not empty **do**
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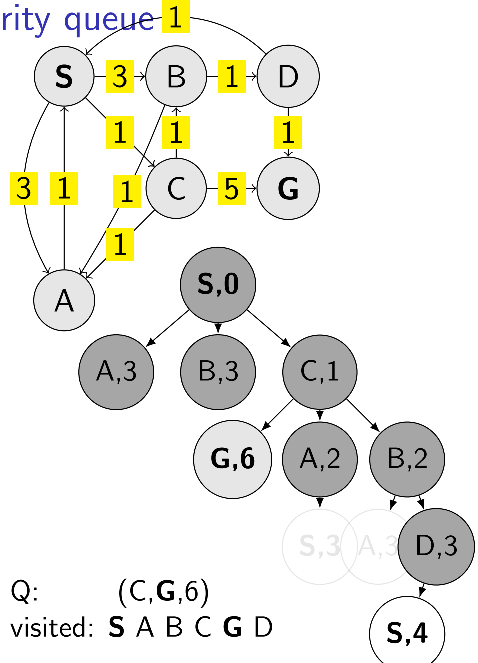

Uniform Cost Search (Dijkstra), Q is priority queue

function FORWARD_SEARCH
  Q.insert(s_0) and mark s_0 as visited
  while Q not empty **do**
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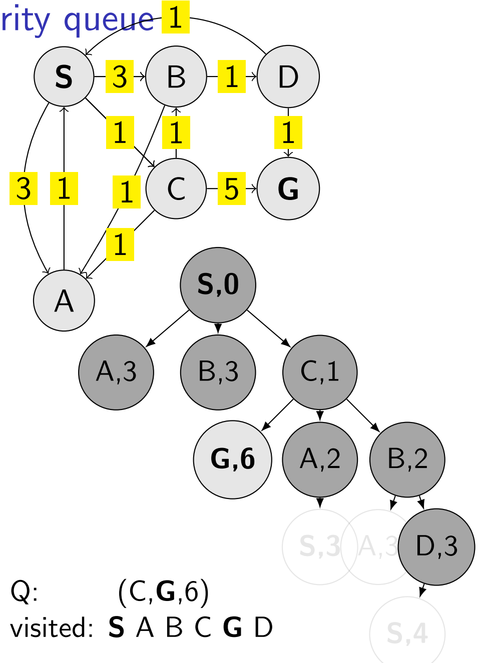

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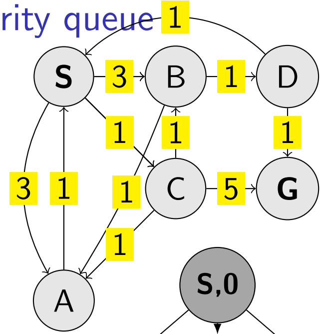

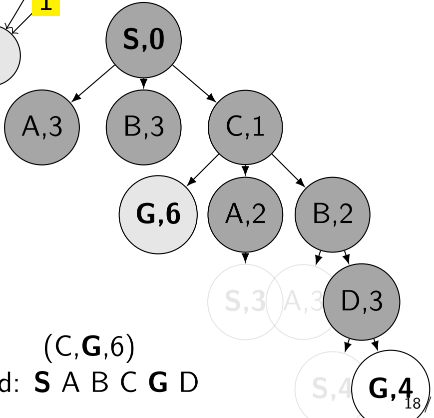

Q: (C,**G**,6)
visited: **S** A B C **G** D

Uniform Cost Search (Dijkstra), Q is priority queue

function FORWARD_SEARCH
    Q.insert(s_0) and mark s_0 as visited
    while Q not empty **do**
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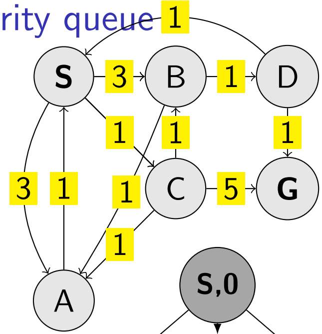

