Fourth homework assignment announced 22. 5. 2025, due 6. 6. 2025 as a zip file in Brute (or day ahead of the exam, if you want to attend the exam on June 2nd)

In this homework assignment, you are expected to collect 10 points, but you can actually collect up to 30 points. The exercises refer to Chapter 10 of the lecture notes: `https://cw.fel.cvut.cz/wiki/_media/courses/b0m36qua/lectures/quantum_computing_via_randomized_algorithms_2025.pdf`

Discrete Quantum Walks

3. Demonstrate that the directional bias of a Hadamard walker, as in Figure 10.1, depends on the initial coin state. (3 points)

4. Verify that Fig. 10.3 is indeed correct (make your own plot). Explain what would we expect to see if we measured after each iteration of U. (3 points)

Quantum Walk on a Complete Graph

5. The trap, as defined in Fig. 10.5 makes no sense for a quantum walk (although this is not quite the case for a Szegedy walk). Explain why. (2 points)

6. Analyze the convergence properties of iterated applications of the unitary operator $U^n |x\rangle$ that implements a quantum walk. Under what conditions does $U^n|x\rangle$ converge, and how does the unitary property of preserving distances in the Hilbert space play a role in this convergence? (4 points)

7. Derive the probability of success at step 3 and step 4 for the quantum walk on K_4. (4 points)

8. Simulate the quantum walk on K_4 for a large number of steps and for $N \sim 1000$. Show that the analogue of Fig. 10.6 shows an oscillatory behavior. (4 points)

Szegedy Walks

9. Simulate a Szegedy walk on your favorite graph. Compare the validity of your results against `QuantumWalk.jl`. (6 points)

Quantum Walk on the Hypercube

11. Below Eq. (10.35) we read: "Note that $U(\pi/2)$ flips every bit of the state ... the opposite vertex of the hypercube." Pick your favorite $n > 2$ and demonstrate this. (3 points)