Quantum Computing

Exercises 3: Real Quantum Theory and Bell states

1. Check that the probabilities of measuring σ_y in the state:

$$|\psi\rangle = \frac{1}{\sqrt{6}}[(1-i)|u\rangle + 2i|d\rangle].$$

are the same restricting ourselves to $\mathbb{R}$ and performing the change

$$|\tilde{\psi}\rangle = \frac{1}{2}[|\psi\rangle \otimes |+i\rangle + |\psi\rangle^* \otimes |-i\rangle]$$

$$\tilde{\mathbf{A}} = \frac{1}{2}[\mathbf{A} \otimes |+i\rangle\langle +i| + \mathbf{A}^* \otimes |-i\rangle\langle -i|]$$

in both the operator and the state, where $|\pm i\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle)$.

2. Given a system of two qubits labeled by A and B, obtain the Bell states,

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) \quad , \quad |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B) \quad , \quad |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$$

by applying a combination of a Hadamard gate and a cNOT gate to the states:

$$|00\rangle \; , \; |01\rangle \; , \; |10\rangle \; , \; |11\rangle$$