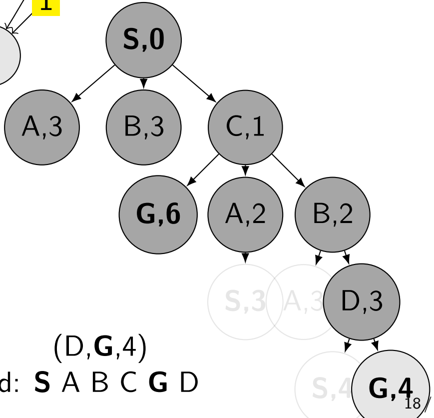

Q: (D,**G**,4)
visited: **S** A B C **G** D

Uniform Cost Search (Dijkstra), Q is priority queue

function FORWARD_SEARCH
    Q.insert(s_0) and mark s_0 as visited
    while Q not empty **do**
        $p, s, \_ \leftarrow$ Q.pop_first()
        parent[s] $\leftarrow s$
        if $s \in \mathcal{S}_G$ **then return** Success
        for all $a \in \mathcal{A}(s)$ **do**
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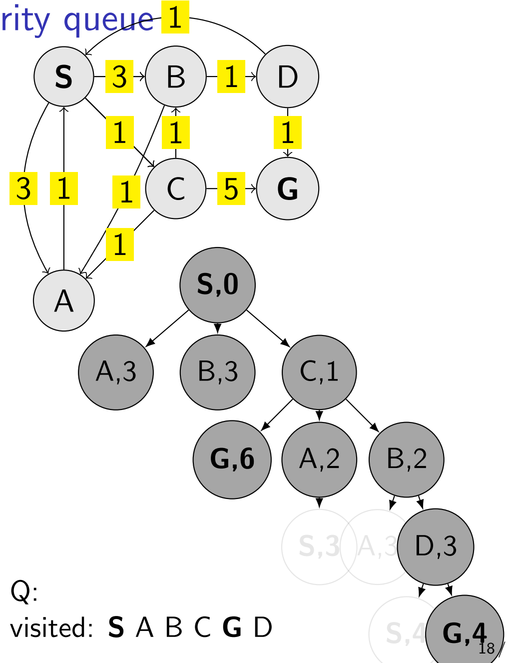
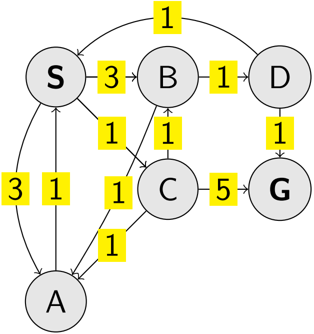


UCS properties

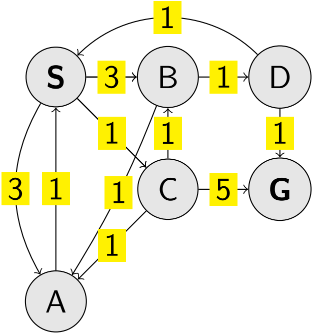


Complete?

Optimal?

Complexities?

UCS properties



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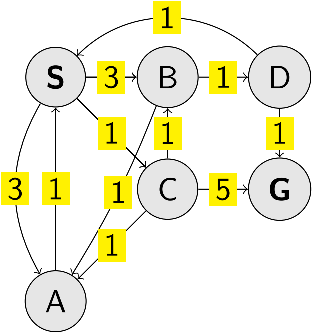

