# Quantum Computing 2026 - Exercise Sheet 2

## Introduction to Quantum Computing

**1** (Basics tensor products). *(a) Compute the following tensor products of states.*

*a)* $|0\rangle \otimes |1\rangle$ *b)* $|1\rangle \otimes |1\rangle$ *c)* $|1\rangle \otimes |0\rangle \otimes |1\rangle$ *d)* $|0\rangle \otimes |1\rangle \otimes |1\rangle$.

*(b) Compute the following tensor products of operators.*

*a)* $\sigma_x \otimes \sigma_y$ *b)* $\sigma_z \otimes \sigma_x$ *c)* $\mathbb{I} \otimes \sigma_z$ *d)* $\sigma_z \otimes \mathbb{I} \otimes \sigma_x$

**2** (Entanglement). *(a) Are these states entangled:*

*(I)* $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

*(II)* $|\psi_2\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$

*(III)* $|\psi_3\rangle = \frac{1}{\sqrt{2}}(|001\rangle + |011\rangle)$.

*(b) Apply the following multipartite measurements*

*a)* $\sigma_x \otimes \sigma_z |\psi_1\rangle$ *b)* $\sigma_y \otimes \mathbb{I} \otimes \sigma_x |\psi_2\rangle$ *c)* $\mathbb{I} \otimes \sigma_z \otimes \sigma_z |\psi_3\rangle$

**3** (Single-qubit gates). *Build the gates that do the following transformations:*

*(a)* $|0\rangle \to |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |1\rangle \to |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

*(b)* $|+\rangle \to |+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle), \quad |-\rangle \to |-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle)$

*(c)* $|0\rangle \to |+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle), \quad |1\rangle \to |-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle)$

**4** (Multi-qubit gate). *Build the gate that implements the following transformations*

$$|00\rangle \to |00\rangle\,, \quad |01\rangle \to |01\rangle\,, \quad |10\rangle \to |11\rangle\,, \quad |11\rangle \to |10\rangle\,.$$

**5** (Creation of Entanglement). *Build the entangled state (Bell state)* $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ *starting with the state* $|00\rangle$*, by using the gates H and CNOT. Draw the circuit implementing this.*

**6** (Teleportation protocol). *Alice and Bob share a state* $|\beta_{00}\rangle$ *initialized as* $|00\rangle$*. They go to separated places and Alice wants to teleport an unknown state* $|\phi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$*. By teleporting here we mean sending the state to Bob only with classical information. The full system's initial state is then* $|\psi_0\rangle = |\phi\rangle\,|\beta_{00}\rangle$.

*(a) Entangle the shared state* $|\beta_{00}\rangle$ *and write the new full state* $|\psi_1\rangle$.

*(b) We now want to correlate Alice's unknown state with the shared state. For this apply a CNOT gate with Alice's unknown state as control and Alice's shared state as the target. Then apply a Hadamard gate to Alice's unknown state. What is the full state* $|\psi_2\rangle$.

*(c) Rearrange* $|\psi_2\rangle$ *in the following form*

$$|\psi_3\rangle = \frac{1}{2}(|00\rangle\,|a_1\rangle + |01\rangle\,|a_2\rangle + |10\rangle\,|a_3\rangle + |11\rangle\,|a_4\rangle).$$

*Find Bob's states* $|a_1\rangle\,, |a_2\rangle\,, |a_3\rangle\,, |a_4\rangle$.

*(d) Alice measures her two qubits with results* $\{|00\rangle\,, |01\rangle\,, |10\rangle\,, |11\rangle\}$ *and sends these 2 bits of information to Bob. If Alice measures* $|00\rangle$ *then Bob will have the original state* $|\phi\rangle$*. What measurements does Bob have to apply to his state to recover* $|\phi\rangle$ *if Alice sends the bits* 01, 10, 11*?*

*(e) Draw the quantum circuit for the full teleportation protocol.*