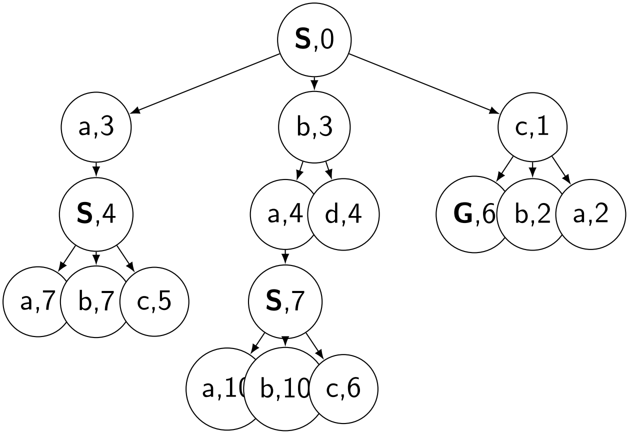

Complete?
Optimal?
Complexities?

Node selection, take argmin $f(n)$. Search Node: $n = (p, s, \text{priority\_value})$

Selecting next node to explore (`pop` operation):

$$\texttt{node} \leftarrow \operatorname*{argmin}_{n \in \texttt{Q}} f(n)$$

What is $f(n)$ for DFS, BFS, and UCS?

- DFS:
- BFS:
- UCS:

- $f(n) = n.\text{cost\_from\_start}$
- $f(n) = n.\text{depth}$
- $f(n) = -n.\text{depth}$

The good: (one) frontier as a priority queue
(I.e., priority queue will work universally. Still, stack (LIFO) and queue (FIFO) are (conceptually) the perfect data structures for DFS and BFS, respectively.)
The bad: All the $f(n)$ correspond to the cost from n to the start cost-to-come (to n).

Node selection, take argmin $f(n)$. Search Node: $n = (p, s, \text{priority\_value})$

Selecting next node to explore (pop operation):

$$\texttt{node} \leftarrow \operatorname*{argmin}_{n \in \texttt{Q}} f(n)$$

What is $f(n)$ for DFS, BFS, and UCS?

- DFS:
- BFS:
- UCS:

- $f(n) = n.\text{cost\_from\_start}$
- $f(n) = n.\text{depth}$
- $f(n) = -n.\text{depth}$

The good: (one) frontier as a priority queue
(I.e., priority queue will work universally. Still, stack (LIFO) and queue (FIFO) are (conceptually) the perfect data structures for DFS and BFS, respectively.)
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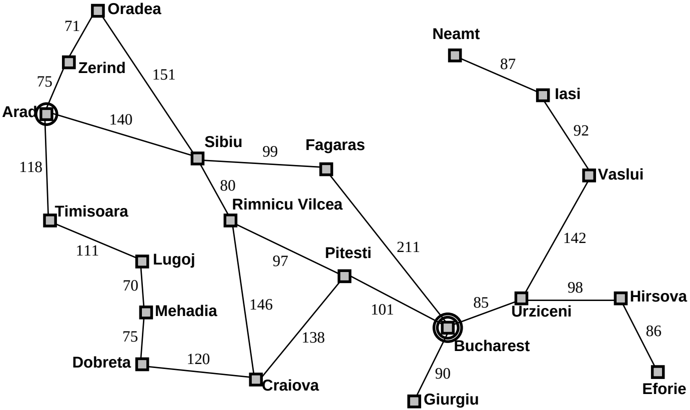
How far are we from the goal cost-to-go ? – Heuristics

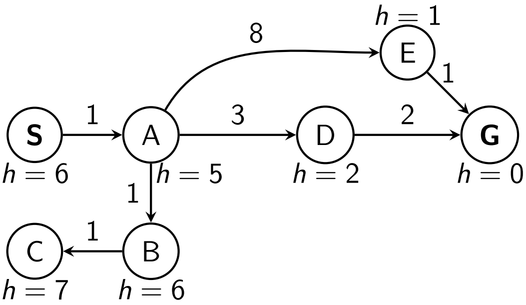
- A function that estimates how close a *state* is to the goal.
- Designed for a particular problem.
- $h(n.state)$ – it is function of the state (attribute of node)
- It is often shortened as $h(n)$ – heuristic value of node n.

Example of heuristics



Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Greedy, take the $n^* = \operatorname{argmin}_{n \in \mathtt{Q}} h(n)$



What is wrong (and nice) with the Greedy?

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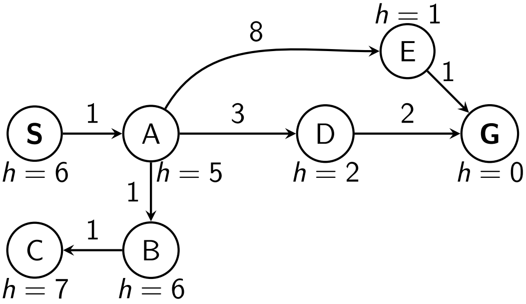

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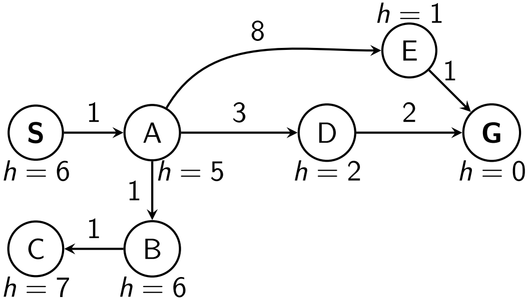
A* combines UCS and Greedy



UCS orders by backward (path) cost $g(n)$
Greedy uses heuristics (goal proximity) $h(n)$

A* orders nodes by: $f(n) = g(n) + h(n)$

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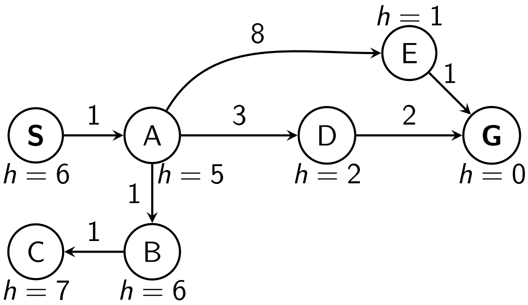

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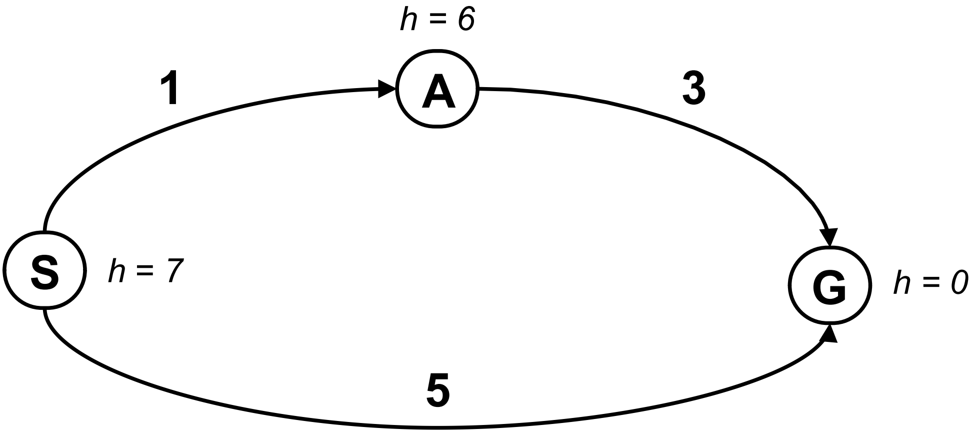
Is A* optimal?

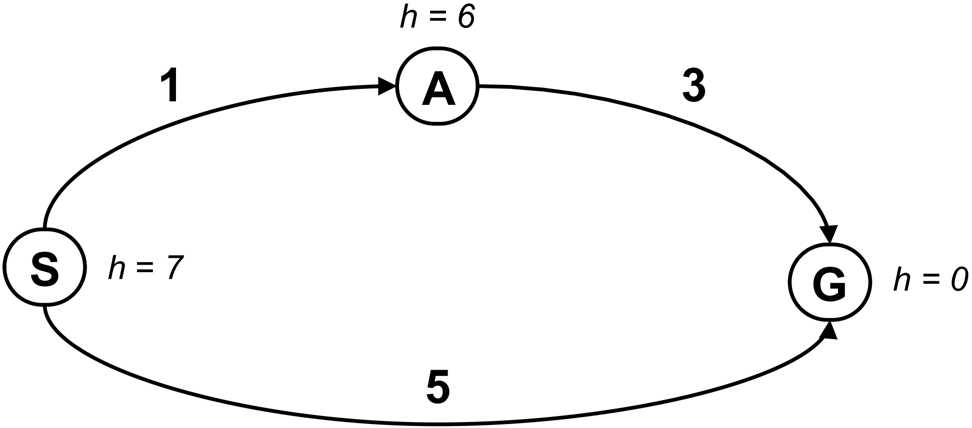


[2]

What is the problem?

[2] Graph example: Dan Klein and Pieter Abbeel

Is A* optimal?



2

What is the problem?

[2] Graph example: Dan Klein and Pieter Abbeel

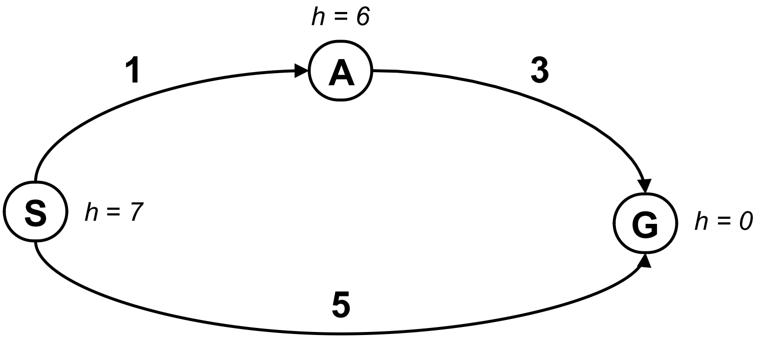
What is the right $h(A)$?

A: $0 \leq h(A) \leq 4$

B: $h(A) \leq 3$

C: $0 \leq h(A) \leq 3$

D: $0 \leq h(A)$



Admissible heuristics

A heuristic function h is admissible if:

$$
\begin{aligned}
h(n) &\leq \text{cost}(n.state, \text{Goal}_{\text{nearest}}) \\
h(\text{Goal}) &= 0
\end{aligned}
$$

Consistent heuristic

function FORWARD_SEARCH
  Q.insert(s_0) and mark s_0 as visited
  while Q not empty **do**
    $p, s, \_ \leftarrow$ Q.pop_first()
    parent[s] $\leftarrow s$
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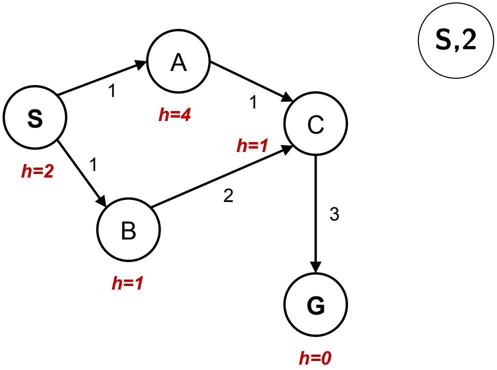

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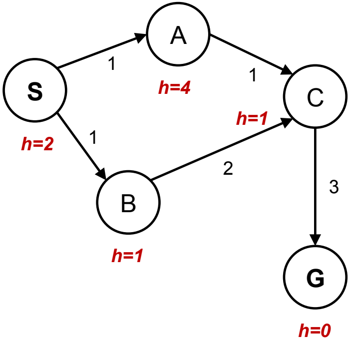

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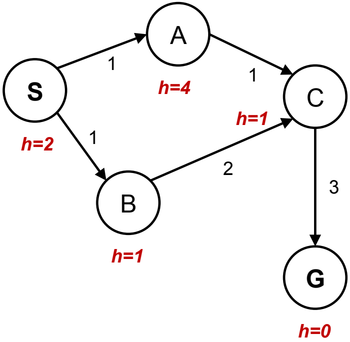

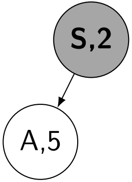

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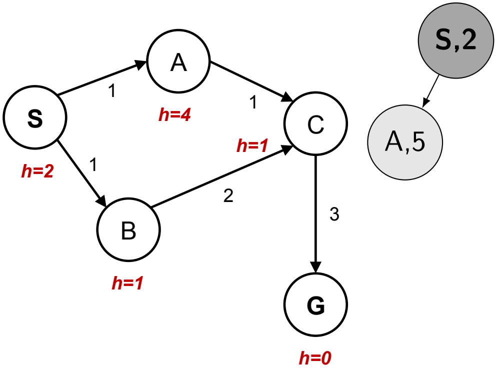

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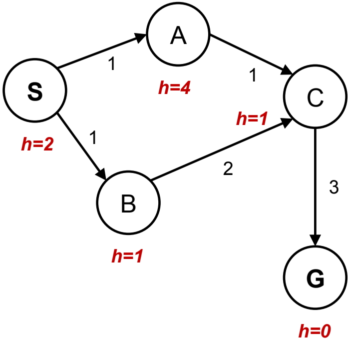

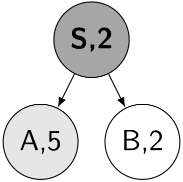

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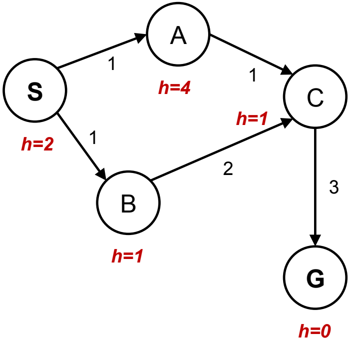

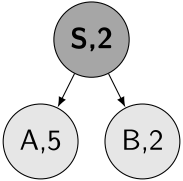

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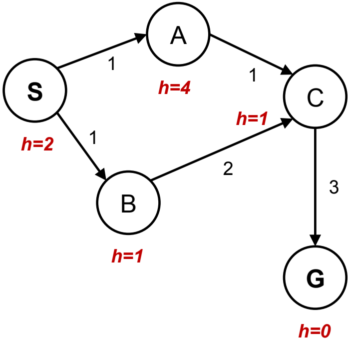

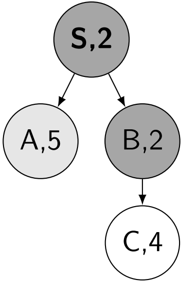

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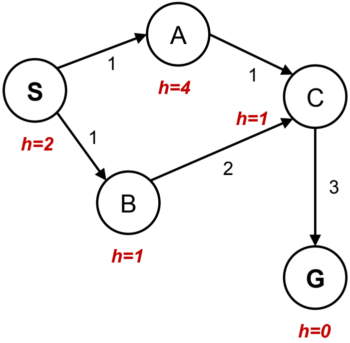

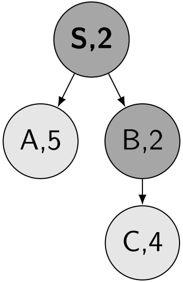

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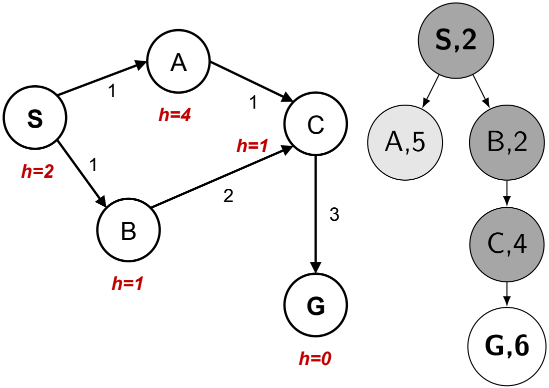

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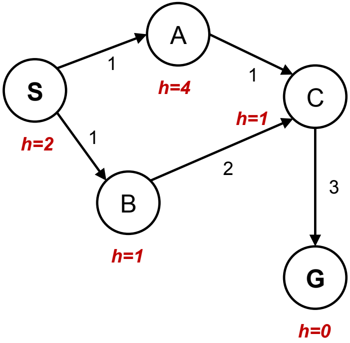

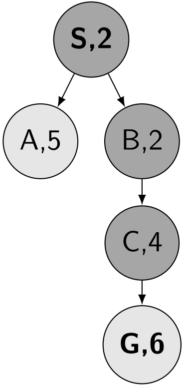

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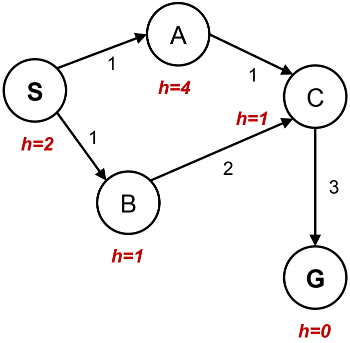

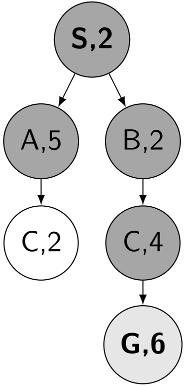

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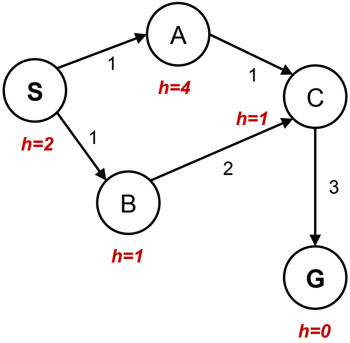
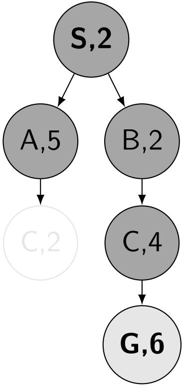




What would be the proper $h(A)$?

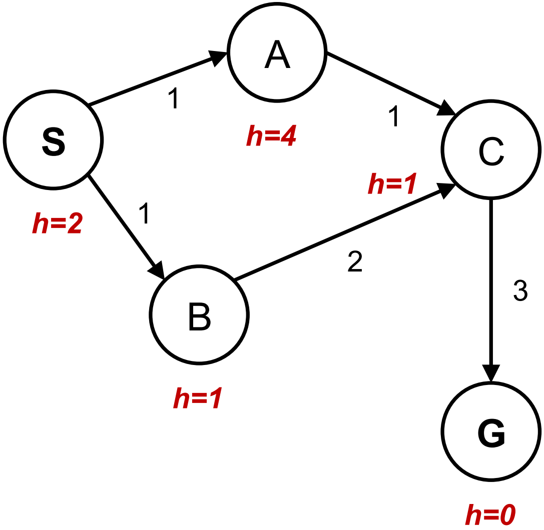
Consider other $h(s)$ fixed.

A: $h(A) = 1$

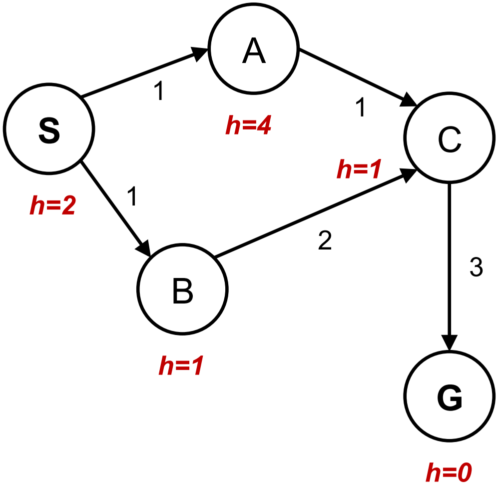
B: $h(A) = 2$

C: $1 \leq h(A) \leq 2$

D: $0 \leq h(A) \leq 1$



Consistent heuristics

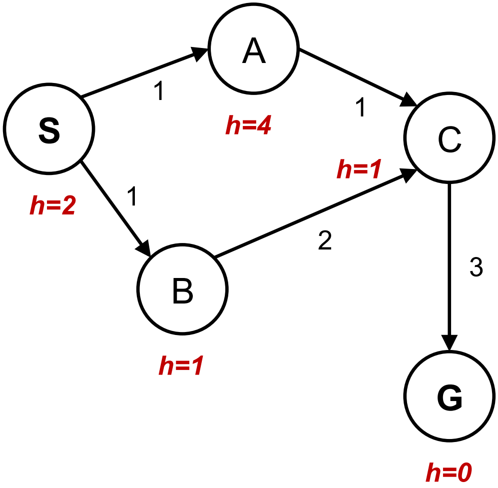


Admissible h:
$h(A) \leq$ true cost $A \to G$

Consistent h:
$h(A) - h(C) \leq$ true cost $A \to C$
in general:
$h(p) - h(s) \leq$ true cost $p \to s$ for any pair: parent p and its successor s

$f(n) = g(n) + h(n)$ along a path never decreases!

Consistent heuristics



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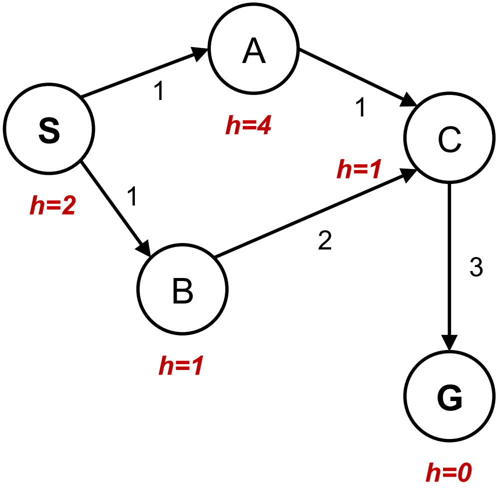

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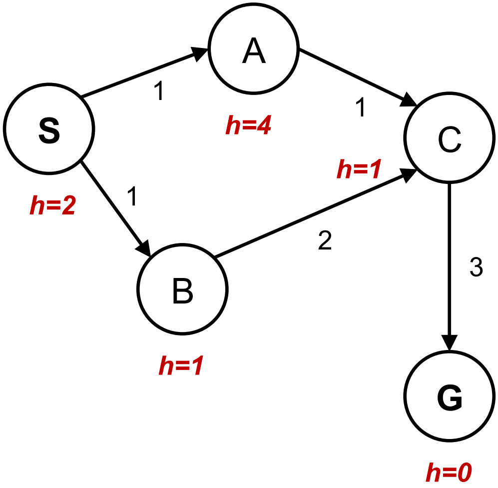

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Summary

- Effectivness – adding heuristic estimates of cost-to-go
- Not all heuristics are equally good (admissibility, consistence, informativeness)

References, further reading

Some figures from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms.

[1] Steven M. LaValle.
Planning Algorithms.
Cambridge, 1st edition, 2006.
Online version available at: http://planning.cs.uiuc.edu.

[2] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
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